

Specially Designed Mathematics Instruction

Grades 4-8



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Introduce yourself.

Describe your role as an educator.

Describe the mathematics you support.



Share your Twitter handle!



Specially Designed Math Instruction Grades 4-8

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Critical Content



Where student IS

Specially
Designed Math
Instruction

Where student NEEDS TO BE



Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...

Fluently multiply multi-digit whole numbers using the standard algorithm.

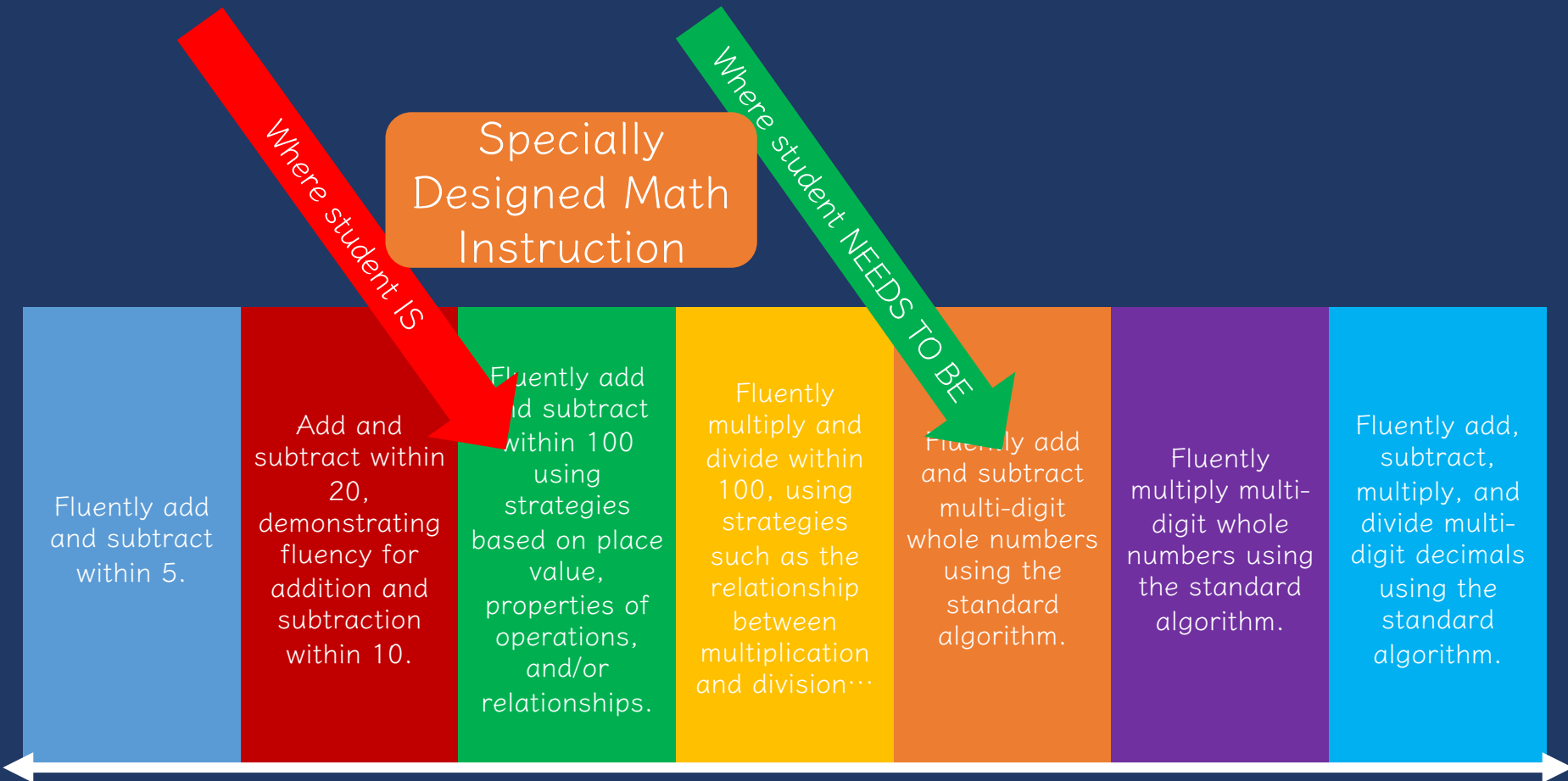
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Fluently add and subtract within 5.

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.





Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right...

Compose and decompose numbers from 11 to 19 into ten ones and some further ones...

Use place value understanding to round whole numbers to the nearest 10 or 100.



Specially
Designed Math
Instruction

Where student IS

Where student NEEDS TO BE

Compose and decompose numbers from 11 to 19 into ten ones and some further ones...

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Use place value understanding to round whole numbers to the nearest 10 or 100.

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right...

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.



Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations...

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20...

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve real-world and mathematical problems leading to two linear equations in two variables.

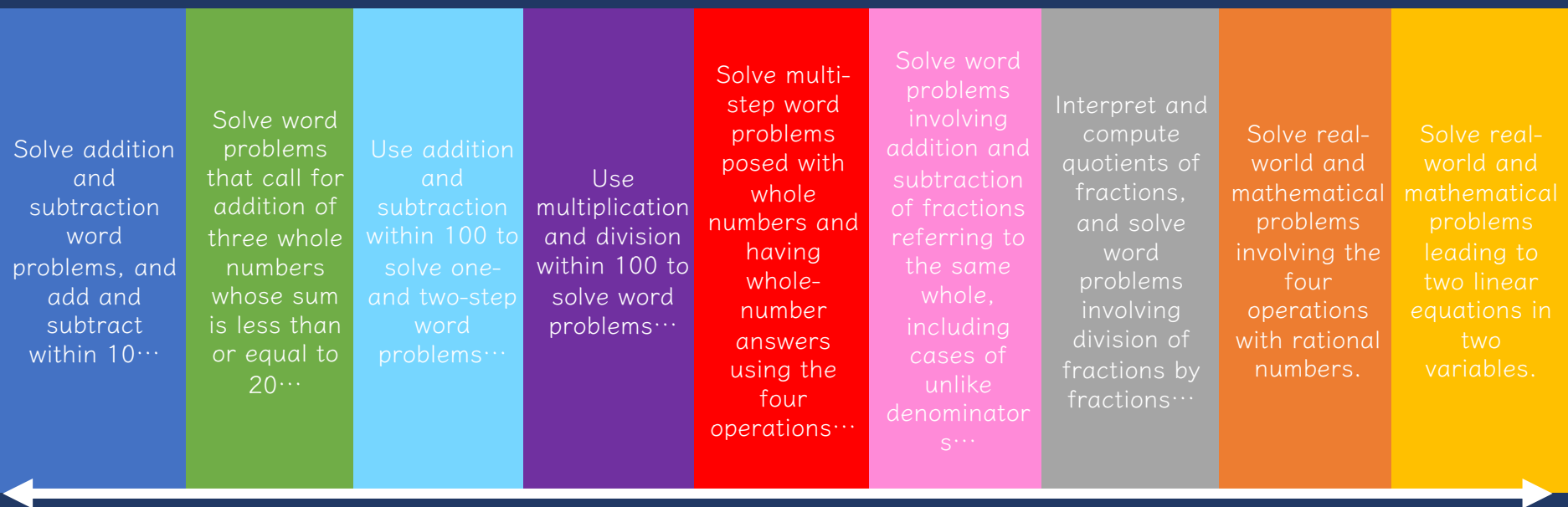
Use multiplication and division within 100 to solve word problems...

Solve addition and subtraction word problems, and add and subtract within 10...

Use addition and subtraction within 100 to solve one- and two-step word problems...

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators...





Where student IS

Use addition and subtraction within 100 to solve one- and two-step word problems...

Where student NEEDS TO BE

Use multiplication and division within 100 to solve word problems...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations...





An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence	Represent and solve problems involving addition and subtraction	Represent and solve problems involving addition and subtraction	Represent & solve problems involving multiplication and division	Use the four operations with whole numbers to solve problems	Understand the place value system	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers	Work with radical and integer exponents
Count to tell the number of objects	Understand and apply properties of operations and the relationship between addition and subtraction	Add and subtract within 20	Understand properties of multiplication and the relationship between multiplication and division	Generalize place value understanding for multi-digit whole numbers	Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of numbers to the system of rational numbers	Analyze proportional relationships and use them to solve real-world and mathematical problems	Understand the connections between proportional relationships, lines, and linear equations**
Compare numbers	Add and subtract within 20	Use place value understanding and properties of operations to add and subtract	Multiply & divide within 100	Use place value understanding and properties of operations to perform multidigit arithmetic	Use equivalent fractions as a strategy to add and subtract fractions	Understand ratio concepts and use ratio reasoning to solve problems	Use properties of operations to generate equivalent expressions	Analyze and solve linear equations and pairs of simultaneous linear equations
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Work with addition and subtraction equations	Measure and estimate lengths in standard units	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Apply and extend previous understandings of arithmetic to algebraic expressions	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Define, evaluate, and compare functions
Work with numbers 11–19 to gain foundations for place value	Extend the counting sequence	Relate addition and subtraction to length	Develop understanding of fractions as numbers	Build fractions from unit fractions by applying and extending previous understandings of operations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Reason about and solve one-variable equations and inequalities	Represent and analyze quantitative relationships between dependent and independent variables	Use functions to model relationships between quantities
	Understand place value		Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects	Understand decimal notation for fractions, and compare decimal fractions	Graph points in the coordinate plane to solve real-world and mathematical problems*			
	Use place value understanding and properties of operations to add and subtract		Geometric measurement: understand concepts of area and relate area to multiplication and to addition					
	Measure lengths indirectly and by iterating length units							

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.

** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

<https://achievethecore.org/category/774/mathematics-focus-by-grade-level>



Table A.3. Grades 6–8 Curriculum Focal Points and Connections Compared with the Expectations of the Content Standards in *Principles and Standards for School Mathematics*

Curriculum Focal Points and Connections	Expectations of the Content Standards
<p>Grade 6 Curriculum Focal Points</p> <p><i>Number and Operations: Developing an understanding of and fluency with multiplication and division of fractions and decimals</i></p> <p>Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.</p> <p><i>Number and Operations: Connecting ratio and rate to multiplication and division</i></p> <p>Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., “If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12”). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.</p> <p><i>Algebra: Writing, interpreting, and using mathematical expressions and equations</i></p> <p>Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that</p>	<p><i>Number and Operations, Grades 6–8</i></p> <ul style="list-style-type: none"> Work flexibly with fractions, decimals, and percents to solve problems Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line Develop meaning for percents greater than 100 and less than 1 Understand and use ratios and proportions to represent quantitative relationships Develop an understanding of large numbers [identified in Grades 4 and 5 Curriculum Focal Points] and recognize and appropriately use exponential, scientific, and calculator notation Use factors, multiples, prime factorization, and relatively prime numbers to solve problems Develop meaning for integers and represent and compare quantities with them Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods



<https://www.nctm.org/curriculumfocalpoints/>





What's the critical math content for your students?

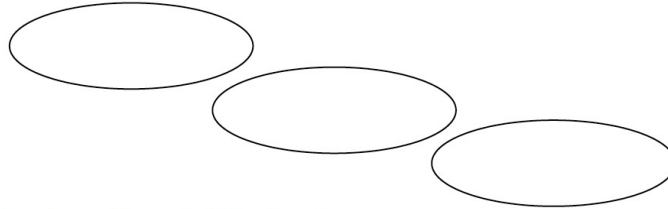


Designing an Instructional Platform

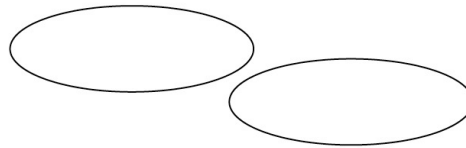


Instructional Platform

Instructional Delivery



Instructional Strategies



Evidence-Based Practices

A large empty rectangular box intended for notes related to Evidence-Based Practices.

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



evidence-based practice

A practice that
has shown
consistent and
positive results



evidence-based practice



evidence-based intervention

An intervention
(i.e., packaged
program) that
has shown
**consistent and
positive** results



evidence-based practice



evidence-based intervention

evidence-based strategy

A method or strategy that has shown **consistent and positive** results



evidence-based practice



evidence-based intervention

evidence-based strategy

promising practice

A method or strategy that has shown a **positive** result



evidence-based practice



evidence-based intervention

evidence-based strategy

promising practice

~~no or negative
evidence~~



evidence-based practice



evidence-based intervention

evidence-based strategy

promising practice

Assessment data to
show results

Improvement from
before intervention

Improvement
compared to no
treatment students

Replication

Multiple researchers

Multiple students

Multiple times

Setting and students
similar to your own



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction





1. Critical math: What is the critical math you need to teach to your students?
2. How will you sequence the critical math across the school year?
3. Which evidence-based practices will you use to teach the critical math?



Evidence-Based Practice: Systematic Instruction



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

INSTRUCTIONAL STRATEGIES



Explicit Instruction

MODELING

PRACTICE

SUPPORTS



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



Modeling is a
dialogue
between the
teacher and
students.

MODELING

Step-by-step
explanation

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Modeling includes a step-by-step explanation of how to do a math problem.

A teacher may do 1 modeled problem or several.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

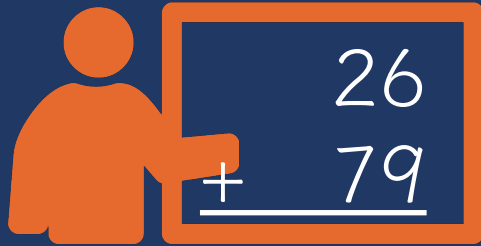
SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

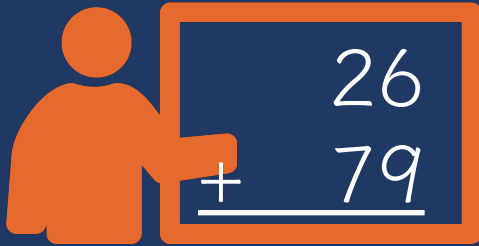
Providing affirmative and corrective feedback





“Today, we are learning about addition. This is important because sometimes you have different amounts – like money – and you want to know how much money you have altogether.”





“Let’s solve this problem. What’s the problem?”

“26 plus 79.”



“To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?”

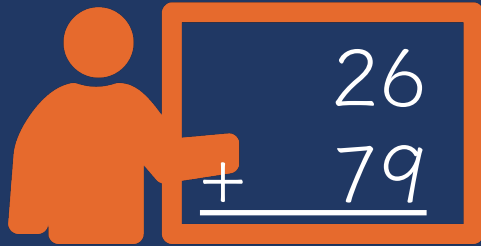
“Add.”



“How did you know we want to add?”

“There’s a plus sign.”





“The plus sign tells us we want to add. To add, let’s use the partial sums strategy. What strategy?”

“Partial sums.”



“With the partial sums strategy, we start adding in the greatest place value. What’s the greatest place value in this problem?”

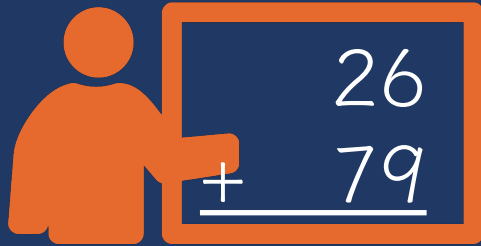
“The tens.”



“So, let’s add the tens. What’s 20 plus 70?”

“90.”





"20 plus 70 equals 90. Let's write 90 right here below the equal line. What will we write?"

"90."



"90 is the partial sum when you add the tens. What does 90 represent?"

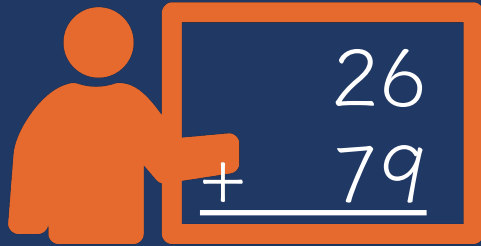
"It's the partial sum of adding 20 plus 70."



"Now, let's add the ones. What should we add?"

"6 plus 9."





“6 plus 9 equals what?”

“15.” 

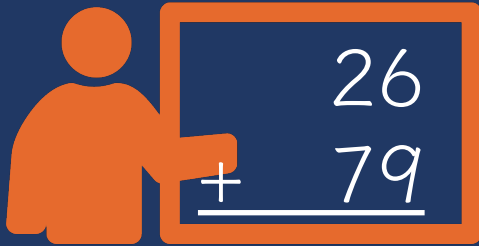
“Let’s write 15 below the 90.
Where do we write the 15?”

“Below the 90.” 

“15 is the partial sum when you add the ones.
Now, let’s add the partial sums together. What will we add?”

“90 plus 15.” 





“What’s 90 plus 15?”

“How did you add those numbers?”

“So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?”

“105.”



“I added 90 plus 10 then added 5 more.”



“We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums.”



Modeling
needs to
include
planned
examples.

MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

These
examples
should be
sequenced so
easier skills
lead to more
difficult
skills.



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

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Providing affirmative and corrective feedback



What's the math that you
model with your students?



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

Practice continues as a dialogue between the teacher and students.

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Guided practice is practice in which the teacher and students practice problems together.



“Let’s work on a problem together.”



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Independent practice is practice in which the students practice independently with teacher support.



“Now, you’ll practice a problem on your own. Use your attack strategy!”



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



How do you engage students
in guided practice?



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

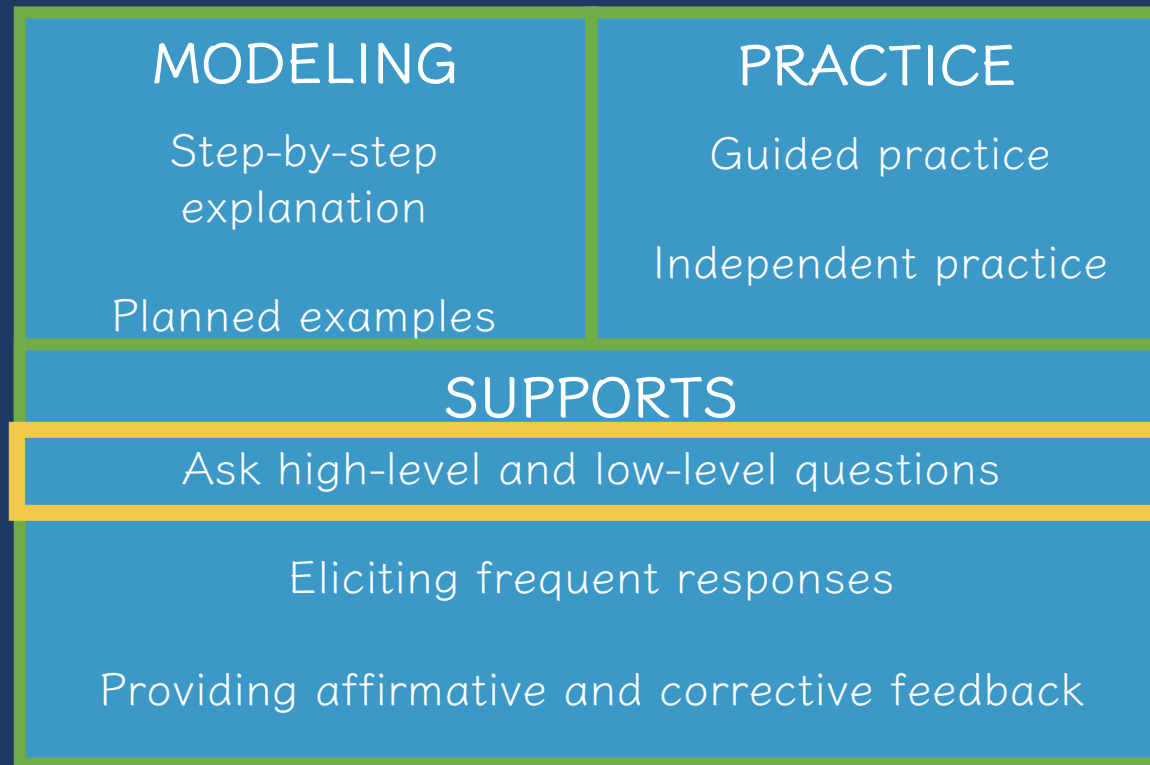
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

These **Supports** should be used in
both **Modeling** and **Practice**.





During **Modeling** and **Practice**, it is essential to engage students and check for understanding.



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



“What is 7 times 9?”

“63.”



