On the Calculation of OFDM Error Performance with Phase Noise in AWGN and Fading Channels

Himal A. Suraweera, Xiaolin Zhou and Jean Armstrong
Department of Electrical and Computer Systems Engineering
Monash University
Melbourne Victoria 3800, Australia
E-mail: {himal.suraweera, xiaolin.zhou, jean.armstrong}@eng.monash.edu.au

***Abstract*— Oscillator phase noise (PN) in orthogonal frequency division multiplexing (OFDM) systems can cause severe performance degradation. In contrary to the assumptions made previously, recent work has discovered that the intercarrier interference (ICI) distribution due to a wide range of PN strengths in fact deviates from Gaussian characteristics. Hence we have compared the accuracy of a theoretical Gaussian symbol error rate (SER) analysis to that of the simulated SER in additive white Gaussian noise (AWGN), frequency flat and frequency selective Rayleigh fading channels. In multipath channels the SER is evaluated jointly considering the fading effects on the OFDM signal and ICI. Our results indicate that modeling ICI due to PN as Gaussian in the AWGN channel can lead to pessimistic theoretical predictions. However in fading channels, the difference between the simulated and the theoretical results is marginal.**

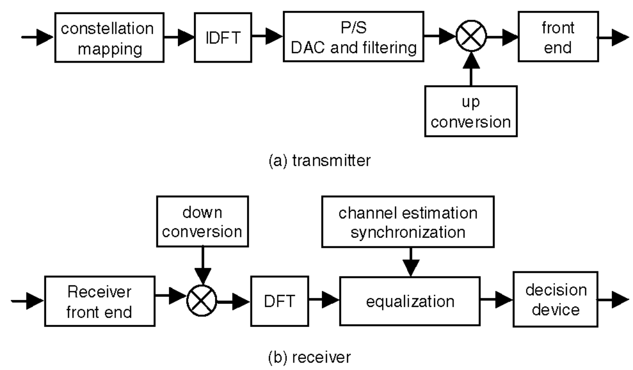

Fig. 1. Block diagram of a general OFDM communication system.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a popular modulation technique used in broadband wireless communication systems. OFDM is robust against multipath fading and has a high spectral efficiency. However OFDM systems are sensitive to synchronization errors including timing/frequency offset and phase noise (PN) [1].

Previous technical literature has extensively analyzed the PN effects in OFDM systems [2]-[6]. PN will rotate all the demodulated subcarriers of an OFDM symbol by a common phase angle. This angle is known as the common phase error (CPE). In the presence of PN, the orthogonality among the subcarriers is lost and this introduces intercarrier interference (ICI). If these two effects are not compensated at the receiver, severe performance degradation and a loss in data capacity is to be expected. In [7] information theoretic studies in terms of the cut-off rate and capacity were reported to highlight the performance limitations due to PN.

Most of the previous papers analyzing the PN effects have concentrated on characterizing the ICI (calculating the ICI power for various PN and OFDM system parameters) and only a few authors have derived theoretical error formulae [8], [9], [10]. A drawback of these efforts is their simplified theoretical formulation at least for the purpose of error prediction in the fading channels. An accurate error analysis for the fading channel must consider the joint effects between the useful OFDM signal and the interference due to PN. Many have also assumed Gaussian interference characteristics for a wide variety of PN which have been questioned in a recently published paper by Petrovic *et al.* [6]. By adopting the relative PN bandwidth (incorporates both PN and OFDM system parameters), they have shown that the ICI cannot be categorized by a Gaussian random variable for a wide range of PN. This brings up several interesting questions such as

1). Does previous work predicting the error performance using Gaussian assumptions for the ICI yield pessimistic symbol error rate (SER) results?

2). In many cases of practical interest, a Gaussian error analysis is mathematically tractable. One can use a plethora of already existing results for the purpose of error analysis. Hence when ICI characteristics deviate from Gaussian conditions, is there a way that we can modify a Gaussian analysis to predict accurate theoretical results?

3). In multipath channels, can we consider joint fading effects for the OFDM signal and the interference due to PN to build a better theoretical error solution?. Note that in this scenario the OFDM system performance is dominated by the bad subcarriers which are subject to deep fading. For these subcarriers the effect of interference is also minimal. Hence for analytical tractability if ICI is assumed as Gaussian even for cases when it is not so quite true, will we reduce the accuracy of the SER results?

