Fundamentals of Liquefaction Under Static and Cyclic Loadings

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Outline:

1. Preamble
2. Basic behavior of cohesionless soils under drained and undrained loadings
3. Field tests and correlations ...
4. And back-calculation of failures
5. Analysis issues
6. The probability of failure (and triggering mechanisms)
7. Conclusions

1. Preamble

In order to put the topic of this lecture in perspective, here are the conclusions of Prof. Norbert Morgenstern in his 6^{th} De Mello Lecture* regarding the causes of failure of tailngs dams:

* http://www.victorfbdemello.com.br/arquivos/Lectures/6TH_VICTOR_DE_MELLO_LECTURE.pdf

“From a technical perspective, it is of interest to note that inadequate understanding of undrained failure mechanisms leading to static liquefaction with extreme consequences is a factor in about 50% of the cases.

Inadequacies in site characterization, both geological and geotechnical, is a factor in about 40% of the cases.

Regulatory practice, considered appropriate for its time and place, did not prevent these incidents.

However, the most important finding is that the dominant cause of these failures arises from deficiencies in engineering practice associated with the spectrum of activities embraced by design,construction, quality control, quality assurance, and related matters. This is a very disconcerting finding.”

This presentation covers only the first of Prof. Morgenstern's points, but his second and third points are equally important and should not be forgotten. For example, a design that assumes that the intended drainage measures will work, is not complete unless measures are in place to ensure their proper construction and long-term operation.

Underlying themes:

- Soils are interesting because they are variable and behave differently with different fines content, different confining / consolidation pressures, and under different styles and rates of loading

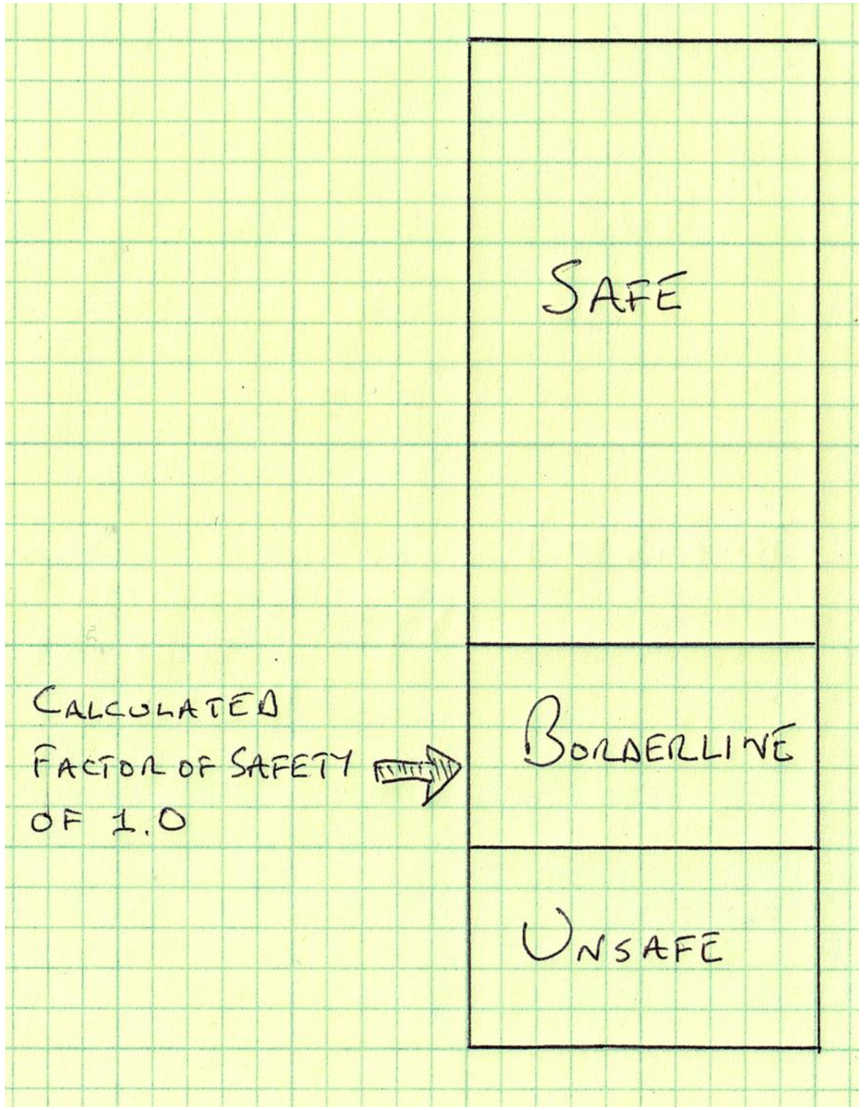
- It is difficult, if not impossible, to model every last detail in a real-world problem and at least some judgment is required

- It would be nice to move away from factors of safety and talk about probabilities of failure, but this just multiplies the challenges

Speaker’s background – just so that you know who I am.

- B.E. in Civil Engineering, University of Sydney, 1963
- Worked for 5 years for Australian Government – principally on investigations, design and construction of Corin Dam – see next slide – on which I supervised site investigations and then served as assistant resident engineer in charge of embankment construction
- Ph.D., University of California, 1973 – advisor H. Bolton Seed
- 4 years with Dames & Moore based in San Francisco – dams, nuclear facilities, ports and offshore structures
- 42 years individual consultant on a variety of dams, tailings dams, levees, ports, bridges, nuclear facilities and other structures

On the basis of this experience … essentially all earth structures fall into one of the three categories shown on the next slide. Those that are obviously safe because they have been designed and constructed with care, those that are unsafe because they have failed or are obviously failing, and the ones in the middle that require careful investigations and analysis …



Relative Safety

Three options if structure is BORDERLINE:

1. Improve so that structure is safe.
2. Monitor and maintain.
3. Collect more data and do an improved analysis to clarify.

Brief history of liquefaction studies

- Term originally referred to static liquefaction, or flow slides - Ft Peck Dam, submarine landslides, etc. (Casagrande)
- Seed and Lee (1966) applied the same term to liquefaction caused by cyclic loading, i.e. earthquakes
- The Harvard school (Casagrande et al.) did not like this usage and wanted to call the phenomenon “cyclic mobility”, but (little known fact) Casagrande made peace with Seed when they worked together on the Teton Dam failure
- Canadians (Nerlerk, CANLEX), and others, have done useful recent work on static liquefaction, i.e. flow slides, especially relative to tailings dams

The Mechanism of Liquefaction 1

- Requires cohesionless soils – sands, silts, and non-cohesive clay sized particles – clayey soils can also generate flow slides as a result of the collapse of a flocculent structure – “quick clays” – sometimes said to be liquefaction, but this is really a different mechanism

- Requires fully saturated undrained conditions – further enabled by existing excess pore pressures

- Requires a loading condition that will cause a decrease in volume in a dry soil

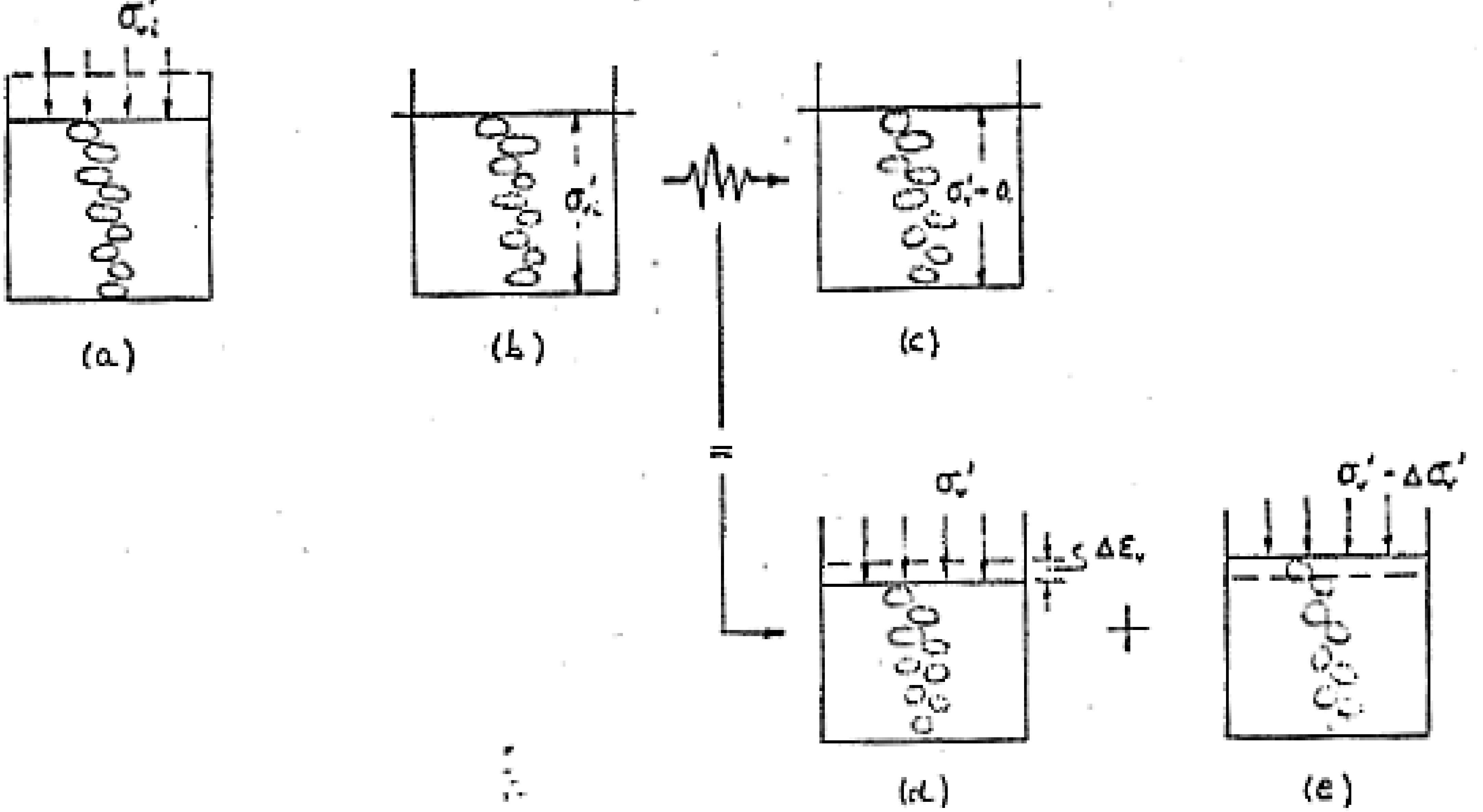
The Mechanism of Liquefaction 2

- This loading can be a “static” or cyclic shear loading in a loose (contractive) soil, [triggered by additional loading, reduction in lateral stresses, excavation or erosion at the toe of a slope, a rise in pore pressures, rapid drawdown, vibrations or a jolt]

- Or a “cyclic” shear loading in loose to medium dense soils. Cyclic loading of medium dense to dense soils may cause “lateral spreading” or limited deformations of embankments, but will normally not trigger flow slides

- Because the volume cannot change with fully saturated undrained conditions, this results in relaxation of stresses between soil particles and load transfer to the water

The following slide, taken from Pyke (1973), illustrates the mechanism that is involved. It is not that the soil particles apply pressure to the water as they try to contract, but that the structure of the soil particles relaxes, and the applied loads are transferred to the relatively incompressible water.



From Pyke (1973) and Martin, Finn and Seed (1975)

$$\Delta u = -\Delta\sigma'_v = E_{unload} \times \Delta\varepsilon_v$$

2. Basic behavior of cohesionless soils under drained and undrained loadings

Laboratory tests rarely if ever capture all the field conditions, but they are the best way to understand the basic behavior of soils …

Drained vs undrained loadings

Whether a loading is undrained depends on:

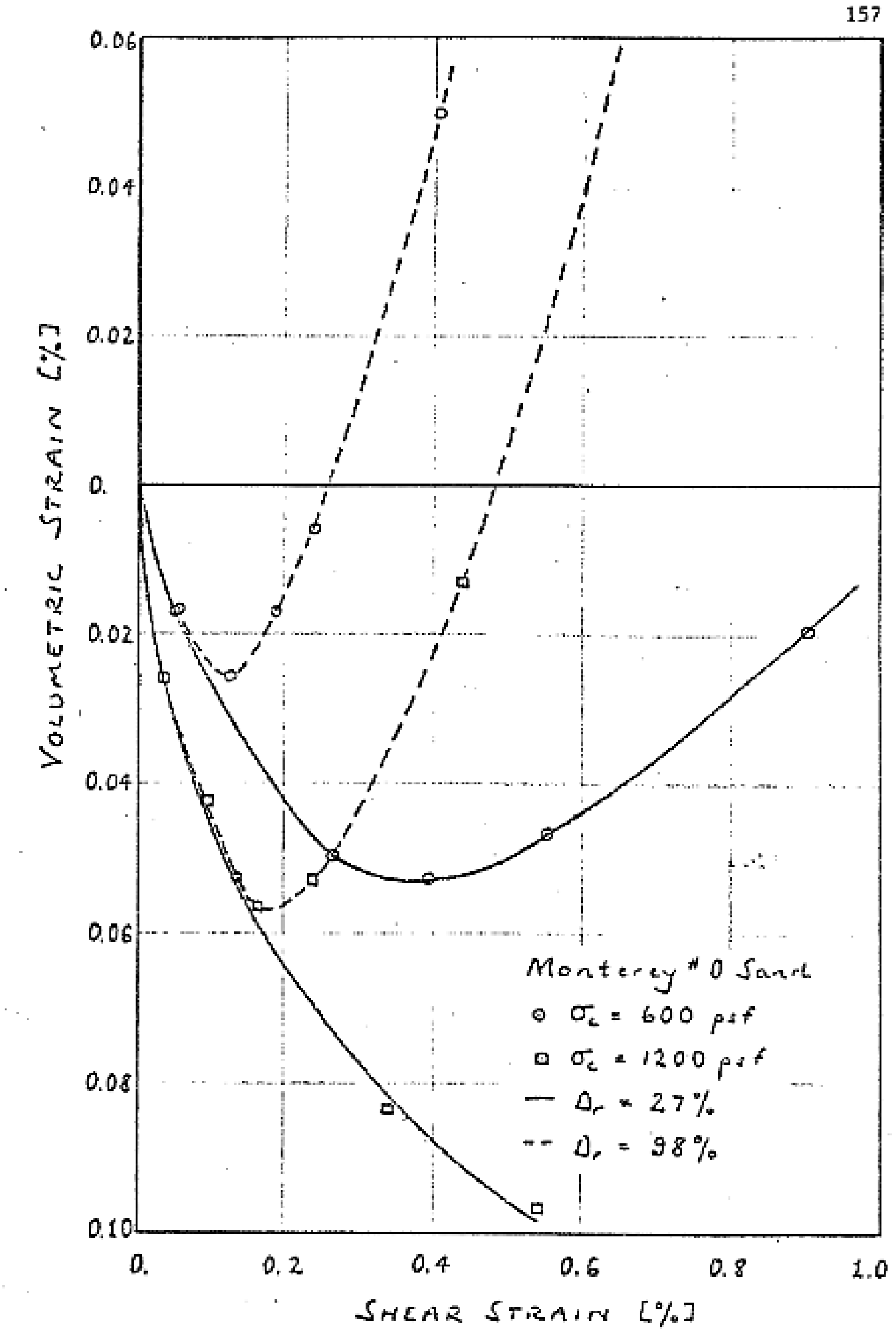
- Hydraulic conductivity
- Boundary conditions
- Presence of lens and stringers of less permeable material
- Rate of loading

Dry or drained loadings

We are ultimately interested in undrained loadings, but it helps to first understand the volume changes of sands under drained loadings. The next two slides, using data from Poul Lade's Ph.D. thesis, replotted in my Ph.D. thesis, shows how the relative density, the confining pressure and the style of loading all affect the volume changes created by shear loadings.