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

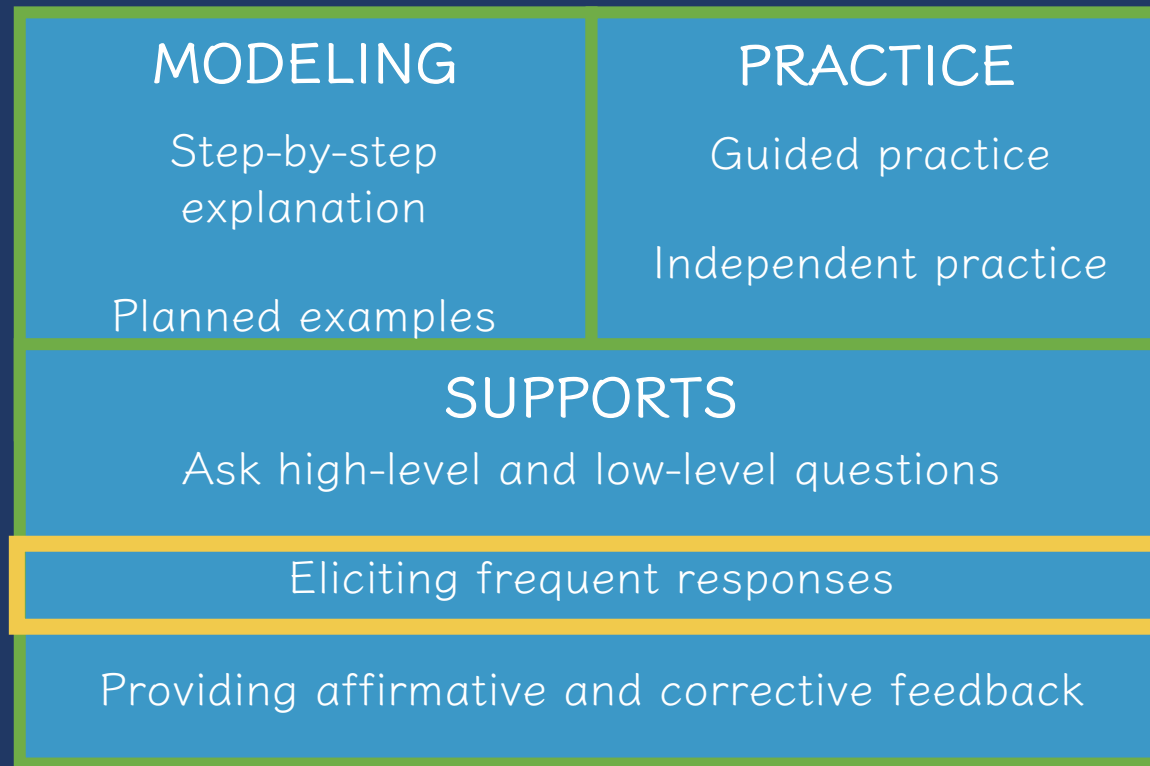
Providing affirmative and corrective feedback



“Why do you use
zero pairs?”

“Because a positive 1
and a negative 1 equal
0. I use the zero pair
to help me subtract.”





During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

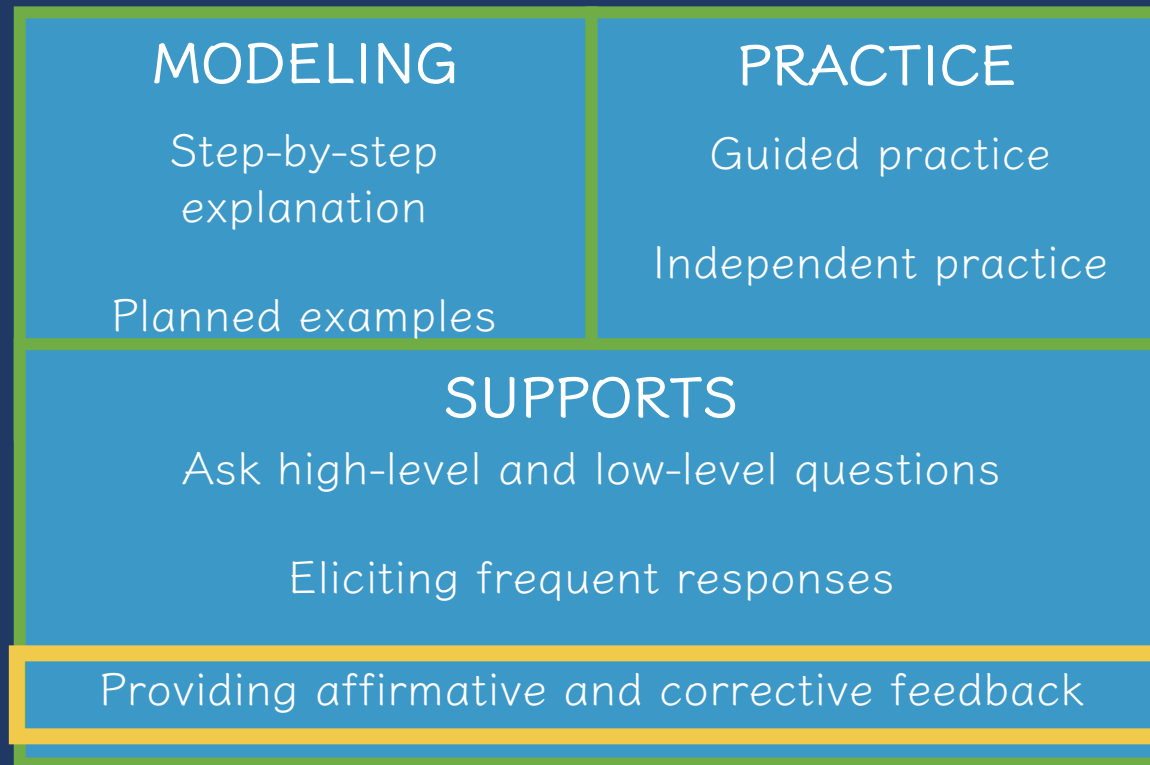
Eliciting frequent responses

Providing affirmative and corrective feedback



- Oral
- Written
- With manipulatives
- With drawings
- With gestures





During **Modeling** and **Practice**, students should receive immediate feedback on their responses.



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



“Nice work using your
word problem attack
strategy.”



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



“Let’s look at that again. Tell me how you added in the hundreds column.”



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



Which of these supports
should you use more often?



Evidence-Based Practice: Mathematical Language



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

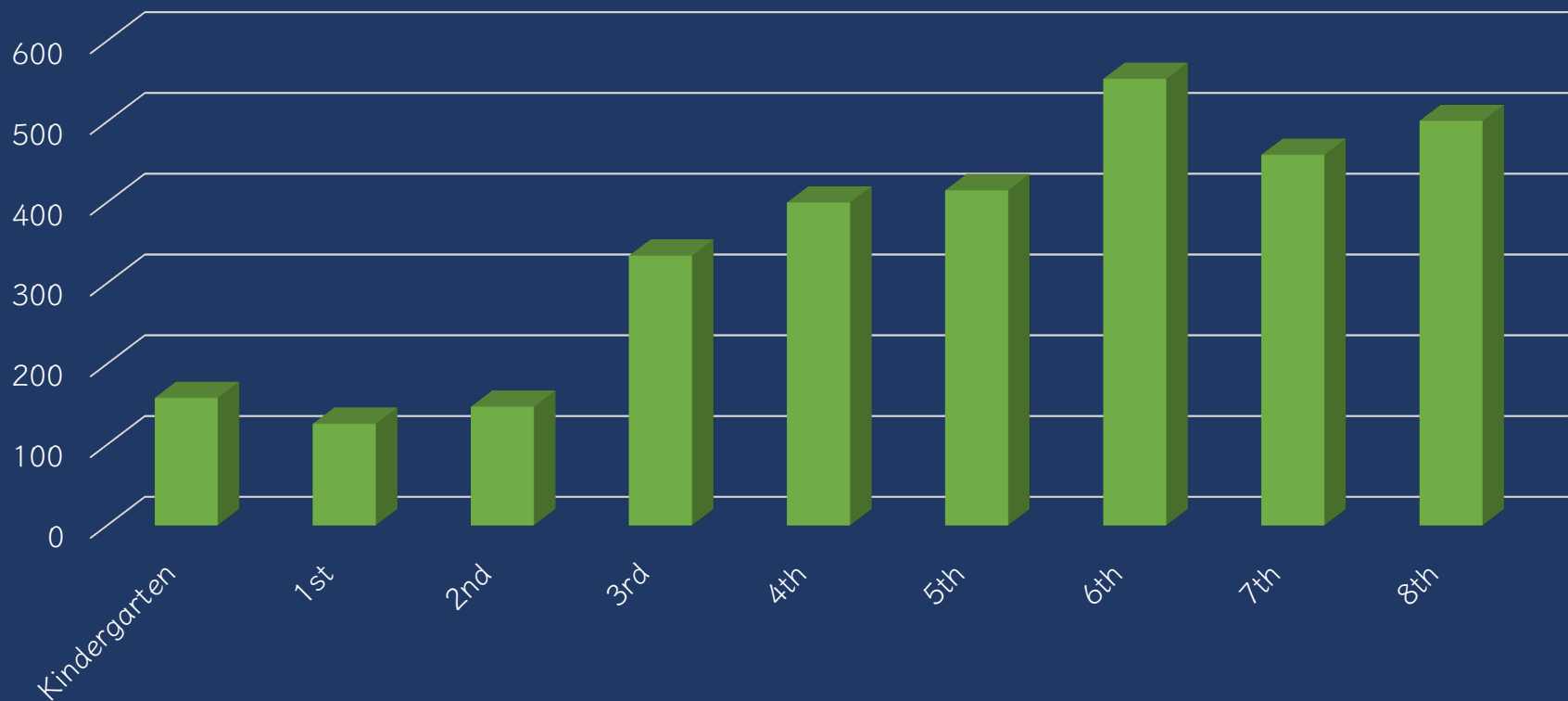
INSTRUCTIONAL STRATEGIES

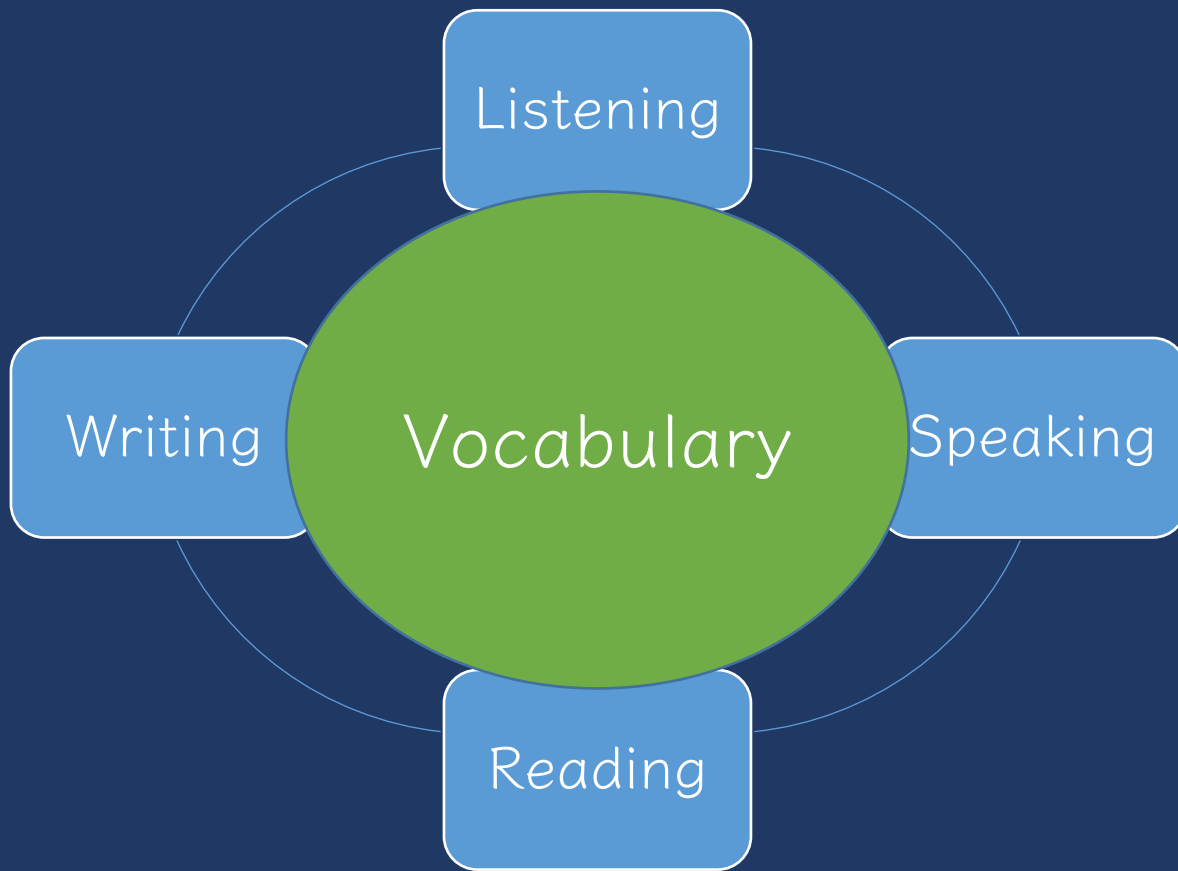


Mathematical Language

Instead of that...	Say this...







1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
(but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings
(but a more precise math meaning)
3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings
(but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings

divide vs.
Continental
Divide

variable vs.
variably
cloudy

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings

factor vs.
multiple

hundreds vs.
hundredths

numerators
vs.
denominator

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings

mesa vs.
tabla

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings
(but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
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6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way

skip count
vs. multiples

one-fourth
vs. one
quarter



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way
11. Informal terms may be used for formal math terms

rhombus vs.
diamond

vertex vs.
corner



Use formal math language

Use terms precisely



Mathematics

Regroup
Value
Hundreds
Less
Ones
Fewer
Greater
Tens
Balance
Digit
Place

Supporting Clear and Concise Mathematics Language

Instead of That, Say This

Elizabeth M. Hughes, Sarah R. Powell, and
Elizabeth A. Stevens

TEACHING Exceptional Children, Vol. 51, No. 4, pp. 286-295, Copyright 2018 The Authors. DOI: 10.1177/0048111718782924

Middle-School Mathematics



Math Language in Middle School


Be More Specific

Sarah R. Powell, Elizabeth A. Stevens, and
Elizabeth M. Hughes


TEACHING Exceptional Children, Vol. 51, No. 4, pp. 286-295, Copyright 2018 The Authors. DOI: 10.1177/0048111718782924

286 COUNCIL FOR EXCEPTIONAL CHILDREN





What number is in
the tens place?




What digit is in the tens place?
What is the value of the digit in
the tens place?

135

Why this is important...


- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.





The alligator eats the
bigger number

is less than
OR
is greater than



Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because $<$ and $>$ are two distinct symbols.





carry OR borrow

regroup OR
trade OR
exchange





$$\begin{array}{r} 167 \\ + 294 \\ \hline \end{array}$$

Why this is important...

- “Carry” or “borrow” is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.





top number and
bottom number



numerator and
denominator

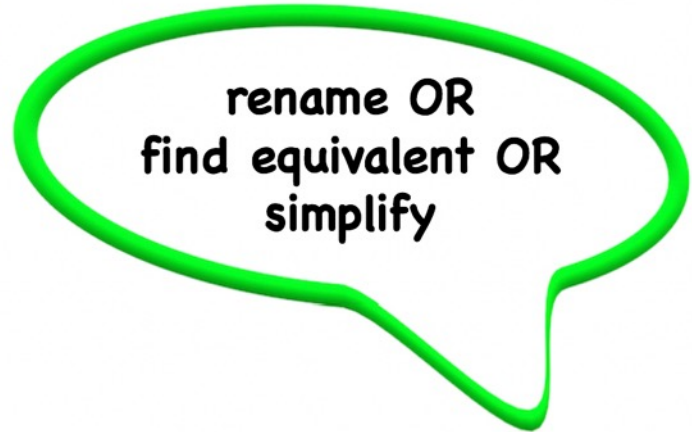

Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.






reduce the fraction



rename OR
find equivalent OR
simplify

Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.



Four point seven
Four point oh seven

Four and seven tenths
Four and seven hundredths



4.7
4.07

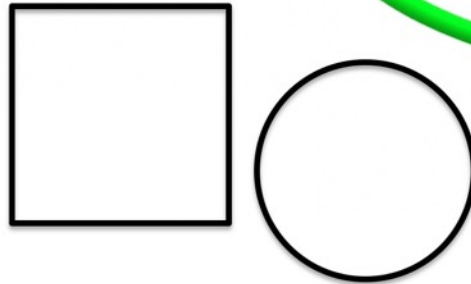
Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



box OR ball

square OR
circle



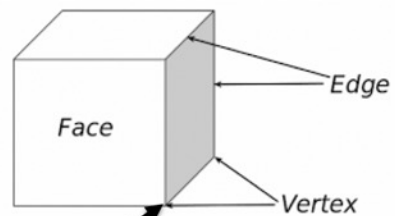
Why this is important...

- Use the formal language of shapes to confirm informal language.



point

vertex



Why this is important...

- This is the endpoint where two or more line segments or rays meet.





flips, slides, turns




reflections,
translations,
rotations

Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.



long hand and
short hand



minute hand and
hour hand

Why this is important...

- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.



What are examples of,
“Instead of ____, Say ____?”



Use formal math language

Use terms precisely



Factor

$$1 \times 8 = 8$$

$$2 \times 4 = 8$$

factor

factor

Multiple

$$8 \times 1 = 8$$

$$8 \times 2 = 16$$

multiples of 8

E



Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

$$4:3$$

Proper fraction

$$\frac{2}{9}$$

Unit fraction

$$\frac{1}{6}$$

D



Coefficient
Constant
Term
Variable

$$\begin{array}{ccc} \text{term} & \text{term} & \text{term} \\ \text{---} & \text{---} & \text{---} \\ 2x^2 + x - 3 \\ \text{coefficient} & \text{variable} & \text{constant} \end{array}$$

A



Equation $9x - 4 = 7x$

Expression $9x - 4$

Formula $a^2 + b^2 = c^2$

Function $f(x)$

Inequality $9x - 4 > 6x$

c



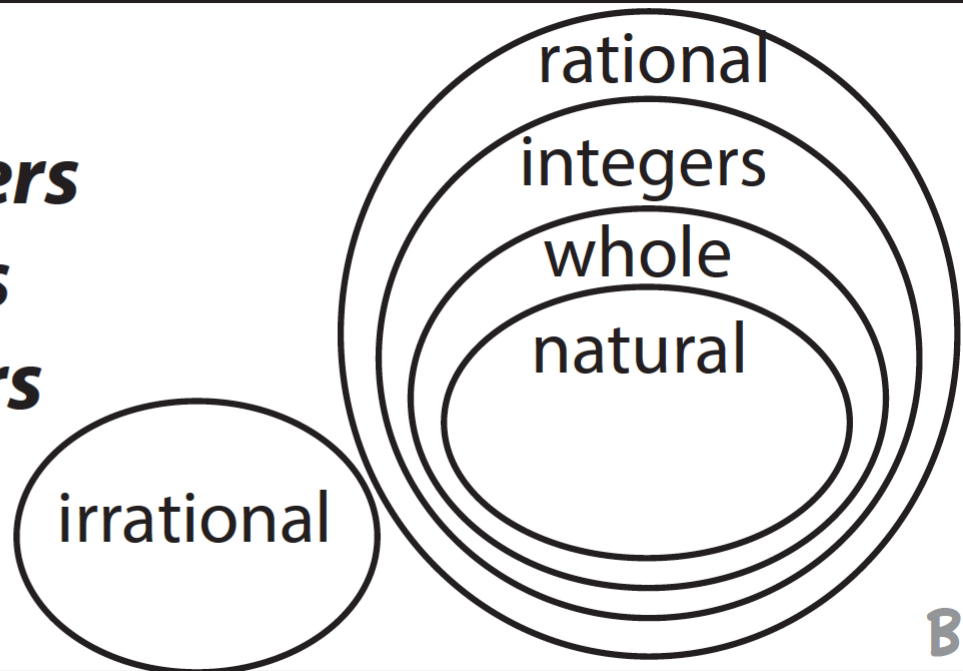
Integers

Irrational numbers

Natural numbers

Rational numbers

Whole numbers



B



Quadrilaterals

Kite



Rhombus



Parallelogram



Square



Rectangle



Trapezoid

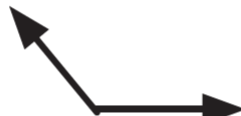


A

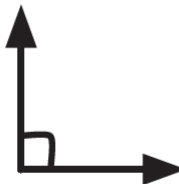
Acute angle



Obtuse angle



Right angle



Straight angle



B

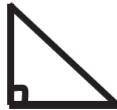
Acute triangle



Obtuse triangle



Right triangle



Equilateral triangle



Isosceles triangle



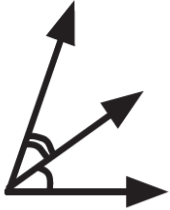
Scalene triangle



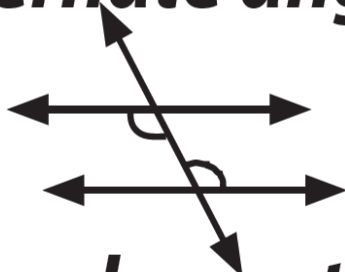
C



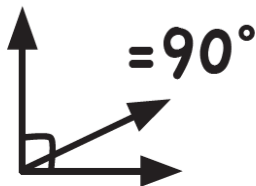
Adjacent angles



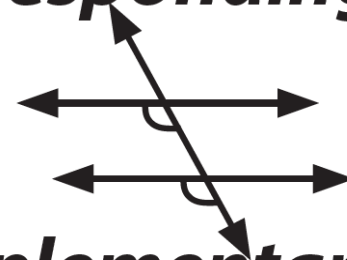
Alternate angles



Complementary angles



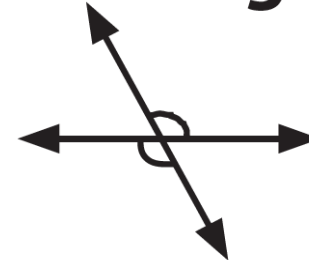
Corresponding angles



Supplementary angles ***= 180°***



Vertical angles



D

Congruent figures

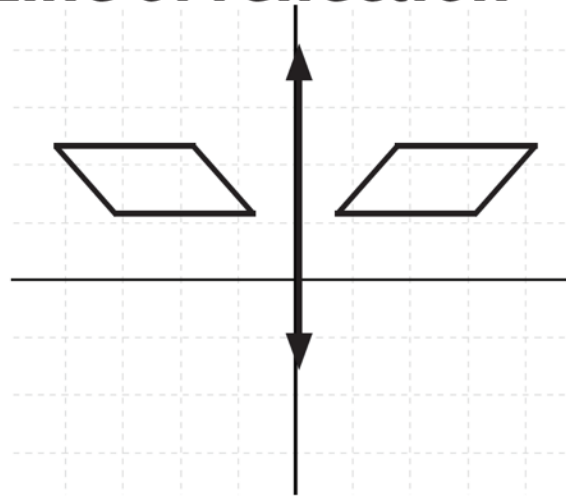


Similar figures

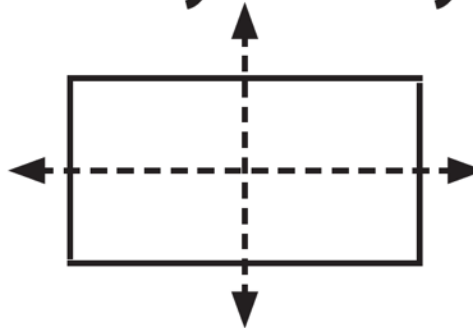


E

Line of reflection



Line of symmetry



f

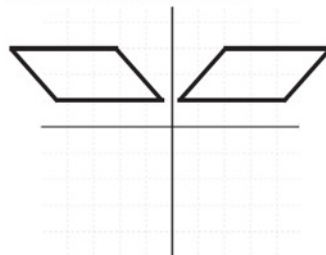
Dilation



Scale factor

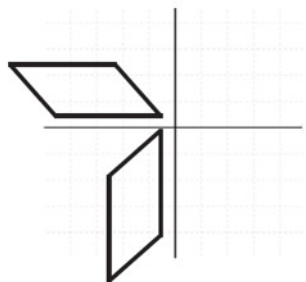
← scale factor
is 1:2

Reflection

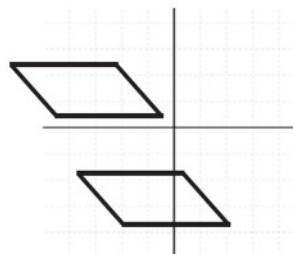


Transformation

Rotation



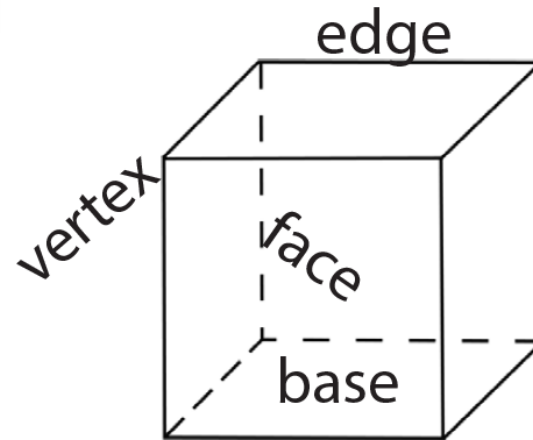
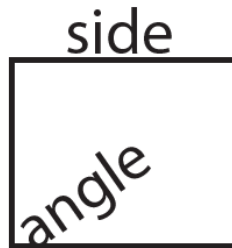
Translation

