# Five Essentials for Math Instruction Within Your MTSS Framework

# Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin

www.sarahpowellphd.com

srpowell@utexas.edu

@sarahpowellphd

Say hello.

Share a bit about yourself and the math you support.

# Objectives

Participants will describe the core components of explicit instruction.

Participants will describe why formal math language is important.

Participants will explain different representations that can help students understand concepts and procedures.

Participants will outline ways to build fluency.

Participants will describe two effective practices for word-problem instruction.

# Five Essentials for Math Instruction Within Your MTSS Framework

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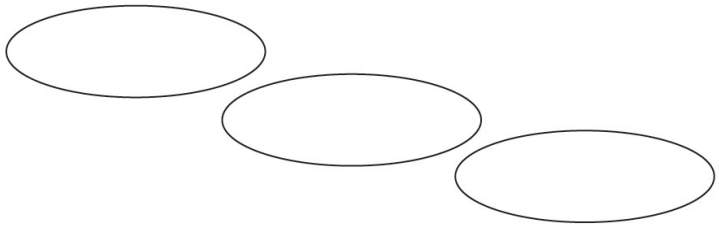
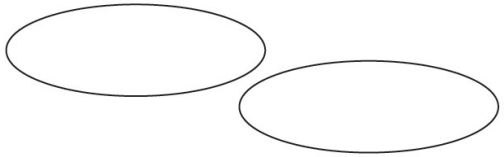
srpowell@utexas.edu
@sarahpowellhd
www.sarahpowellphd.com

Instructional Platform

Instructional Delivery



Instructional Strategies



# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

# Objectives

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# Explicit Instruction

# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit instruction

## INSTRUCTIONAL STRATEGIES

Explicit Instruction

MODELING

PRACTICE

SUPPORTS

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

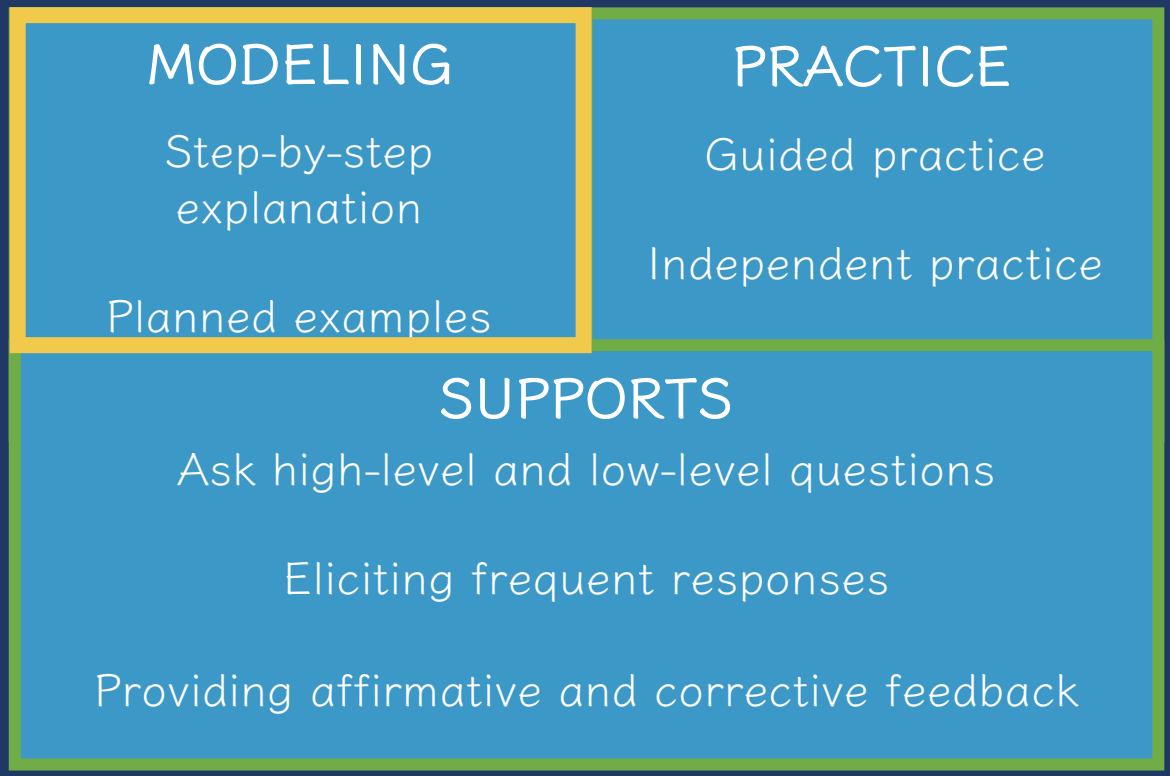
## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

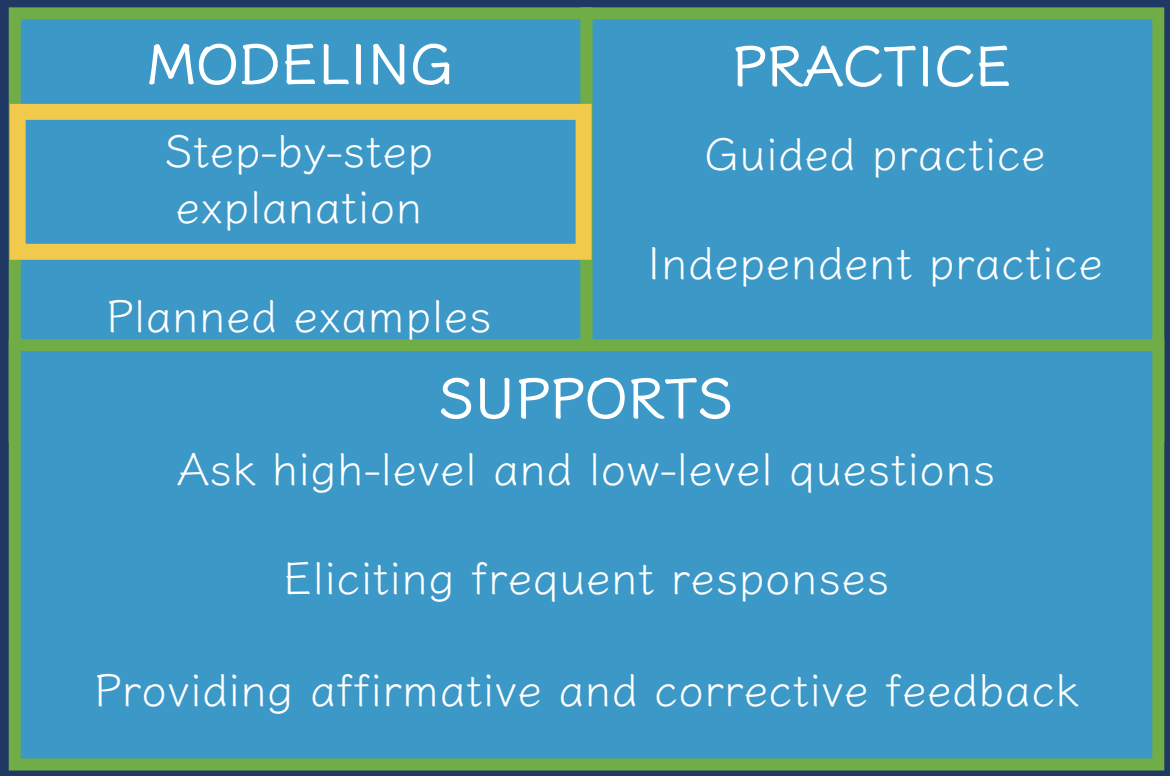
Providing affirmative and corrective feedback

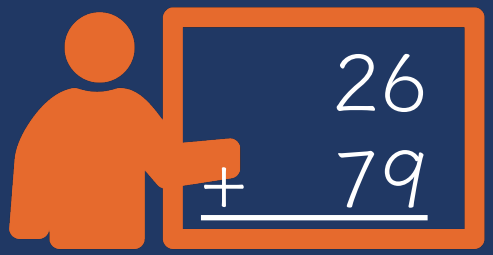
**Modeling** is a dialogue between the teacher and students.



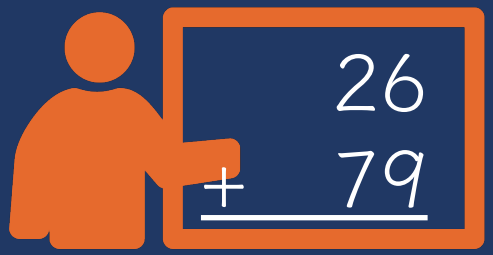
**Modeling** includes a step-by-step explanation of how to do a math problem.

A teacher may do 1 modeled problem or several.






"Today, we are learning about addition. This is important because sometimes you have different amounts – like money – and you want to know how much money you have altogether."




"Let's solve this problem. What's the problem?"

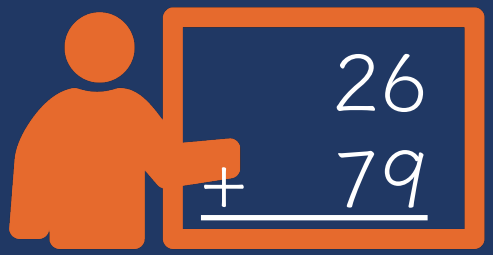
"26 plus 79."

"To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?"

"Add."

"How did you know we want to add?"

"There's a plus sign."




"The plus sign tells us we want to add. To add, let's use the partial sums strategy. What strategy?"

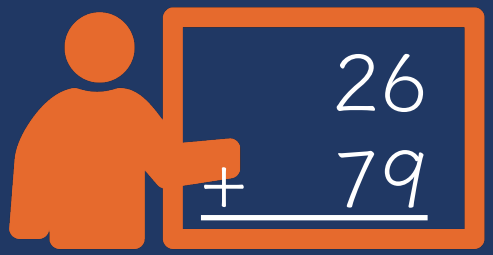
"Partial sums."

"With the partial sums strategy, we start adding in the greatest place value. What's the greatest place value in this problem?"

"The tens."

"So, let's add the tens. What's 20 plus 70?"

"90."




"20 plus 70 equals 90. Let's write 90 right here below the equal line. What will we write?"

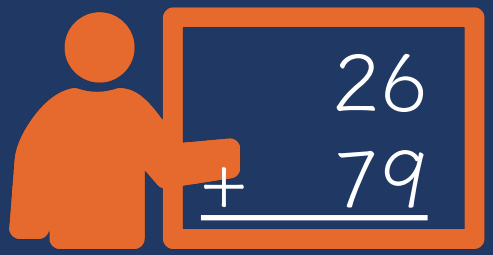
"90."

"90 is the partial sum when you add the tens. What does 90 represent?"

"It's the partial sum of adding 20 plus 70."

"Now, let's add the ones. What should we add?"

"6 plus 9."




"6 plus 9 equals what?"

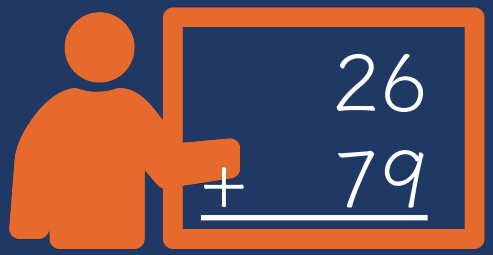
"15."

"Let's write 15 below the 90. Where do we write the 15?"

"Below the 90."

"15 is the partial sum when you add the ones. Now, let's add the partial sums together. What will we add?"

"90 plus 15."




"What's 90 plus 15?"

"How did you add those numbers?"

"So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?"