In this paper our aim is to answer these questions. The results presented here indicate that modeling the PN as Gaussian and evaluating the SER in the AWGN channel leads to appreciable discrepancies between the theoretical and simulated results especially at high values of signal-to-noise

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ratio. However in fading channels, provided that the channel correlation effect between the useful OFDM signal and the ICI due to PN is considered, the theoretical results closely agree with the simulations. In summary our contributions in this paper are:

1). In the presence of PN, accurate theoretical error performance evaluation techniques are presented for frequency flat and selective fading channels. It uses the existing error performance analysis concepts, however to our best knowledge their applicability to the PN problem in OFDM is novel.

2). The consequences of assuming Gaussian ICI characteristics (when it is not the case) for the analytical error performance prediction in AWGN channels are addressed.

This paper is organized as follows. In Section II we introduce the OFDM system model with PN, and ICI effects are analyzed. In Section III and IV the SER performance of the OFDM system is investigated over AWGN and fading channels. The simulated results are presented in Section V. Finally we summarize the main findings of this paper and some conclusions are drawn in Section VI.

Notation: $E(\cdot)$ denotes the statistical expectation operator. Lower/upper case letters are used to denote time/frequency variables. $(\cdot)^*$ is the complex conjugate.

II. OFDM System with Phase Noise

In this Section we describe the OFDM system model in the presence of PN. The OFDM transmissions consist of symbol blocks each with a period of T_s seconds. In this time N subcarriers $X(k)$ for $k = 0, 1, ..., (N-1)$ are modulated by a signal alphabet $\mathcal{A}$ and used for carrying information. For 4-QAM $\mathcal{A} \in \{\pm 1 \pm j\}$ and $j = \sqrt{-1}$. The sampled m-th transmitted time domain OFDM symbol to the channel is

$$x_m(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_m(k) e^{\frac{j2\pi nk}{N}} \tag{1}$$

where the constant $1/\sqrt{N}$ normalizes the power and $n = -N_c, ..., (N-1)$. Next a cyclic prefix greater than the maximum channel delay is appended and the composite signal is transmitted to the channel. N_c is the cyclic prefix length. In this paper we assume a tap delay-line multipath channel model given by

$$h(t,\tau) = \sum_{l=0}^{L-1} h_l(t) \delta\left(t - \tau_l\right) \tag{2}$$

In (2) L signifies the number of channel paths, $\{h_l(t)\}_{l=0}^{L-1}$ denotes zero mean complex Gaussian random variables and τ_l is the l-th delay in OFDM sample spacing Δt. For simplicity and without loss of generality delays $\{\tau_l\}_{l=0}^{L-1}$ are assumed to be in integer multiples of Δt. Note that (2) is generic and the frequency flat/selective conditions could be simulated using this model with varying parameters.

The sampled received OFDM signal $y(n)$ in the presence of PN impairments is given by

$$y_m(n) = e^{j\theta(n)} \{x_m(n) \otimes h(n)\} + w(n) \tag{3}$$

The PN $\theta(n)$ produces a multiplicative distortion on the OFDM signal as indicated by (3). The random variable $w(n)$ represents complex AWGN with zero mean and variance $N_0/2$ per real, imaginary dimensions. The transmitted information on the k-th subcarrier is obtained by passing $y_m(n)$ through an N-point discrete Fourier transform (DFT). The demodulated signal for the k-th subcarrier $Y_m(k)$ is given by

$$Y_m(k) = I_m(0) H_m(k) X_m(k) + I_m(k) + W(k) \tag{4}$$

where $I_m(0)$ is the CPE and $W(k)$ is the k-th subcarrier AWGN. $I_m(k)$ is the ICI component for the k-th subcarrier respectively given by

$$I_m(0) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\theta_m(n)} \tag{5}$$

and

$$I_m(k) = \frac{1}{N} \sum_{l=0}^{N-1} e^{j\left(\theta_m(l) + \frac{2\pi lk}{N}\right)} \tag{6}$$

A. Phase Noise Models

Many parameters are required for accurate PN modeling [11]. Past literature have relied both on practical measurements and theoretical assumptions for the purpose. For the free running oscillator, a popular model is to define PN a sampled Brownian motion process. Hence is the discrete time PN is given by $\theta(n+1) = \theta(n) + \epsilon(n)$, where $\epsilon(n)$ is a Gaussian random variable with zero mean and variance $\sigma_\epsilon^2 = 4\pi\beta T_s$. T_s is the sample time and β refers to the 3 dB bandwidth of the corresponding oscillator power spectral density. A detailed description of the PN process and parameters involved in the free-running can be found in [6].