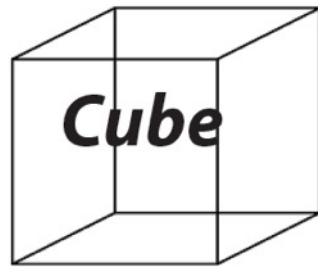


G



Angle
Base
Edge
Face
Side
Vertex





Cube



Prism



Pyramid



Cylinder



Sphere

I

Coordinate plane

Ordered pair Quadrant 2

Quadrants

x

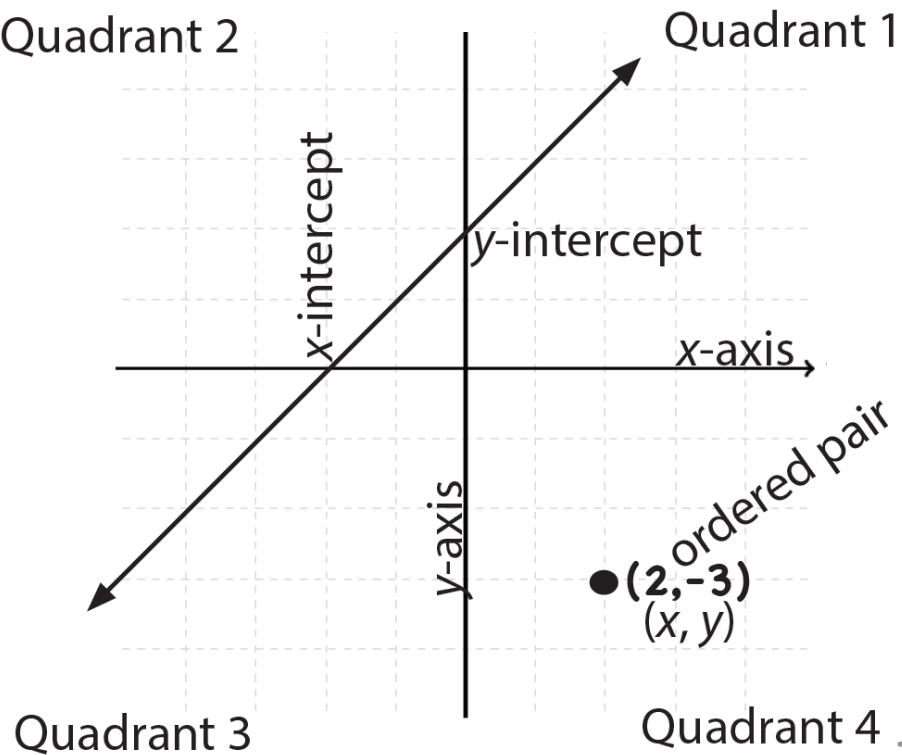
x-axis

x-intercept

y

y-axis

y-intercept





Which terms do your students not use precisely?

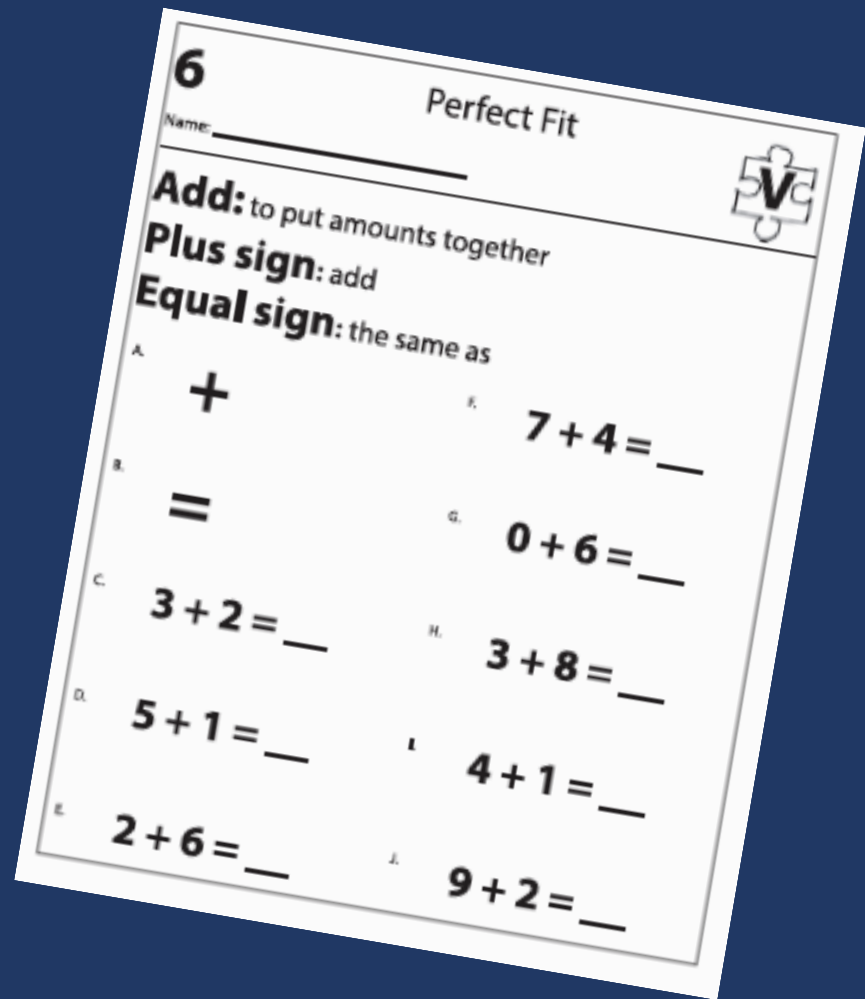
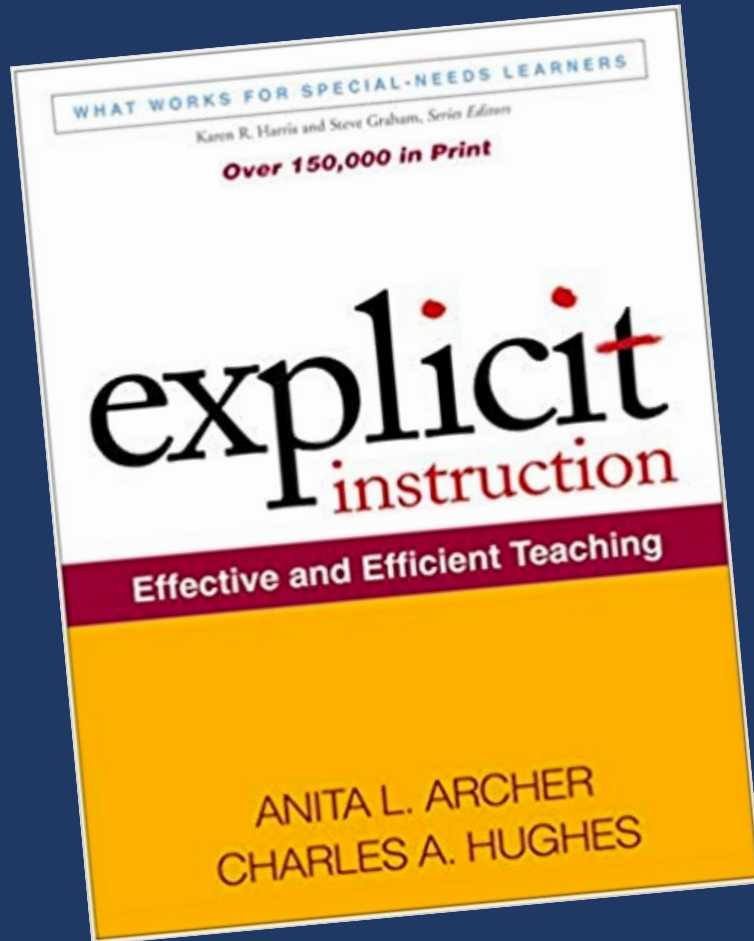


Use formal math language

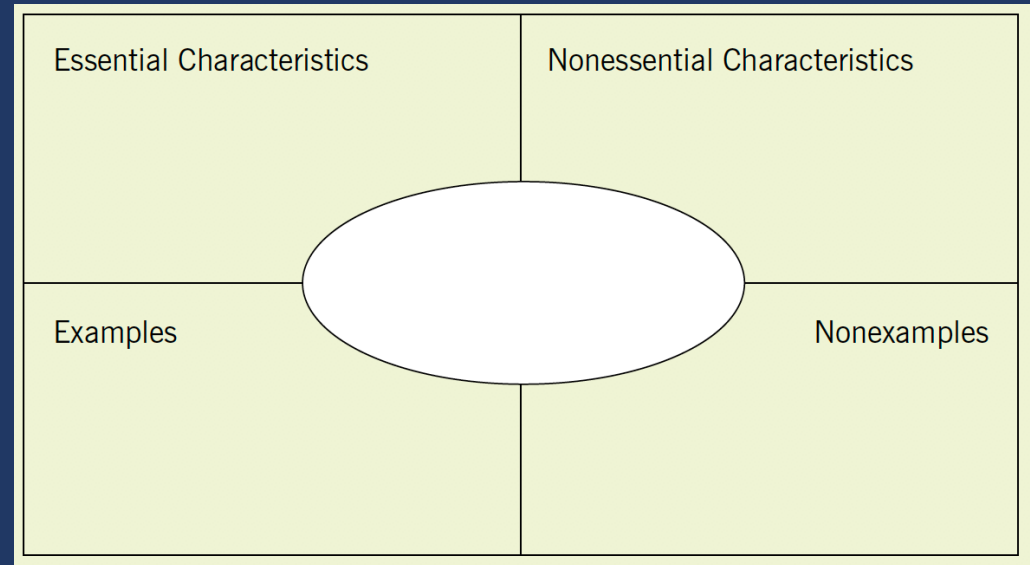
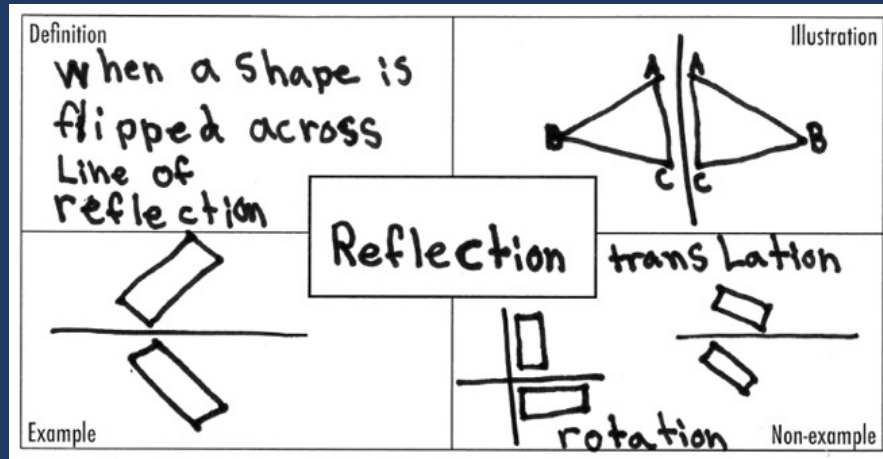
Use terms precisely



1. Use explicit instruction



2. Use graphic organizers



Dunston & Tyminski (2013)



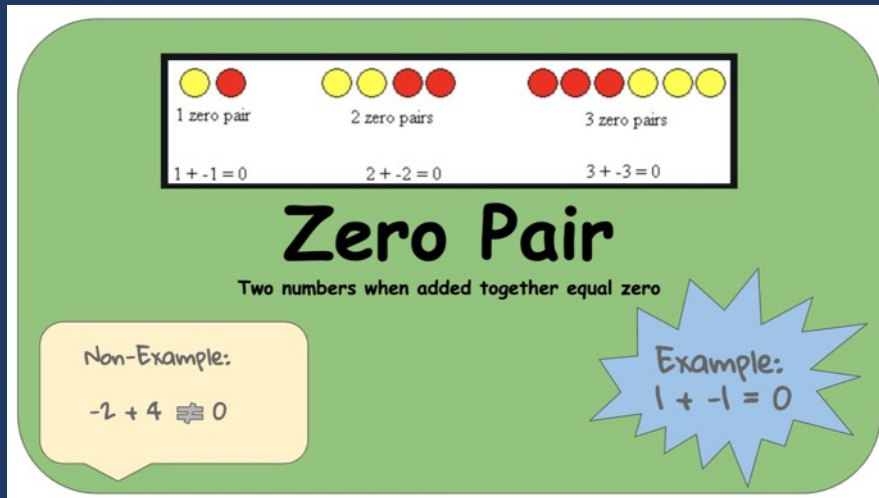
2. Use graphic organizers

Word	Lightbulb Word
Definition	Picture

Dunston & Tyminski (2013)



3. Have students create vocabulary cards



A green-bordered card titled "Zero Pair" with the subtitle "Two numbers when added together equal zero". At the top, it shows three examples of zero pairs: 1 pair (1 yellow, 1 red), 2 pairs (2 yellow, 2 red), and 3 pairs (3 yellow, 3 red). Below each pair is the equation $1 + -1 = 0$, $2 + -2 = 0$, and $3 + -3 = 0$ respectively. A yellow box on the left contains a "Non-Example:" $-2 + 4 \neq 0$. A blue starburst on the right contains an "Example:" $1 + -1 = 0$.

1 zero pair 2 zero pairs 3 zero pairs

$1 + -1 = 0$ $2 + -2 = 0$ $3 + -3 = 0$

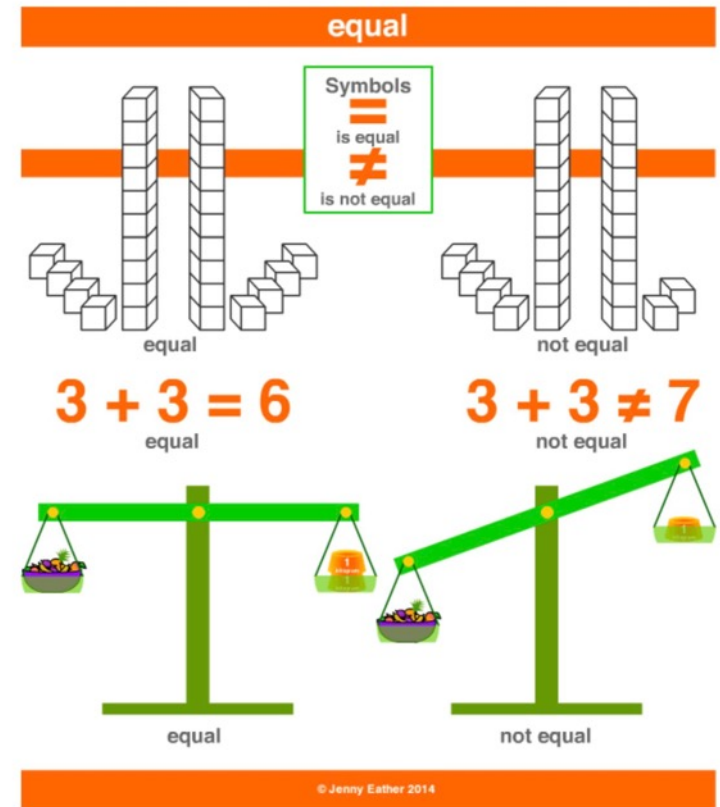
Zero Pair

Two numbers when added together equal zero

Non-Example:
 $-2 + 4 \neq 0$

Example:
 $1 + -1 = 0$

6. **Equal**: having the same amount or value.



A white card with orange borders illustrating the concept of equality. At the top, an orange bar says "equal". Below it, two sets of blocks are shown. The first set has two equal stacks of 3 blocks each, labeled "equal" with the equation $3 + 3 = 6$. The second set has two unequal stacks (one of 3, one of 4), labeled "not equal" with the equation $3 + 3 \neq 7$. A green box in the center lists symbols: "=" is equal, and " \neq " is not equal. At the bottom, two balance scales are shown. The first is balanced, labeled "equal". The second is tilted, labeled "not equal". An orange bar at the very bottom says "© Jenny Esther 2014".

equal

Symbols
= is equal
 \neq is not equal

equal not equal

$3 + 3 = 6$ $3 + 3 \neq 7$

equal not equal

© Jenny Esther 2014

4. Have students create glossaries


Integer Definitions

Zero Pairs
A positive and negative cancel one another;

Positive
A number that is greater than zero.

Absolute Value
The distance of a number from zero on a number line; shown as $||$

Negative
A number that is less than zero. Identified by a minus sign.



Numerator: how many parts of the whole

- Ex. $\frac{4}{10}$

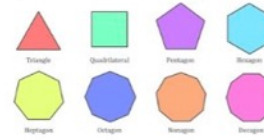
Odd number: a number not divided evenly by 2

- Ex. 1, 3, 5, 7, 9....