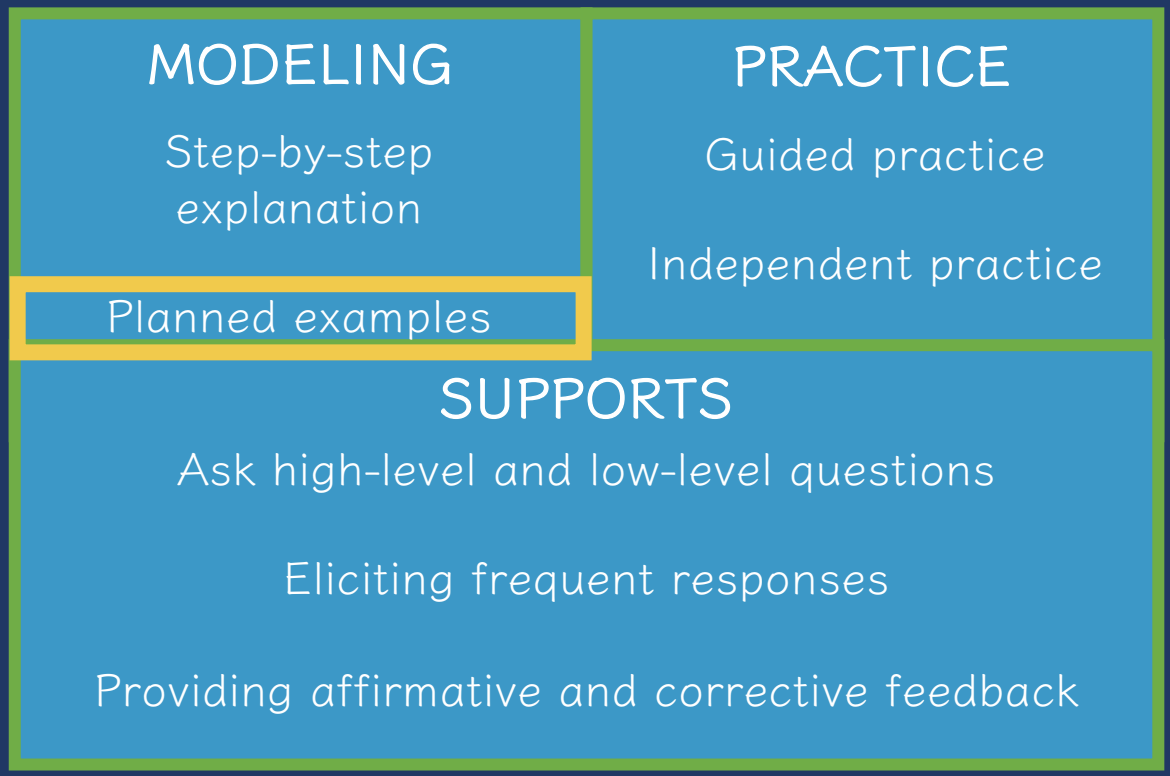
"105."

"I added 90 plus 10 then added 5 more."

"We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums."

**Modeling** needs to include planned examples.

These examples should be sequenced so easier skills lead to more difficult skills.



## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

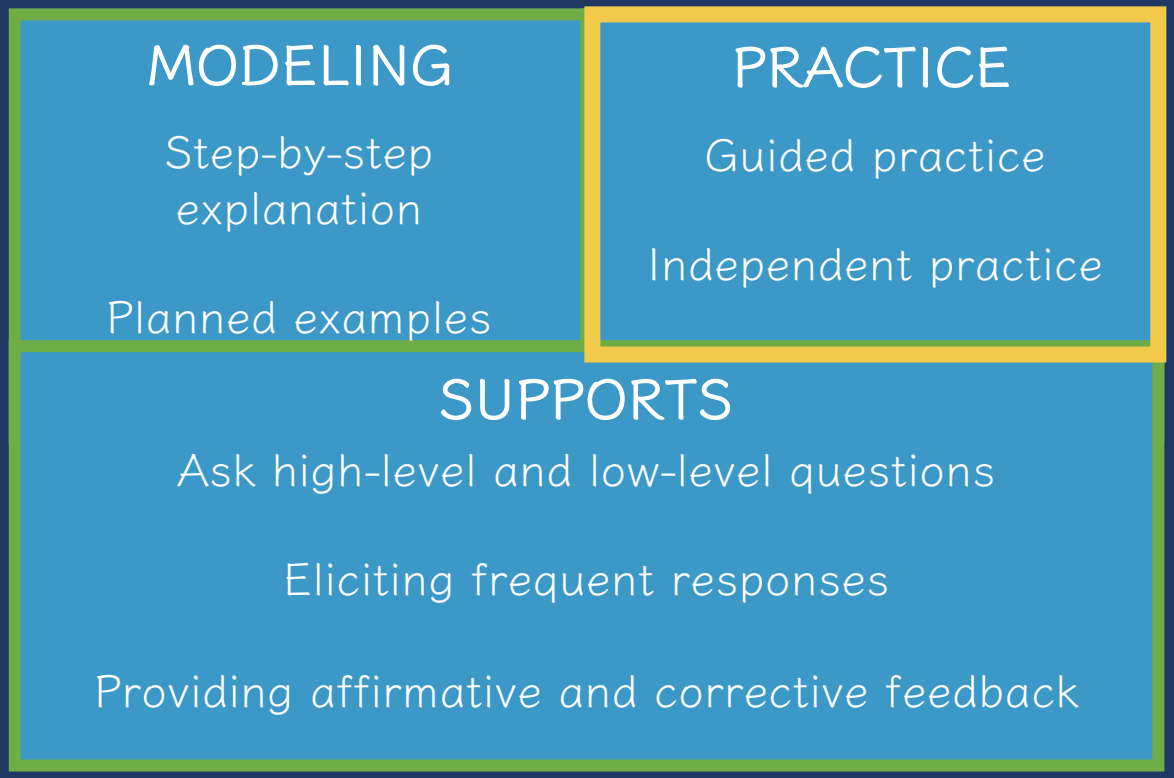
Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback




**Practice** continues as a dialogue between the teacher and students.

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## SUPPORTS

Ask high-level and low-level questions

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Guided practice is practice in which the teacher and students practice problems together.

"Let's work on a problem together."

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Independent practice is practice in which the students practice independently with teacher support.

"Now, you'll practice a problem on your own. Use your attack strategy!"

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Step-by-step explanation

Planned examples

## PRACTICE

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Ask high-level and low-level questions

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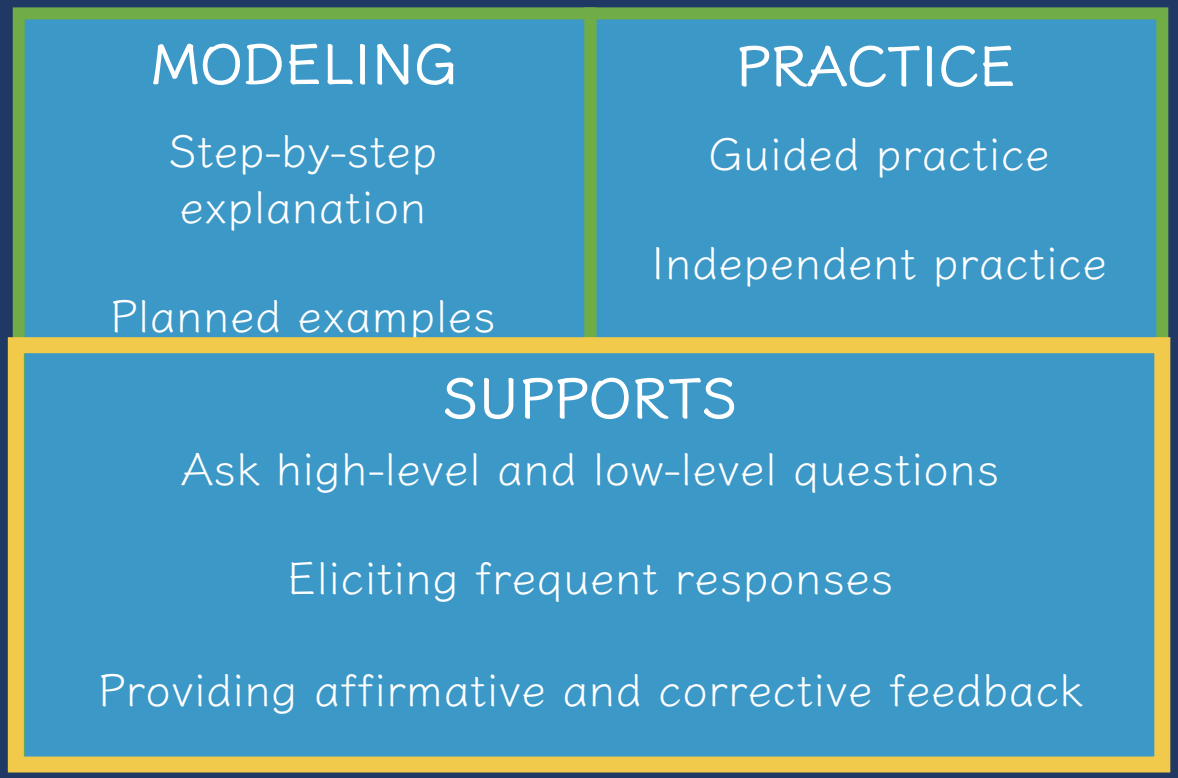
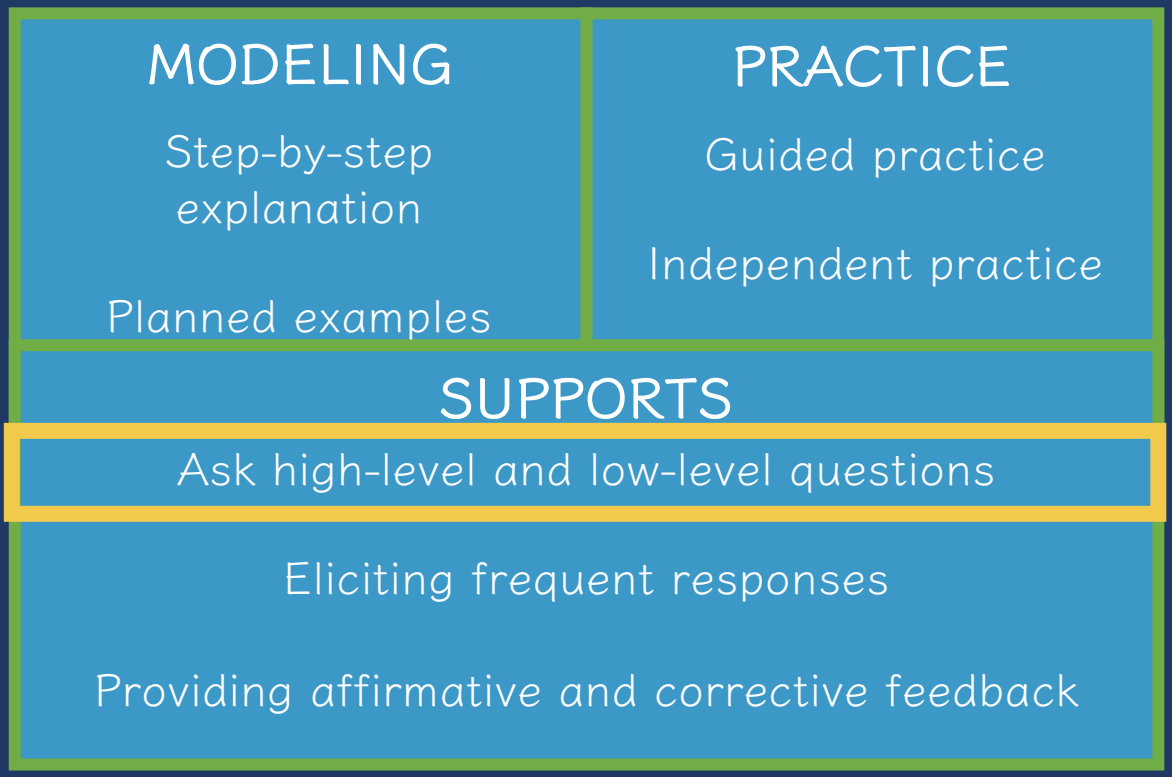



These **Supports** should be used in both **Modeling** and **Practice.**




During **Modeling** and **Practice**, it is essential to engage students and check for understanding.

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"What is 7 times 9?"

"63."