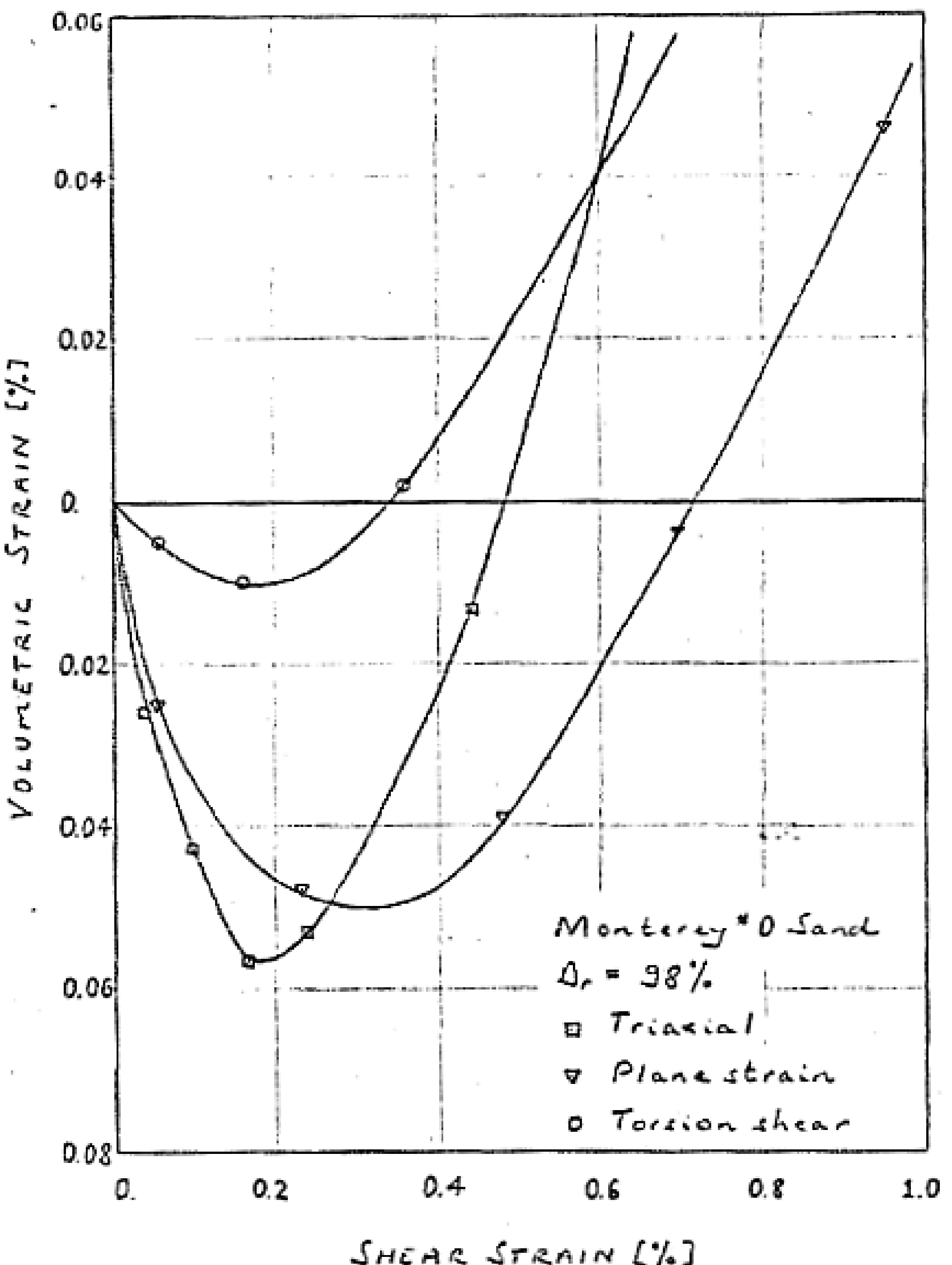
Monotonic triaxial tests from Lade (1972) showing the effect of relative density and confining pressures at relatively small strains.

Note that at the lower confining pressure, even the loose sand become dilatant after 0.5% shear strain.



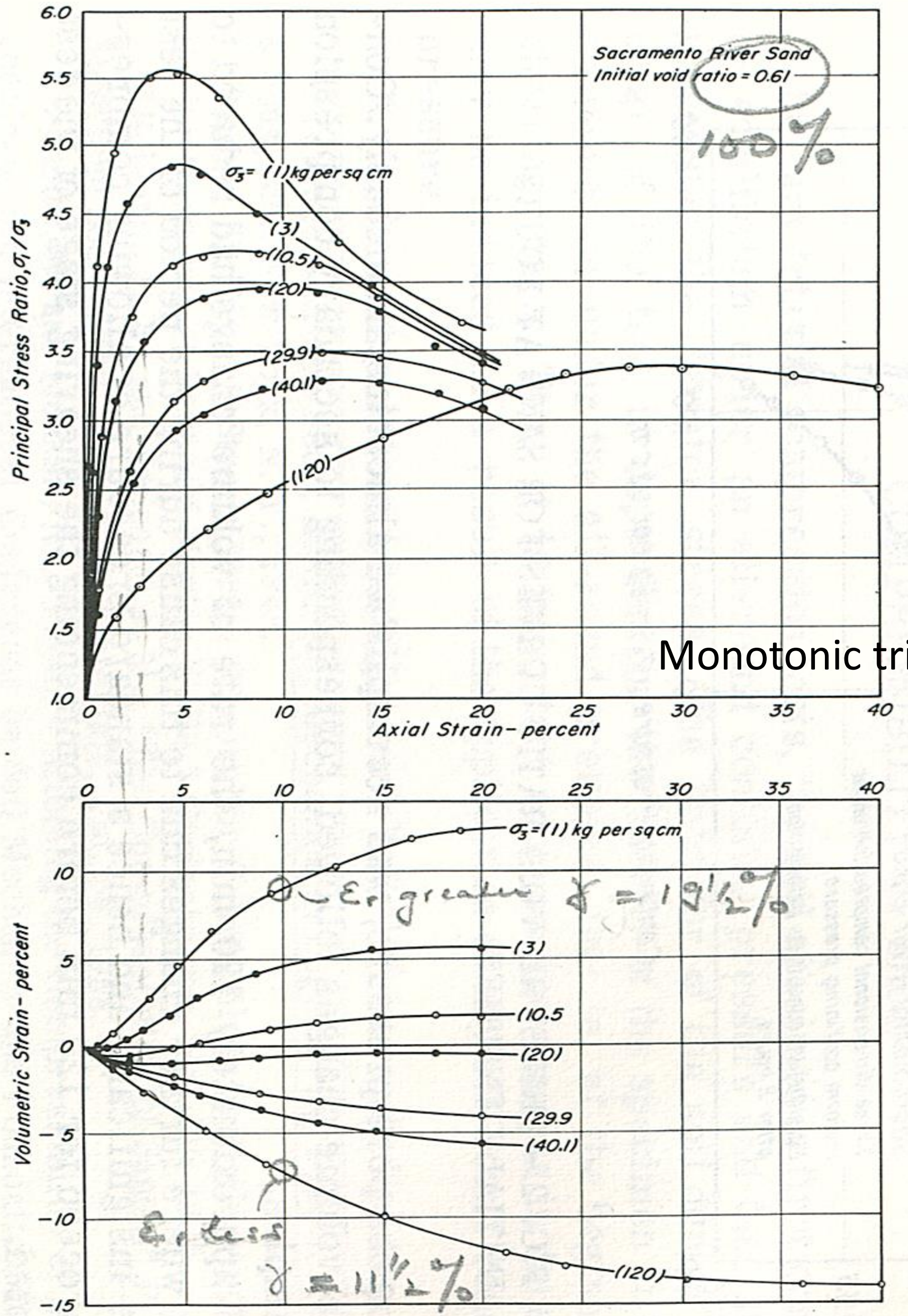
Monotonic triaxial, plane strain and torsion shear tests from Lade (1972).

Note the big effect of the style of loading on the volume change. So, any one laboratory test is unlikely to be the whole truth.

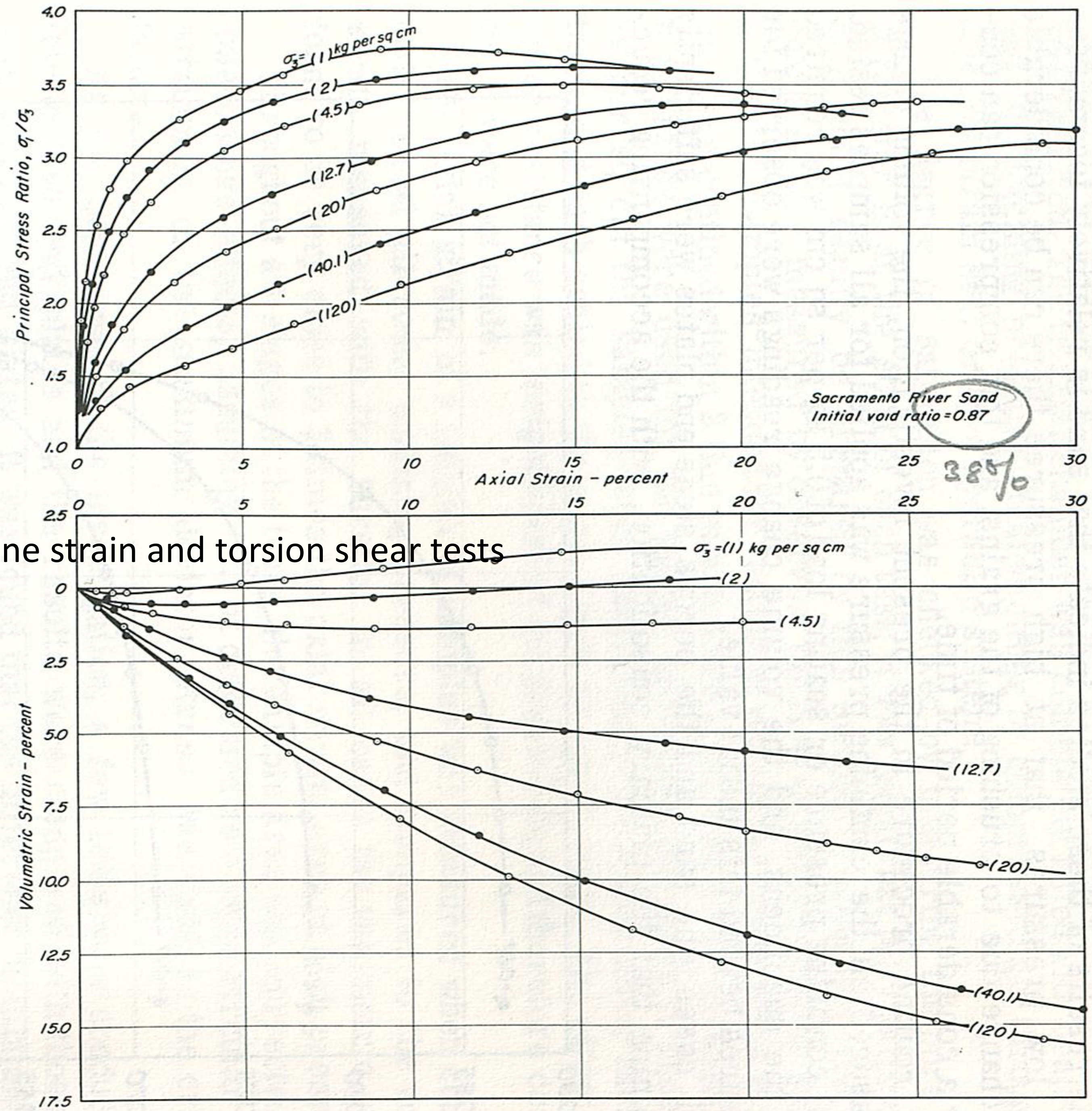


The next slide, taken from Lee and Seed (1967), shows the behavior at larger strains under drained loading in triaxial tests. The two panels on the left-hand side show results for very dense specimens and the two panels on the right-hand side show results for loose specimens.