Percent: a specific number in comparison to 100

- 74%

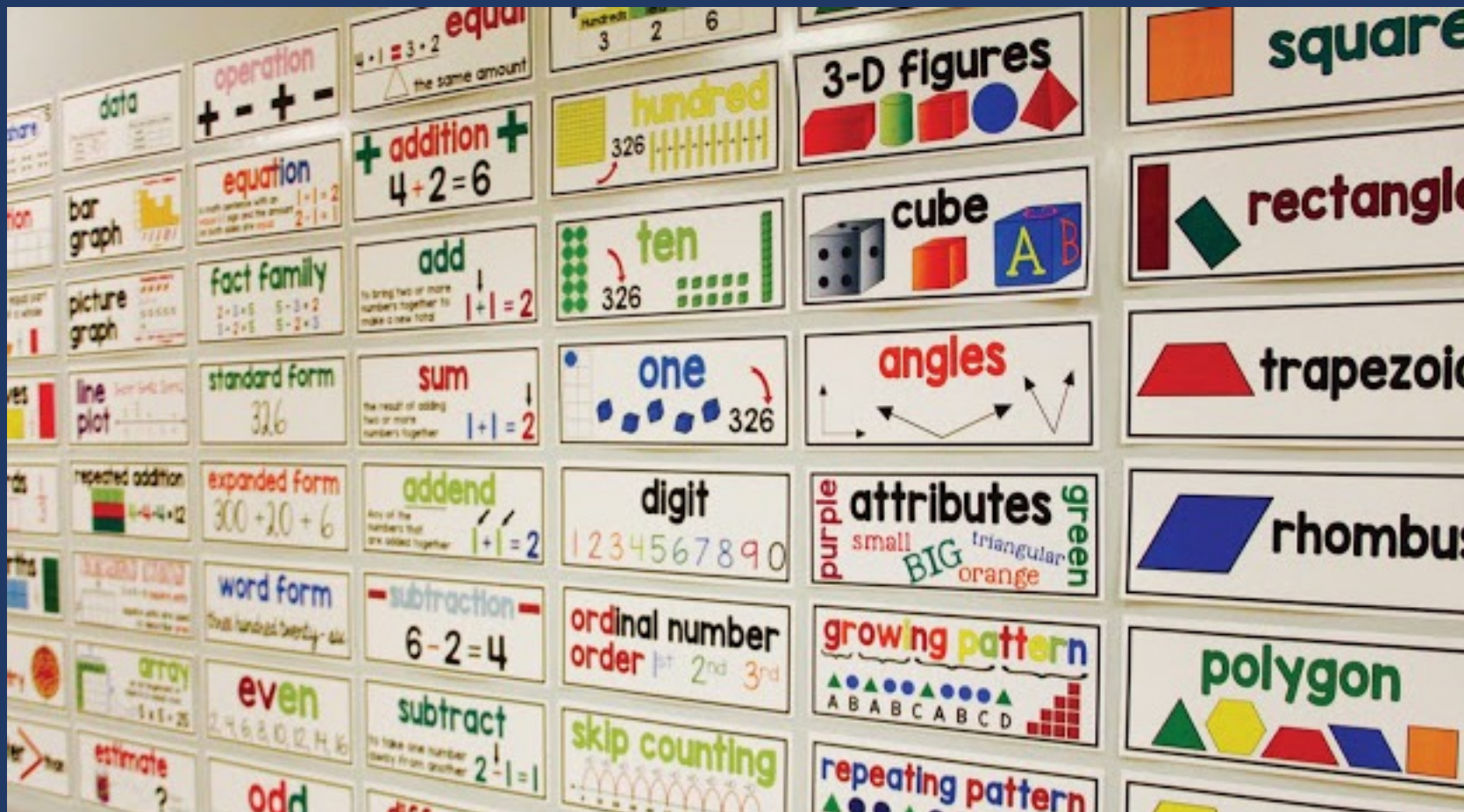
Polygon: any enclosed shape that is made up of 3 or more straight lines



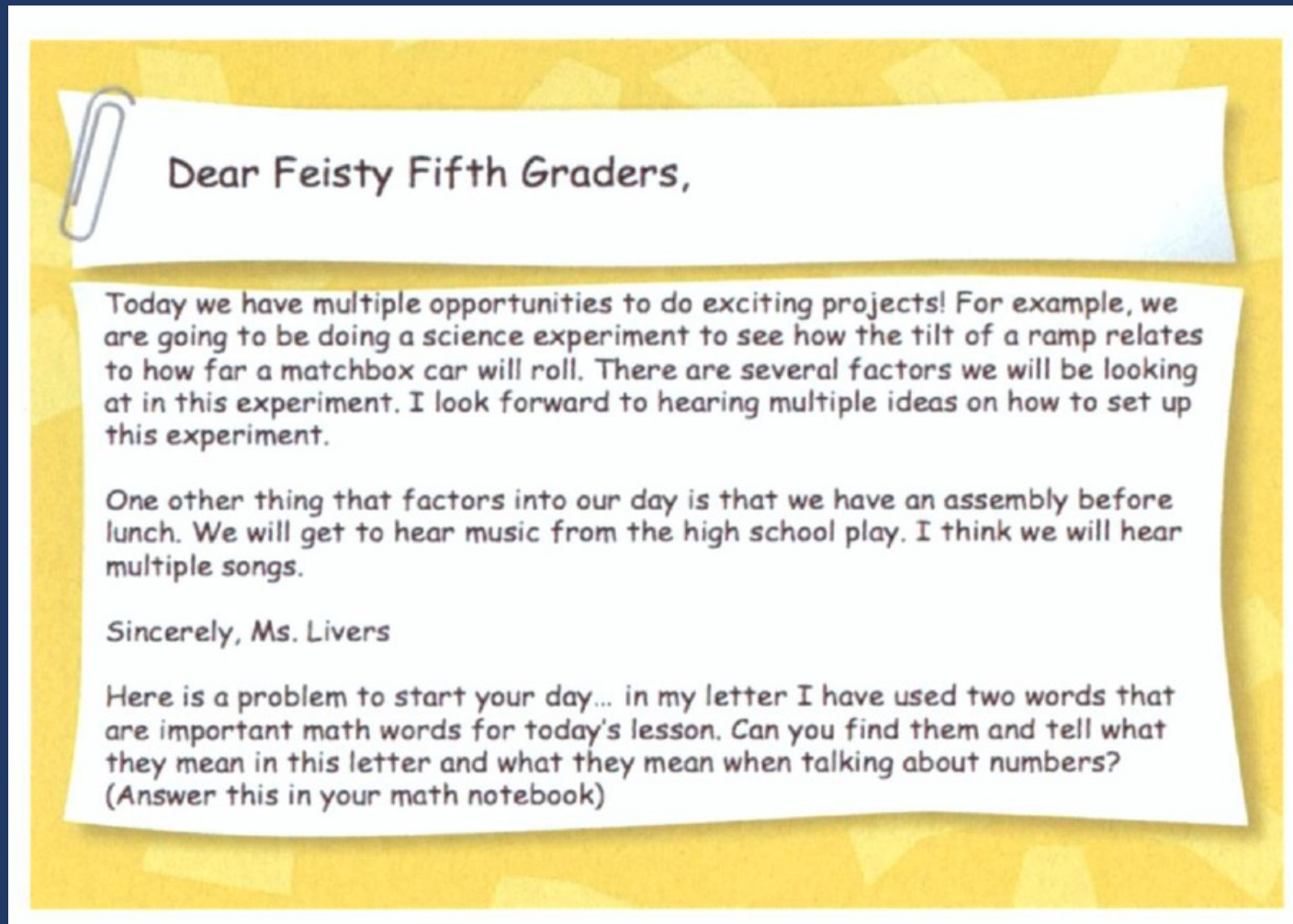
- Ex.



5. Create a word wall



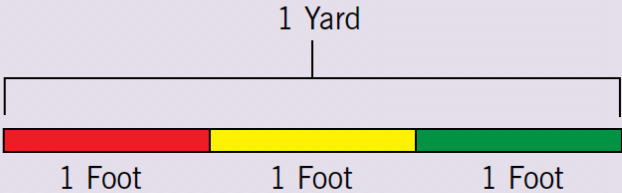
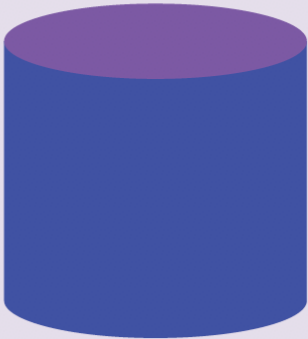
6. Preview vocabulary



Bay-Williams & Livers (2009)



7. Cluster vocabulary

	Length	Weight
Meaning	How long something is	How heavy something is
Visual	<p>1 Yard</p>  <p>1 Foot 1 Foot 1 Foot</p>	<p>2000 pounds = 1 ton</p> 

Livers & Bay-Williams (2014)



7. Cluster vocabulary

Rating	Word	Definition	Synonym(s)	Example	Sample Problem
2	expression	a mathematical phrase combining operations, numbers and/or variables.	phrase algebraic expression	6 $6n$ $6+n$ <div>no equal sign</div>	Lucia earns \$8 per hour for babysitting and gets a \$5 tip. Write an <u>expression</u> to represent the amount she would earn if she worked for x hours.
2	variable	a quantity that can change or take many values. (refers to the letter or symbol representing the quantity)	unknown	x D y T	The <u>variable</u> x represents the number of hours Charlie works in a week. Write an expression to represent his earnings if he earns \$9 per hour.
1	product	the result when two or more numbers are multiplied	total answer	$3 \times 2 = 6$ ↑ product	The <u>product</u> of 6 and a number is 24. What is the number?
3	quotient	the result of a division (refers to the number of times the divisor divides the dividend)	answer	$18 \div 2 = 9$ $2 \overline{)18}$ <div>9 ← quotient</div>	Estimate the <u>quotient</u> when 365 is divided by 12.

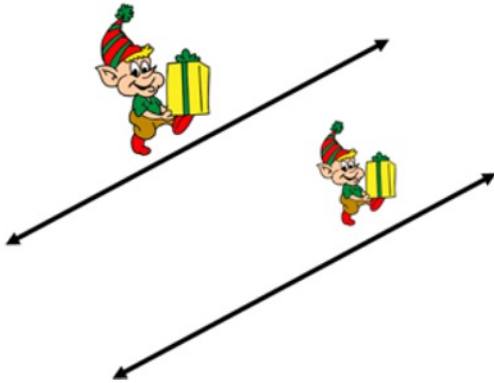
Marin (2018)



8. Use mnemonics

Parallel Lines (Pair of Elves)

Lines that are the same distance apart and will never intersect



The Pair of Elves are the same distance apart and will never intersect.

The Pair of Elves are on Parallel Lines

Ray (Run Away)

A line that has a starting point but no endpoint

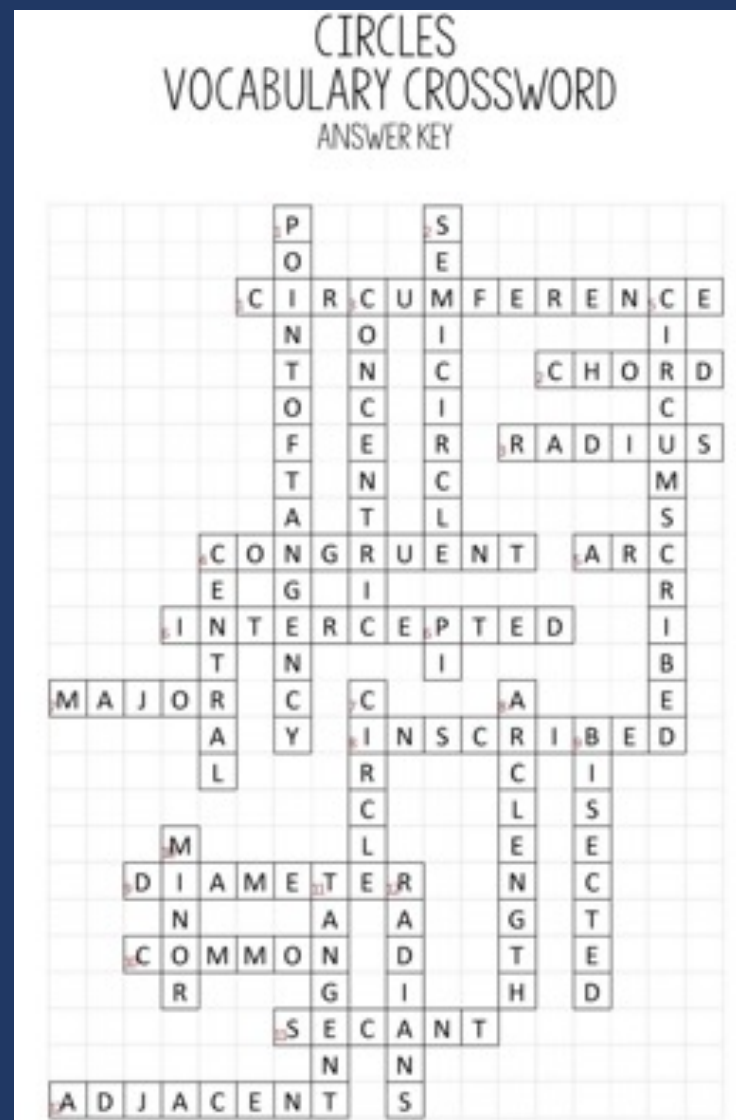
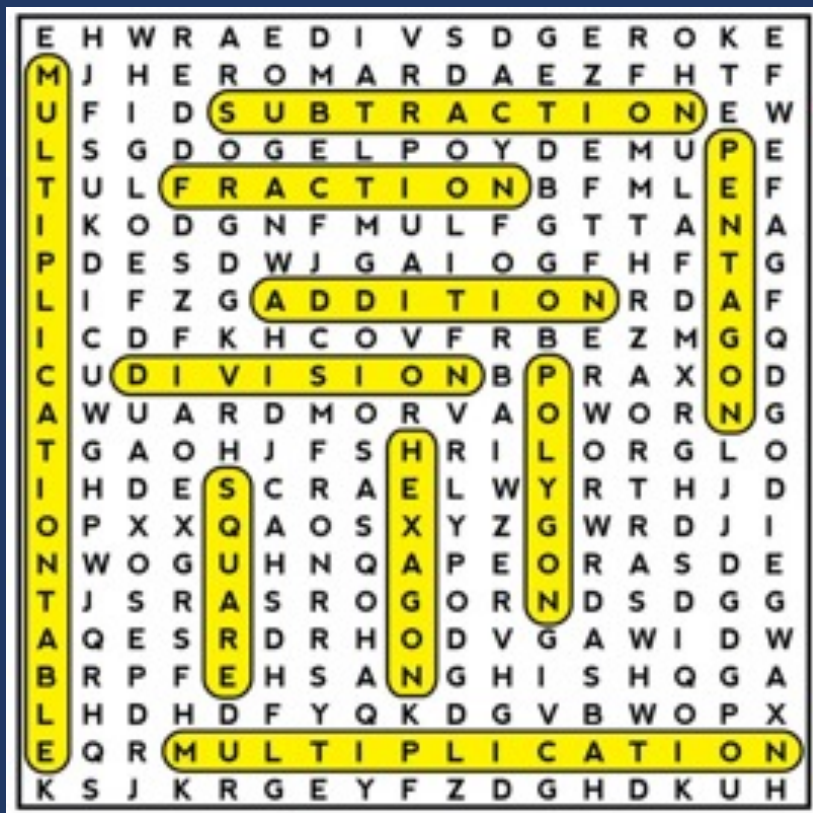


Start here!! Run away and never stop running Ray.

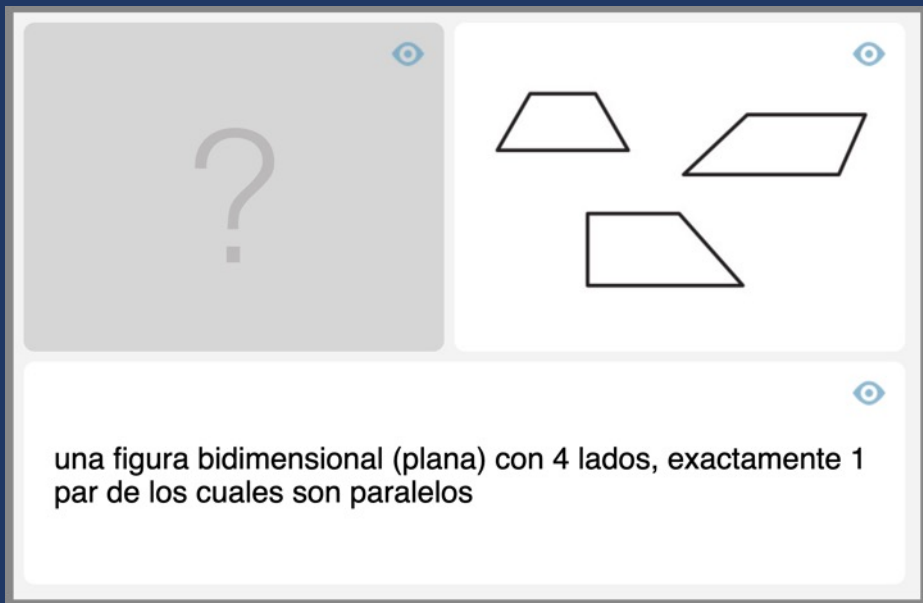
Riccomini et al. (2015)



9. Do word games



10. Use technology



una figura bidimensional (plana) con 4 lados, exactamente 1 par de los cuales son paralelos

Math Learning Center



Houghton Mifflin Math eGames

Math Lingo

60 minutes = 1 ____

change	minute hand	hour hand
hour	equal amounts	second
quarter-hour	half-hour	minute

Math Lingo





1. Discuss how you will use formal math language.
2. Discuss how you will help students distinguish among similar math terms.
3. Describe your strategy for focusing more on math language in your math instruction.



Evidence-Based Practice: Multiple Representations



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

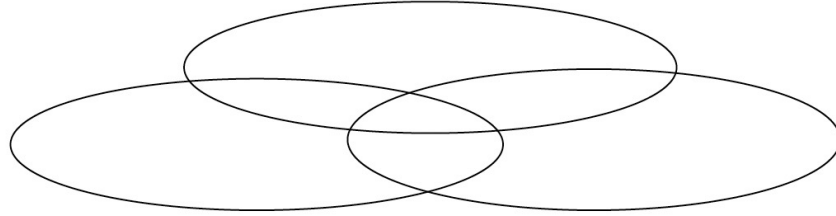
Precise
language

Multiple
representations

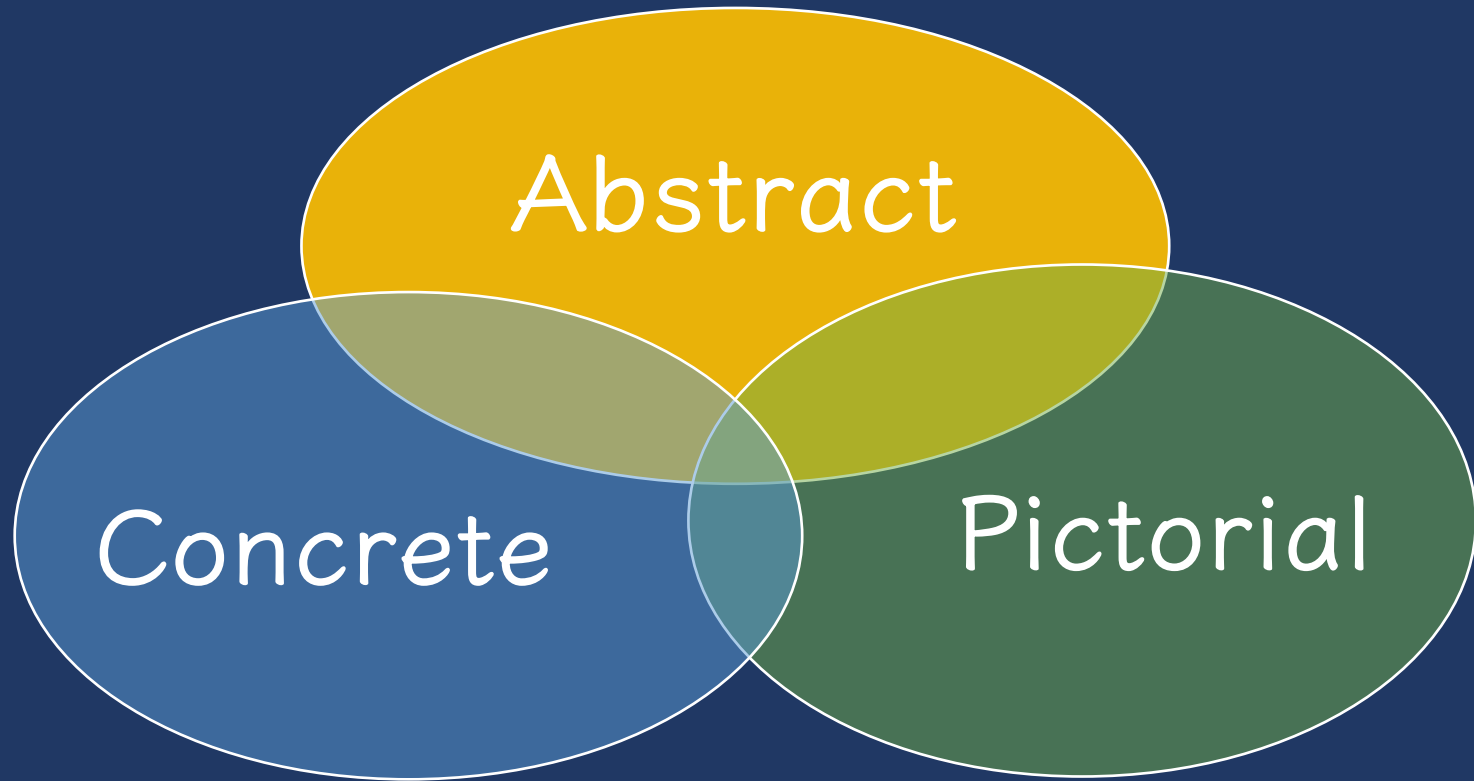
INSTRUCTIONAL STRATEGIES

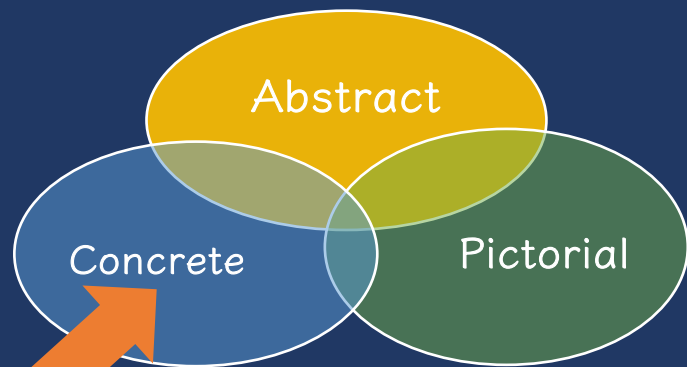


Multiple Representations

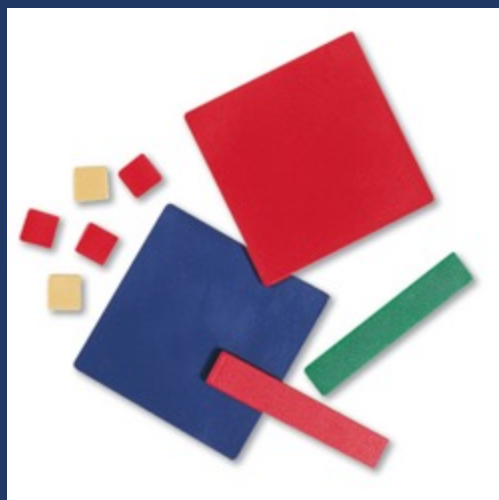
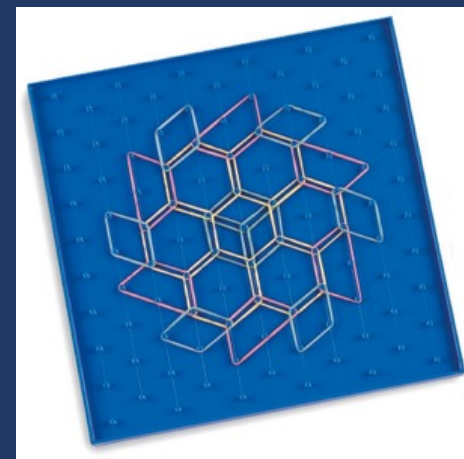