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

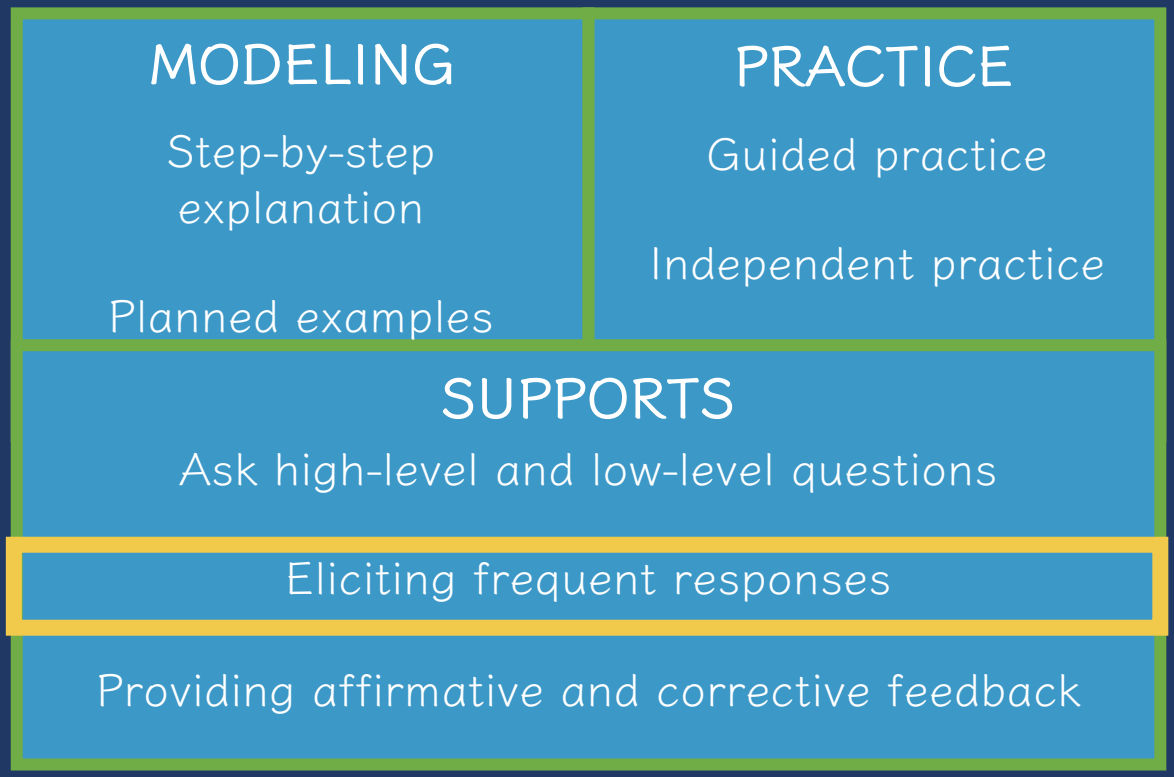
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Why do you use zero pairs?"

"Because a positive 1 and a negative 1 equal 0. I use the zero pair to help me subtract."




During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

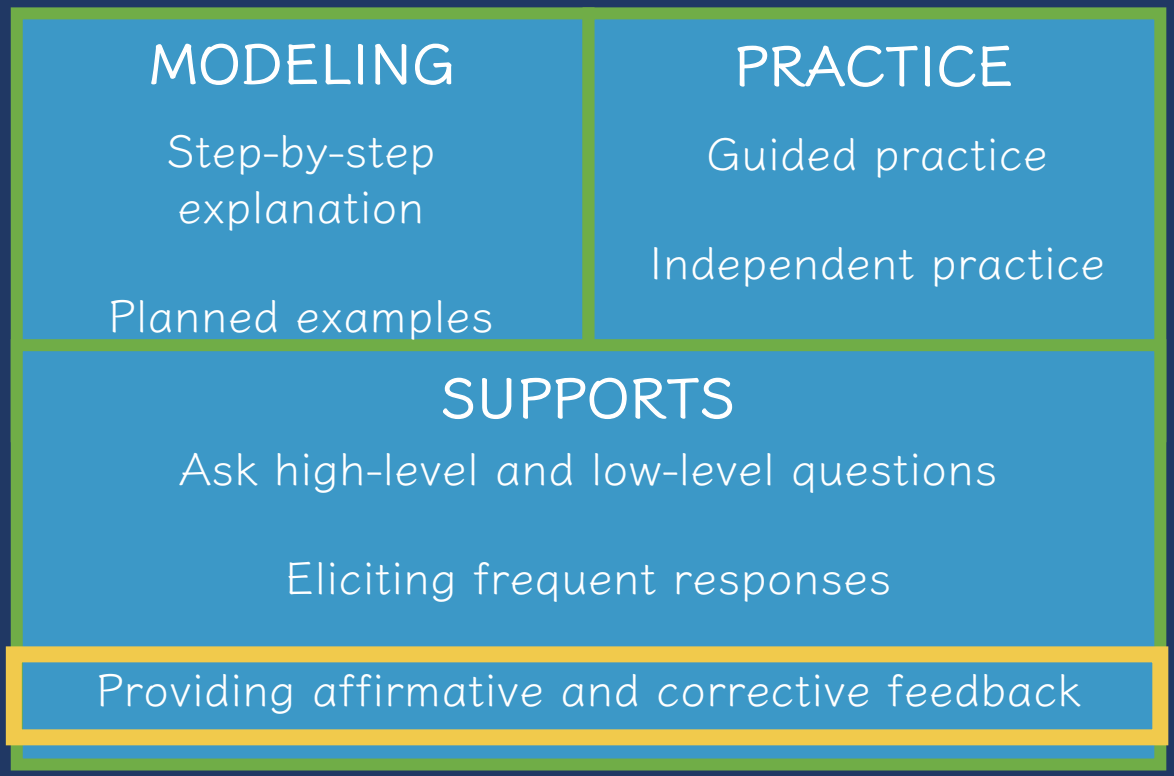
## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

- Oral
- Written
- With manipulatives
- With drawings
- With gestures




During **Modeling** and **Practice**, students should receive immediate feedback on their responses.

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

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Providing affirmative and corrective feedback

"Nice work using your word problem attack strategy."

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Let's look at that again. Tell me how you added in the hundreds column."

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

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Ask high-level and low-level questions

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Ask high-level and low-level questions

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Providing affirmative and corrective feedback

What are your strengths with explicit instruction?

What are opportunities for growth with explicit instruction?

# Objectives

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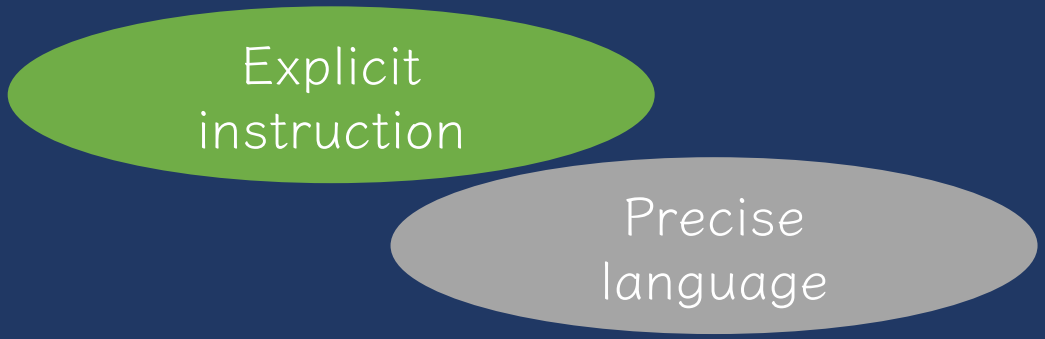
# Mathematical Language

# Instructional Platform

## INSTRUCTIONAL DELIVERY




## INSTRUCTIONAL STRATEGIES

## Mathematical Language

| Instead of that... | Say this... |
|---|---|
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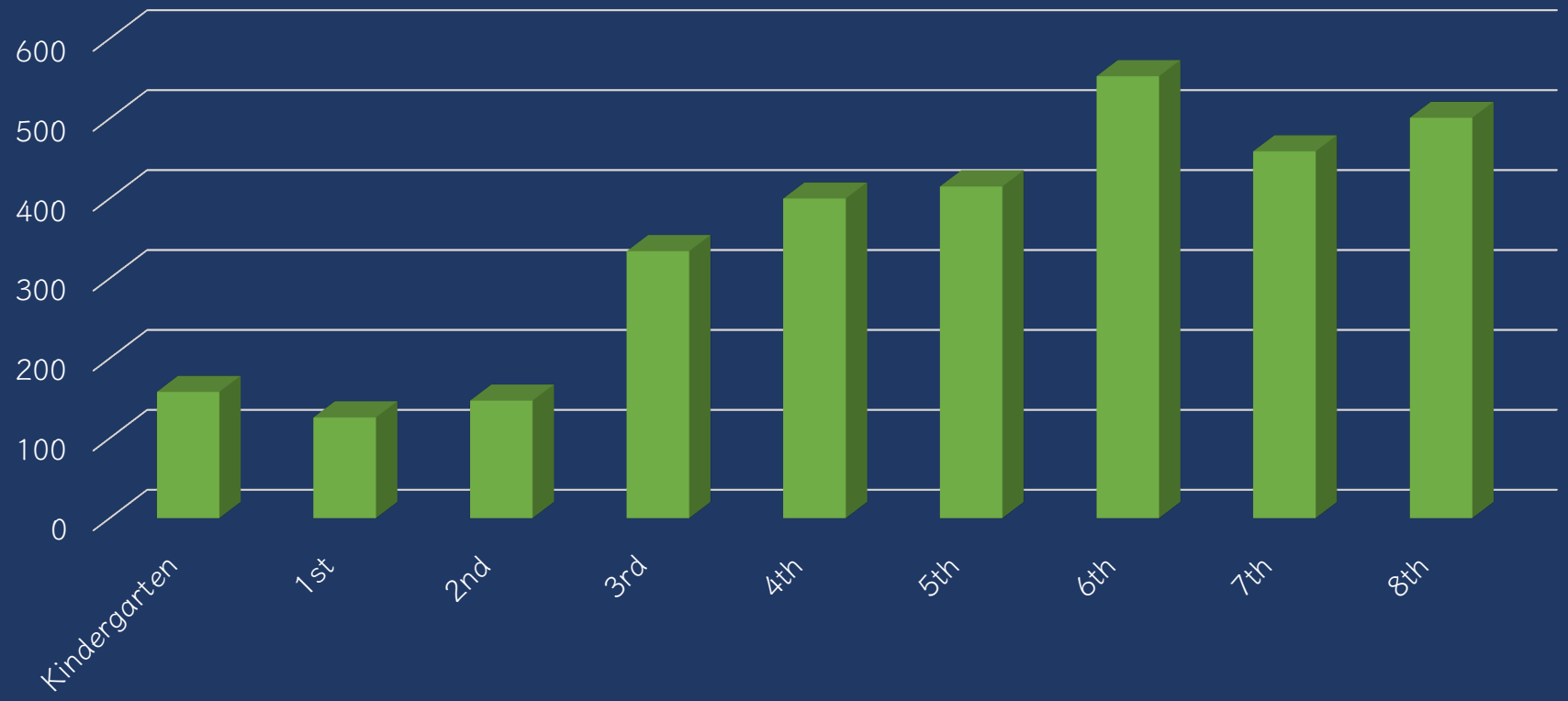
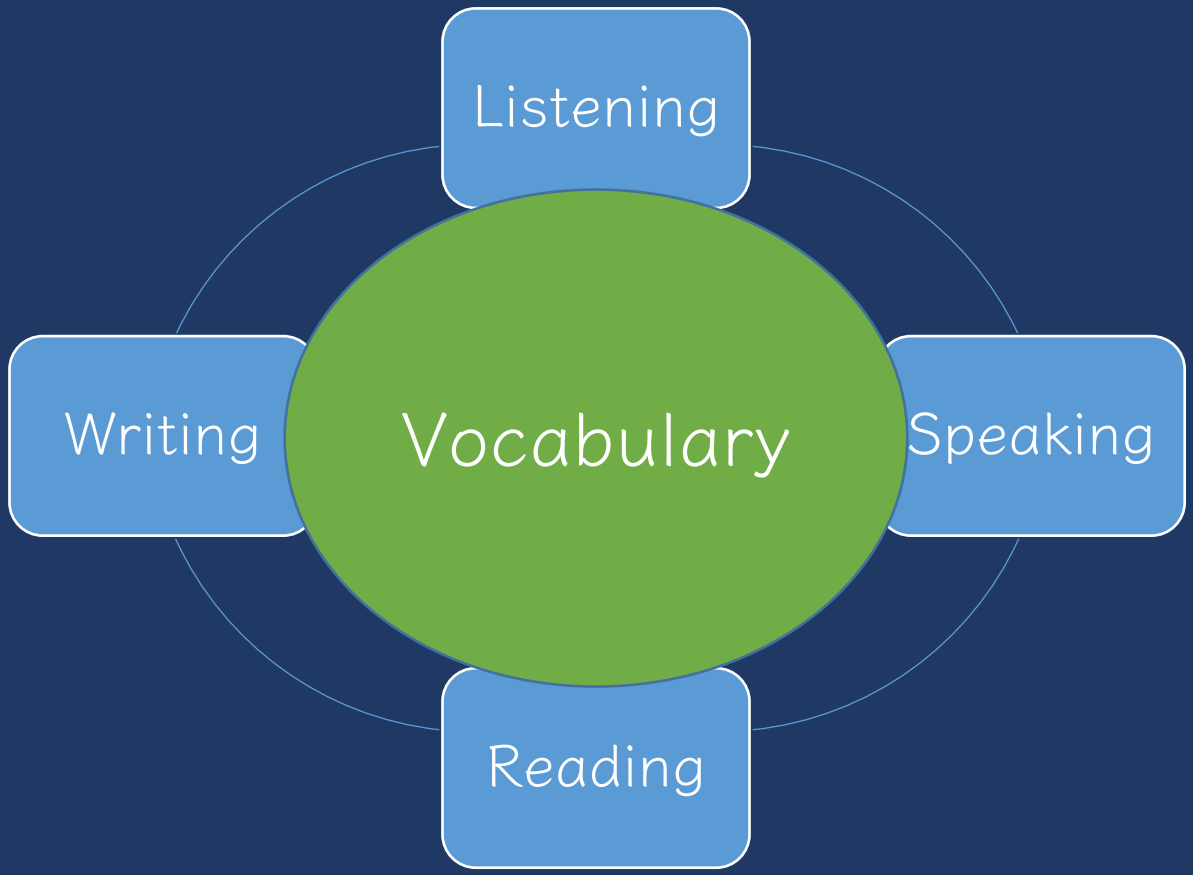