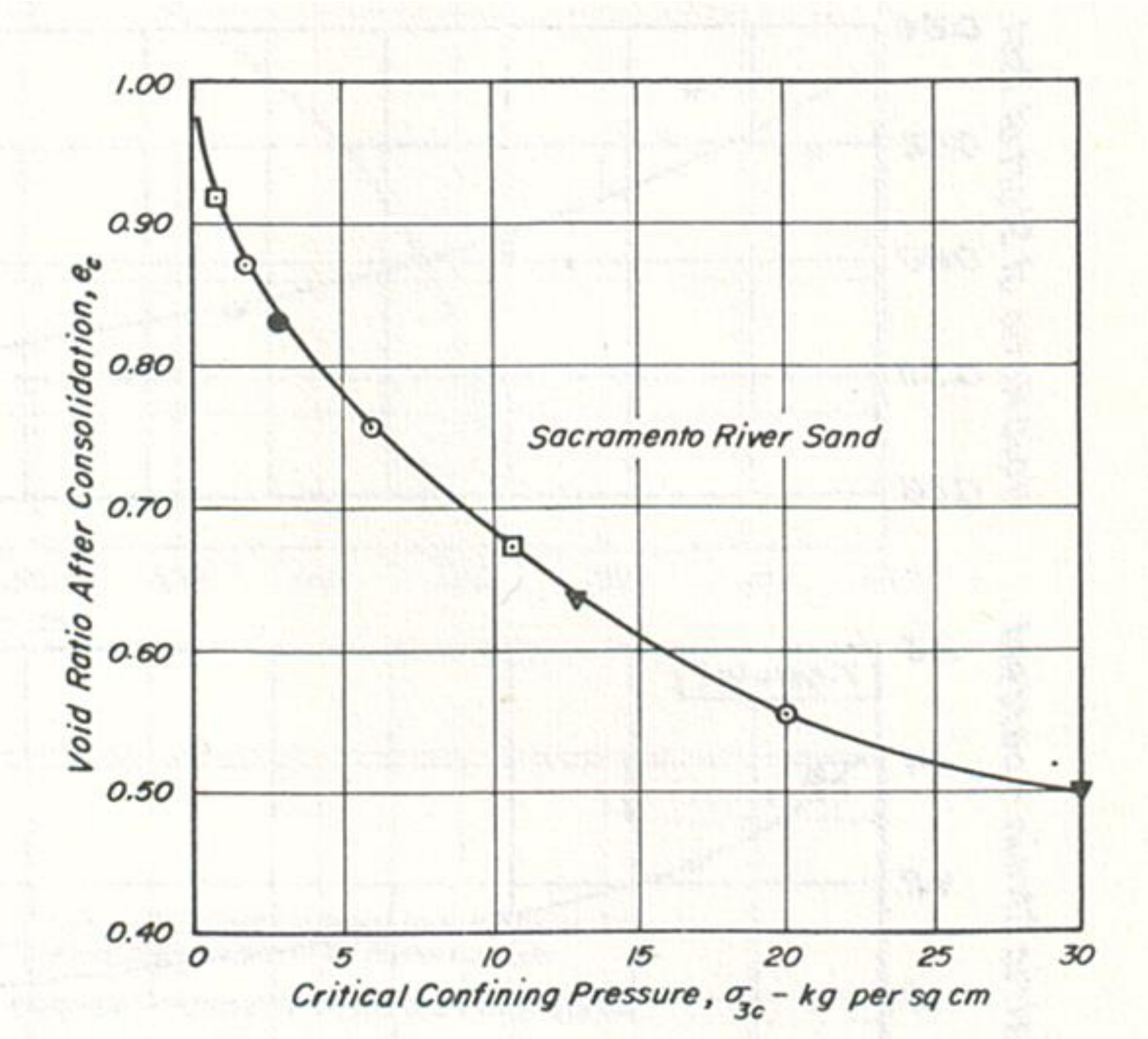
Note that at large strains, all specimens tend to converge to more or less the same strength for a given initial density or void ratio, regardless of the confining pressure.



Monotonic triaxial, plane strain and torsion shear tests



From data such as that shown on the previous slide, it is possible to construct a plot of the critical confining pressure, the confining pressure for which there will be no volume change at large strains, as a function of the initial void ratio.

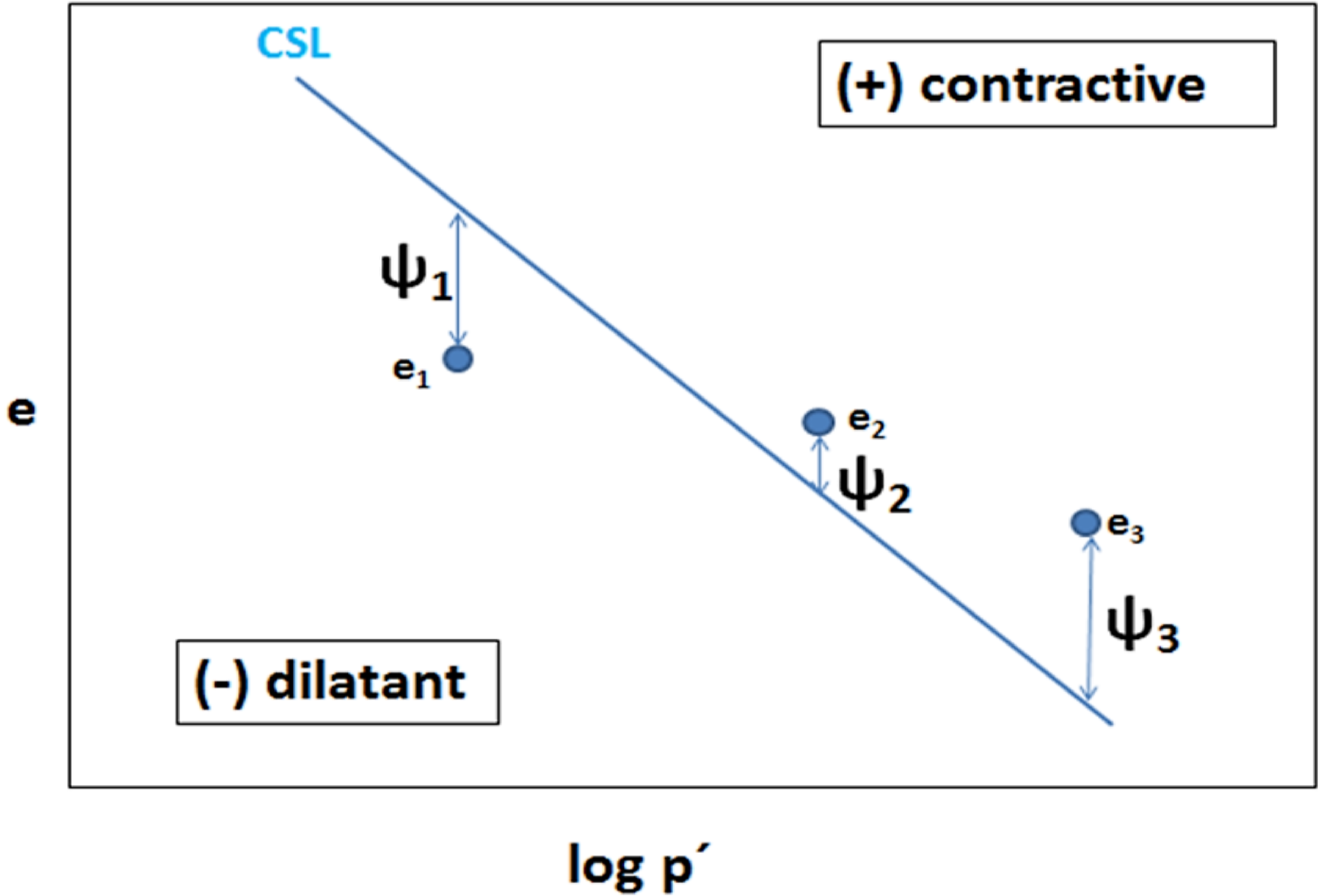


Such a curve is sometimes called the "critical state line", as shown on the following plot taken from the report on the investigation into the failure of the Fundao tailings dam http://fundaoinvestigation.com/.

The following plot also illustrates a "state parameter" defined as the difference between the initial void ratio and the critical void ratio at the initial confining pressure.

The critical state line and the state parameter are useful concepts for illustrating, for instance, the effect of confining pressure on whether a sand is dilatant or contractive. In this plot the same soil is shown to be contractive at high confining pressures and dilative at low confining pressures.

However, the critical state is a concept rather than a fundamental law of nature and, while it can be argued that the initial fabric of a sand is reset by shearing to large strains, the style of loading can still impact the location of the critical state line.


CSL
(+) contractive
(-) dilatant
ψ1
ψ2
ψ3
e1
e2
e3
e
log p´

Compaction of dry sands at all relative densities can also occur under cyclic loading and an extensive set of data on the compaction of one sand under both uni-directional and bi-directional shearing is given by Pyke (1973).

Additionally, if even a symmetrical cyclic loading is superimposed on an element that is subjected to an initial shear stress. that element will accumulate deformation in the direction of the initial shear stress - see Pyke (1979) – and this can cause excessive deformations.

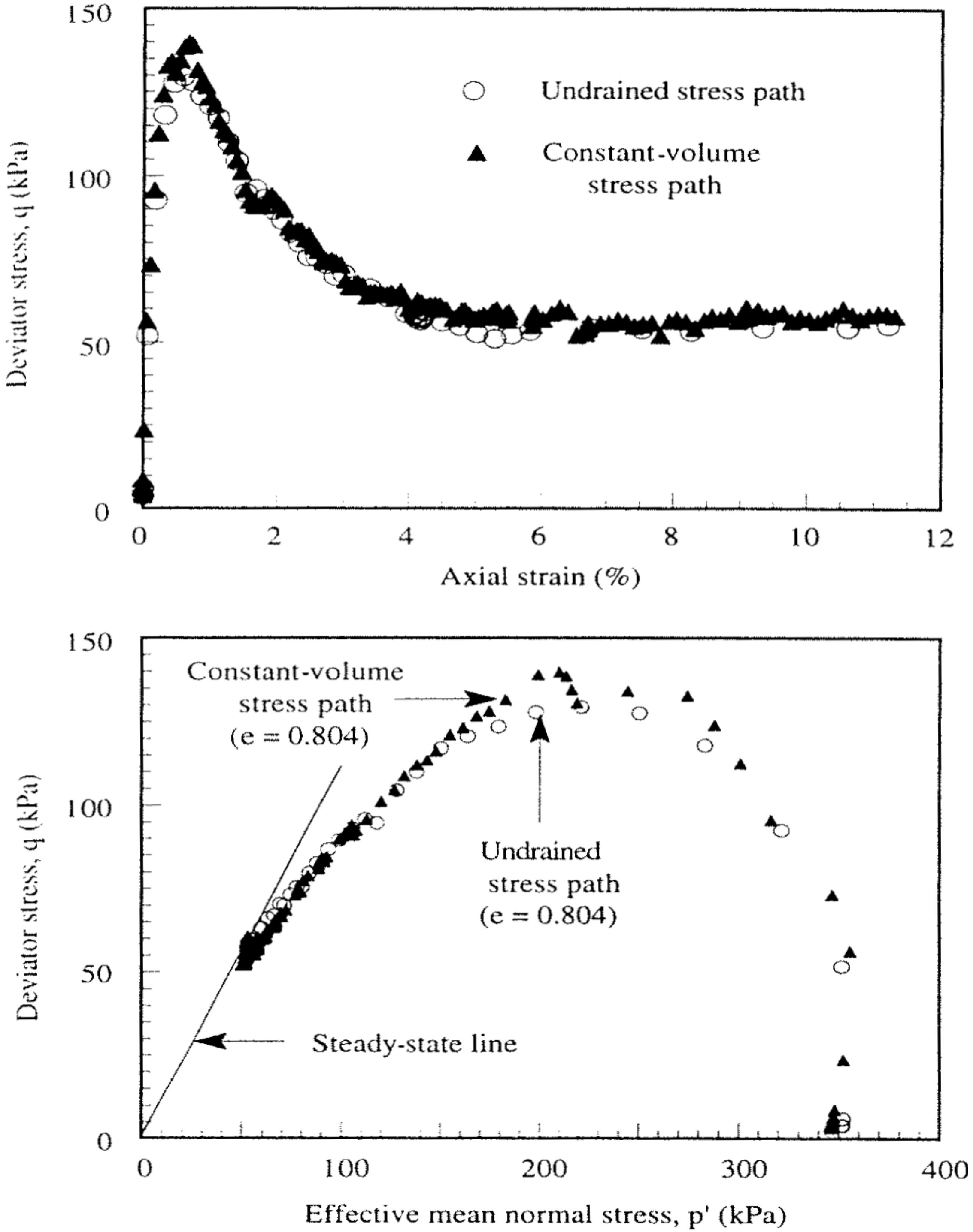
But, this presentation focusses more on the behavior of loose or contractive sands and the question of what kind of static loadings or relatively low vibrational or blast loadings can trigger a flow slide.

Undrained loadings

Even though a loose sand is ductile when loaded under drained conditions, because positive excess pore pressures are generated and the effective confining stresses decrease, loose sands exhibit brittle behavior under undrained loading conditions with a significant drop from a peak strength to a residual strength.

Again, a single soil can behave quite differently under different effective confining stresses. This cannot be emphasized enough!

The behavior of loose sands is illustrated in the following slide, taken from Sasitharan et al. (1993), and now showing the data in the lower part of the figure in effective stress space, rather than void ratio v. pressure space as before.



150
100
50
0
Deviator stress, q (kPa)
Undrained stress path
Constant-volume stress path
0 2 4 6 8 10 12
Axial strain (%)
150
100
50
0
Deviator stress, q (kPa)
Constant-volume stress path (e = 0.804)
Undrained stress path (e = 0.804)
Steady-state line
0 100 200 300 400
Effective mean normal stress, p' (kPa)

What is marked as the “steady state line” in this figure is the same as the CSL or critical state line shown in previous figure..

What can get confusing in all of this, is that we are talking about undrained strengths but, at least temporarily, are showing results in an effective stress space. This helps understand the mechanisms involved and is essential if one is going to conduct more sophisticated effective-stress analyses, but that requires accurately predicting the excess pore pressures under varying styles of loading, and so by the time we get to back-calculating undrained strengths from case histories of failure, we go back to directly talking about undrained shear strengths!

