Core Concept

Solving Quadratic Equations

By graphing Find the x-intercepts of the related function $y = ax^2 + bx + c$.

Using square roots Write the equation in the form $u^2 = d$, where u is an algebraic expression, and solve by taking the square root of each side.

By factoring Write the polynomial equation $ax^2 + bx + c = 0$ in factored form and solve using the Zero-Product Property.

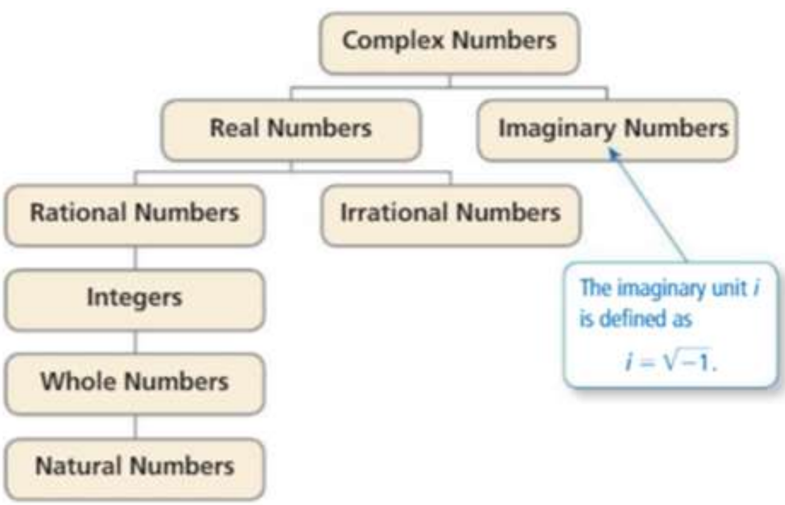
When the left side of $ax^2 + bx + c = 0$ is factorable, you can solve the equation using the *Zero-Product Property*.

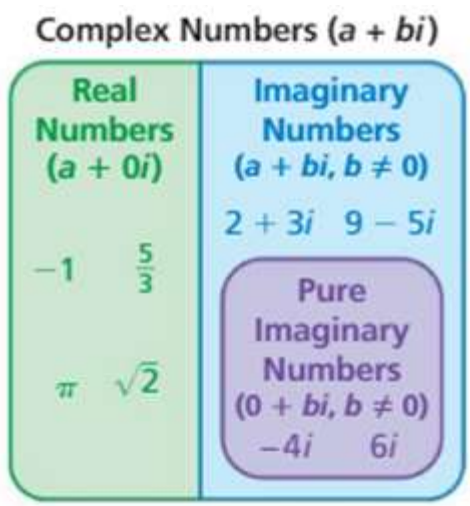
Core Concept

Zero-Product Property

Words If the product of two expressions is zero, then one or both of the expressions equal zero.

Algebra If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.





Core Concept

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.	$\sqrt{-3} = i\sqrt{3}$
2. By the first property, it follows that $(i\sqrt{r})^2 = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$

Core Concept

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

Square-Root Property

WORDS	NUMBERS	ALGEBRA
To solve a quadratic equation, you can take the square root of both sides. Be sure to consider the positive and negative square roots.	$x^2 = 15$ $\sqrt{x^2} = \pm\sqrt{15}$ $x = \pm\sqrt{15}$	If $x^2 = a$ and a is a nonnegative real number, then $x = \pm\sqrt{a}$.

Solving Quadratic Equations $ax^2 + bx + c = 0$ by Completing the Square

1. Collect variable terms on one side of the equation and constants on the other.
2. As needed, divide both sides by a to make the coefficient of the x^2-term 1.
3. Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
4. Factor the variable expression as a perfect square.
5. Take the square root of both sides of the equation.
6. Solve for the values of the variable.

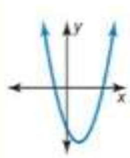
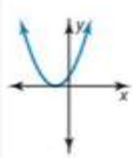
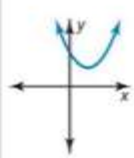
The Quadratic Formula

If $ax^2 + bx + c = 0$ $(a \neq 0)$, then the solutions, or roots, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Core Concept

Analyzing the Discriminant of $ax^2 + bx + c = 0$

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$	 Two x-intercepts	 One x-intercept	 No x-intercept

Core Concept

Graphing a Quadratic Inequality in Two Variables

To graph a quadratic inequality in one of the forms above, follow these steps.

Step 1 Graph the parabola with the equation $y = ax^2 + bx + c$. Make the parabola *dashed* for inequalities with $<$ or $>$ and *solid* for inequalities with $\leq$ or $\geq$.

Step 2 Test a point (x, y) inside the parabola to determine whether the point is a solution of the inequality.

Step 3 Shade the region inside the parabola if the point from Step 2 is a solution. Shade the region outside the parabola if it is not a solution.

Concept Summary

Methods for Solving Quadratic Equations

Method	When to Use
Graphing	Use when approximate solutions are adequate.
Using square roots	Use when solving an equation that can be written in the form $u^2 = d$, where u is an algebraic expression.
Factoring	Use when a quadratic equation can be factored easily.
Completing the square	Can be used for *any* quadratic equation $ax^2 + bx + c = 0$ but is simplest to apply when $a = 1$ and b is an even number.
Quadratic Formula	Can be used for *any* quadratic equation.