Powell, Bos, & Lin (2019)



Use formal math language

Use terms precisely

What number is in the tens place?

What digit is in the tens place?
What is the value of the digit in the tens place?

135

**Why this is important...**

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.

The alligator eats the bigger number

**is less than**
OR
**is greater than**

**Why this is important...**

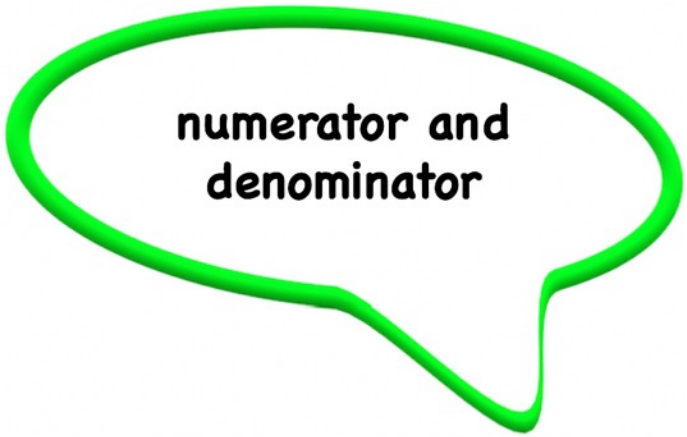
- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.



$$\begin{array}{r} 167 \\ +\ 294 \\ \hline \end{array}$$

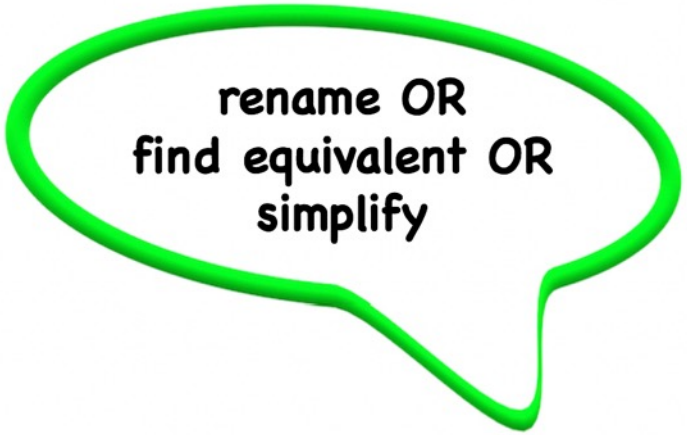
**Why this is important...**

- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.

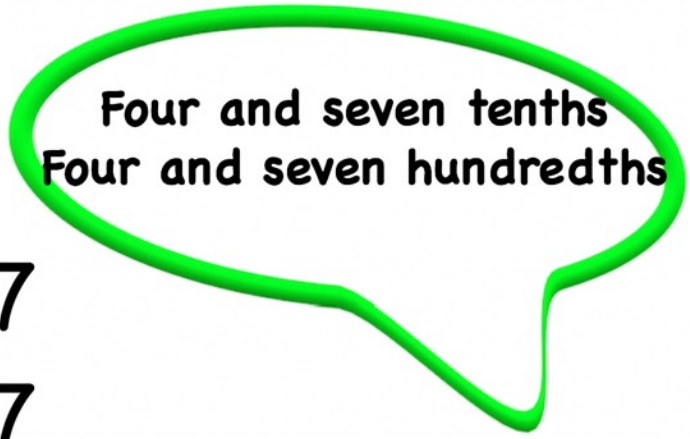

**Why this is important...**

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.


**Why this is important...**

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.


4.7

4.07

**Why this is important...**

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.

box OR ball

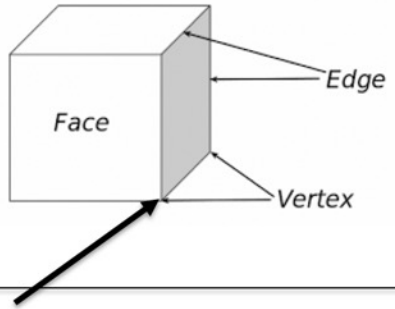
square OR circle

**Why this is important...**

- Use the formal language of shapes to confirm informal language.

point

vertex




**Why this is important...**

- This is the endpoint where two or more line segments or rays meet.

Mathematical Language

| Instead of that... | Say this... |
|---|---|
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| | |

Identify examples of "Instead of ____, say ____."

Use formal math language

Use terms precisely

## Factor

$1 \times 8 = 8$

$2 \times 4 = 8$

factor factor

## Multiple

$8 \times 1 = 8$

$8 \times 2 = 16$

multiples of 8

E

## Improper fraction

$\frac{8}{5}$

## Proportion

$\frac{2}{5} = \frac{8}{20}$

## Mixed number

$1\frac{3}{5}$

## Ratio

4:3

## Proper fraction

$\frac{2}{9}$

## Unit fraction

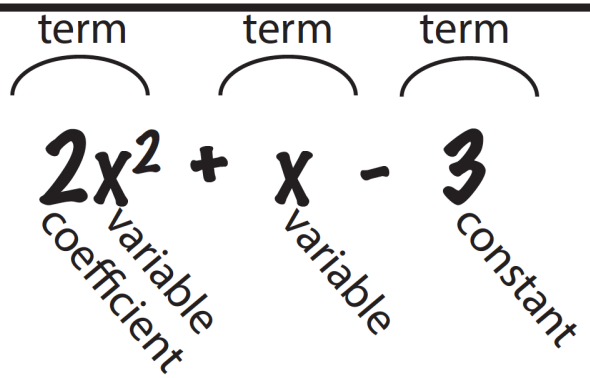
$\frac{1}{6}$

D

| | |
|---|---|
| ***Equation*** | $9x - 4 = 7x$ |
| ***Expression*** | $9x - 4$ |
| ***Formula*** | $a^2 + b^2 = c^2$ |
| ***Function*** | $f(x)$ |
| ***Inequality*** | $9x - 4 > 6x$ |

C

***Coefficient***
***Constant***
***Term***
***Variable***




A

## Quadrilaterals

**Kite**

**Rhombus**

**Parallelogram**

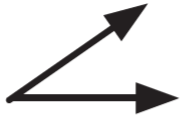
**Square**

**Rectangle**

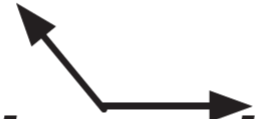
**Trapezoid**

A

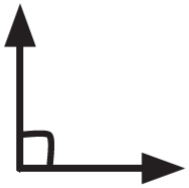
**Acute angle**



**Obtuse angle**



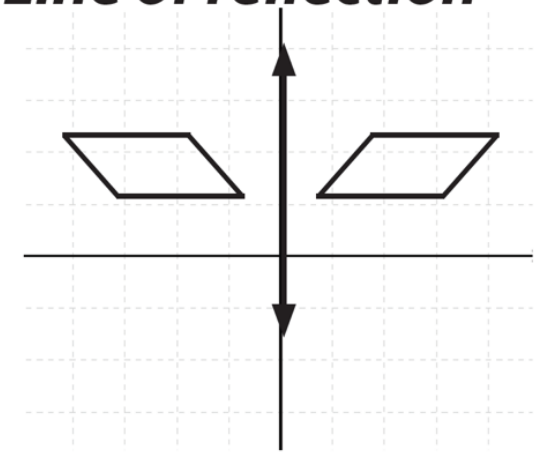
**Right angle**



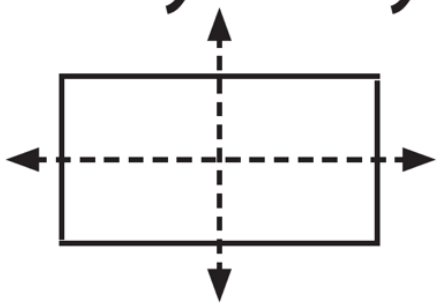
**Straight angle**

B

## Line of reflection

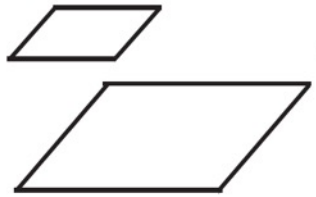


## Line of symmetry



F

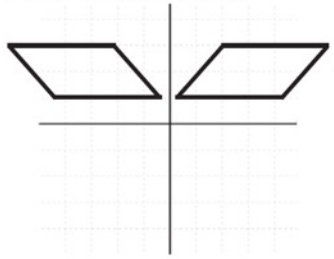
## Dilation



## Scale factor

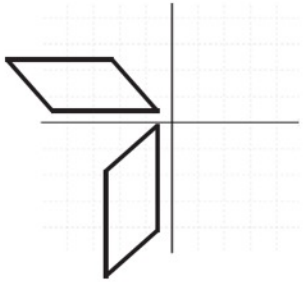
scale factor is 1:2

## Reflection

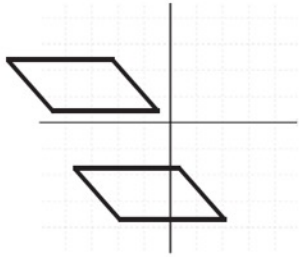


## Transformation

## Rotation

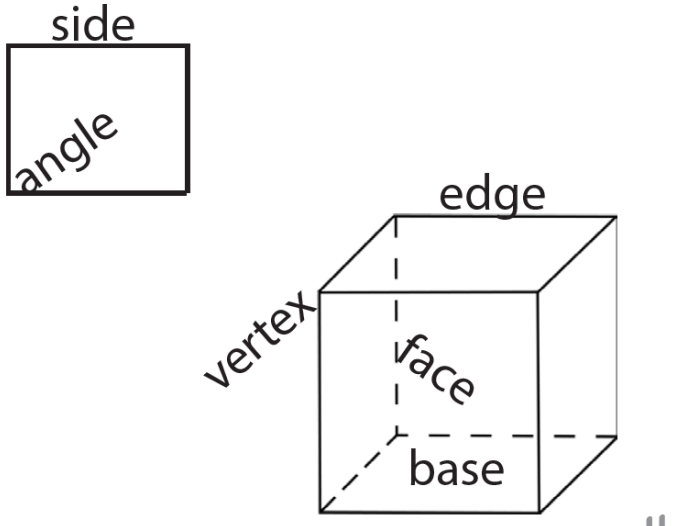


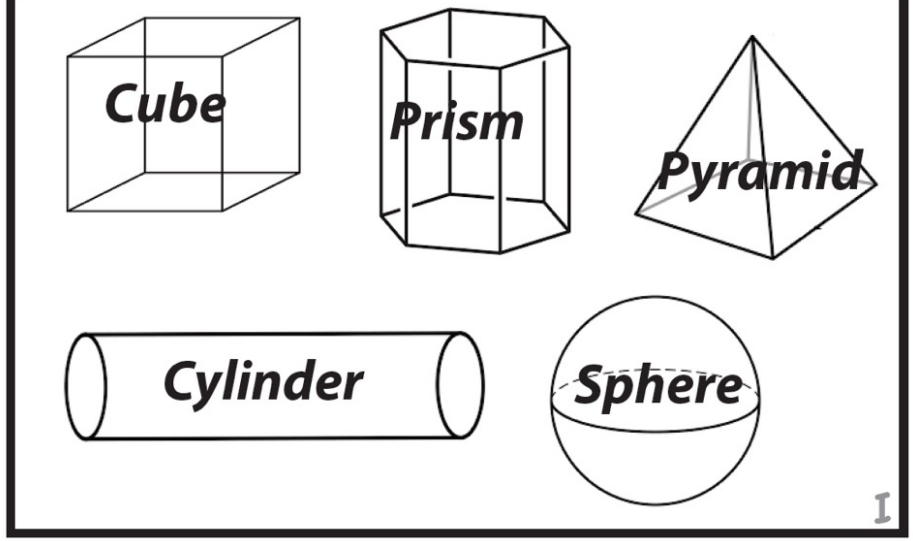
## Translation



G

***Angle***
***Base***
***Edge***
***Face***
***Side***
***Vertex***



Use formal math language

Use terms precisely

What are your strategies for focusing on math language?