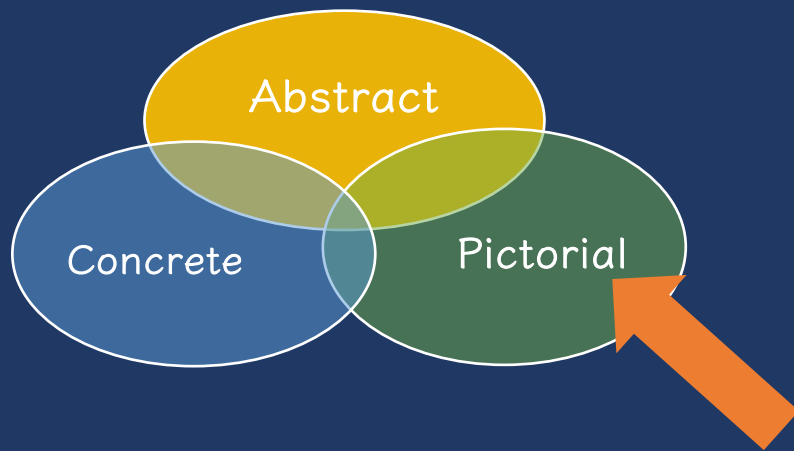
Multiple Representations



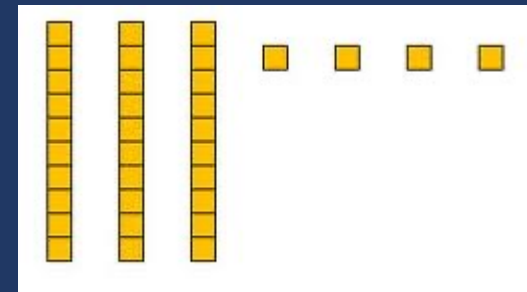
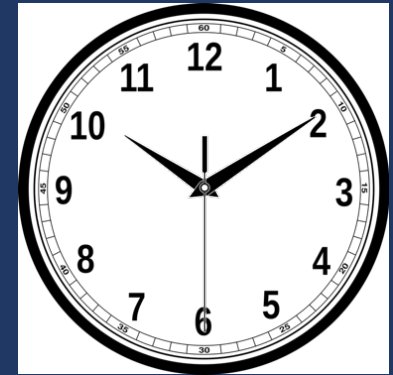
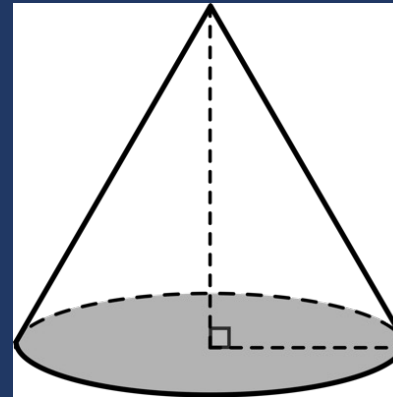


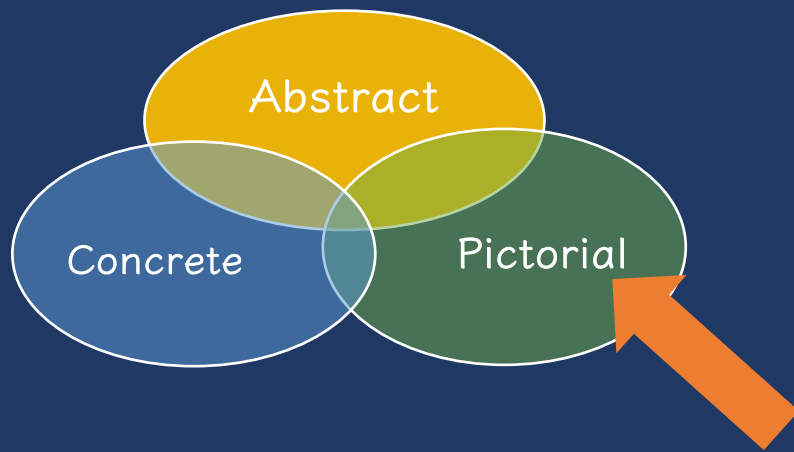
Three-dimensional objects



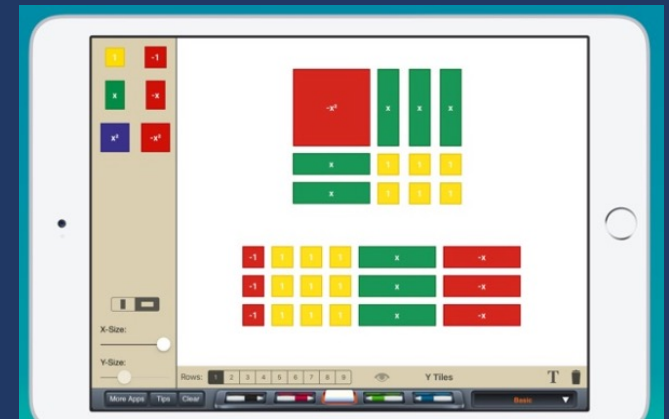
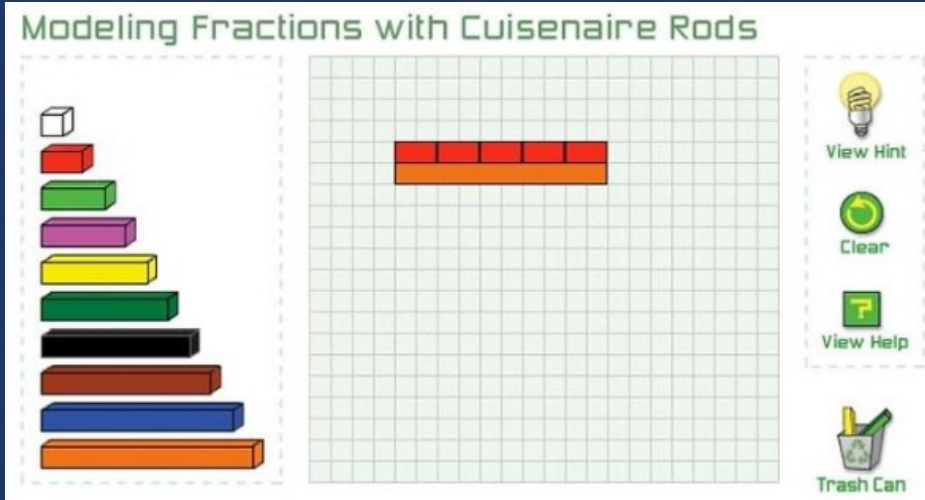
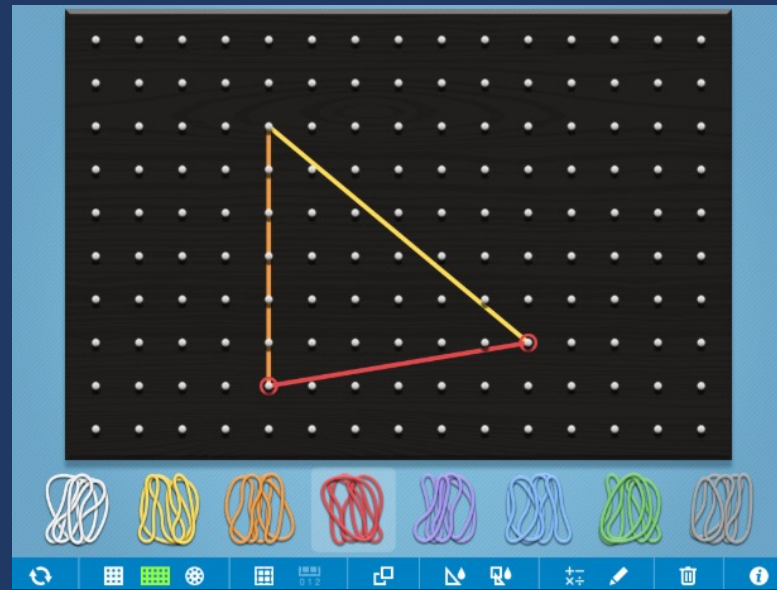


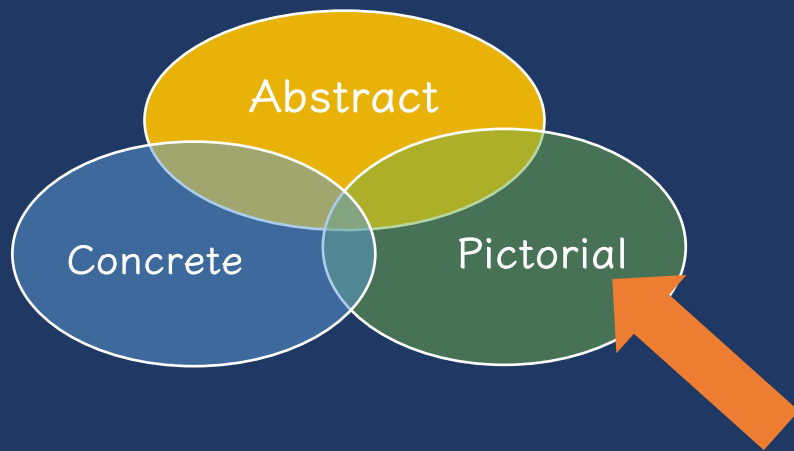
Two-dimensional images



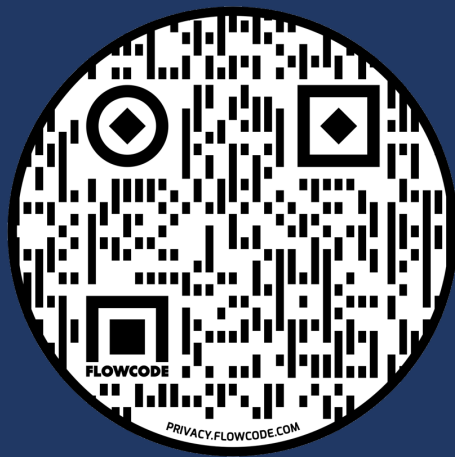


Two-dimensional images





Two-dimensional images



Virtual Manipulatives

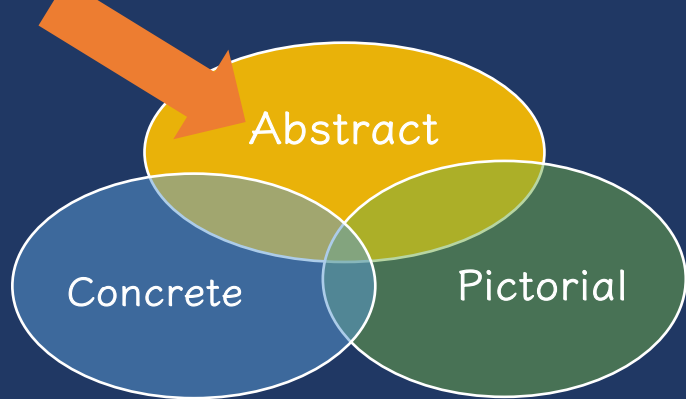
Help students see and learn math using different tools!

Number & Operations	Place Value
Fractions & Decimals	Integers & Algebra
Geometry	Time & Money
Data & Probability	Extras

Sarah R. Powell, Ph.D.
srpowell@utexas.edu
www.sarahpowellphd.com
@sarahpowellphd

Fractions & Decimals				





Numerals and symbols and words

$$2 + 8 = 10$$

$$34 = 3 \text{ tens and } 4 \text{ ones}$$

$$x - 6 = 8$$

$$\begin{array}{r} 4,179 \\ + \quad 569 \\ \hline \end{array}$$





If you are left handed:

What's one of your favorite hands-on manipulatives?

If you are right handed:

What's one of your favorite virtual manipulatives?



Evidence-Based Practice: Building Fluency with Facts and Computation



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

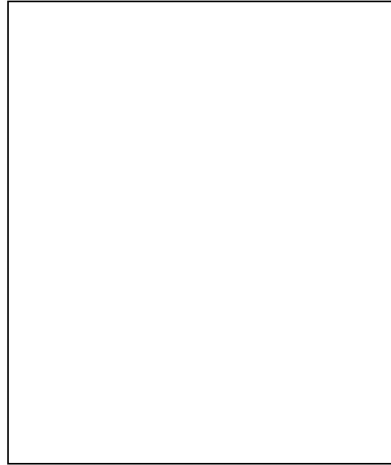
INSTRUCTIONAL STRATEGIES

Fluency building

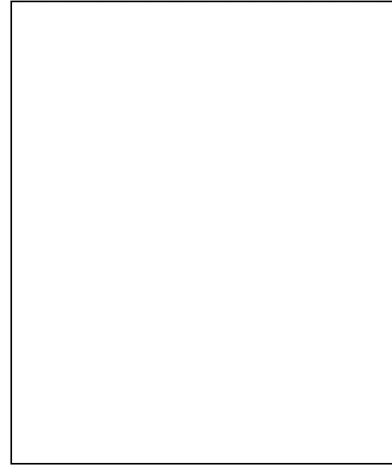


Fluency

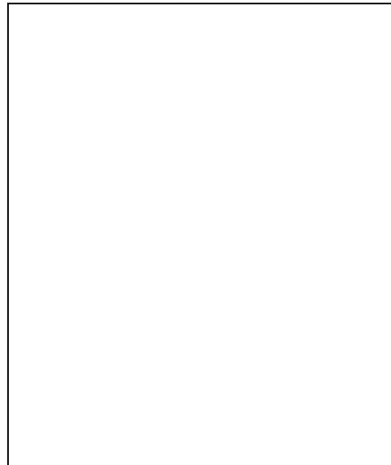
Addition



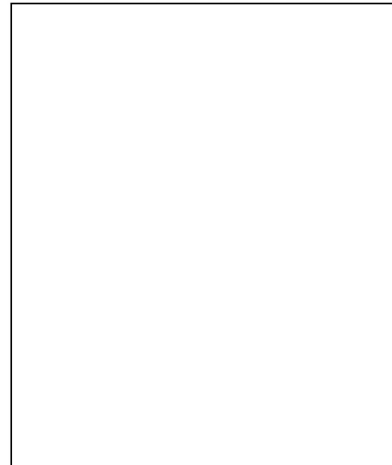
Subtraction



Multiplication



Division



Building Fluency

Addition	Subtraction
Multiplication	Division

- Fluency is doing mathematics easily and accurately.
- Fluency in mathematics makes mathematics easier.
- Fluency provides less stress on working memory.
- Fluency helps students build confidence with mathematics.



Addition	Subtraction
Multiplication	Division

- With fluency, it is important to emphasize both conceptual learning and procedural learning.
- Fluency is not strictly procedural!



Addition

100 addition basic facts

- Single-digit addends sum to a single- or double-digit number

$$\begin{array}{r} 5 \\ + 4 \\ \hline 9 \end{array}$$

(addend)
(addend)
(sum)



Addition: Total (Part-Part-Whole, Combine)

Count one set, count another set, put sets together, count sum

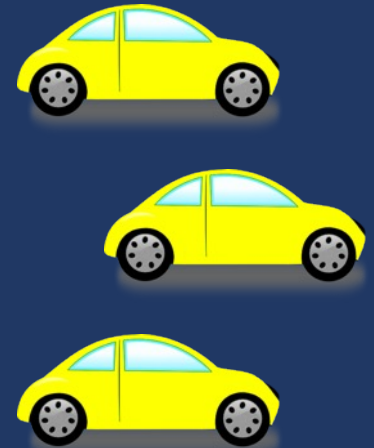
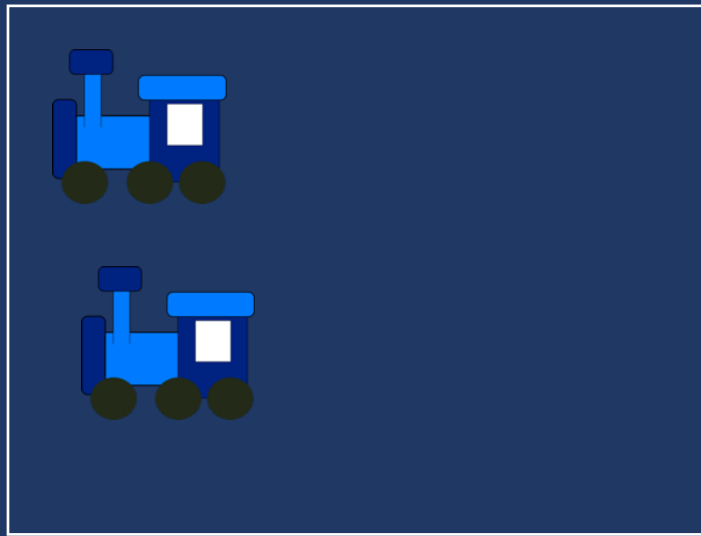


$$2 + 3 = 5$$



Addition: Join (Change Increase)

Start with a set, add the other set, count sum



$$2 + 3 = 5$$

Total

Parts put together into a **total**

- Karly saw **4** cardinals and **5** blue jays. How many birds did Karly see?

$$4 + 5 = ?$$



Change

An amount that **increases** or decreases

- Premila had \$**4**. Then they earned \$**3** for cleaning their room. How much money does Premila have now?

$$4 + 3 = ?$$



Total Versus Change (Join)

$$3 + 9 = \underline{\quad}$$



If you have brown eyes:

What's a Total story to show addition?

If you don't have brown eyes:

What's a Change/Join story to show addition?



Subtraction

100 subtraction basic facts

- Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

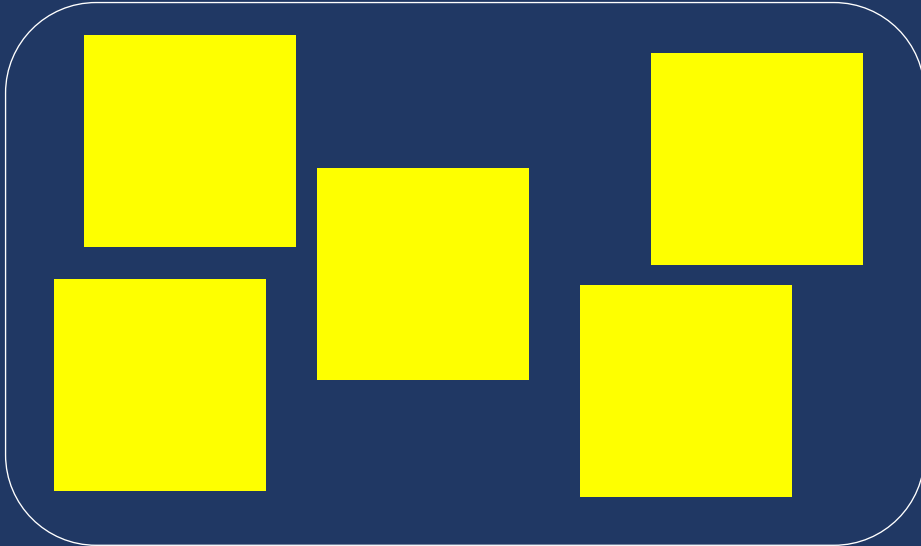
$$\begin{array}{r} 16 \\ - 8 \\ \hline 8 \end{array}$$

(minuend)
(subtrahend)
(difference)



Subtraction: Separate (Change Decrease)

Start with a set, take away from that set, count difference

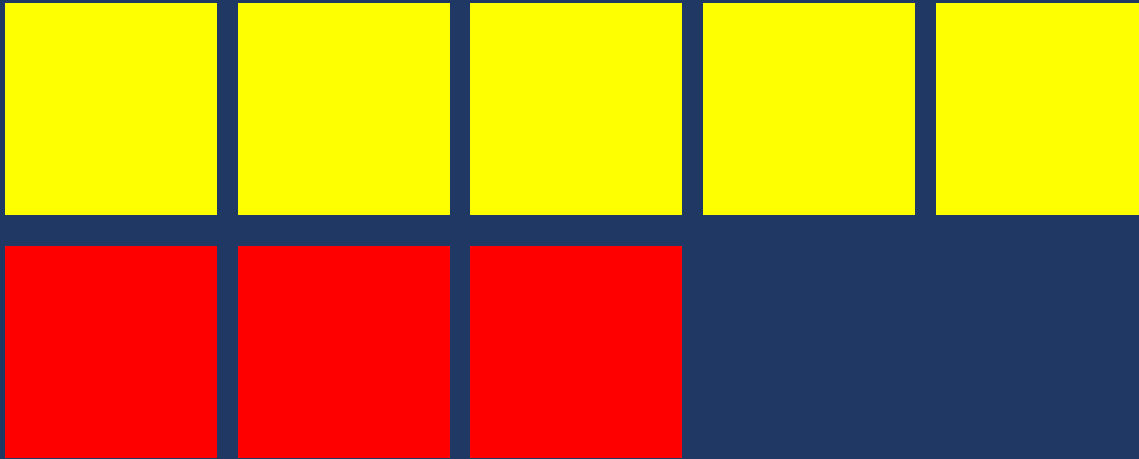


$$5 - 3 = 2$$



Subtraction: Difference (Compare)

Compare two sets, count difference



$$5 - 3 = 2$$



Change

An amount that increases or decreases

- Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

$$9 - 2 = ?$$



Difference

Greater and less amounts compared for a difference

- Rachel has 9 apples. Jodie has 4 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)

$$9 - 4 = ?$$



Change (Separate) versus Difference

$$9 - 5 = \underline{\quad}$$



If you weren't born in Virginia:

What's a Change/Separate story to show subtraction?