In [6] the similarity of the ICI to a Gaussian distribution is compared using a statistical measure called the *kurtosis* $\mathcal{K}$ defined by

$$\mathcal{K} = \frac{E\{\nu^4\}}{\sigma_n^4} - 3 \tag{7}$$

The random variable ν corresponds to ICI and σ_n^2 is the ICI power. In other words if $\mathcal{K} = 0$, then ICI due to PN follows a normal distribution.

III. SER Analysis in AWGN Channels

In this Section we use the Gaussian ICI model (often assumed in the previous technical literature) to evaluate the SER of an OFDM system in the presence of PN. The validity of the results obtained is discussed in Section V.

In the AWGN channel, the channel gains $H_m(k)$ are set to unity and (4) is expressed by $Y_m(k) = I_m(0)X_m(k) + N_m(k)$. The total noise observed at the receiver is $N_m(k)$ and is given by

$$N_m(k) = \sum_{l=0, l\neq k}^{N-1} I_m(l-k) X_m(k) + W(k) \tag{8}$$

The SER in AWGN channel is evaluated by expressing the effective SINR including PN and using the error expression for

M-QAM modulation. Assuming coherent detection the SINR γ is written as [5]

$$\gamma = \frac{E[|I_m(0)|^2]\sigma_x^2}{\sigma_n^2 + N_0} \tag{9}$$

where σ_x^2 is the average signal power and the ICI power is given by [5]

$$\begin{aligned}\sigma_n^2 &= E\left(\left|\sum_{l=0,l\neq k}^{N-1} I_m(l-k)X_m(l)\right|^2\right) \\ &= \sigma_x^2\left(\sum_{l=0,l\neq k}^{N-1} E\{|I_m(l-k)|^2\}\right)\end{aligned} \tag{10}$$

However noting that the received signal is only multiplied by the phase factor $e^{j\theta(n)}$, σ_n^2 can also be written as

$$\sigma_n^2 = \sigma_x^2(1 - E\{|I_m(0)|^2\}) \tag{11}$$

Since in the previous literaturee ICI due to PN is assumed as Gaussian, average SER P_E is simple to calculate. For M-QAM $P_E(k)$ is expressed by

$$P_E = 2a \times \text{erfc}\left(\sqrt{\frac{\gamma}{b}}\right) - a^2 \times \text{erfc}^2\left(\sqrt{\frac{\gamma}{b}}\right) \tag{12}$$

where $a = 4(1 - 1/\sqrt{M})$, $b = 2(M-1)/(3\log_2 M)$ and the complementary error function $\text{erfc}(x) = 2/\sqrt{\pi}\int_x^\infty e^{-t^2}dt$.

IV. SER Analysis in Fading Channels

OFDM is robust against multipath fading. This fact is the main reason for its adoption in many broadband communication systems. In the following the performance evaluation of OFDM over frequency flat/selective fading channels due to PN impairments is studied.

A. Frequency Flat Fading Channels

When the wireless channel has frequency flat fading characteristics, all OFDM subcarriers experience same severity of fading. Hence we drop the subscript denoting the subcarriers for simplicity, i.e., $H_m(k) = H_m$ and $|H_m(k)|^2 = \alpha^2$ for $k = 0, 1, ..., (N-1)$. The received signal $Y_m(k)$ is

$$\begin{aligned}Y_m(k) &= I_m(0)H_mX_m(k) \\ &+ H_m\left(\sum_{l=0,l\neq k}^{N-1} I_m(l-k)X_m(l)\right) + W(k)\end{aligned} \tag{13}$$

The instantaneous SINR at the receiver decision device conditioned on the channel coefficient is given by

$$\gamma_h = \frac{E[|I_m(0)|^2]\alpha^2\sigma_x^2}{\alpha^2\sigma_n^2 + N_0} \tag{14}$$

In order to evaluate the SER in the flat fading channel, we note that the instantaneous SINR is a function of the Rayleigh distributed random variable α. Hence the average SER in the flat fading channel is obtained by averaging the AWGN error expression over channel Rayleigh pdf, $p(\alpha)$ of α.