But to further complicate matters …

… there are:

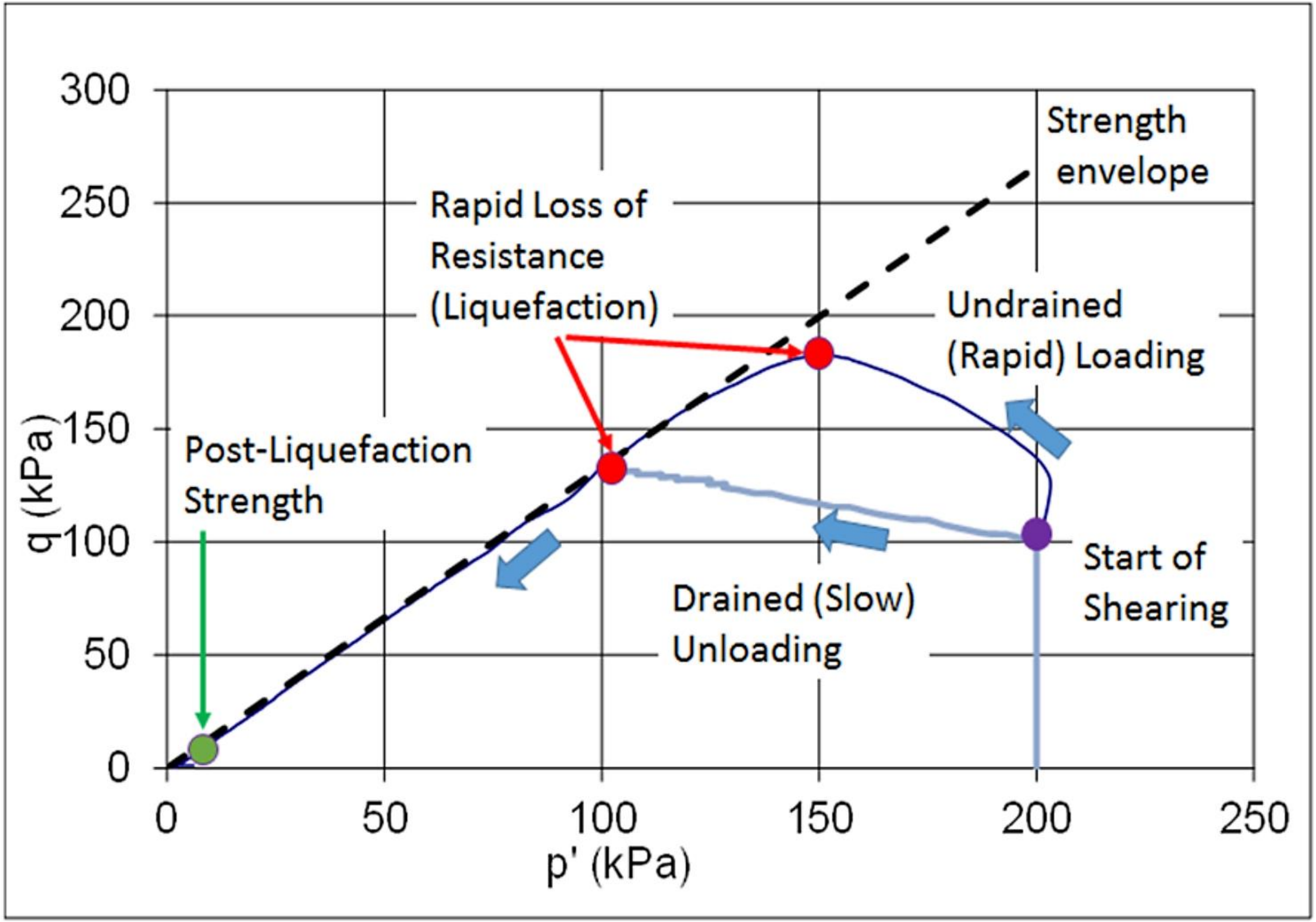
(1) some situations in which portions of the loading can be drained and other portions undrained;

(2) big differences that can result from the style of loading (as shown previously in the test results for dry sands)

This is illustrated in the following slide, also taken from the Fundao Investigation report.

The plot shows two stress paths leading to a “collapse” in the undrained shear strength.

One is a conventional monotonic loading, and the other is a drained loading, postulated in the case of Fundao to have been caused by the squeezing of thin layers or lenses of slimes, which in turn caused lateral spreading and a decrease in the horizontal confining stresses in the overlying sands.



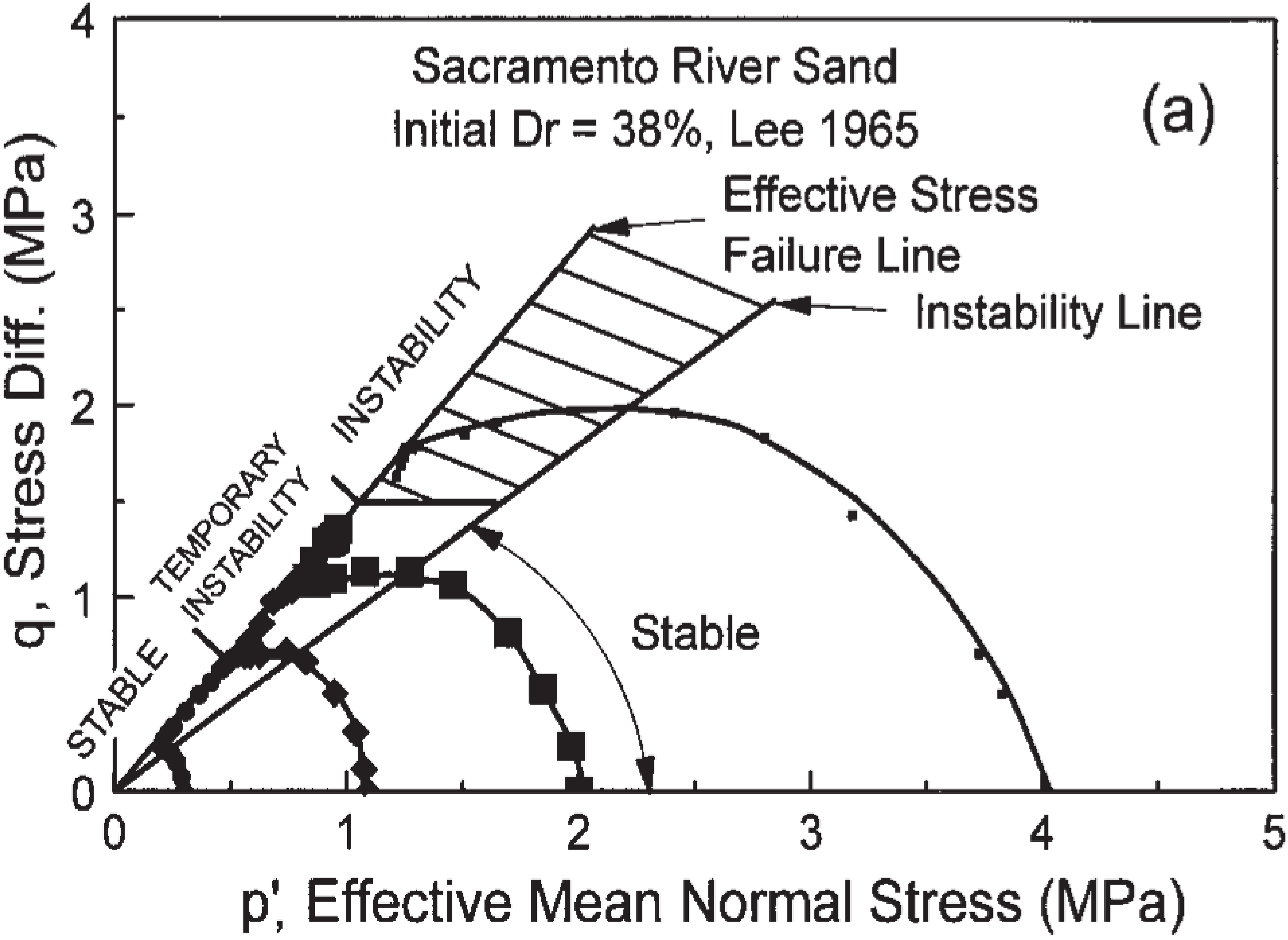
Strength envelope
Rapid Loss of Resistance (Liquefaction)
Undrained (Rapid) Loading
Post-Liquefaction Strength
Start of Shearing
Drained (Slow) Unloading
q (kPa)
p' (kPa)
0
50
100
150
200
250
300

Note that while this kind of loading is sometimes called an “extension loading”, its effect of the fabric of a loose sand is likely even worse than the classic “triaxial extension” loading in which the axial stress on the test specimen is reduced. In this case, the loading is simulated in the laboratory by increasing the back-pressure inside the specimen, thus reducing the effective lateral stress.

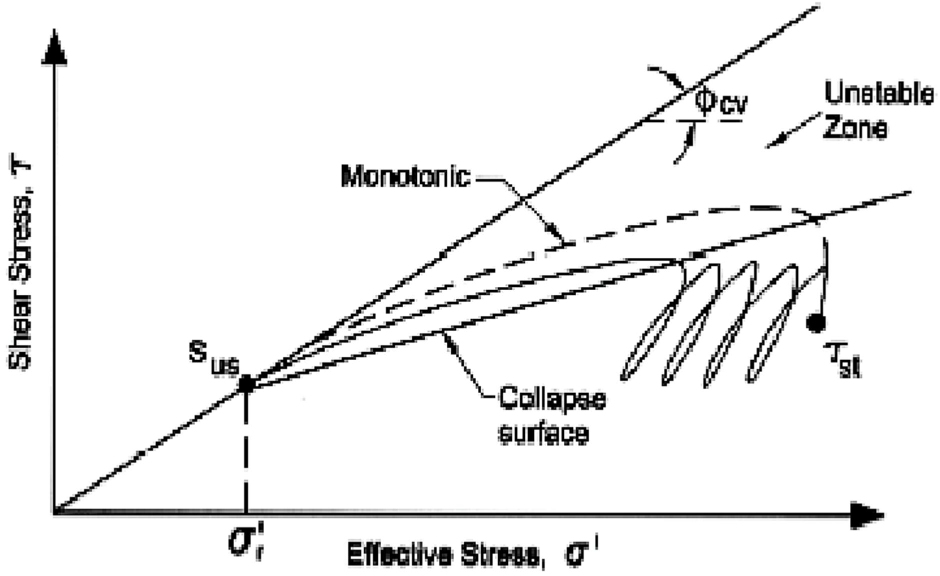
More generally, the mechanism of the undrained strengths of loose sands can be seen in the next slide, again using data from Seed and Lee (1967), as re-plotted by Yamamuro and Lade (1997).

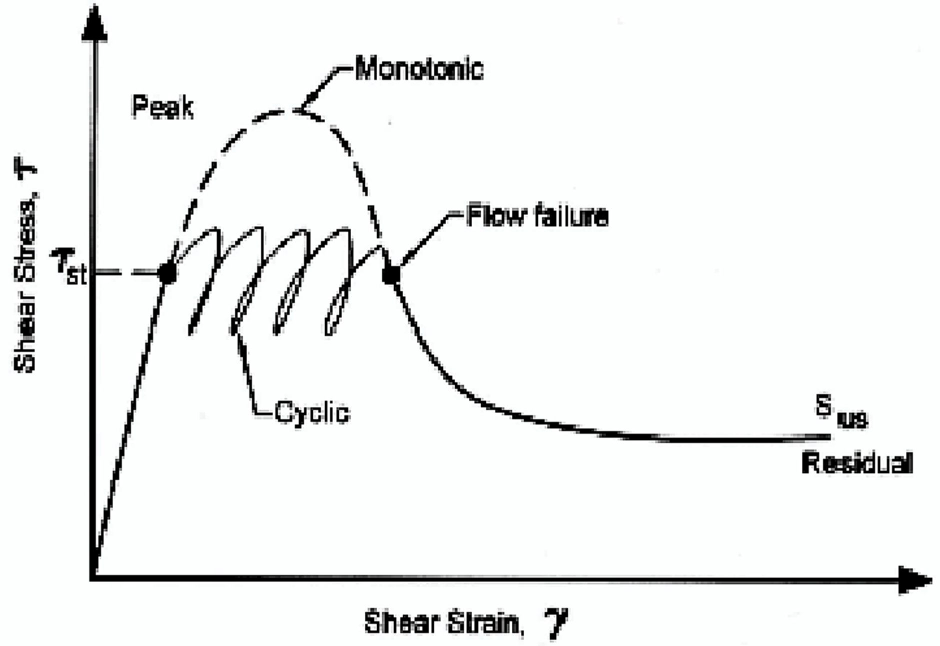
Results are shown for the same sand subjected to undrained triaxial tests starting at three different confining pressures. At the lowest confining pressure the sand is dilatant and is stable; at the middle confining pressures that sand initially tends to contract and is temporarily unstable, but at larger strains it tend to dilate and so negative excess pore pressures are generated; at higher confining pressures, the sand is contractive and potentially unstable.

The “instability line” in this plot is drawn through the origin, but it is commonly drawn to join the effective stress failure line at the residual strength.


Sacramento River Sand
Initial Dr = 38%, Lee 1965
(a)
Effective Stress
Failure Line
Instability Line
INSTABILITY
TEMPORARY
INSTABILITY
STABLE
Stable
q, Stress Diff. (MPa)
p', Effective Mean Normal Stress (MPa)
0
1
2
3
4
5

The next slide, taken from Davies, McRoberts and Martin (2002), shows conceptually how a drop to residual strength and potentially a flow slide, can also be triggered by cyclic loadings.

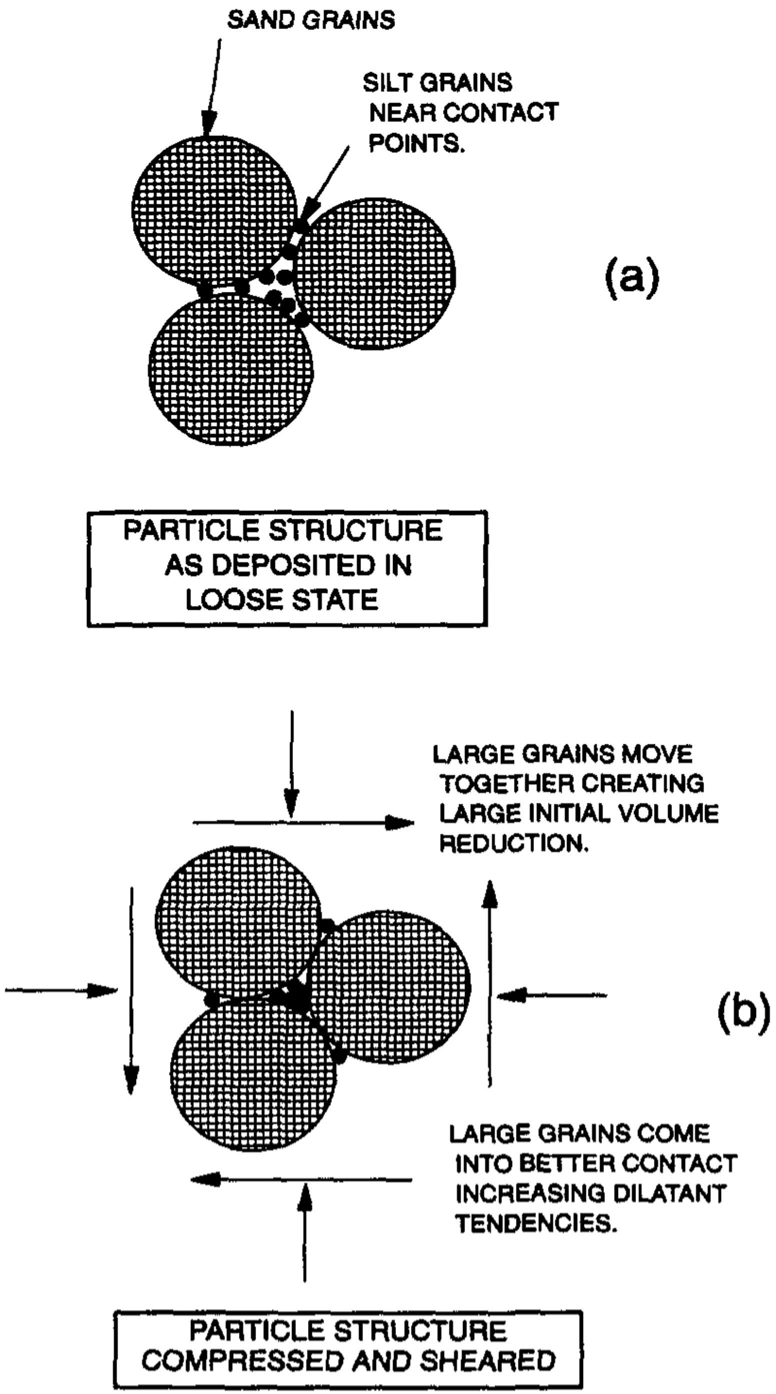

Shear Stress, τ
φcv
Unstable
Zone
Monotonic
Sus
τst
Collapse
surface
σ'r
Effective Stress, σ'
(a)