If you were born in Virginia:

What's a Difference story to show subtraction?



Multiplication

100 multiplication facts

- Multiplication of single-digit factors results in a single- or double-digit product

$$\begin{array}{r} 2 \\ \times 3 \\ \hline 6 \end{array}$$

(factor)
(factor)
(product)



Multiplication: Equal Groups

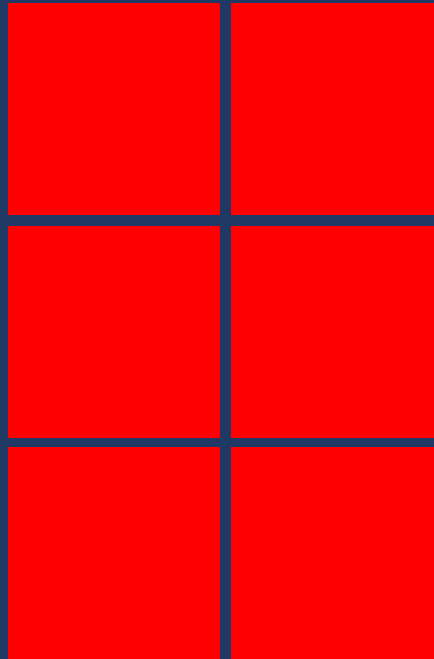
Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

Multiplication: Equal Groups

Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$



Multiplication: Comparison

Show a set, then multiply the set



$$3 \times 2 = 6$$

Equal Groups

Groups multiplied by **number in each group** for a **product**

Rhiannon has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Rhiannon have altogether?



Comparison

Set multiplied by a number of **times** for a **product**

Vivienne picked **6** apples. Jessica picked **2** times as many apples as Vivienne. How many apples did Jessica pick?



Equal Groups versus Comparison

$$2 \times 5 = \underline{\quad}$$



If you have glasses on:

What's an Equal Groups story to show multiplication?

If you don't have glasses on:

What's a Comparison story to show multiplication?



Division

90 division basic facts

Divisor and quotient are single-digit numbers
and dividend is single- or double-digit number

$$8 \div 4 = 2$$

(dividend) (divisor) (quotient)



Division: Equal Groups (Partitive Division)

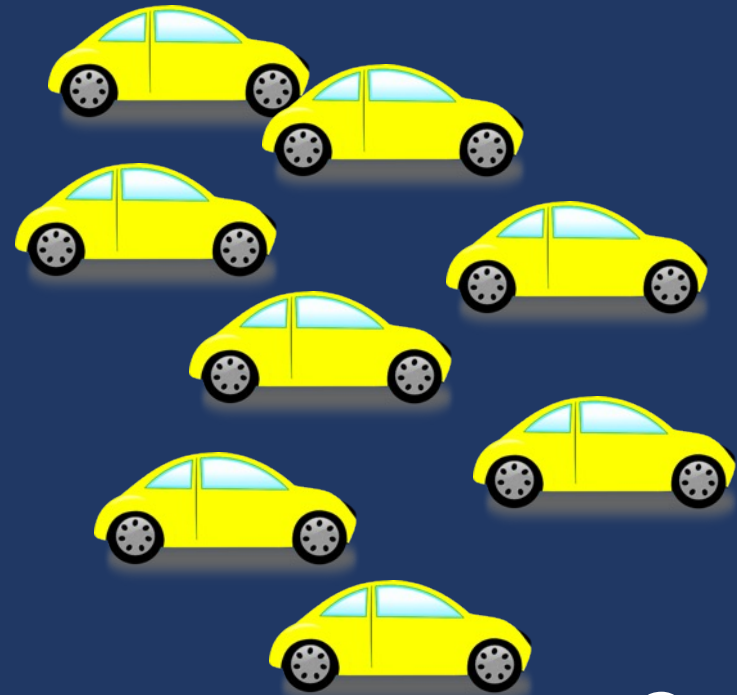
Show the dividend, divide equally among divisor, count quotient



$$8 \div 2 = 4$$

Division: Equal Groups (Quotative Division)

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$

Equal Groups

Groups multiplied by **number in each group** for a **product**

Stefanie has **12** apples. She wants to share them equally among her **2** friends. How many apples will each friend receive?

Nicole has **12** apples. She put them into bags containing **6** apples each. How many bags did Nicole use?



Partitive versus Quotative

$$12 \div 4 = \underline{\quad}$$



If you watched *Stranger Things*:

What's a Partitive story to show division?

If you watched *Ted Lasso*:

What's a Quotative story to show division?



Addition	Subtraction
Multiplication	Division

- Build fluency with math facts.
 - Addition: single-digit addends
 - Subtraction: single-digit subtrahend
 - Multiplication: single-digit factors
 - Division: single-digit divisor

$$\begin{array}{r}
 5 \\
 + 8 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 9 \\
 - 4 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 6 \\
 \times 7 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 56 \\
 \div 8 \\
 \hline
 \end{array}$$



Cover, Copy, Compare

$$\begin{array}{r} 9 \\ \times 6 \\ \hline 54 \end{array}$$

$$\begin{array}{r} 7 \\ \times 8 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 9 \\ \times 9 \\ \hline 81 \end{array}$$

$$\begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array}$$

$$\begin{array}{r} 8 \\ \times 8 \\ \hline 64 \end{array}$$

$$\begin{array}{r} 8 \\ \times 6 \\ \hline 48 \end{array}$$

$$\begin{array}{r} 6 \\ \times 5 \\ \hline 30 \end{array}$$

$$\begin{array}{r} \times \\ \hline 6 + 4 = \end{array}$$

$$7 + 3 =$$

$$2 + 7 =$$

$$5 + 6 =$$

$$4 + 7 =$$

$$7 + 8 =$$

$$6 + 7 =$$

$$7 + 9 =$$

$$7 + 6 =$$

$$8 + 7 =$$

$$7 + 0 =$$

$$9 + 6 =$$

$$6 + 0 =$$

$$6 + 8 =$$

File Folder

$$6 + 3 =$$

$$1 + 7 =$$

Taped Problems

$$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$$

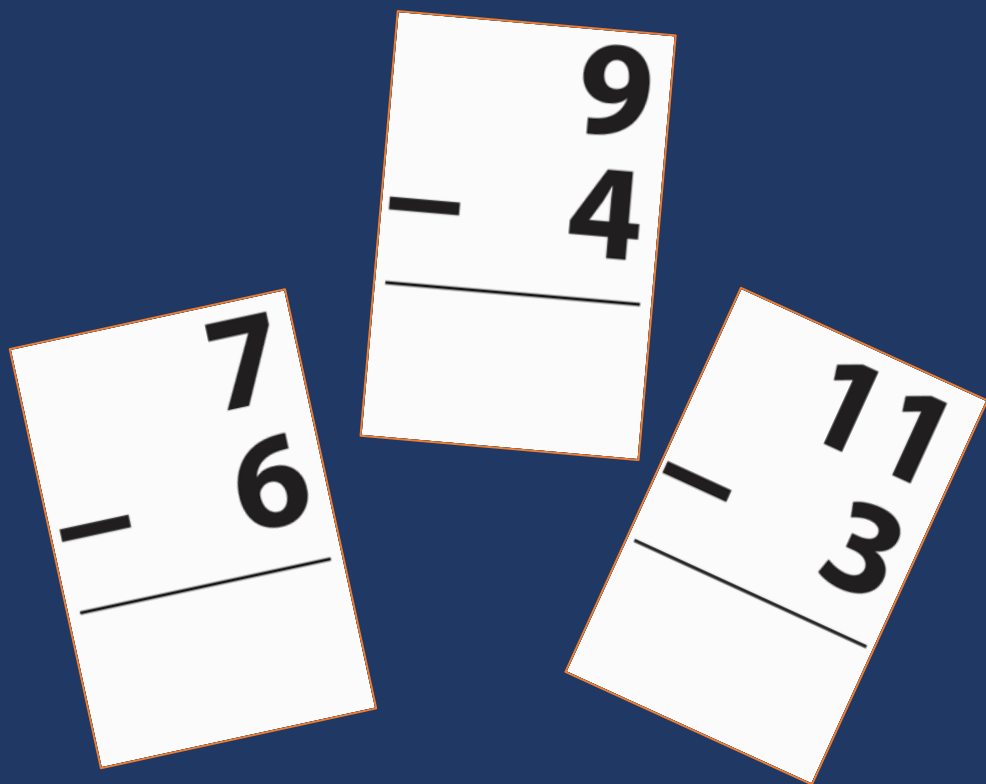
$$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$$

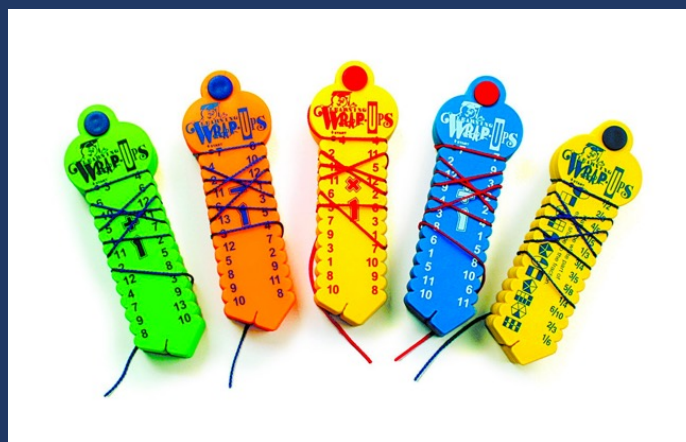
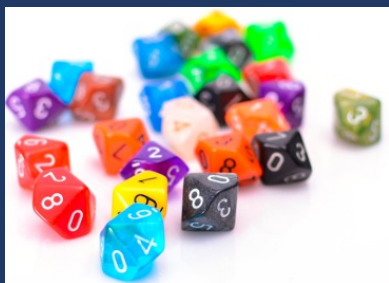
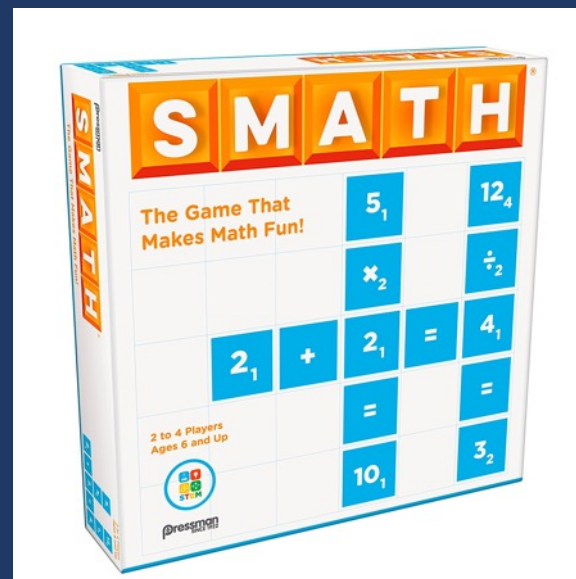
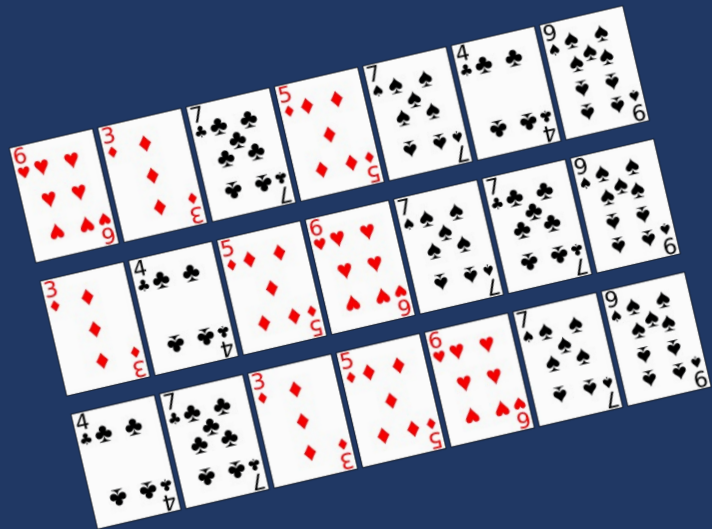
$$\begin{array}{r} 4 \\ \times 8 \\ \hline \end{array}$$

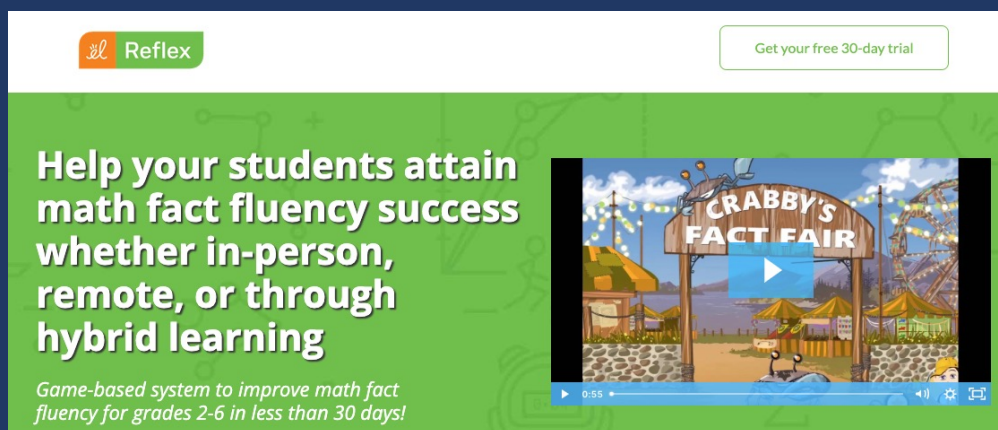
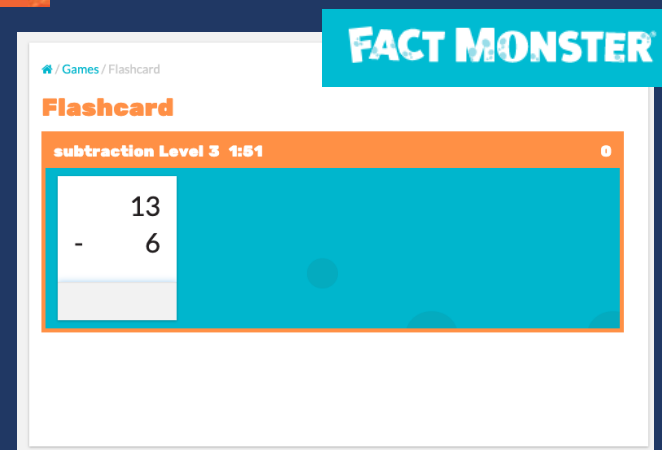
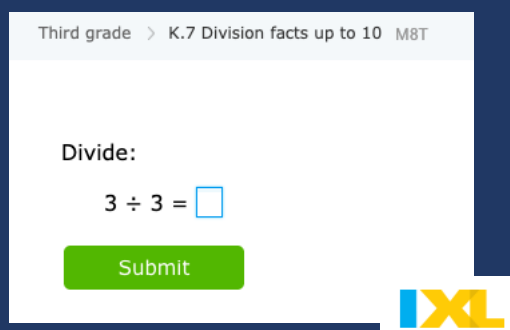
$$\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$$





Flash Card Graph		Name: _____											
38													
37													
36													
35													
34													
33													
32													
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4													
3													
2													
1													
	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10	Day 11	Day 12	





DAILY and
BRIEF



Addition	Subtraction
Multiplication	Division



What are other ways to practice fluency?



Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ - 724 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div 15 \\ \hline \end{array}$$



Addition	Subtraction
Multiplication	Division

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ + 3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 7.892 \\ \div 0.14 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$



Addition	Subtraction
Multiplication	Division

Build fluency with integer computation

$$-135 \div 2 =$$

$$\begin{array}{r} 6 \\ \times -12 \\ \hline \end{array}$$

$$-14 - (-7) =$$

$$\begin{array}{r} 1.4 \\ + -3.9 \\ \hline \end{array}$$





1. Describe how you will help students learn their mathematics facts.
2. Describe how you will support students with other types of math fluency.



Evidence-Based Practice: Word-Problem Solving



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20

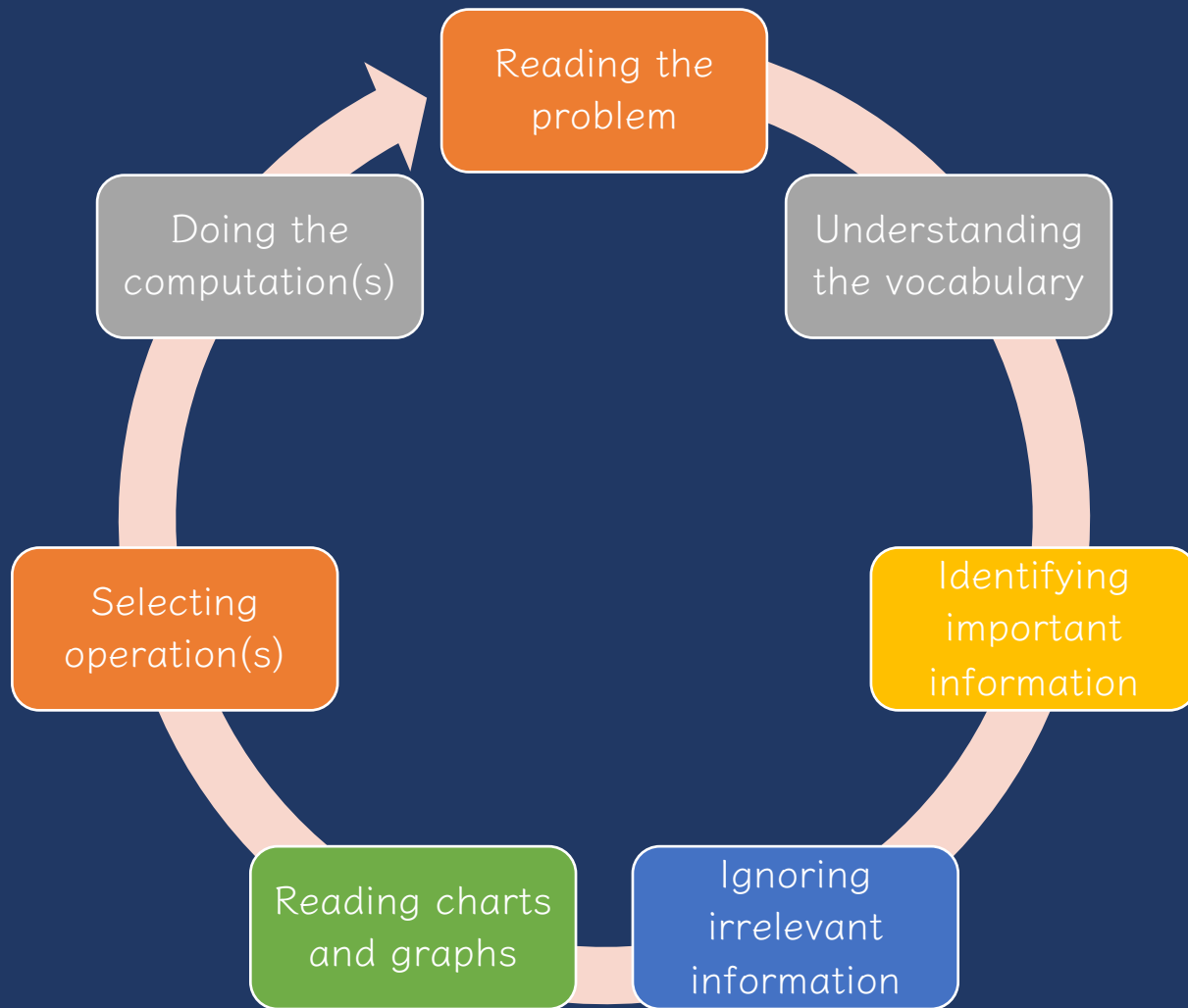


How would you solve this problem? What skills are necessary to solve this problem?



Word-Problem Solving







What are additional areas of difficulty that we should add to this list?



~~1. Keywords tied to operations~~





Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?



Math Words Used in Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> + add + all together + and + both + combined + how many in all + how much + in all + increased by + plus + sum + together + total 	<ul style="list-style-type: none"> - change - decreased by - difference - fewer or fewer than - how many are left (or have) - how many did not have - how much - how much taller, heavier, less or less - lost - minus - need to - reduce - remain - subtract - take away

Multiplication Words	Division Words
<ul style="list-style-type: none"> x by (dimension) x double x each group x every x factor of x increased by x multiplied by x of x product x times x triple 	<ul style="list-style-type: none"> ÷ as much ÷ cut up ÷ each group ÷ equal share ÷ half (or at) ÷ how many parts ÷ per ÷ percent ÷ quotient ÷ ratio of ÷ separated ÷ share some

Division

Taking a total amount and sharing equally amongst groups to find the total amount.