# Objectives

Participants will describe the core components of explicit instruction.

Participants will describe why formal math language is important.

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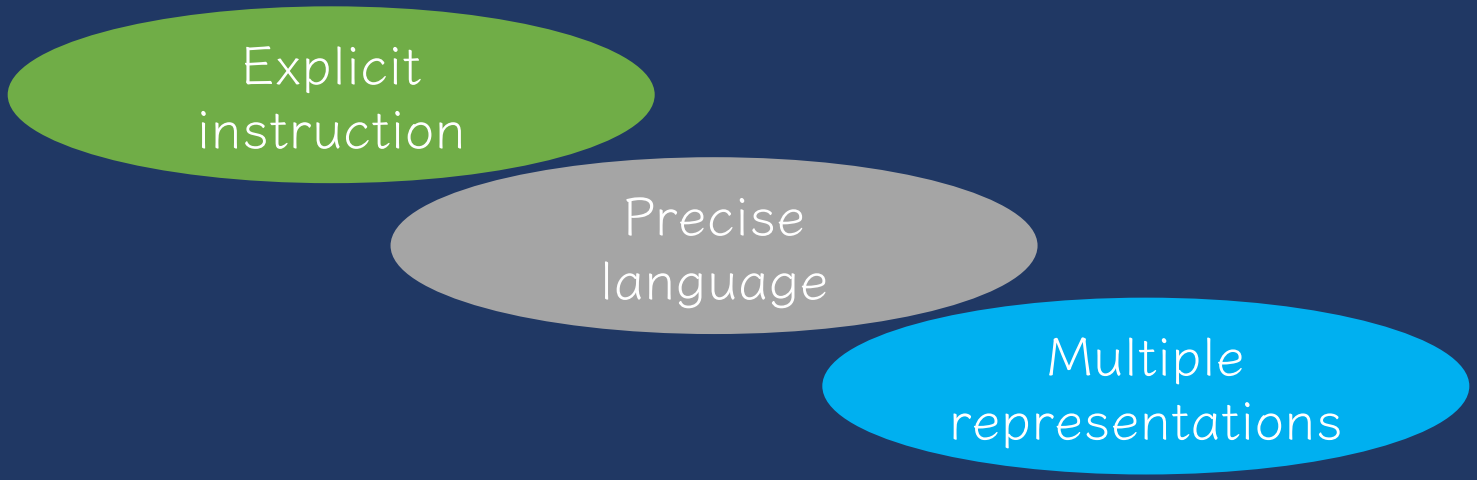
Participants will outline ways to build fluency.

Participants will describe two effective practices for word-problem instruction.