$$p(\alpha) = \frac{2\alpha}{\bar{\alpha}}e^{-\alpha^2/\bar{\alpha}}, \ \alpha > 0 \tag{15}$$

where $\bar{\alpha} = E\{|H|^2\}$. This is a well known method of calculating the error performance in fading channels [12]. P_E is expressed by

$$\begin{aligned}P_E =& \frac{4a}{\bar{\alpha}}\int_0^\infty \text{erfc}\left(\sqrt{\frac{\gamma_h}{b}}\right)\alpha e^{-\alpha^2/\bar{\alpha}}d\alpha \\ &- \frac{2a^2}{\bar{\alpha}}\int_0^\infty \text{erfc}^2\left(\sqrt{\frac{\gamma_h}{b}}\right)\alpha e^{-\alpha^2/\bar{\alpha}}d\alpha\end{aligned} \tag{16}$$

The integrals in (16) can be evaluated as infinite summations. These summations can also be truncated without compromising much accuracy. The first integral of (16) is given by [15]

$$\begin{aligned}I_1 &= \frac{4a}{\bar{\alpha}}\int_0^\infty \text{erfc}\left(\sqrt{\frac{\gamma_h}{b}}\right)\alpha e^{-\alpha^2/\bar{\alpha}}d\alpha \\ &= a - \frac{ac}{\sqrt{2}d^3}e^{\frac{-c^2}{2d^2}}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{c^2}{2d^4}\right)^k U\left(k+\frac{3}{2}, k'; \frac{1}{d^2}\right)\end{aligned} \tag{17}$$

The confluent hypergeometric function of the second kind

$$U(x, y; z) = 1/\Gamma(x)\int_0^\infty e^{-zt}t^{x-1}(1+t)^{y-x-1}dt \tag{18}$$

$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\ dt$ and the second integral involved in (16) is simplified as

$$\begin{aligned}I_2 &= -\frac{2a^2}{\bar{\alpha}}\int_0^\infty \text{erfc}^2\left(\sqrt{\frac{\gamma_h}{b}}\right)\alpha e^{-\alpha^2/\bar{\alpha}}d\alpha \\ &= \frac{a^2}{2} - \frac{a^2c}{\sqrt{2}d^3}e^{\frac{-c^2}{2d^2}}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{c^2}{2d^4}\right)^k U\left(k+\frac{3}{2}, k', \frac{1}{d^2}\right) \\ &+ \frac{4\sqrt{2}a^2}{\sqrt[3]{\pi}}e^{\frac{1-2c^2}{2d^2}}\left(\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{d^{-3k}c^{(2k+2n+4)}\,\Gamma(n+2)}{(2n+1)n!(2d)^{2n}}\right. \\ &\left.\times(-1)^n \times W_{-n-2-\frac{k}{2},\frac{k+1}{2}}\left(\frac{1}{d^2}\right)\right)\end{aligned} \tag{19}$$

where $k' = (k+2)$, $c^2 = E[|I_m(0)|^2]\sigma_x^2/bN_0$ and $d^2 = \sigma_n^2/N_0$. $W_{(\cdot)}$ is the Whittaker function [16]. $\bar{\alpha}$ is set to unity.

B. Frequency Selective Fading Channels

In general, most of the wireless fading channels encountered in practice are frequency selective. In this case the received signal is given by (4). Since the fading channel coefficients are different across the subcarrier spectrum, $H_m(k)$ can not be factored out of the ICI summation. See (13). However in order to perform a theoretical analysis similar to flat fading, we consider the adjacent channel frequency domain correlation. Hence the k-th subcarrier $H_m(k)$ could be written as [15],

$$H_m(k) \approx \lambda_{\Delta k}H_m(l) + \left(\sqrt{1-\lambda_{\Delta k}^2}\right)Z_m \tag{20}$$

The correlation coefficient $\lambda_{\Delta k}$ depends on the distance or separation between k-th and l-th frequency bins. Z_m is modeled as a Gaussian random variable, $Z_m \sim \mathcal{N}(0, 1)$. In other terms given that k-th and l-th bins are sufficiently apart, the channel correlation is negligible.

$$\begin{aligned}\lambda_{\Delta k} &= E\{H(k)H^*(k+\Delta k)\} \\ &= E\left\{\Big(\sum_{l=0}^{L-1} h(l)e^{\frac{-j2\pi kl}{N}}\Big)\Big(\sum_{m=0}^{L-1} h^*(m)e^{\frac{-j2\pi(k+\Delta k)m}{N}}\Big)\right\} \\ &= \sum_{l=0}^{L-1} \sigma_l^2 e^{\frac{j2\pi\Delta kl}{N}}\end{aligned} \tag{21}$$