Keywords
 how many will each receive
 How many in each group
 Shared
 Divided
 Equal/Equally
 Quotient

Multiplication

Putting things/objects into equal groups to find the total amount.

Keywords
 Altogether
 In all
 Product
 Equal groups
 At this rate
 Doubled, tripled (and so on)

Subtraction

Taking away from a total amount.

Keywords
 Different
 More than
 Less than
 How many more
 Fewer than
 How many left over

Addition

Putting two or more things/amounts together.

Keywords
 Total
 Altogether
 In all
 Sum
 More than
 Combined

Problem Solving Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add altogether both combined How many in all? in all increase join plus sum total together 	<ul style="list-style-type: none"> are not decrease difference fewer, larger, shorter How much more? left less than minus remain take away

Multiplication	Division
<ul style="list-style-type: none"> area as much by factor multiple multiplied product in all 	<ul style="list-style-type: none"> average evenly equal parts divisor divided distribute quotient ratio some separate split

Key Words

addition: sum, both, total, together, plus, perimeter, more than

subtraction: difference, fewer, less than, take away, minus, how many more...

multiplication: product, times, each group, every, factor, multiple, multiplied, product, in all

division: quotient, split, equal groups, half, shared equally, each, distribute

OPERATION clue words

ADDITION	SUBTRACTION
<ul style="list-style-type: none"> total join more than in all altogether sum increased 	<ul style="list-style-type: none"> less than decreased remaining change left fewer take away difference minus

MULTIPLICATION	DIVISION
<ul style="list-style-type: none"> product times twice as many as by of groups 	<ul style="list-style-type: none"> quotient each per into divided by split share equally

When they say... They mean...

Addition	Subtraction
<ul style="list-style-type: none"> Sum Total Plus In all And Altogether Perimeter Together 	<ul style="list-style-type: none"> Fewer Less than Exceed Remain Are not Minus Difference How many more Take away Left over

Multiplication	Division
<ul style="list-style-type: none"> Times Each Twice Per Product Multiple Groups led by 	<ul style="list-style-type: none"> Half Separate Split Quotient Divisor Cut up Dividend Same Divided by Cut up

Key Words for All Operations

addition: added to, plus, join, more than, add, altogether, and, increase, sum, together, total

subtraction: decrease, minus, less than, gave, subtract, difference, share, less, take away

multiplication: times, multiple, twice, equal groups, multiplied, increased by, product, of, total

division: divided by, parts, goes into, percent, quotient, share equally, split up, divided into, half

Math Operation - Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add altogether and both in all sum total increase 	<ul style="list-style-type: none"> difference fewer than gave/take away decreased by how many more show much longer/smaller/shorter minus remaining

Multiplication	Division
<ul style="list-style-type: none"> area product Each by - of - per Times double, twice, triple total increase 	<ul style="list-style-type: none"> quotient divide into equal parts/share equally per amount of each

KEY WORD Posters

ADDITION	MULTIPLICATION
<ul style="list-style-type: none"> sum total more than plus both combined increased by perimeter 	<ul style="list-style-type: none"> product double area times per every each by

SUBTRACTION	DIVISION
<ul style="list-style-type: none"> difference remain left less than minus how many more fewer than decrease give away reduce discount 	<ul style="list-style-type: none"> quotient divide by into split out of shared per every each evenly equal groups half

Math Keywords

Addition
 +
 add, altogether, and, both, how many, how much, in all, increased by, plus, sum, together, total

Subtraction
 -
 are not, change, decreased by, difference, fewer, have left, how many did not have, how many more, how much more, less than, remain, subtract, take away, taller/shorter

Division
 ÷
 as much, cut up, divided by, each group has half (or other fractions), how many in each parts, quotient of, separated, share something equally, split

Multiplication
 ×
 by (dimension), double, each group, multiplied by, of, product of, times, triple

Math Key Words

Addition	Subtraction	Multiplication	Division
plus, sum, add, total, all together, increase, more, combine	subtract, minus, difference, left, left over, decrease, take away, fewer	times, product, factor, double, groups, each, area, rows	quotient, split, share, divide, separate, each, average, equal groups



Word-Problem Words Poster Set

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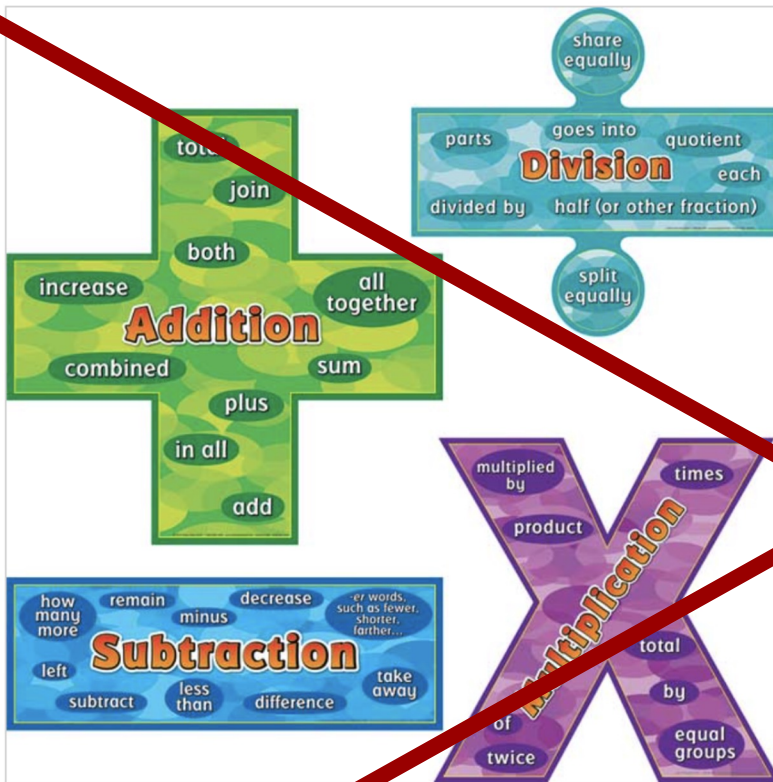
1 poster set

Quantity

Ready to Ship

[Add to Cart](#)

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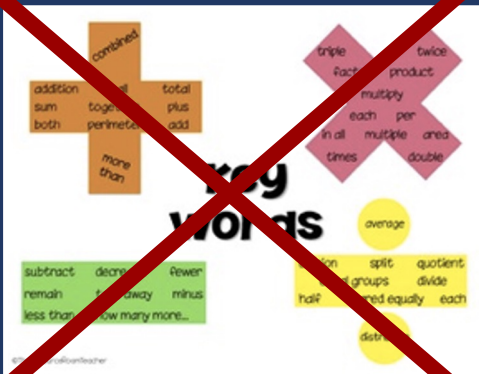


Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords ^a		Multiple keywords ^a		Keyword(s) led to correct solution ^a	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

^aWhen a problem featured a keyword.





Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema ^a		Any keyword		Keyword(s) led to correct solution ^b	
	n	%	n	%	n	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

^aSum across schemas does not equal 100 because each word problem featured more than one schema.

^bWhen a problem featured a keyword.



Mr. Rivera's taxable income is \$20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?



Important notes about keywords

Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords
("What does *more than* tell you about?")



But, *do not* tie a keyword to a specific operation!



2. Presenting problems by operation



Name: _____

Date: _____

Addition Word Problems



Solve the word problems. Show your work.

- Noah had 12 books. He got 5 more books. How many books does Noah have in all?
- Bonnie found 8 rocks on her sidewalk and 7 rocks in her backyard. How many rocks did Bonnie find in all?
- Edward had 5 toy cars. He got 8 more toy cars. How many toy cars did Edward have in all?
- Mariela collected 11 feathers. Then she found 3 more feathers. How many feathers did Mariela have in all?
- LaMonte made 14 cookies. Then he made 5 more cookies. How many cookies did LaMonte have in all?

Name: _____

Subtraction

- Martha has 5 teddy bears. She gave 2 of them to her sister, Alyssa. How many teddy bears does Martha have now?

- There are 7 students in a classroom. The art teacher took 3 students to the art room and hand-covers a table to each child. How many paintbrushes are left?

- Ben built 9 sandcastles on the beach. 6 got washed away by waves. How many of Ben's sandcastles are left on the sand?

- Billie has 4 jump ropes. 2 among them have adjustable ropes. How many jump ropes remain non-adjustable?

- Ms. Hollister had 10 jack-o'-lanterns displayed on her front porch. She gives away 4 to Sam. How many jack-o'-lanterns remain on Ms. Hollister's porch?

- Kim and Mimi did go in to a musical store to buy a trumpet. They find a total of 8 trumpets on display but 1 of them have been booked by another customer. How many trumpets can Kim choose from now?


LONG DIVISION WORD PROBLEMS

- Zookeeper Al wants to give each monkey in the zoo an equal number of bananas. There are 37 monkeys in the zoo and 567 bananas. How many bananas does each monkey get? And how many are left over for him to eat himself?
- Betty has 100 oranges and needs to pack them up equally in 23 boxes. How many oranges go in each box and how much does she have left over?
- Miss King has 1376 pages of scrap paper. She wants to make them into scrap paper packets for her 32 students. How many pages will each packet have? How many extra pages will she have left over?
- Mr. Chong has 1,440 pages of scrap paper. He instead wants to make packets of 40 pages each but forgets to check if that will be enough for his 37 students. Will there be enough packets per student? If not how much more scrap paper does he need?



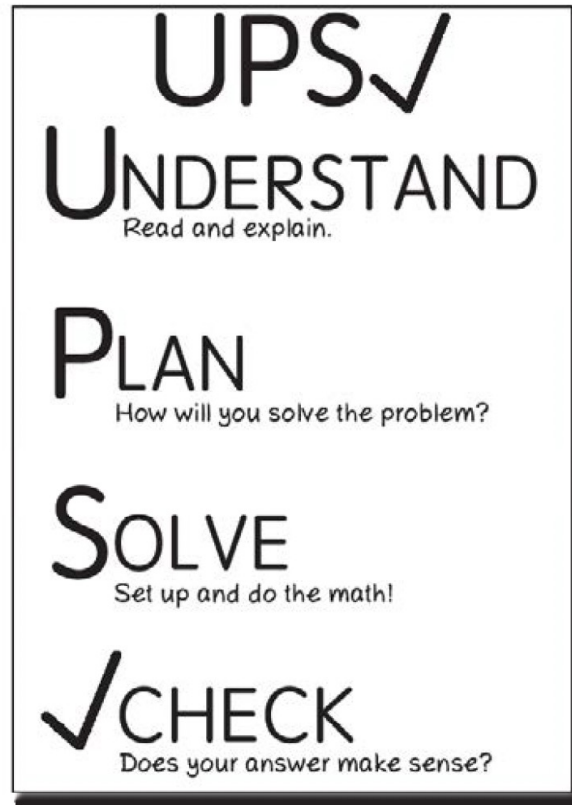
Teaching Problem Solving

Have an attack strategy

Teach word-problem schemas



Attack Strategy



Have an attack strategy

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.



Have an attack strategy

STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve the problem.

Review your answer.

RICE

Read and record the problem.

Illustrate your thinking.

Compute.

Explain your thinking.



Have an attack strategy

SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.



Have an attack strategy

SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

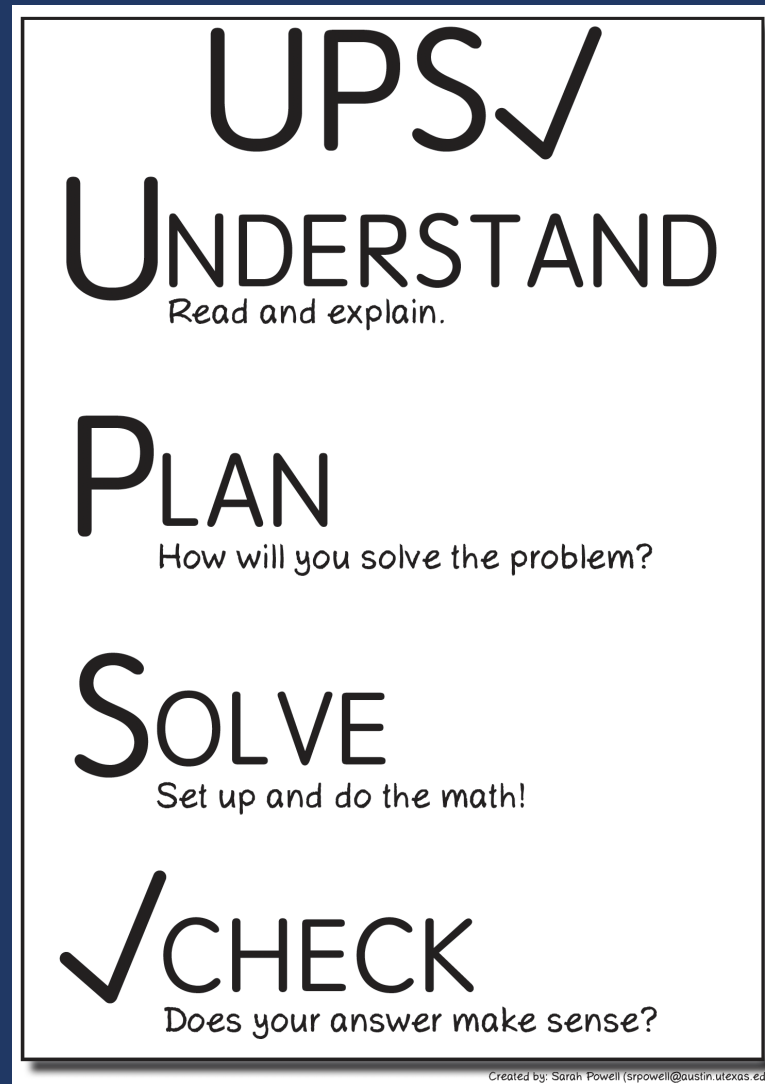
Box action words.

Evaluate steps.

Solve and check.



Have an attack strategy





Share your favorite attack strategy.



Teach word-problem schemas

Total

Difference

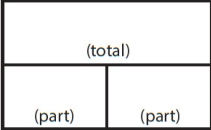
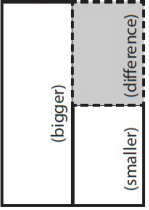
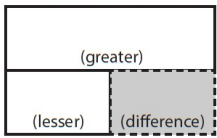
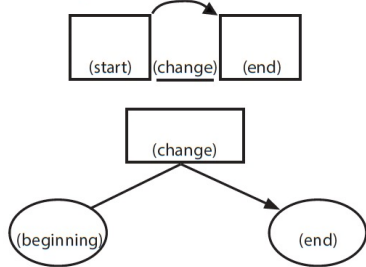
Change

Equal Groups

Comparison

Ratios/Proportions



Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	$P1 + P2 = T$ (part + part = total) 	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	$B - s = D$ (bigger - smaller = difference)  $G - L = D$ (greater - less = difference) 	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	$ST + / - C = E$ (start + / - change = end) 	End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?	Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?	Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?	Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?
		End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?	Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?	Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?	



Total

Part-part-whole
Combine

Parts put together into a **total**

Daniela saw **3** canoes and **8** kayaks.
How many boats did Daniela see?

Total

Daniela saw **11** boats. If **3** of the boats
were canoes, how many were kayaks?

Part

Daniela saw **11** boats. **8** of the boats
were kayaks, how many were canoes?

Part



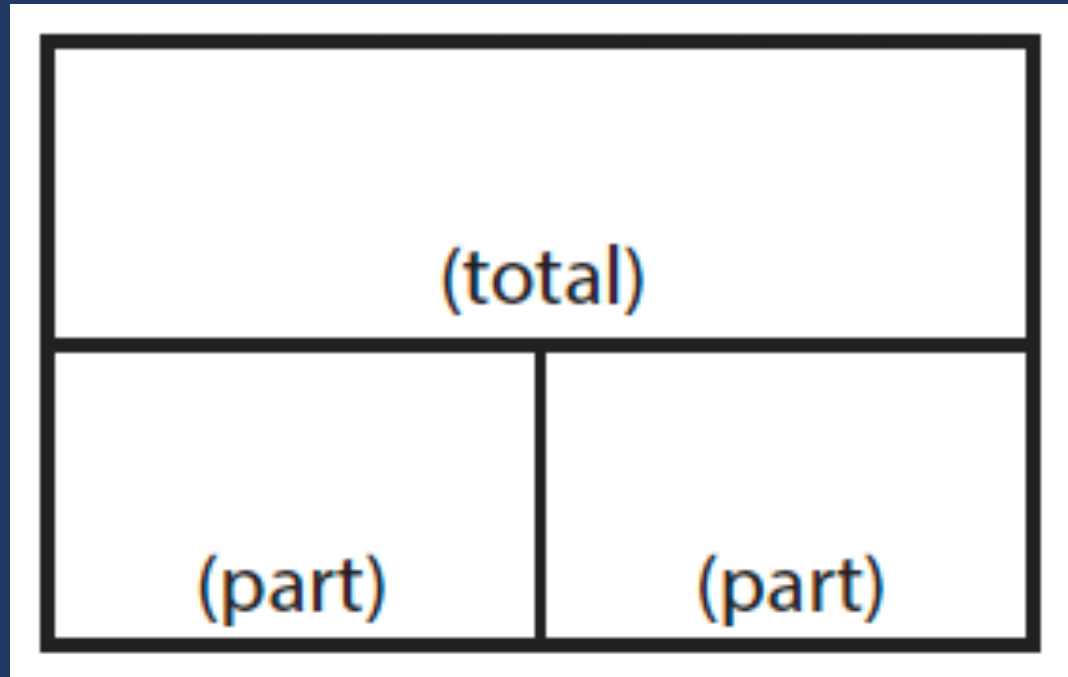
Total

“Are parts put together for a total?”



Total

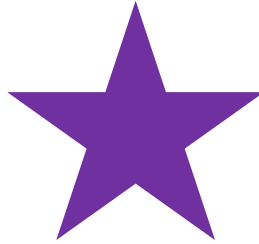
$$P1 + P2 = T$$



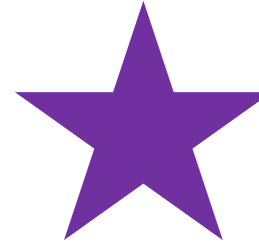
Total

Additive Word Problems

A.
Megan baked 38 sugar cookies and 24 chocolate chip cookies. Enter the total number of cookies Megan baked in all.



B.
In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?



C.
Jana has 162 wooden beads and 95 glass beads. How many more wooden beads than glass beads does Jana have?

D.
The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?



Total



What's an example Total problem?



Difference

Compare

Greater and **lesser** amounts compared for a **difference**

Adrianna has **10** pencils. Tracy has **4** pencils.
How many more pencils does Adrianna have?

Difference

Adrianna has **6** more pencils than Tracy. If Tracy has **4** pencils, how many does Adrianna have?

Greater
amount

Tracy has **6** fewer pencils than Adrianna.
Adrianna has **10** pencils. How many pencils does Tracy have?

Lesser
amount



Total

“Are parts put together for a total?”

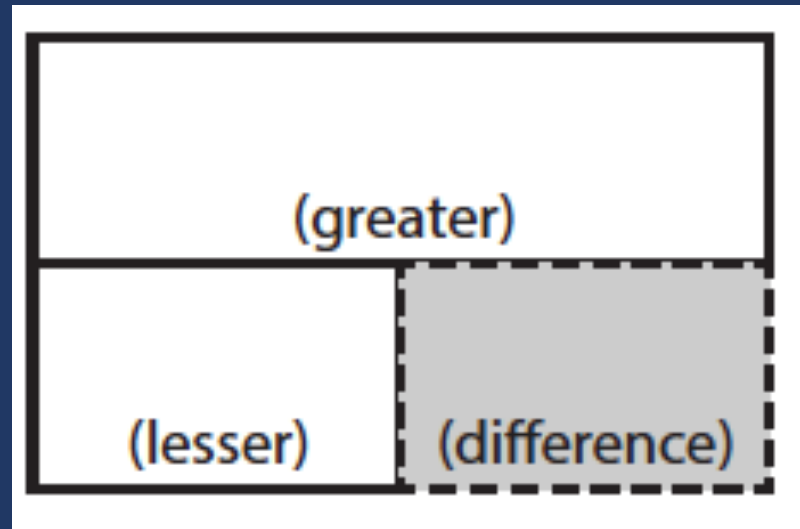
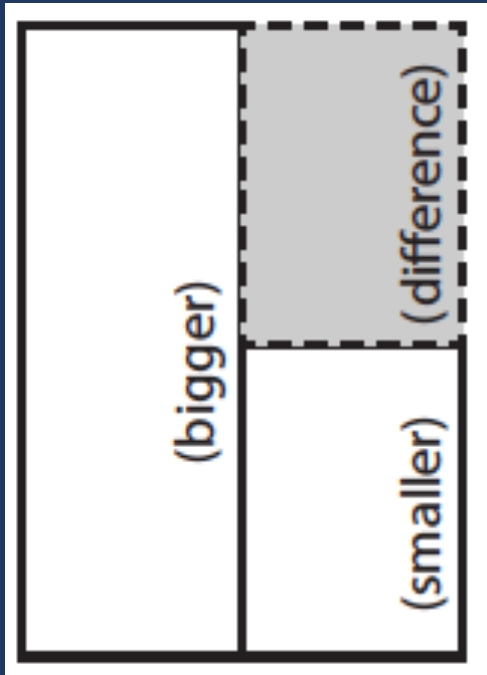
Difference

“Are amounts compared for a difference?”