# Multiple Representations

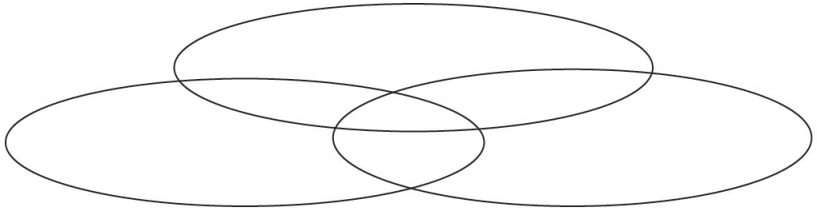
# Instructional Platform

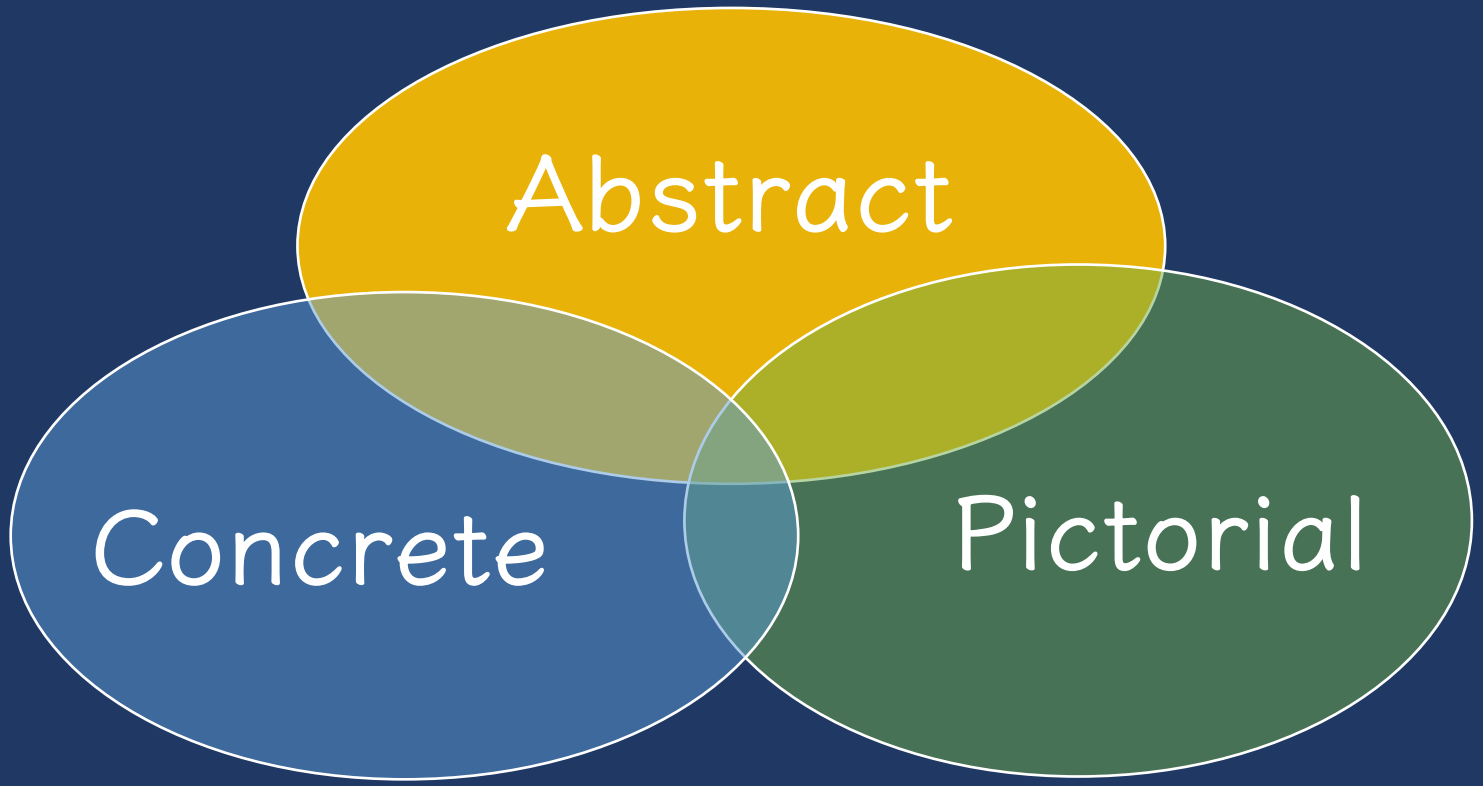
## INSTRUCTIONAL DELIVERY

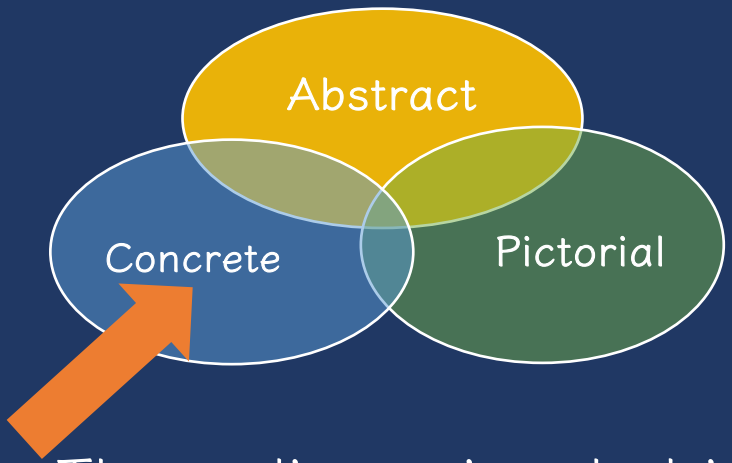



## INSTRUCTIONAL STRATEGIES

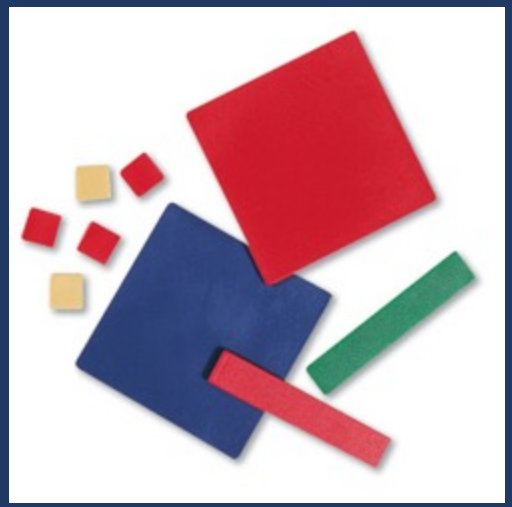
Multiple Representations

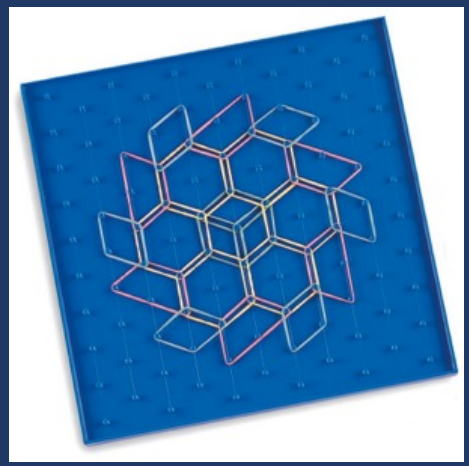


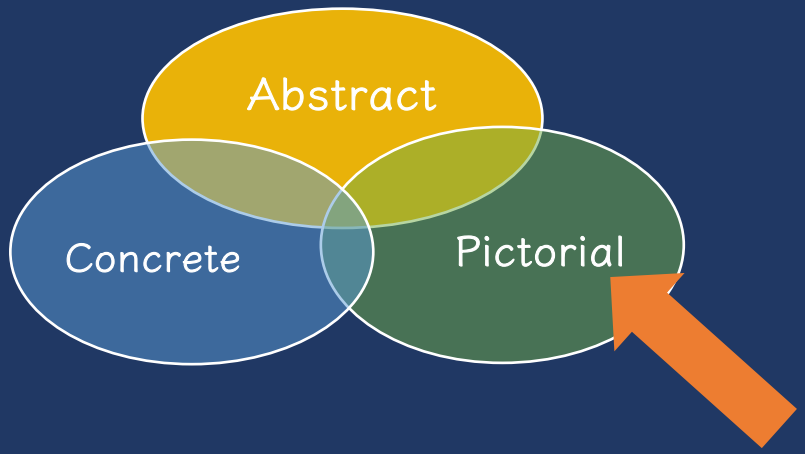




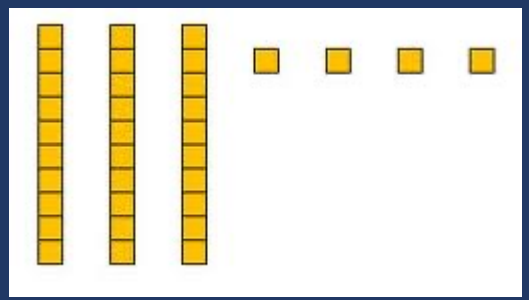

Three-dimensional objects

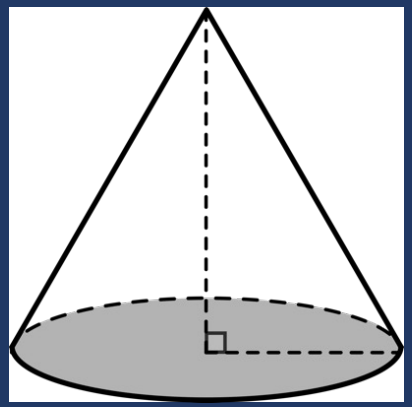


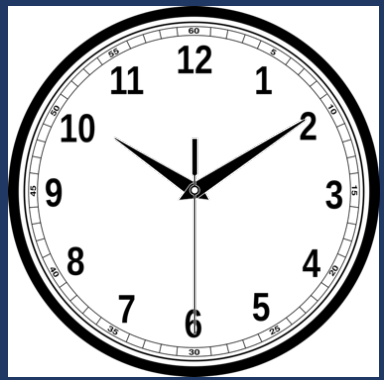


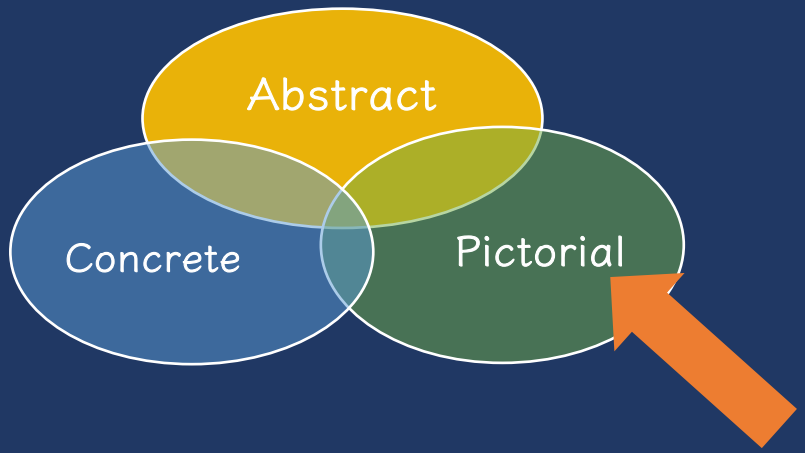



Two-dimensional images







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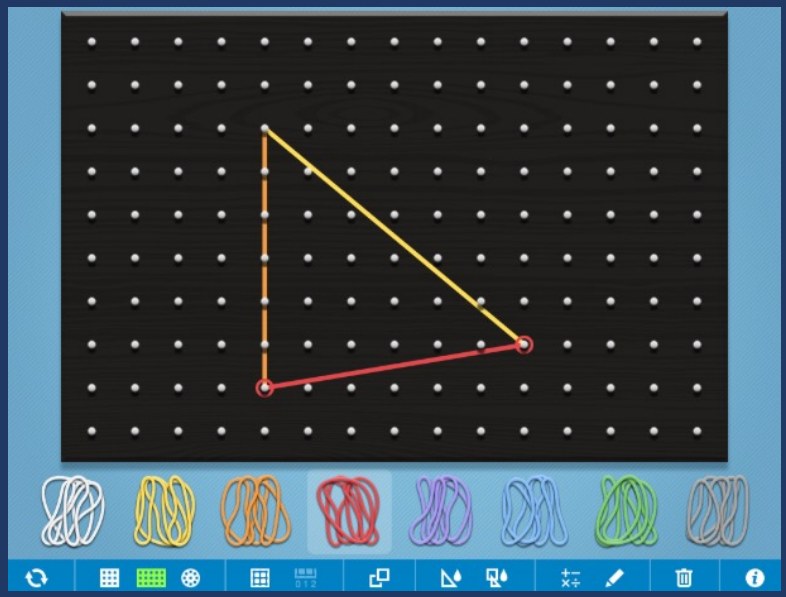

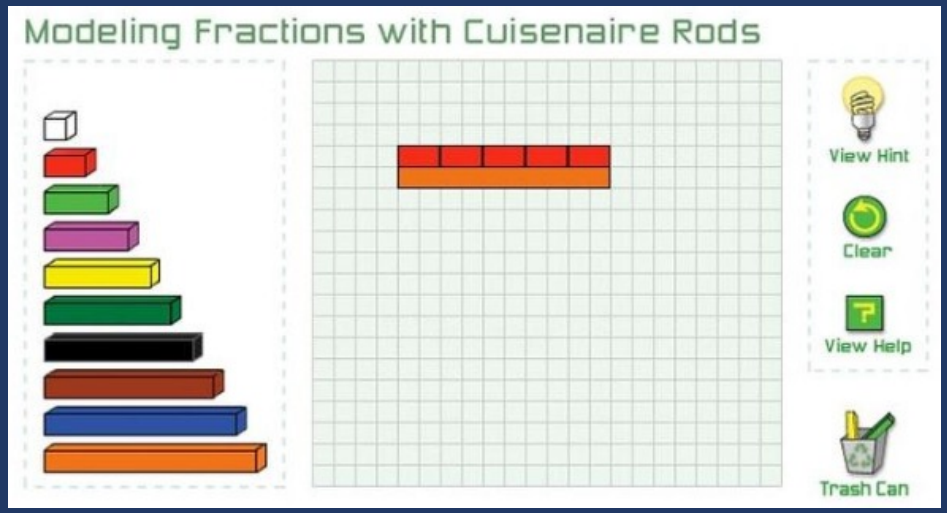


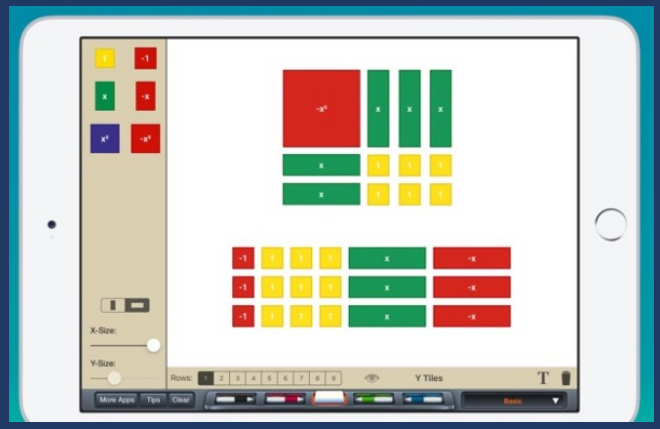

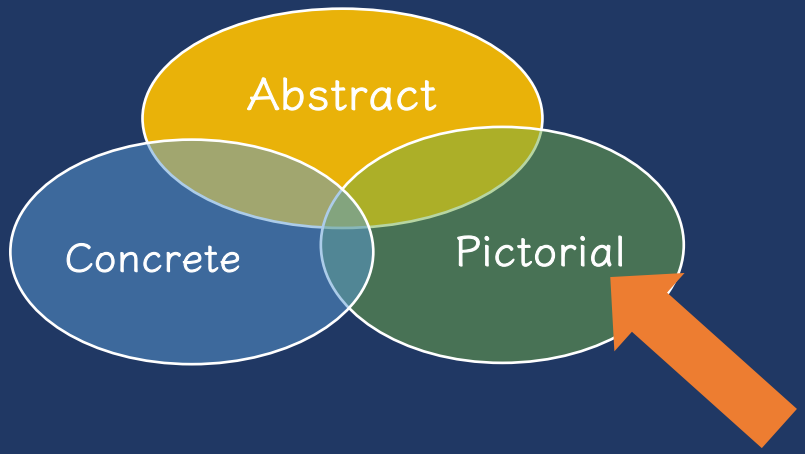


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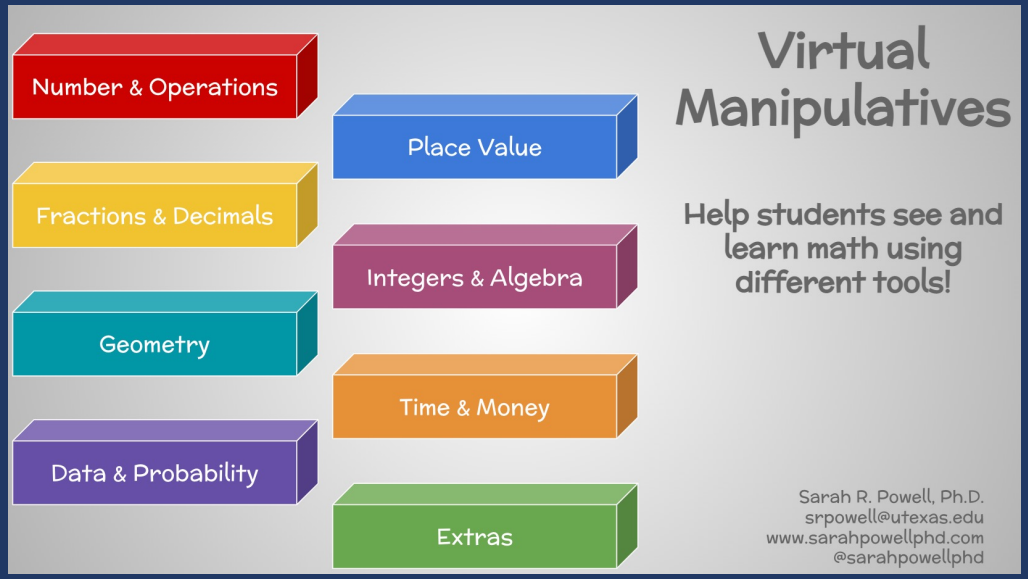
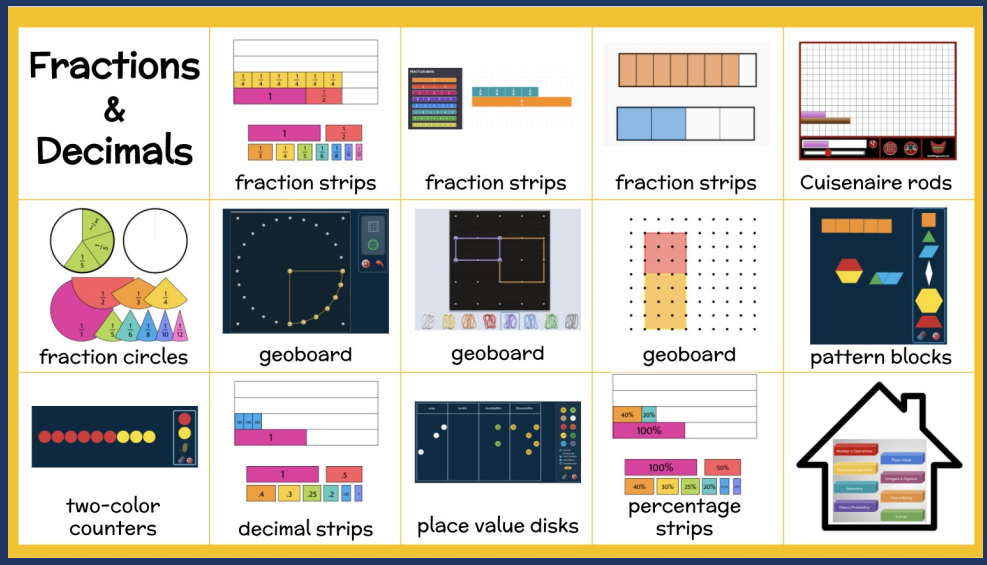
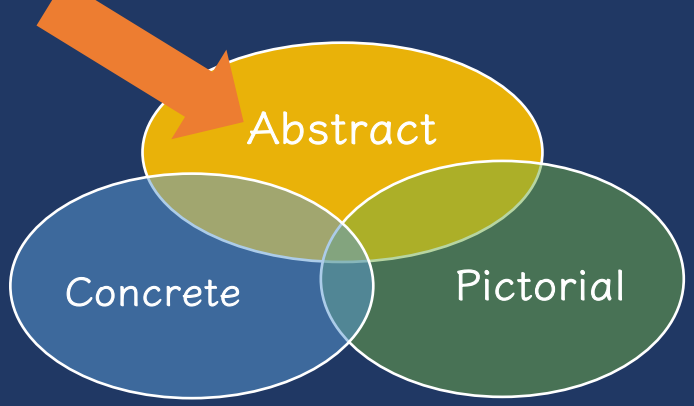




bit.ly/srpowell






Numerals and symbols and words

$2 + 8 = 10$

34 = 3 tens and 4 ones

$x - 6 = 8$

$$\begin{array}{r} 4{,}179 \\ +\quad 569 \\ \hline \end{array}$$

Explore 3 virtual manipulatives.