In (21) σ_l^2 is the power of the l-th tap. For example, assuming a uniform power delay profile, i.e., $\sigma_l^2 = 1/L$ (21) can be further simplified as [14]

$$\lambda_{\Delta k} = \frac{1}{L}\frac{\sin\left(\frac{\pi\Delta kL}{N}\right)}{\sin\left(\frac{\pi\Delta k}{N}\right)} e^{j\pi\Delta k(L-1)/N} \tag{22}$$

By substituting (20), into (4) to obtain

$$\begin{aligned}Y_m(k) &= I_m(0)H_m(k)X_m(k) \\ &+ H_m(k)\left(\sum_{l=0,l\neq k}^{N-1} \lambda_{|l-k|} I_m(l-k)X_m(l)\right) \\ &+ \sum_{l=0,l\neq k}^{N-1} I_m(l-k)X_m(l)Z_m(l) + W(k)\end{aligned} \tag{23}$$

Hence the instantaneous k-th subcarrier SINR in the general frequency selective case can be established from

$$\gamma_h = \frac{E[|I_m(0)|^2]\alpha^2\sigma_x^2}{\alpha^2\sigma_{n_1}^2 + \sigma_{n_2}^2 + N_0} \tag{24}$$

where $\sigma_{n_1}^2$ and $\sigma_{n_2}^2$ are given by

$$\sigma_{n1}^2 = E\left(\left|\sum_{l=0,l\neq k}^{N-1} \lambda_{|l-k|} I_m(l-k)X_m(l)\right|^2\right) \tag{25}$$

and

$$\sigma_{n2}^2 = E\left(\left|\sum_{l=0,l\neq k}^{N-1} I_m(l-k)X_m(l)Z_m(l)\right|^2\right) \tag{26}$$

To calculate the error performance, one can now follow the exact steps similar to the case of flat fading. The above analysis (16) – (19) also leads to similar integrals found in (16) and hence the theoretical SER in closed form can be derived. The total SER P_E of the OFDM system (for frequency selective case) is calculated as the mean of $P_E(k)$. Hence assuming that all the N subcarriers are for carrying data,

$$P_E = \frac{1}{N}\sum_{k=0}^{N-1} P_E(k) \tag{27}$$

In above SER derivations we assumed perfect channel state information at the receiver. However consider a practical OFDM implementation, it is worth mentioning the following:

1). The performance of an OFDM system will depend on the accuracy of the channel estimation output and detection technique employed.

2). In the presence of PN, the channel estimator output is $\hat{H}_m(k) = 1/H_m(k)e^{-j\arg[I_m(0)]}$. However since the CPE is varying per OFDM symbol basis, this variation must also be tracked. The value for $\hat{H}_m(k)$ also depends on the channel estimation method employed (for example, pilots multiplexed with the data subcarriers or block channel estimation). In block channel estimation, further complications can occur due to the use of outdated values. Analyzing these effects are beyond the scope of this paper.

Note that the use of the frequency correlation function to derive an accurate theoretical error formula in OFDM systems involving ICI was first reported in [15]. However the work reported in [15] addresses the ICI problem due to the carrier frequency offset (CFO). The nature of ICI due to PN and CFO are different. While the adjacent channel ICI in CFO is more significant (hence has some kind of regularity), the ICI due to PN has a random structure via PN samples. Hence our approach for the ICI error analysis due to PN is sufficiently novel. Besides, to our best knowledge previous authors have only followed a channel independent approach for error performance evaluation due to PN, which results in inferior theoretical predictions. In this approach rather than treating the signal and ICI fading effects jointly, a separate estimate is made for the faded ICI power. Hence it overestimates the ICI contribution for some subcarriers.

V. Simulation Results

We have performed Matlab simulations to investigate the accuracy of the theoretical analysis provided in previous Sections. In all cases $N = 64$ and uncoded 16-QAM. For frequency selective scenario, a block fading channel was assumed. In other words an independent channel realization was generated per transmitted OFDM symbol. The channel power delay profile is uniform with three paths. Tap spacing is in integer multiples of OFDM sample spacing. The PN process described in [13] is considered. The PN samples are generated by filtering white Gaussian noise with zero mean and unit variance. The mask is $H(z) = a_1/(1 - b_1 z^{-1})$ where $a_1 = 0.0316$, $b_1 = 0.9999$ and PN variance is σ_θ^2 [13].