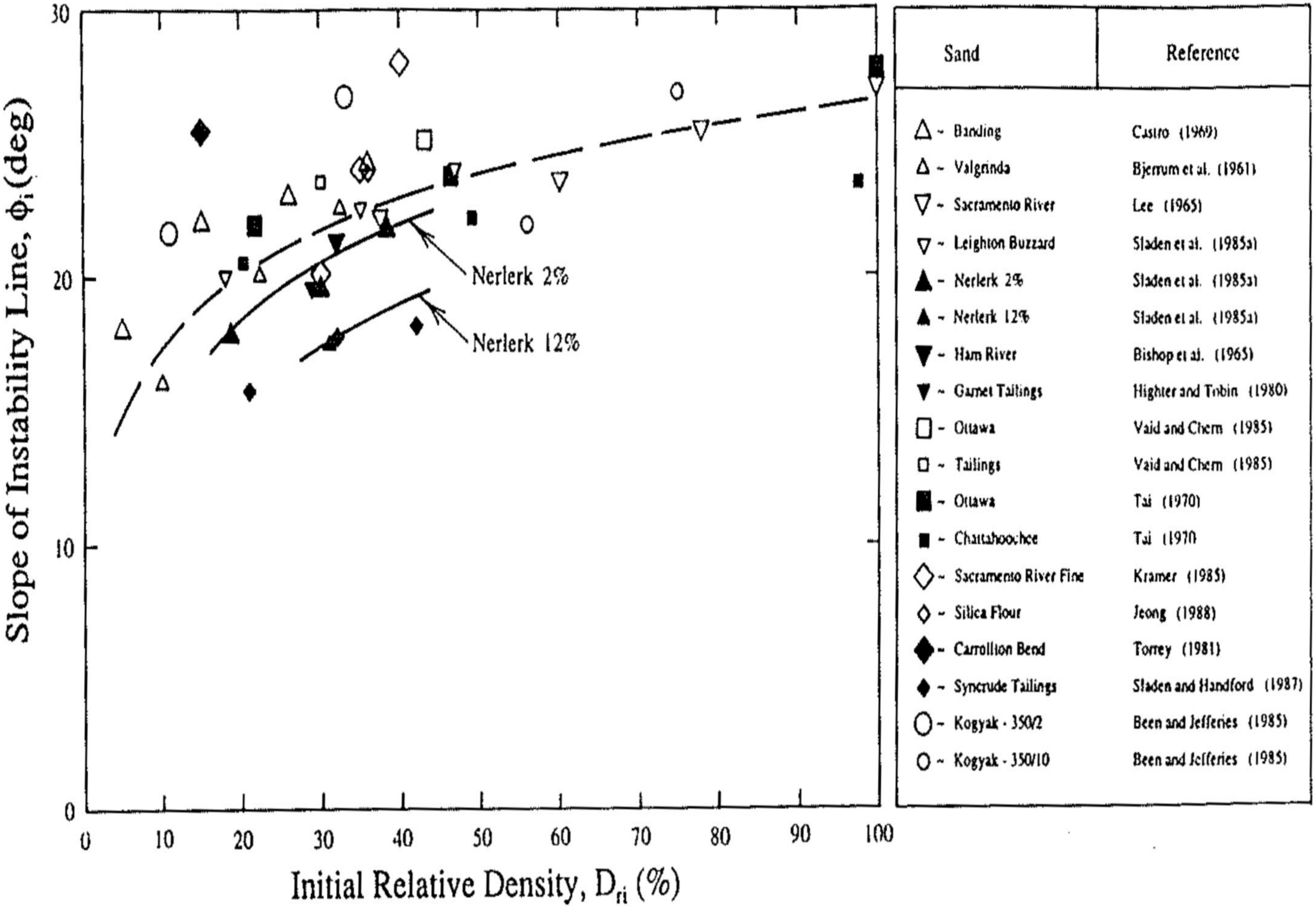

Monotonic
Peak
Shear Stress, τ
τst
Flow failure
Cyclic
Sus
Residual
Shear Strain, γ
(b)

Yet another factor is the presence of finer particles within a matrix of coarser particles.

The next two slides, taken from Lade (1993), show that if there are enough finer particles to interpose themselves between the larger particles, that can facilitate collapse of the fabric.

The second of these two slides shows how the fines in the sand used to construct the Nerlerk berm resulted in a lower slope of the “instability line”. This plot shows the slope of the “instability line” as a function of the estimated initial relative density and introduces the subjects of field tests and back-calculation of apparent shear strengths from failures.

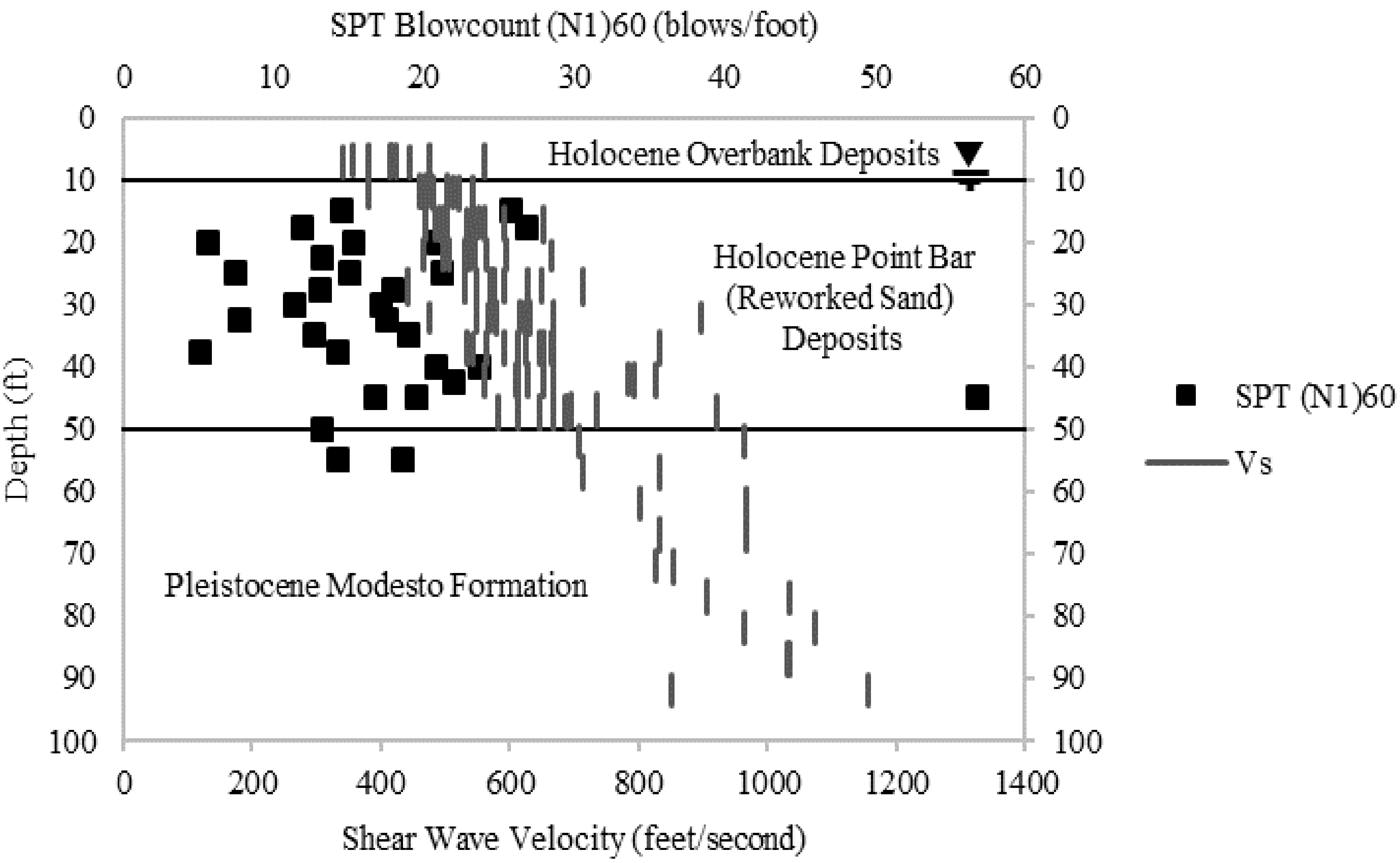

SAND GRAINS
SILT GRAINS NEAR CONTACT POINTS.
(a)
PARTICLE STRUCTURE AS DEPOSITED IN LOOSE STATE
LARGE GRAINS MOVE TOGETHER CREATING LARGE INITIAL VOLUME REDUCTION.
(b)
LARGE GRAINS COME INTO BETTER CONTACT INCREASING DILATANT TENDENCIES.
PARTICLE STRUCTURE COMPRESSED AND SHEARED


Slope of Instability Line, ϕ_i (deg)
Initial Relative Density, D_{ri} (%)
Nerlerk 2%
Nerlerk 12%
0
10
20
30
40
50
60
70
80
90
100
Sand
Reference
Banding — Castro (1969)
Valgrinda — Bjerrum et al. (1961)
Sacramento River — Lee (1965)
Leighton Buzzard — Sladen et al. (1985a)
Nerlerk 2% — Sladen et al. (1985a)
Nerlerk 12% — Sladen et al. (1985a)
Ham River — Bishop et al. (1965)
Garnet Tailings — Highter and Tobin (1980)
Ottawa — Vaid and Chern (1985)
Tailings — Vaid and Chern (1985)
Ottawa — Tai (1970)
Chattahoochee — Tai (1970)
Sacramento River Fine — Kramer (1985)
Silica Flour — Jeong (1988)
Carrollton Bend — Torrey (1981)
Syncrude Tailings — Sladen and Handford (1987)
Kogyak - 350/2 — Been and Jefferies (1985)
Kogyak - 350/10 — Been and Jefferies (1985)