Share with a partner.

# Objectives

Participants will describe the core components of explicit instruction.

Participants will describe why formal math language is important.

Participants will explain different representations that can help students understand concepts and procedures.

Participants will outline ways to build fluency.

Participants will describe two effective practices for word-problem instruction.

# Building Fluency

# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Fluency

# Building Fluency

Fluency is doing mathematics *easily and accurately.*

Fluency in mathematics *makes mathematics easier.*

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both *conceptual learning and procedural learning.*

| | |
|---|---|
| Addition | Subtraction |
| Multiplication | Division |

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

Knowing multiples

Identifying shapes

Knowing formulas

| | |
|---|---|
| Addition | Subtraction |
| Multiplication | Division |

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \div\ 8 \\ \hline \end{array}$$

| | |
|---|---|
| Addition | Subtraction |
| Multiplication | Division |

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ +\ 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ -\ 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div\ 15 \\ \hline \end{array}$$

| | |
|---|---|
| Addition | Subtraction |
| Multiplication | Division |

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ +\ 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div\ 0.14 \\ \hline \end{array}$$

| | |
|---|---|
| Addition | Subtraction |
| Multiplication | Division |

Build fluency with integer computation

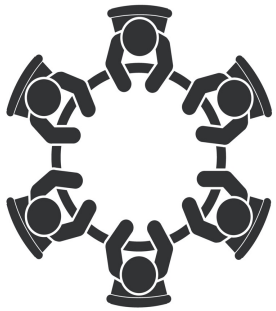
$$\begin{array}{r} 1.4 \\ +\ -3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times\ -12 \\ \hline \end{array}$$

$-14 - (-7) =$

$-135 \div 2 =$

| | |
|---|---|
| Addition | Subtraction |
| Multiplication | Division |



Describe the fluency needs of your students.

# Objectives

Participants will describe the core components of explicit instruction.

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Participants will explain different representations that can help students understand concepts and procedures.

Participants will outline ways to build fluency.

Participants will describe two effective practices for word-problem instruction.

# Word-Problem Solving

# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

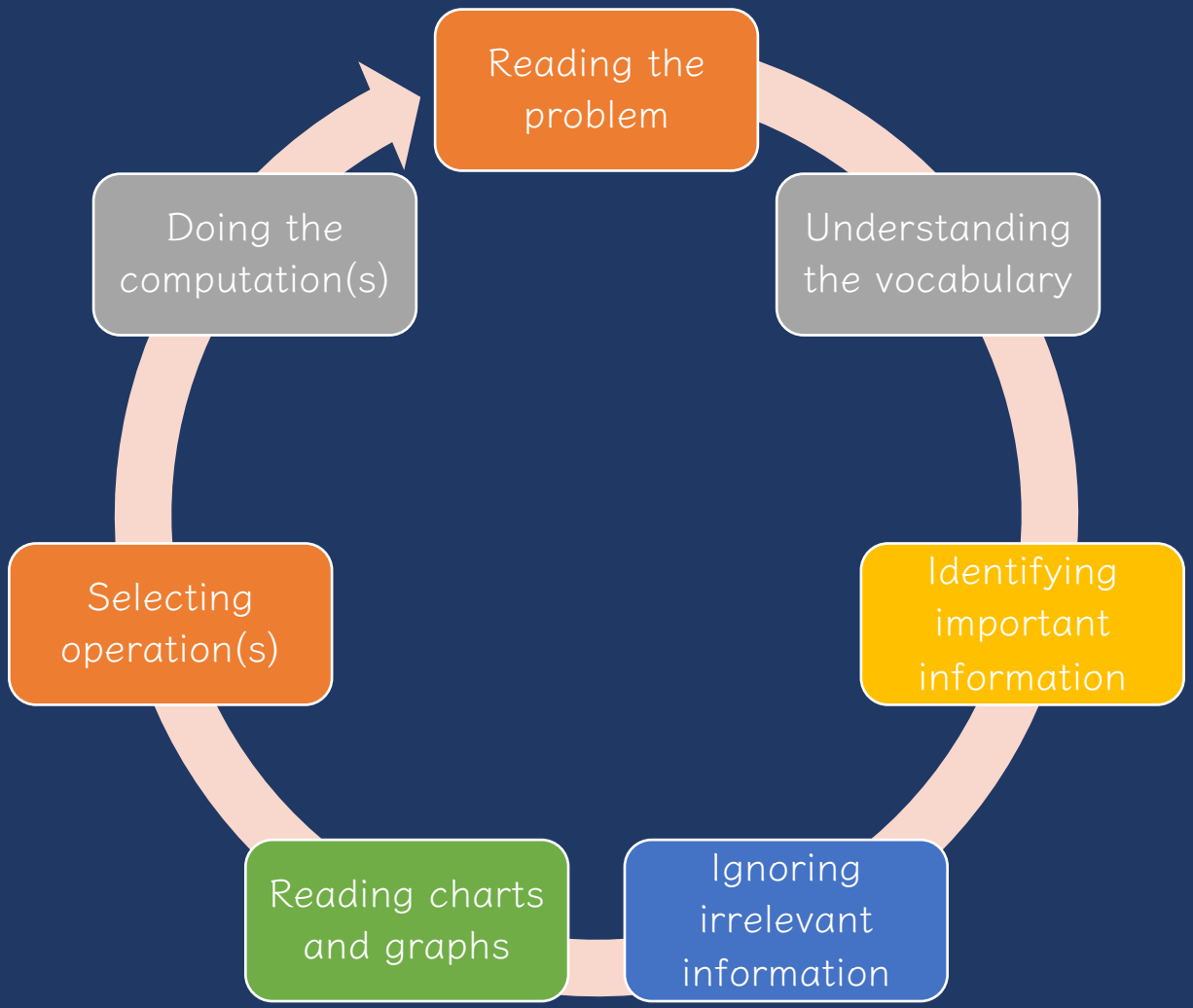
Multiple representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

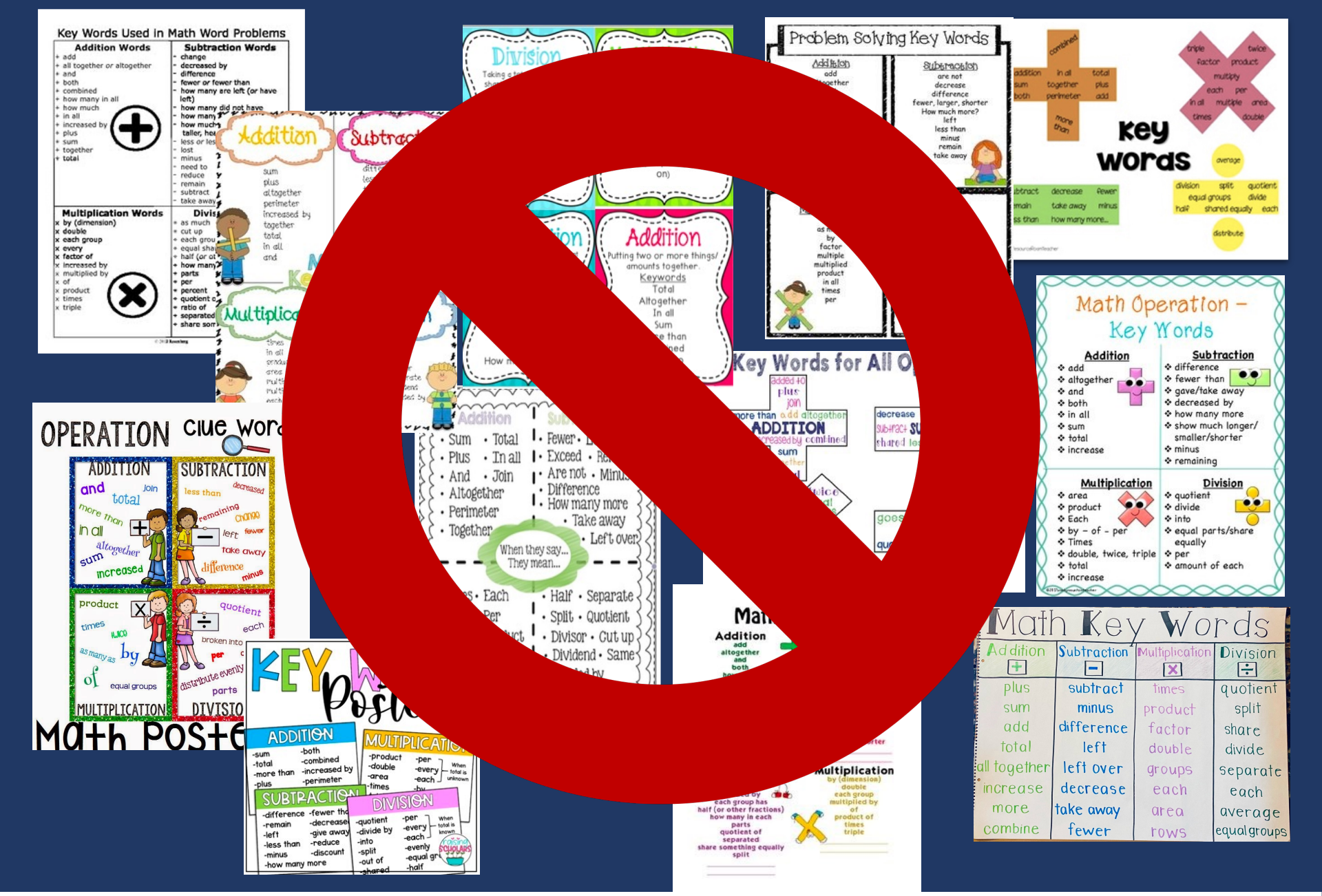
Word-Problem Solving



# 1. Keywords tied to operations


Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?



*Description of Single-Step Word Problems (n = 132)*

| Schema | Occurrence of schema | | Any keyword | | Schema-specific keywords[a] | | Multiple keywords[a] | | Keyword(s) led to correct solution[a] | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *n* | % | *n* | % | *n* | % | *n* | % | *n* | % |
| Total | 27 | 20.5 | 26 | 96.3 | 23 | 88.5 | 5 | 19.2 | 21 | 80.8 |
| Difference | 17 | 12.9 | 17 | 100.0 | 14 | 82.4 | 2 | 11.8 | 12 | 70.6 |
| Change | 11 | 8.3 | 7 | 63.6 | 5 | 71.4 | 5 | 71.4 | 2 | 28.6 |
| Equal groups | 29 | 22.0 | 26 | 89.7 | 22 | 84.6 | 18 | 69.2 | 8 | 30.8 |
| Comparison | 10 | 7.6 | 9 | 90.0 | 9 | 100.0 | 4 | 44.4 | 5 | 55.6 |
| Ratios or proportions | 29 | 22.0 | 23 | 79.3 | 9 | 39.1 | 9 | 39.1 | 6 | 26.1 |
| Product of measures | 9 | 6.8 | 9 | 100.0 | 8 | 88.9 | 1 | 11.1 | 5 | 55.6 |

[a]When a problem featured a keyword.