Fig. 2 shows the simulated SER performance in the AWGN channel. For comparison reasons, we have also plotted the theoretical curves assuming a Gaussian distribution for the PN. Clearly the two sets of results deviate substantially. The predicted theoretical results are lower than the simulations in all cases, however their difference reduce as σ_θ^2 is increased.

Fig. 3 illustrates the theoretical and simulated SER performance for the frequency selective Rayleigh fading channel. Note that in this case the predicted results from theory are closer to the actual simulated values (compared to the AWGN channel). In fading channels, one has to carefully consider the joint effects of fading on the OFDM signal and ICI to accurately calculate the error rate. The error performance of an OFDM system is mainly influenced by bad subcarriers

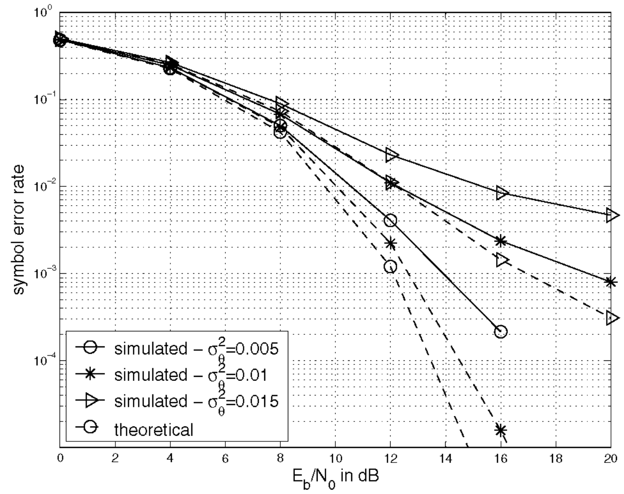

Fig. 2. SER against E_b/N_0 for the AWGN channel. Theoretical results are depicted with dashed lines. Their marker patterns are matched to the corresponding simulation results.

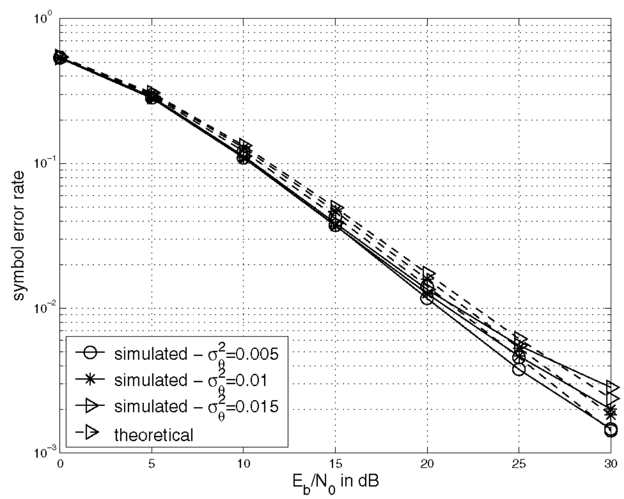

Fig. 3. Theoretical and simulated SER against E_b/N_0 for the frequency selective fading channel. Theoretical results are depicted with dashed lines. Their marker patterns are matched to the corresponding simulation results.

subjected to deep fading nulls in this scenario. For these subcarriers, the effective faded ICI acting as noise is also small. In order to apply the findings of this paper for a wide variety of oscillators and hence types of PN, additional simulations must be performed. For example, the theoretical and simulated error performance evaluation using the free running oscillator model (parameterized by the relative PN bandwidth [6] which incorporates both PN and OFDM system parameters) in AWGN and fading channels is worth mentioning.

The theoretical conceptual framework employed in this analysis (with some modifications) also holds useful in several cases of OFDM performance evaluation such as clipped OFDM and systems experiencing co-channel and adjacent channel interference. Multiband OFDM and OFDM systems affected by existing communication systems are some examples of the later type. In these scenarios, by applying the concepts discussed, more accurate theoretical predictions can be made.

VI. Conclusions

In this paper we have analyzed the validity of the well known Gaussian ICI assumption due to PN for predicting the SER performance of an OFDM system. The results reported are for AWGN and frequency flat/selective channels. Considering the fading effects on the OFDM signal and the ICI, a theoretical SER expression was calculated in closed form. Our simulation results indicate that modeling ICI due to PN as Gaussian in AWGN channel leads to pessimistic theoretical predictions. However in the fading scenario, the difference between the simulated and the theoretical result is marginal. In the fading channel, the error performance is governed by subcarriers in deep nulls. For these subcarriers ICI effects are minimal as it also is exposed to fading.

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