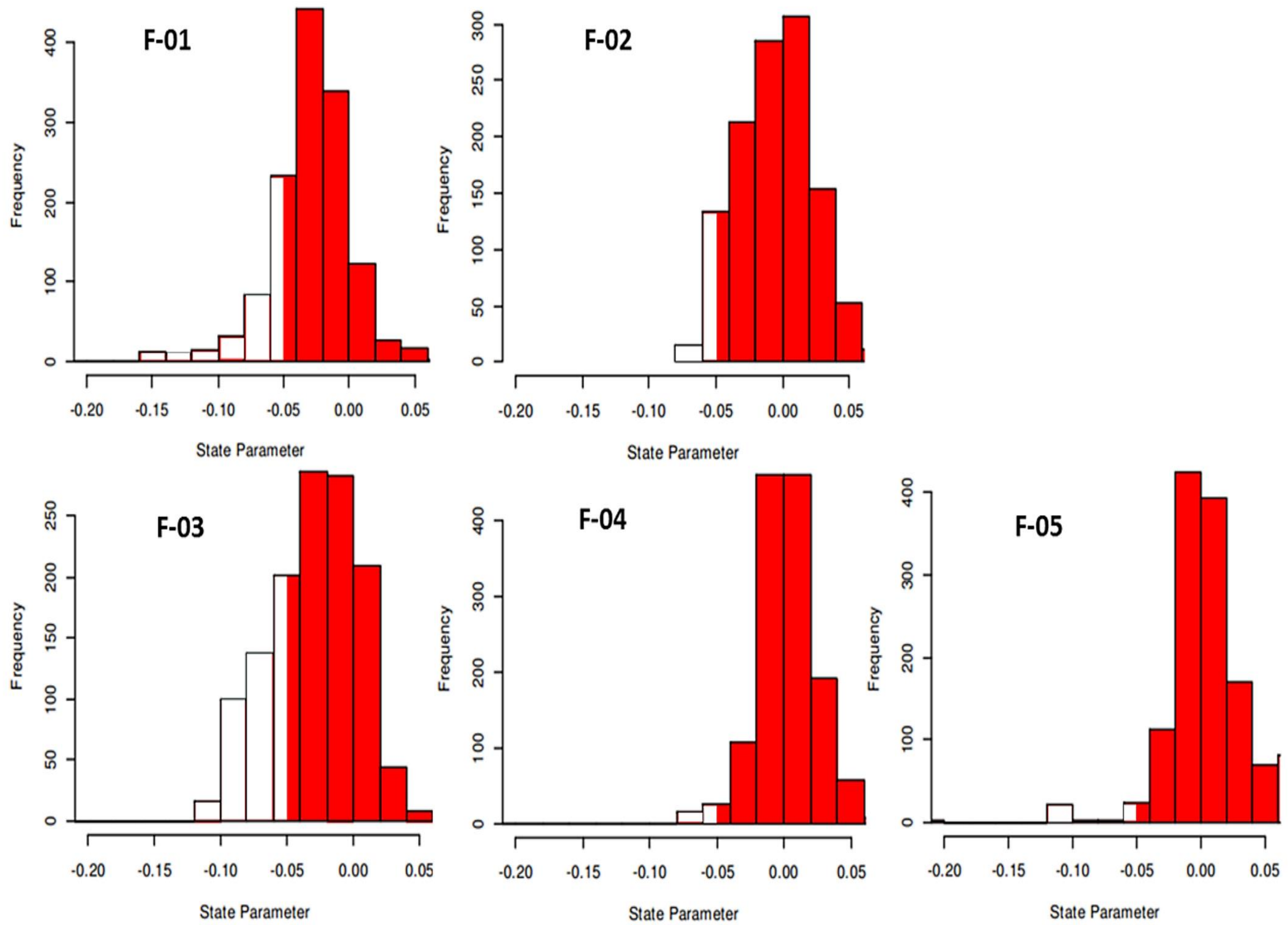
3. Field Tests

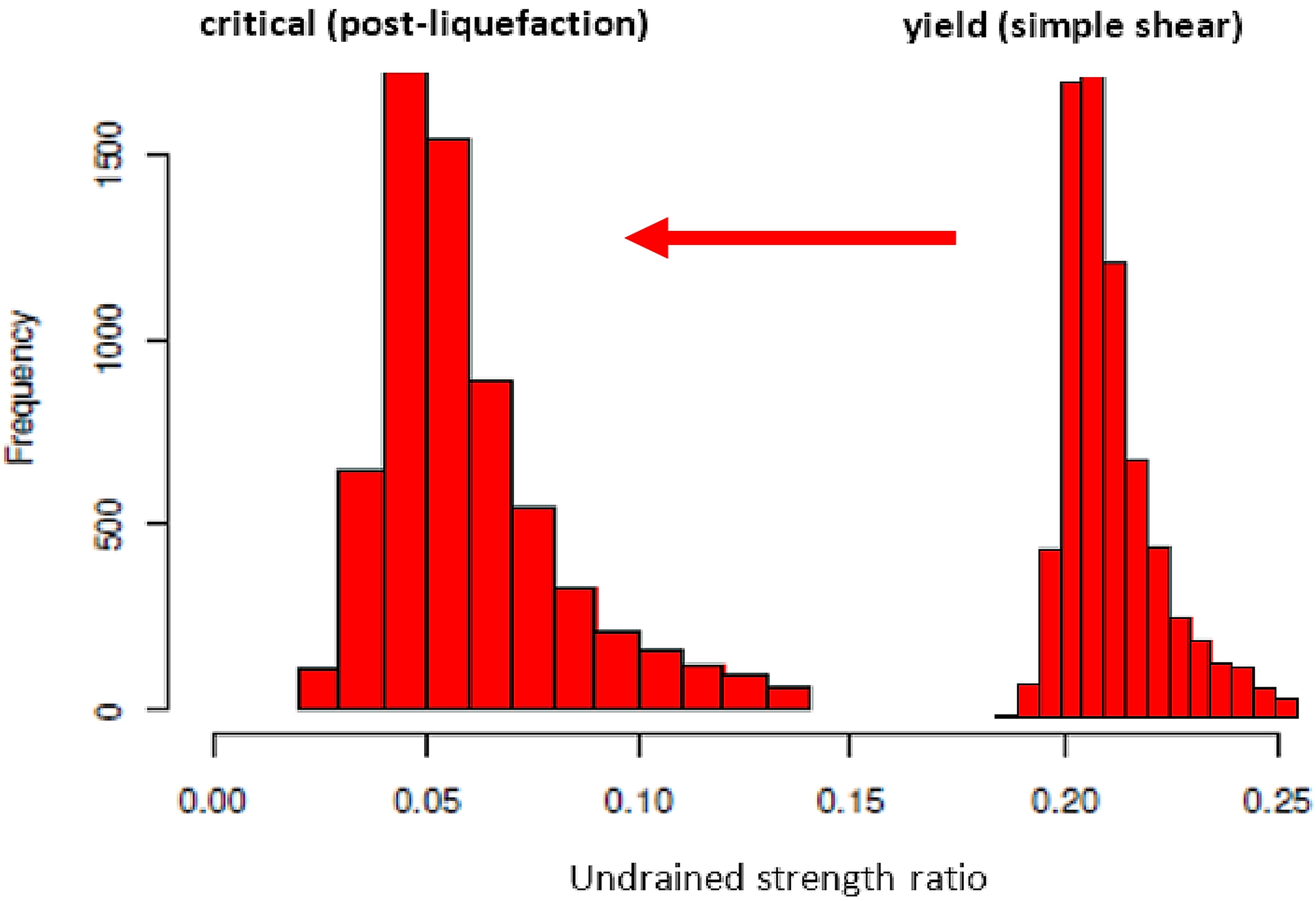
A further issue is that even what we normally think of as “uniform” sand deposits, are often far from uniform. This is most easily seen in field test results.

The next slide shows the variability of measured SPT blowcounts and shear wave velocity in natural sand deposits which by eye appear to be relatively uniform (from Crawford et al., 2019). One of the major findings of the CANLEX research project (Robertson et al, 2000) was that presumed uniform sand deposits are not so uniform.


SPT Blowcount (N1)60 (blows/foot)
0
10
20
30
40
50
60
Holocene Overbank Deposits
Holocene Point Bar
(Reworked Sand)
Deposits
Pleistocene Modesto Formation
Depth (ft)
0
10
20
30
40
50
60
70
80
90
100
0
200
400
600
800
1000
1200
1400
Shear Wave Velocity (feet/second)
SPT (N1)60
Vs

The next two slides, taken from the Fundao Investigations report show, first, the variability of the Fundao tailings expressed in terms of the state parameter interpreted from CPT soundings, and second, the peak and residual undrained shear strengths, again interpreted from CPT soundings using Sadrekarimi (2014).


F-01
F-02
F-03
F-04
F-05
Frequency
State Parameter
-0.20
-0.15
-0.10
-0.05
0.00
0.05



critical (post-liquefaction)
yield (simple shear)
Frequency
0
500
1000
1500
0.00
0.05
0.10
0.15
0.20
0.25
Undrained strength ratio

However …

- CPT measurements reflect both strength and compressibility and thus are impacted by fines content

- And, are strongly impacted by the layer thicknesses (the cone has a nose!)

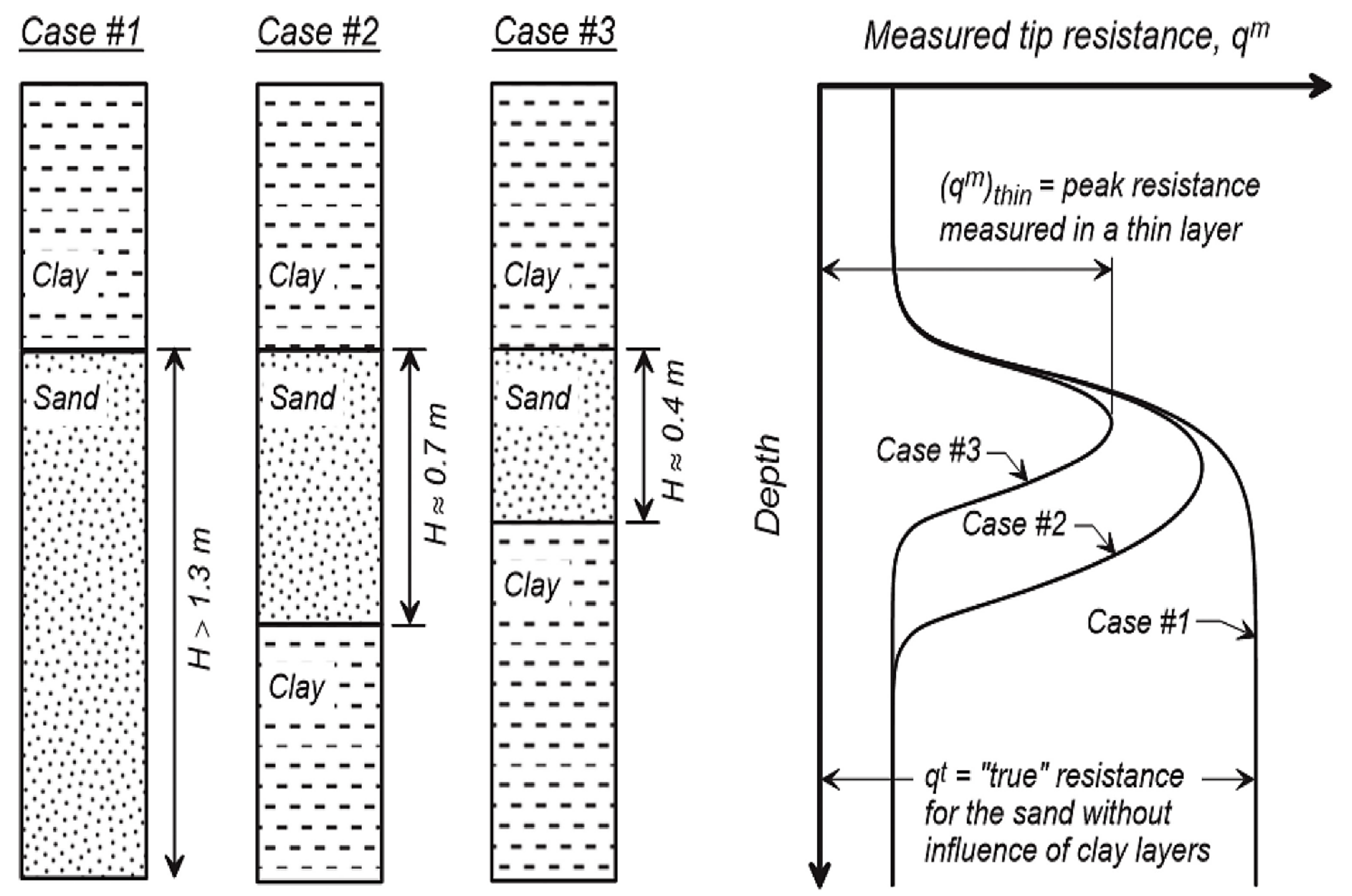


Figure 1. Schematic of thin layer effect for a sand layer embedded in a clay layer (modified from Robertson and Fear 1995).

So, beware of taking correlations of soil properties with CPT data, or other field test data, as anything like the single correct answer. They should rather be taken with a grain of salt.

In summary, there are a number of bothersome practical issues …

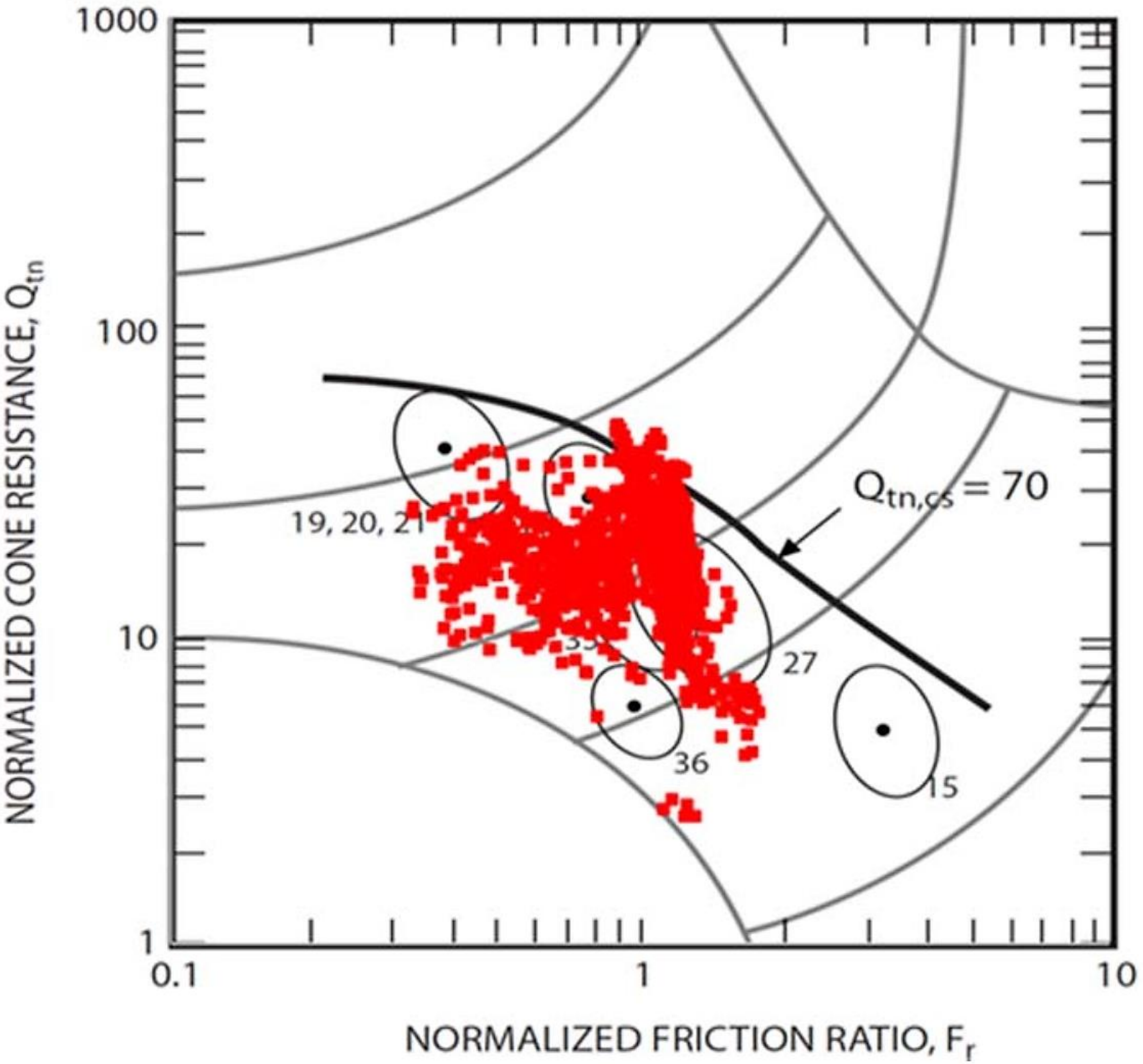
- Even “uniform” materials are variable
- Field measurements all have limitations and may require corrections
- Undisturbed sampling and laboratory testing is difficult and time consuming
- All of which leads to the next topic …