*Description of Multi-Step Word Problems (n = 84)*

| | Occurrence of schema[a] | | Any keyword | | Keyword(s) led to correct solution[b] | |
|---|---|---|---|---|---|---|
| Schema | *n* | % | *n* | % | *n* | % |
| Total | 40 | 47.6 | 39 | 97.5 | 3 | 7.7 |
| Difference | 11 | 13.1 | 11 | 100.0 | 1 | 9.1 |
| Change | 21 | 23.8 | 19 | 95.0 | 1 | 5.3 |
| Equal groups | 49 | 58.3 | 48 | 98.0 | 1 | 2.1 |
| Comparison | 7 | 8.3 | 7 | 100.0 | 0 | 0.0 |
| Ratios or proportions | 22 | 25.0 | 16 | 76.2 | 1 | 6.3 |
| Product of measures | 7 | 8.3 | 7 | 100.0 | 2 | 28.6 |

[a]Sum across schemas does not equal 100 because each word problem featured more than one schema.

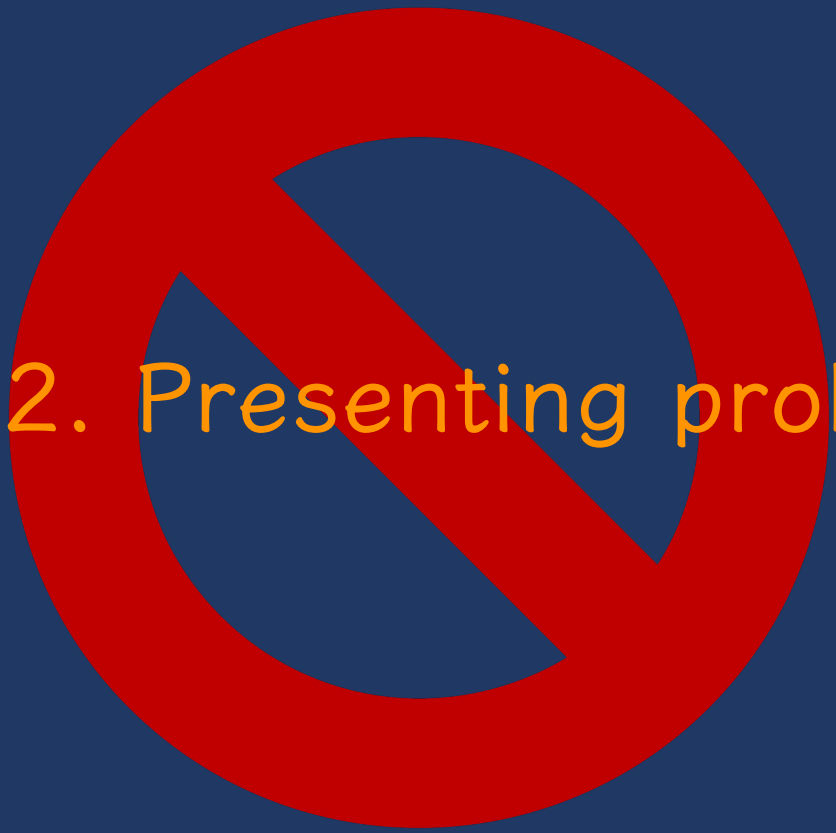
[b]When a problem featured a keyword.

Keywords are important to identify and understand

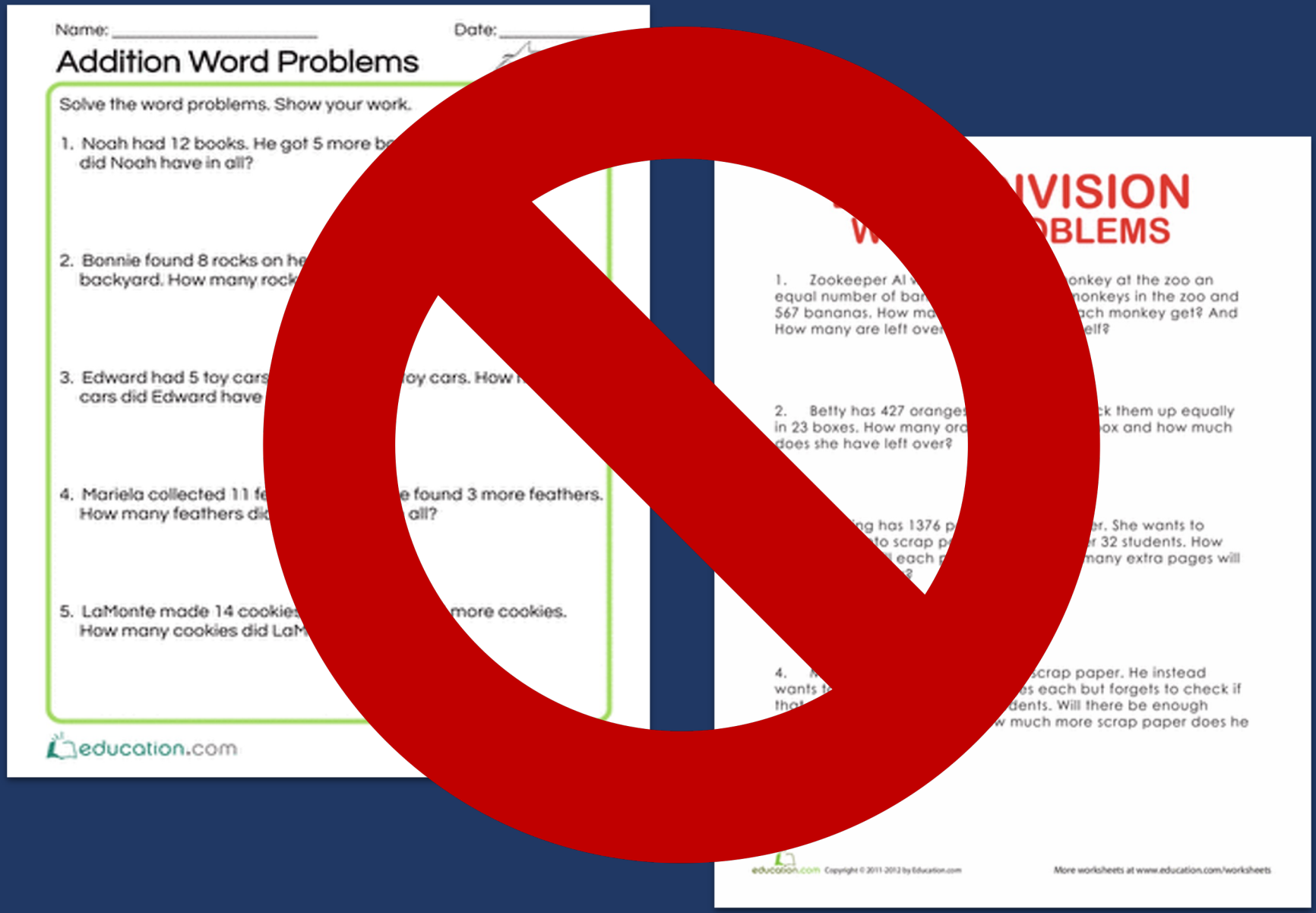
Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")

But, *do not* tie a keyword to a specific operation!



# 2. Presenting problems by operation



Teach an attack strategy

Teach about schemas

# RIDE

**R**ead the problem.

**I**dentify the relevant information.

**D**etermine the operation and unit for the answer.

**E**nter the correct numbers and calculate, then check the answer.

# RIDGES

**R**ead the problem.
**I** know statement.
**D**raw a picture.
**G**oal statement.
**E**quation development.
**S**olve the equation.

## RICE

Read and record the problem.
Illustrate your thinking.
Compute.
Explain your thinking.

## STAR

Stop and read the problem carefully.
Think about your plan and the strategy you will use.
Act. Follow your plan and solve the problem.
Review your answer.

## SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

## SHINES

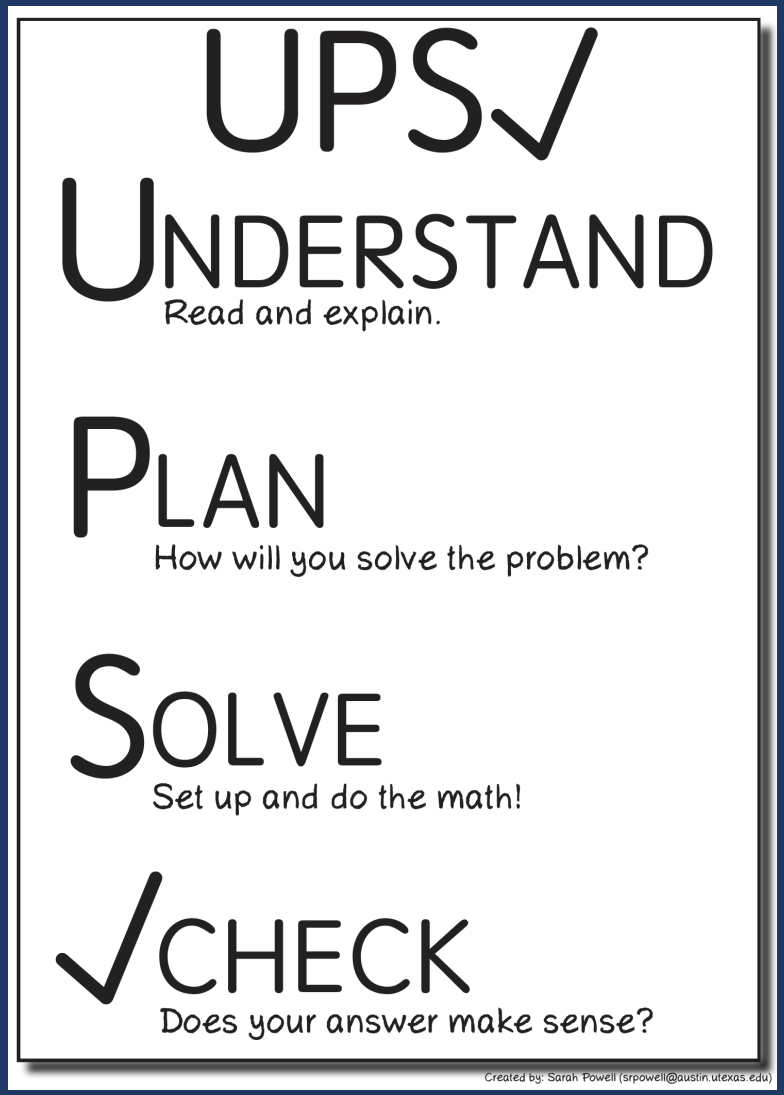
Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.

## R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.

## SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.



Share your favorite attack strategy.

Teach an attack strategy

Teach about schemas

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

# Total

Part-part-whole
Combine

Parts put together into a total

Daniela saw 3 canoes and 8 kayaks. How many boats did Daniela see?

Total

Daniela saw 11 boats. If 3 of the boats were canoes, how many were kayaks?

Part

Daniela saw 11 boats. 8 of the boats were kayaks, how many were canoes?

Part

# Difference

Compare

Greater and lesser amounts compared for a difference

Adrianna has 10 pencils. Tracy has 4 pencils. How many more pencils does Adrianna have?

Difference

Adrianna has 6 more pencils than Tracy. If Tracy has 4 pencils, how many does Adrianna have?

Greater amount

Tracy has 6 fewer pencils than Adrianna. Adrianna has 10 pencils. How many pencils does Tracy have?

Lesser amount

# Change

Join

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

End amount

Change amount

Start amount

# Change

Separate

An amount that increases or **decreases**

Samantha baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Samantha eat?

Change amount

Samantha baked some cookies. She ate 3 of the cookies and has 17 cookies left. How many cookies did Samantha bake?

Start amount

# Equal Groups

Array
Vary

Groups multiplied by number in each group for a product

Toni has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Toni have altogether?

Product

Toni has 24 crayons. They want to place them equally into 2 boxes. How many crayons will Toni place in each box?

Number in each group

Toni has 24 crayons. They put them into boxes with 12 crayons each. How many boxes did Toni use?

Groups

# Comparison

Set multiplied by a number of times for a product

Brooke ran 6 minutes. Shaleeni ran 4 times longer than Brooke. How many minutes did Shaleeni run?

Set

Number of times

Product

# Ratios/Proportions

Description of relationships among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

Xin et al. (2005)

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

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Teach about schemas

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# Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin

www.sarahpowellphd.com

srpowell@utexas.edu

@sarahpowellphd