Difference

$$G - L = D$$



Difference

Additive Word Problems

A.

Megan baked 38 sugar cookies and 24 chocolate chip cookies. Enter the total number of cookies Megan baked in all.

B.

In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.

Jana has 162 wooden beads and 95 glass beads. How many more wooden beads than glass beads does Jana have?



D.

The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?



Difference



What's an example
Difference problem?



Change

Join

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

End amount

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Change
amount

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

Start
amount



Change

Separate

An amount that increases or **decreases**

Samantha baked **20** cookies. Then, she ate **3** of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked **20** cookies. Then, she ate some of the cookies. Now, she has **17** cookies. How many cookies did Samantha eat?

Change
amount

Samantha baked some cookies. She ate **3** of the cookies and has **17** cookies left. How many cookies did Samantha bake?

Start
amount



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

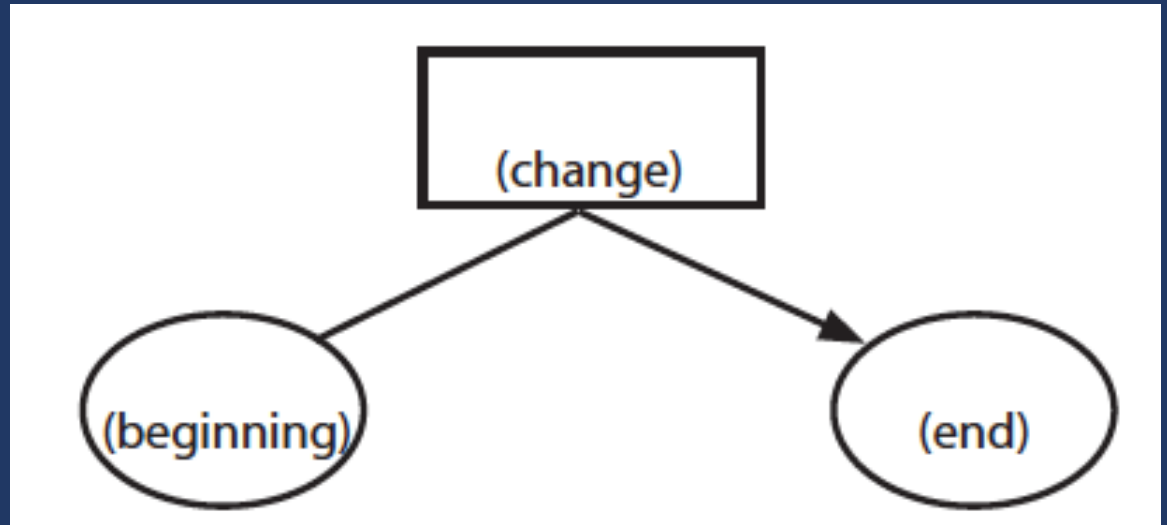
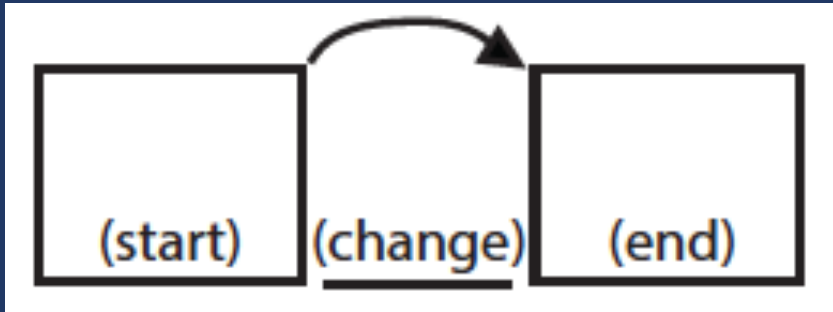
Change

“Does an amount increase or decrease?”



Change

$$ST + / - C = E$$



Change

Additive Word Problems

E.

A plant was $3\frac{3}{4}$ inches tall at the beginning of June. By the end of July, the plant was $9\frac{1}{8}$ inches tall. How many inches did the plant grow in 2 months?



F.

Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?



G.

Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

H.

Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?



Change



What's an example Change problem?



Additive Word Problems

E.

A plant was $3\frac{3}{4}$ inches tall at the beginning of June. By the end of July, the plant was $9\frac{1}{8}$ inches tall. How many inches did the plant grow in 2 months?

F.

Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?

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Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?



H.

Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?



Total



Which schema?

G.

Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

$$P1 + P2 + P3 + P4 = T$$



Change



Which schema?

H.

Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?

$$ST - C + C = E$$



Teach word-problem schemas

Total

Difference

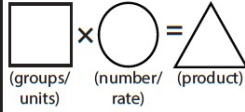
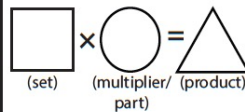
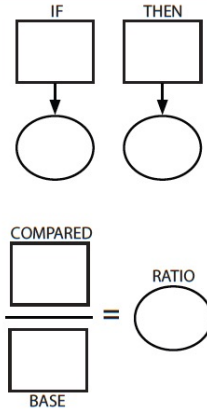
Change

Equal Groups

Comparison

Ratios/Proportions



Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		<i>Product unknown:</i> Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	<i>Groups unknown:</i> Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	<i>Number unknown:</i> Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	<i>With rate:</i> Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		<i>Product unknown:</i> Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	<i>Set unknown:</i> Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	<i>Times unknown:</i> Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	<i>With fraction:</i> Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions (Percentages; Unit Rate) Relationships among quantities Ratio		<i>Subject unknown:</i> Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	<i>Object unknown:</i> Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?	<i>Ratio unknown:</i> Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	<i>With percentage:</i> Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly? <i>With unit rate:</i> Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?

Material collected from: Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.



Equal Groups

Array
Vary

Groups multiplied by **number in each group**
for a **product**

Toni has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Toni have altogether?

Product

Toni has **24** crayons. They want to place them equally into **2** boxes. How many crayons will Toni place in each box?

Number in
each group

Toni has **24** crayons. They put them into boxes with **12** crayons each. How many boxes did Toni use?

Groups



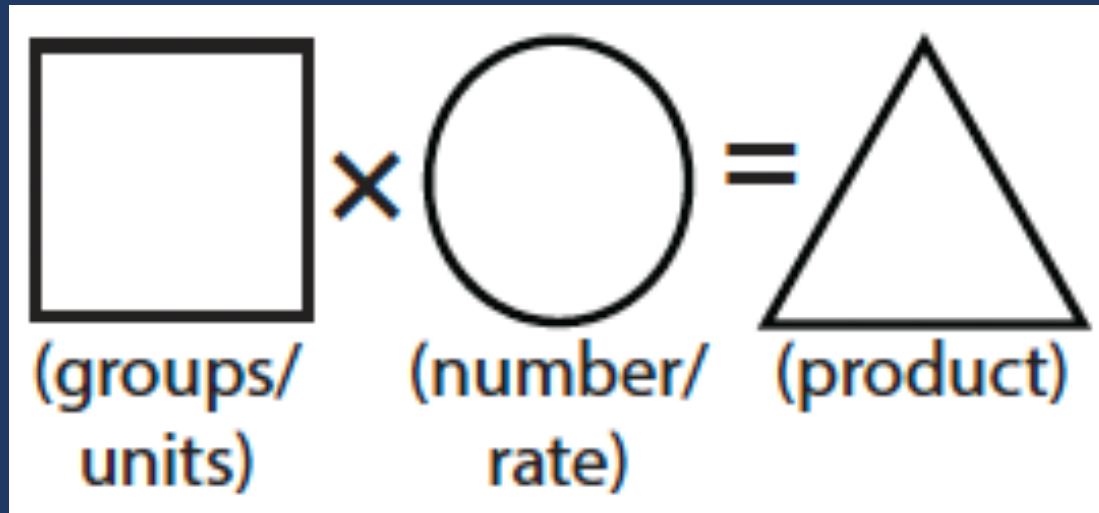
Equal Groups

“Are there groups with an equal number in each group?”



Equal Groups

$$GR \times N = P$$



Equal Groups

Multiplicative Word Problems

A.

Ms. Thompson sold 6 cartons of cherries at the Farmers' Market. Each carton holds 25 cherries. How many cherries did she sell?



B.

Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light bulbs did Jane buy?



C.

Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?

D.

Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?



Equal Groups



What's an example Equal Groups problem?



Comparison

Set multiplied by a number of **times**
for a **product**

Brooke ran **6** minutes. Shaleeni ran **4** times longer than Brooke. How many minutes did Shaleeni run?

Set

Number of
times

Product



Equal Groups

“Are there groups with an equal number in each group?”

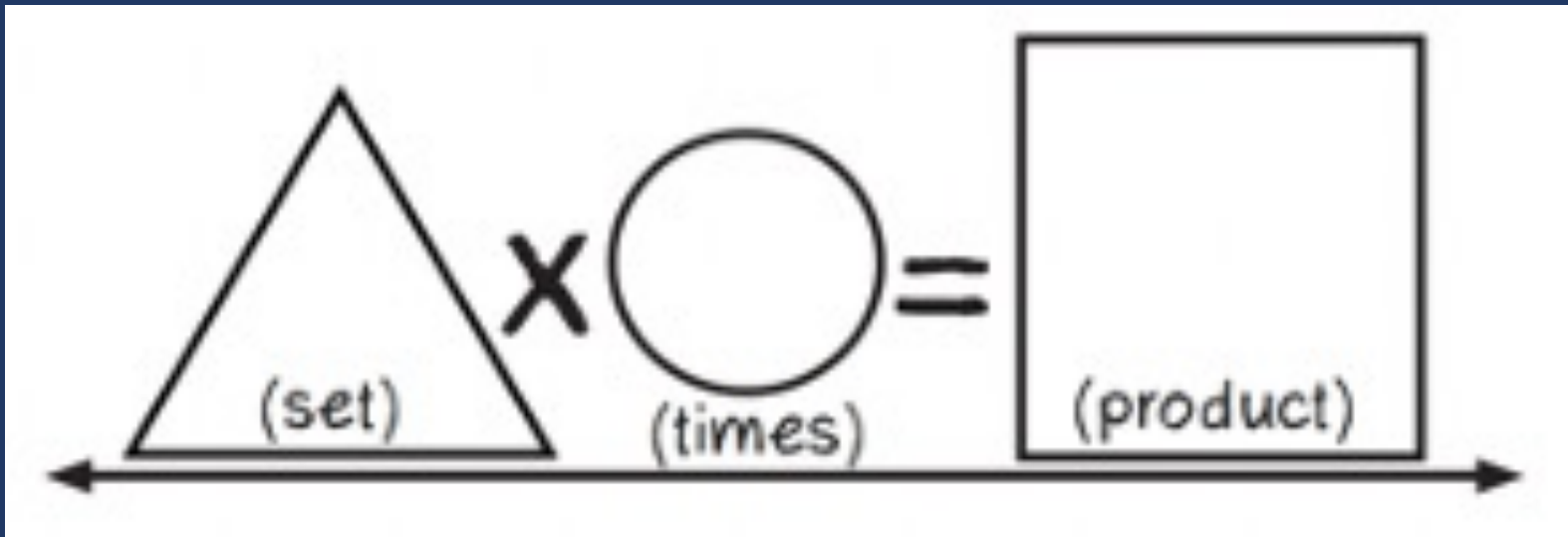
Comparison

“Is a set compared a number of times?”



Comparison

$$S \times T = P$$



Comparison

Multiplicative Word Problems

A.

Ms. Thompson sold 6 cartons of cherries at the Farmers' Market. Each carton holds 25 cherries. How many cherries did she sell?

B.

Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light bulbs did Jane buy?

C.

Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?



D.

Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?



Comparison



What's an example
Comparison problem?



Ratios/Proportions

Description of **relationships** among quantities

Emma typed **56** words in **2** minutes. At this rate, how many words could Emma type in **7** minutes?

Melissa baked cookies and brownies. The ratio of cookies to brownies was **3:5**. If she baked **25** brownies, how many cookies did she bake?

Equal Groups

“Are there groups with an equal number in each group?”

Comparison

“Is a set compared a number of times?”

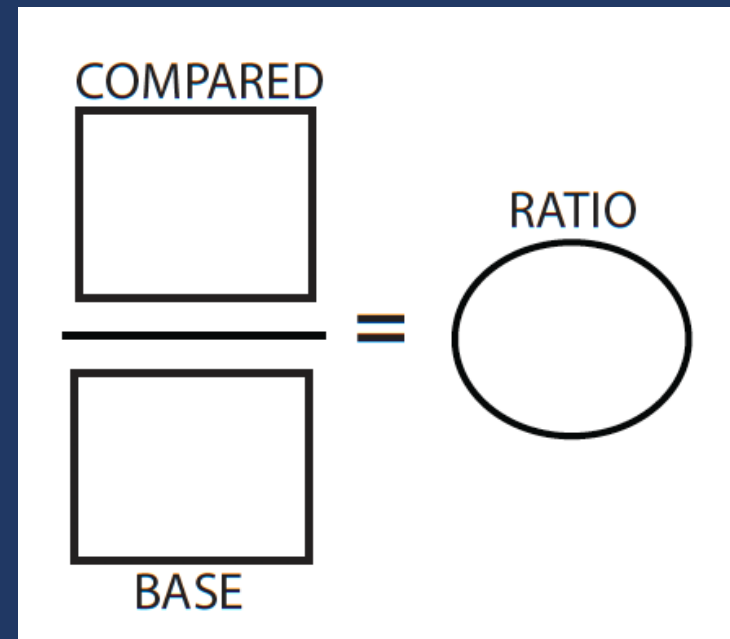
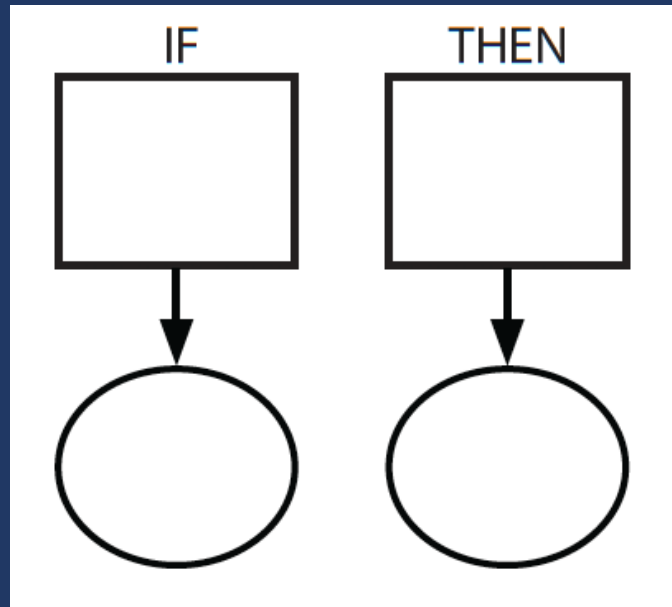
Ratios/Proportions

“Are there relationships among quantities – if this, then this?”



Ratios/Proportions

Description of relationships among quantities



Xin et al. (2005)



Ratios/Proportions

Multiplicative Word Problems

E.
The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?



F.
An airplane's altitude changed -378 feet over 7 minutes. What was the mean change of altitude in feet per minute?



G.
Sara buys a sweater at a department store. The sweater costs \$30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.



H.
Sam's two new aquariums each hold exactly 200 gallons of water. One aquarium will hold small fish and the other will hold large fish. Now he needs new fish for his aquarium. He will buy 5 small fish for every 10 gallons of water in the aquarium. He will buy 8 large fish for every 40 gallons of water in the aquarium. What is the total number of fish Sam will have? What will be the ratio of Sam's small fish to large fish?



Ratios/Proportions



What's an example
Ratio/Proportion problem?



Teach word-problem schemas

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions





Pirate Math Equation Quest

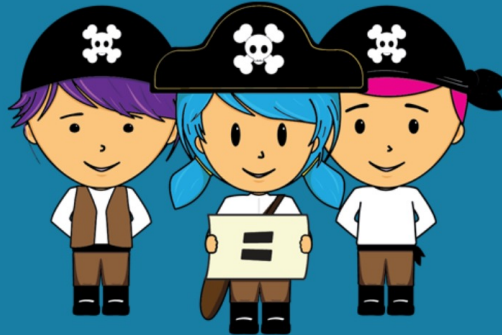
[About](#)[Research](#)[Individual](#)[Small Group](#)[STAAR](#)[Videos](#)

Welcome to Pirate Math Equation Quest!

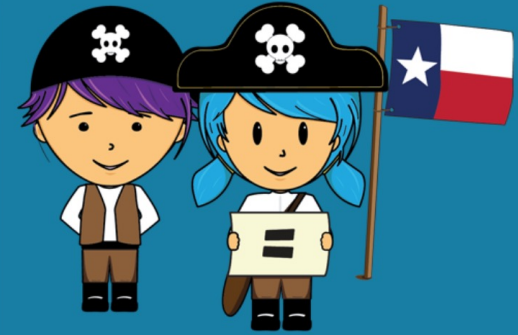
Individual Word-Problem Intervention



Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



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Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

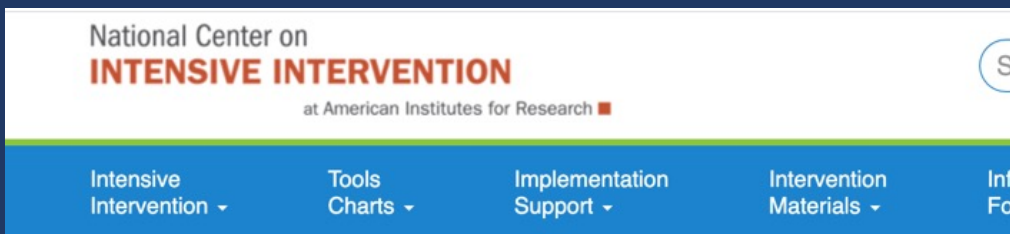
Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction





MODULE 4: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL DELIVERY



MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES

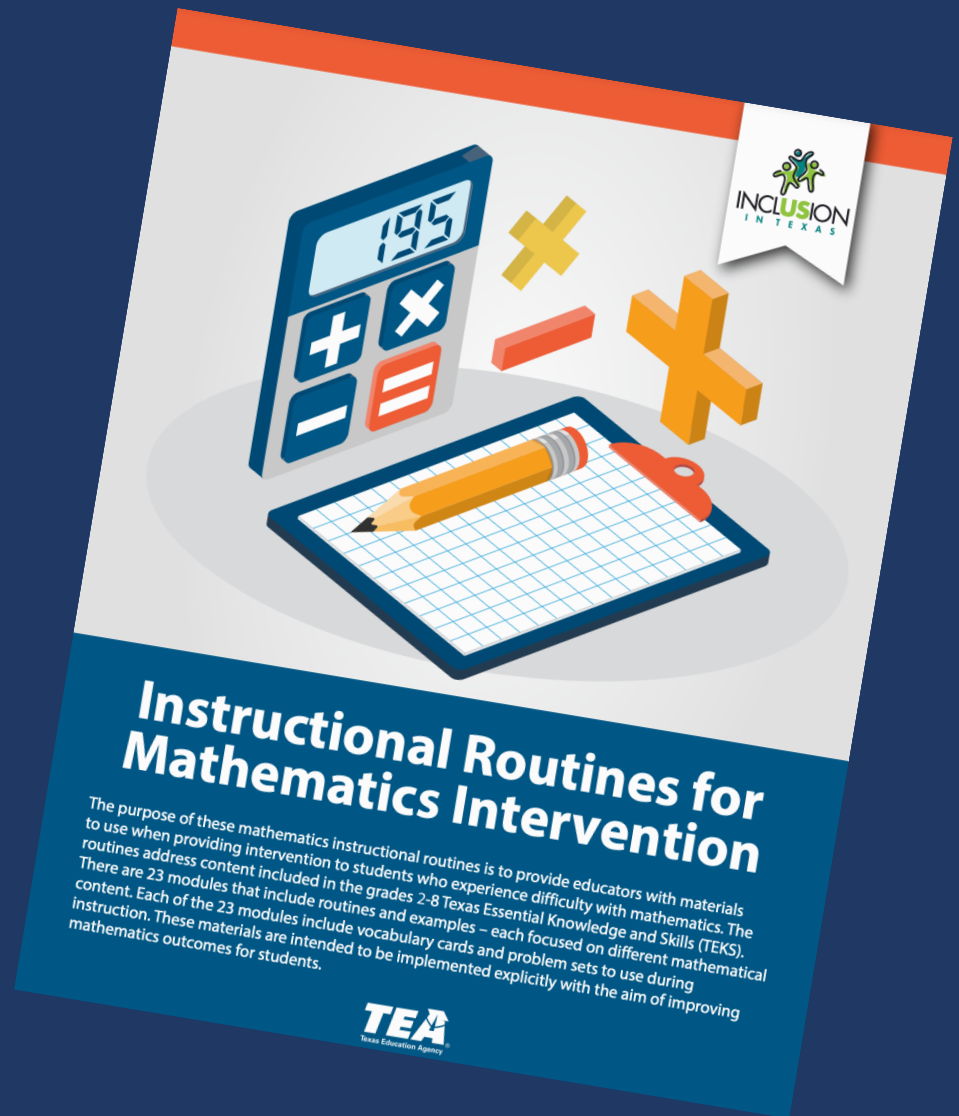
Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.





https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859



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