4. Use of Case Histories

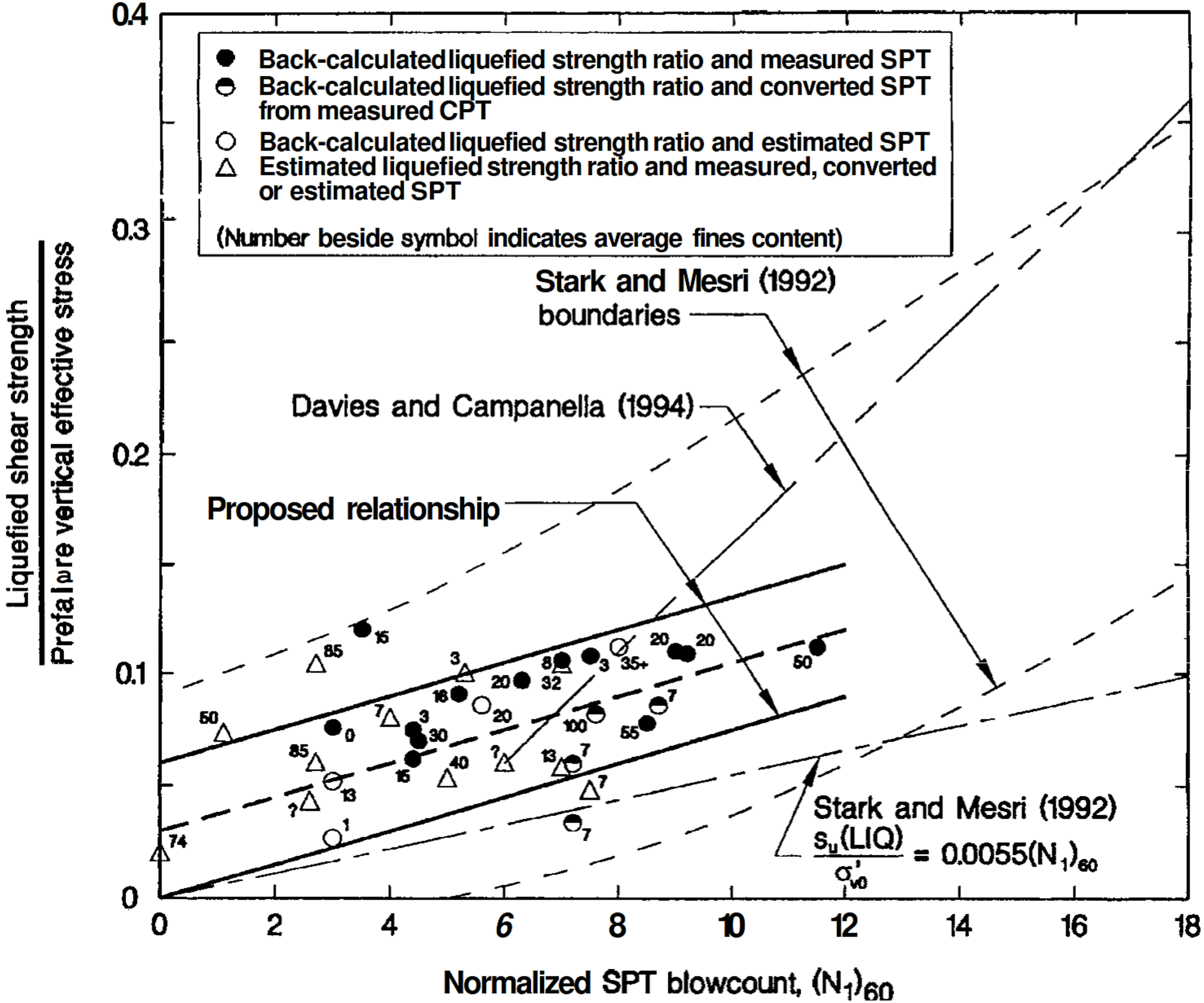
- Some emphasis has been placed on back-calculation of conditions causing failure from case histories for evaluating the potential for both static liquefaction and liquefaction, settlement and lateral spreading of denser materials under earthquake loadings

- But … don’t forget that these may not include all necessary corrections to field measurements and may also involve some gross averaging, so that they also have limitations

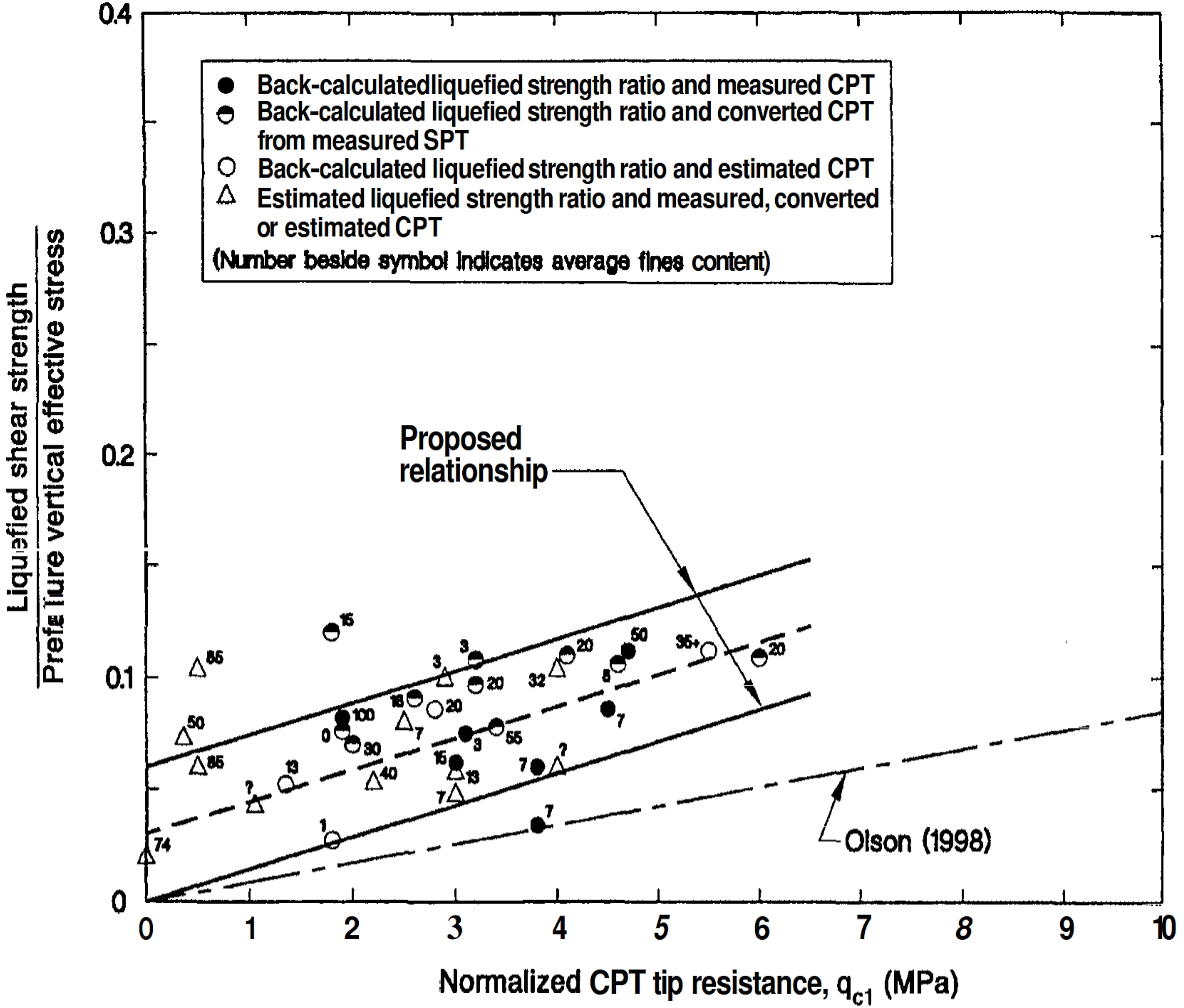
To start with, here is an example of data from the Fundao Investigations report plotted on P.K. Robertson’s chart showing susceptibility to static liquefaction using CPT soundings. This is a quick way of determining that a particular deposit, if fully saturated, may be susceptible to a flow slide.


1000
100
10
1
0.1
1
10
NORMALIZED CONE RESISTANCE, Q_{tn}
NORMALIZED FRICTION RATIO, F_r
$Q_{tn,cs} = 70$
19, 20, 21
33
27
36
15

The next two slides show plots from Olson and Stark (2002) of the residual strengths interpreted from case histories as a function of normalized SPT blowcounts and CPT tip resistance. Note that, in spite of the fact that some scatter remains, the suggested relationships have less uncertainty than earlier attempts.

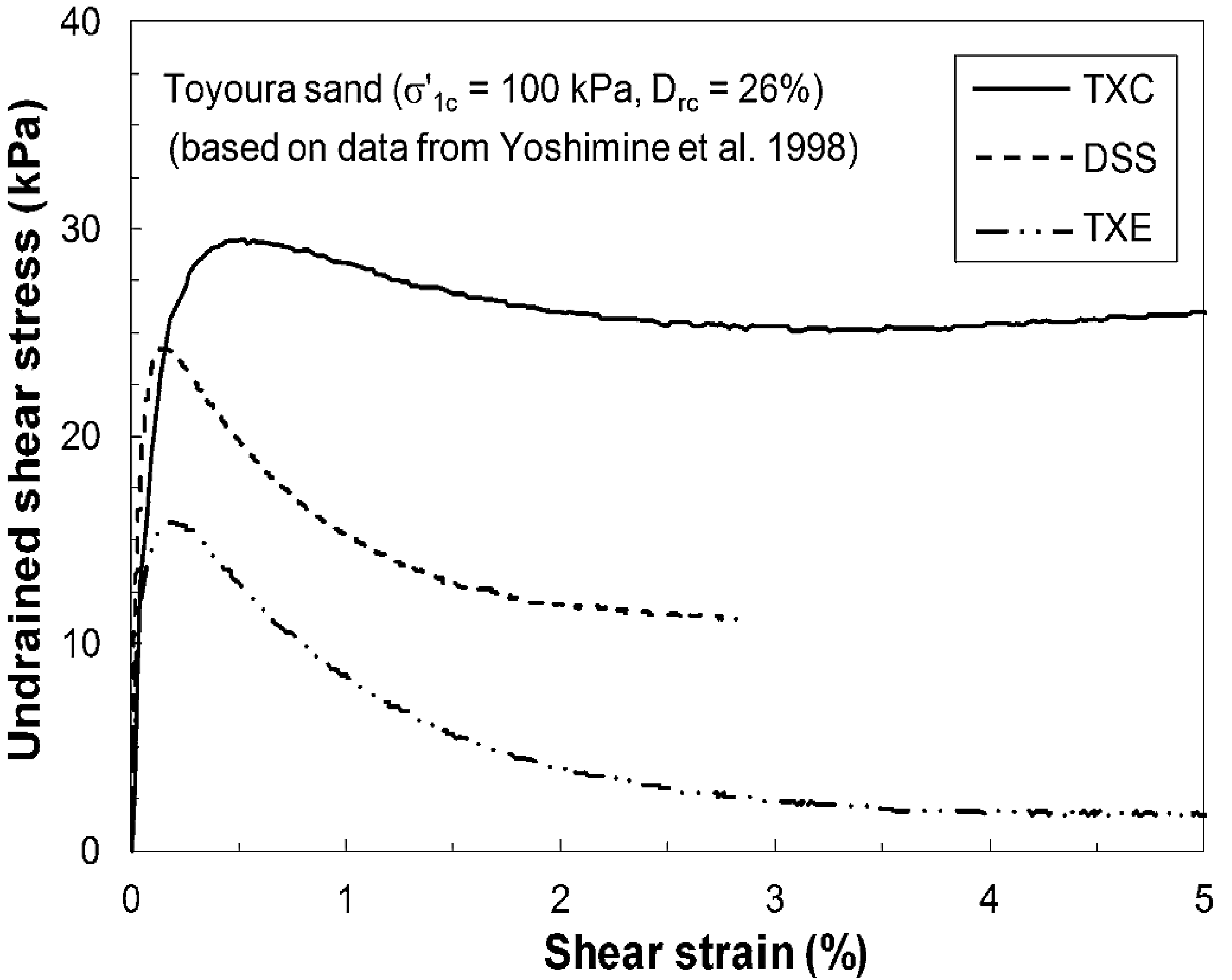


Back-calculated liquefied strength ratio and measured SPT
Back-calculated liquefied strength ratio and converted SPT from measured CPT
Back-calculated liquefied strength ratio and estimated SPT
Estimated liquefied strength ratio and measured, converted or estimated SPT
(Number beside symbol indicates average fines content)
Stark and Mesri (1992) boundaries
Davies and Campanella (1994)
Proposed relationship
Stark and Mesri (1992)
$\frac{s_u(LIQ)}{\sigma'_{v0}} = 0.0055(N_1)_{60}$
Liquefied shear strength / Prefailure vertical effective stress
Normalized SPT blowcount, $(N_1)_{60}$



Back-calculatedliquefied strength ratio and measured CPT
Back-calculated liquefied strength ratio and converted CPT from measured SPT
Back-calculated liquefied strength ratio and estimated CPT
Estimated liquefied strength ratio and measured, converted or estimated CPT
(Number beside symbol indicates average fines content)
Proposed relationship
Olson (1998)
Liquefied shear strength / Prefailure vertical effective stress
Normalized CPT tip resistance, q_{c1} (MPa)

However, some of the scatter must result from the failure to separate out data by the style of loading. This was done subsequently by Sadrekarimi (2014), who used the data from Yoshimine et al. (1998) that is shown on the following slide as a guide.



Toyoura sand (σ'1c = 100 kPa, Drc = 26%)
(based on data from Yoshimine et al. 1998)
TXC
DSS
TXE
Undrained shear stress (kPa)
Shear strain (%)
0
10
20
30
40
0
1
2
3
4
5

Sadrekarimi (2014) then developed the following plots which show both the peak and residual undrained shear strengths as a function of the style of loading. Clearly there are some approximations involved, but these charts likely represent the easiest way to get a handle on the peak and residual undrained shear strengths using either SPT or CPT data at the present time.

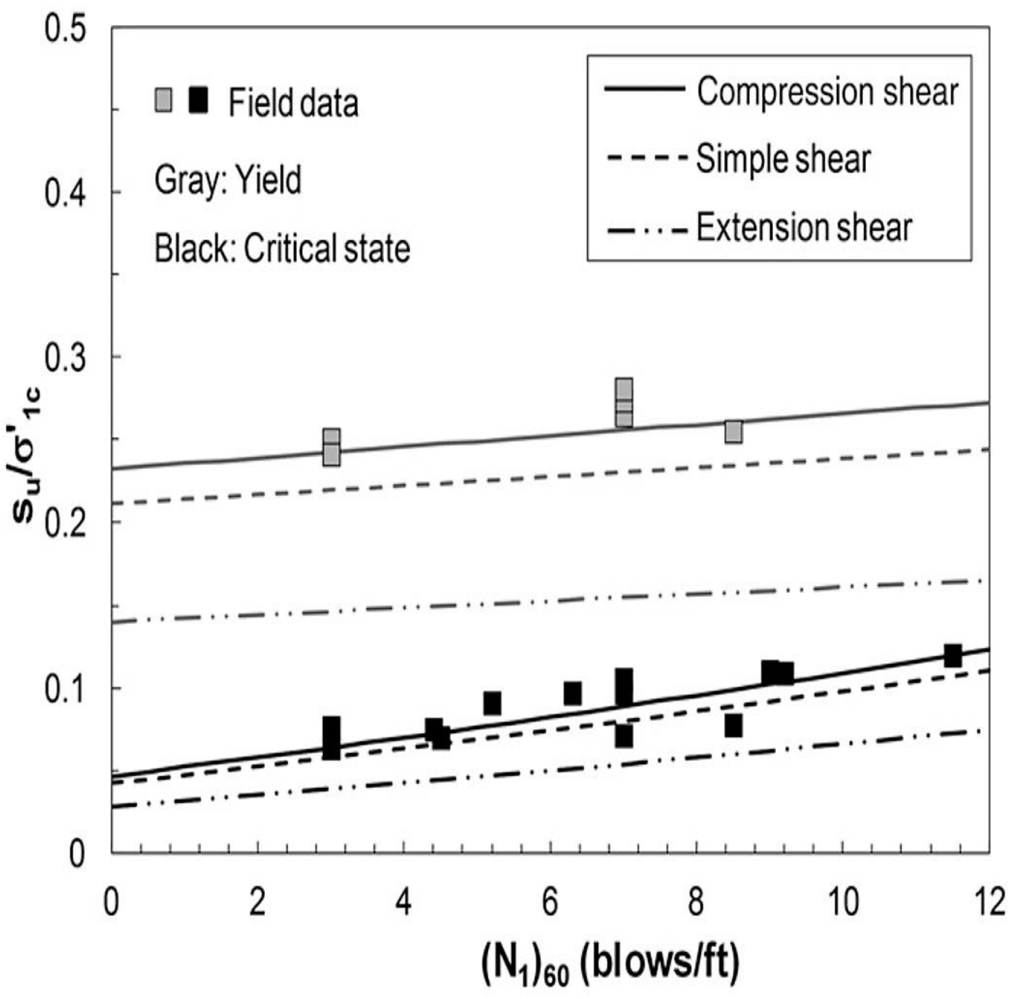


Fig. 9. Comparisons of $s_u(\text{yield})/\sigma'_{vo}$ and $s_u(\text{critical})/\sigma'_{vo}$ mobilized in liquefaction flow failures with Eqs. (2)–(7) based on $(N_1)_{60}$; 1 ft = 0.3 m

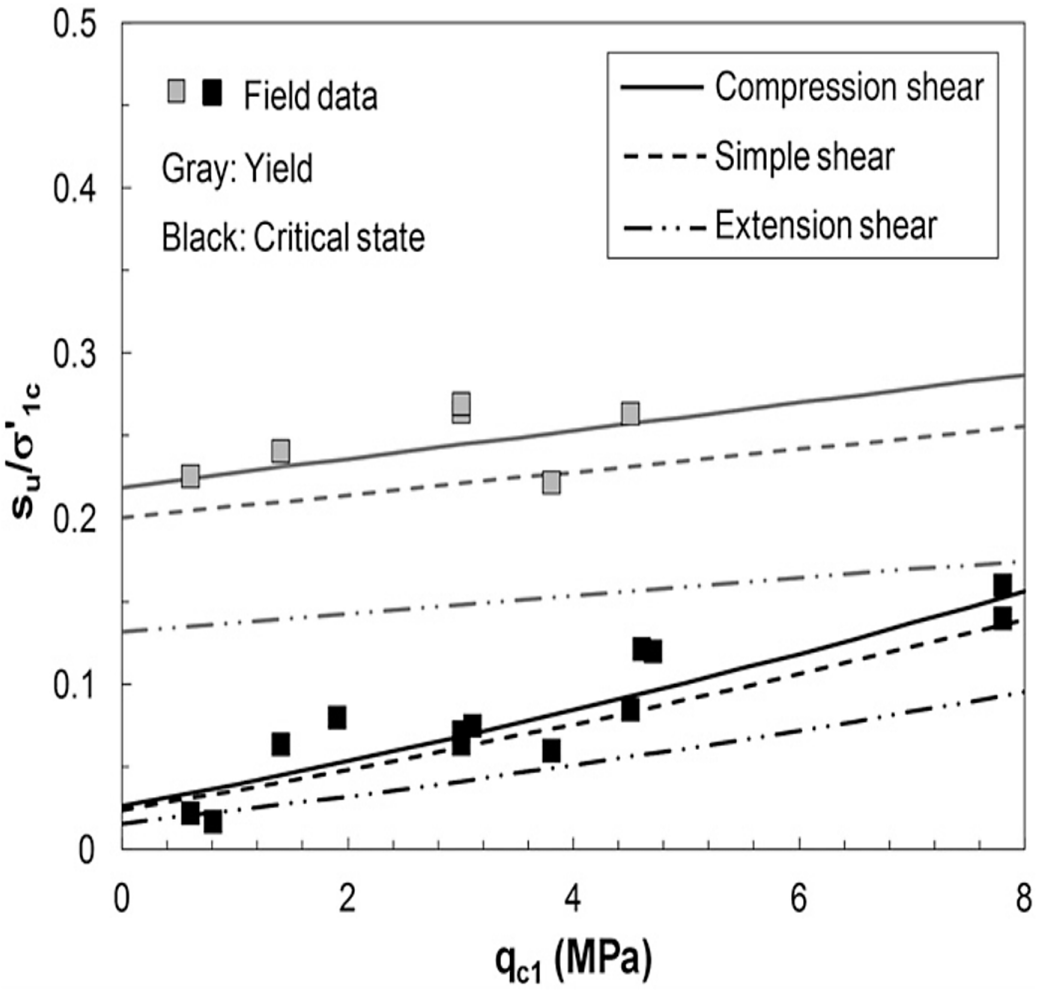


Fig. 10. Comparisons of $s_u(\text{yield})/\sigma'_{vo}$ and $s_u(\text{critical})/\sigma'_{vo}$ mobilized in liquefaction flow failures with Eqs. (2)–(7) based on q_{c1}

However, before moving on to analysis issues, let us also note the useful summary of properties using a somewhat different approach and terminology that was provided by Davies et al. (2002), and is shown on the following slide.

Table 1 – Static Liquefaction General Design Guidelines

$(N_1)_{60}$	ψ**	$S_u/(p')$	γ_F (%)	
			$F_{Trigger \approx 1.0}$	$F_{Trigger \approx 0.5}$
0 – 4	+0.12	0.05 – 0.10	25 – 50	>100
4 – 10	+.05	0.10 – 0.20	10 –25	30 – 100
10 –15	0	0.15 – 0.4	8 –15	20 – 35
15 –20	-0.08	0.3 – 0.5	5 – 10	15 –25
>20	<-0.10	>0.5 (< ~0.68)*	<5	<15

*Dilatancy ignored regardless of state.

**Approximate only, depends on steady state compressibility of tailings, λ

Table 1 implies that materials with a state more dilatant than about Ψ<-0.1 will not be a concern for undrained shear phenomena. This is purely from the author's experience and has been noted elsewhere (e.g. Davies, 1999). It is interesting to note that independent experience of others, e.g. Been (1999) and Jefferies (1999), suggest values of –0.08 and –0.10, respectively, as their practical limits of minimum state to ensure satisfactory engineering performance provided the drained strength of the material is sufficient for all loading conditions. In more simple language, the best way to deal with a problem is to avoid creating it in the first place.

However, note that the SPT blowcounts cited here might be conservative, because of the effect of fines or in order to account for material variability. Ishihara (1993) in his Rankine lecture, while focused more on earthquakes as the triggering mechanism, concludes that a normalized SPT blow count of 9 is the upper bound for triggering a flow slide in a uniform material.

5. Instability analyses!

- Rather than doing conventional stability analyses, maybe we should call them “instability analyses”?
- Instability will not occur everywhere at once, therefore we may have to do a “progressive” analysis of potential failures
- And, account for the style of loading to failure
- And, maybe we should think about 3D effects
- And, recognizing that the soil will not be uniform, we might have to do multiple trials with different assumed distributions of properties

Further analytical considerations:

- How well do we know the detailed stratiraphy and the state of consolidation?

- How well do we know zone of full saturation?

- Even for a single material, can we define median strengths and distribution around the mean?

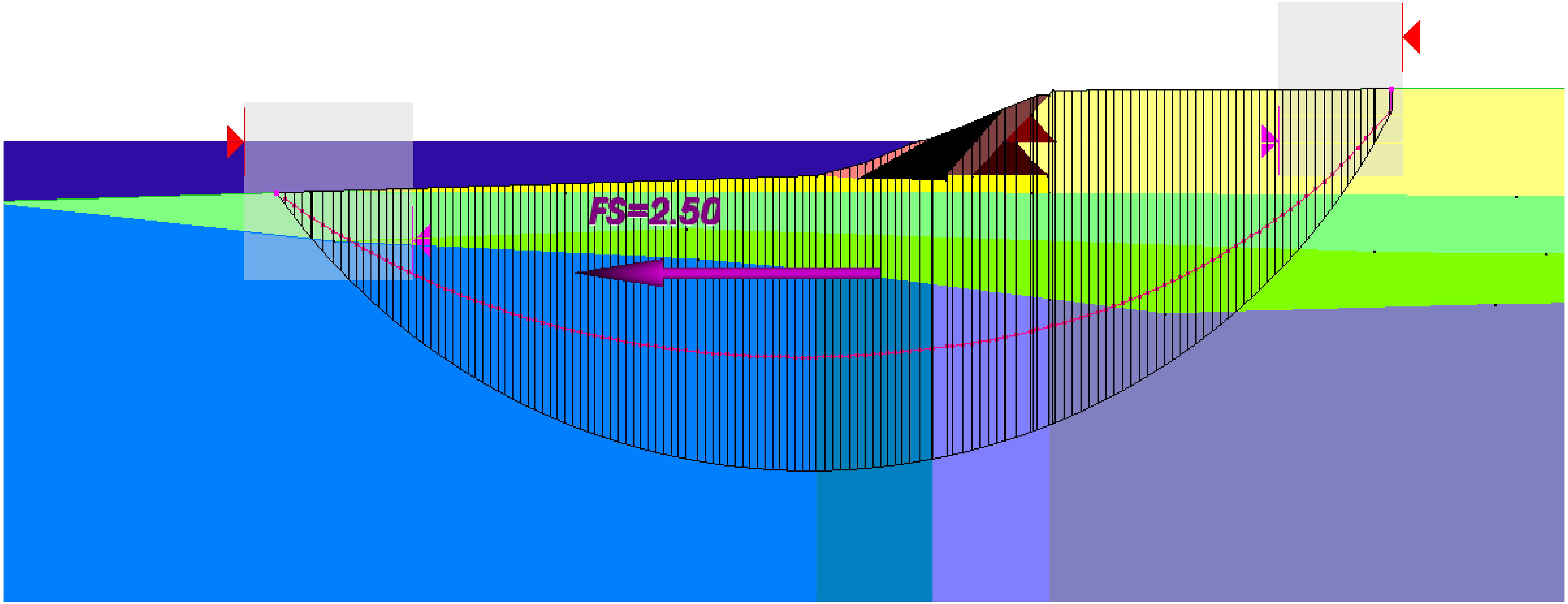
- And finally, limit equilibrium or finite element / difference?

A brief digression on 3D analyses:

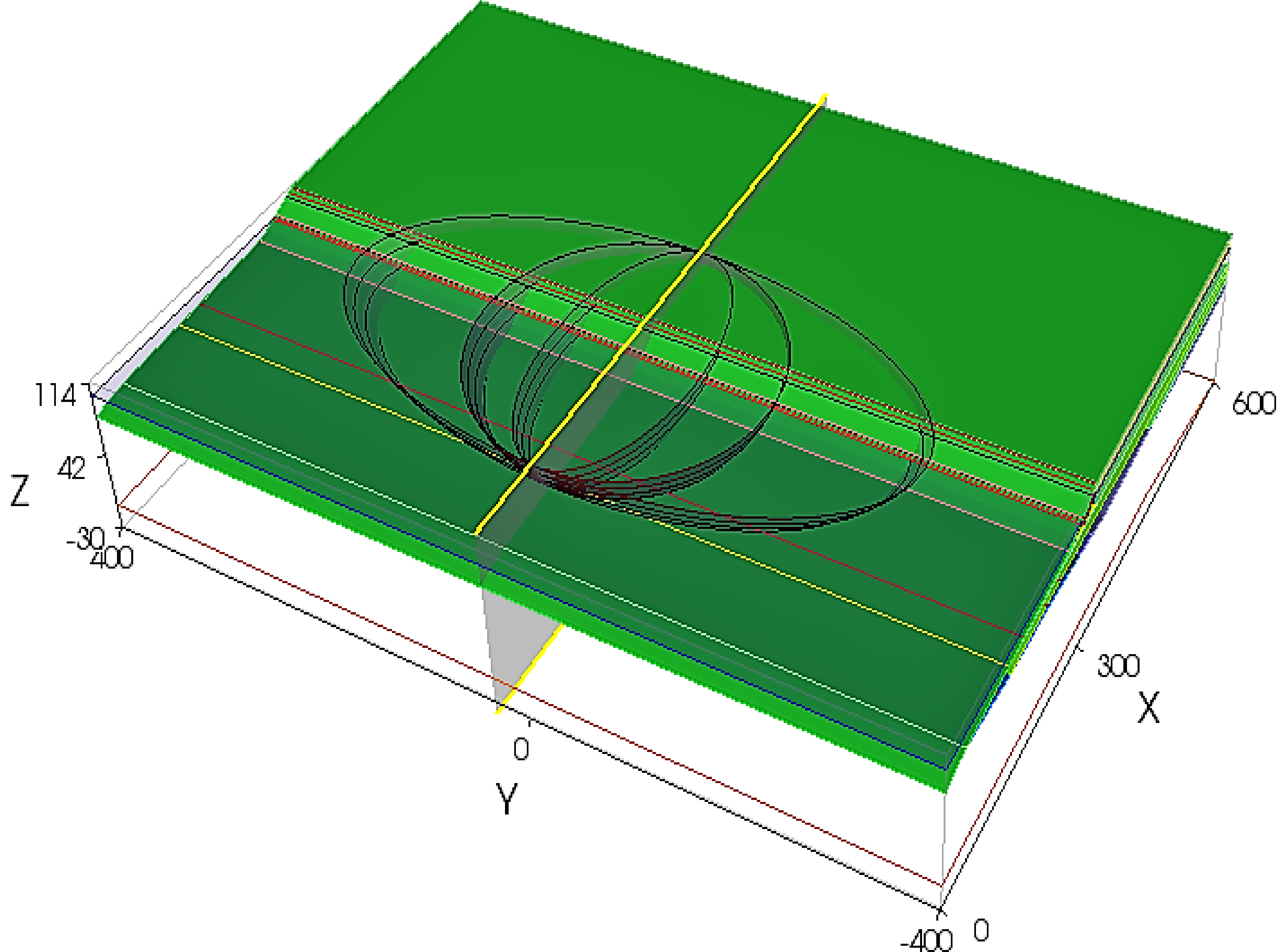
Almost all slides have 3D rather than 2D geometry, but this is generally not taken into account in analyses and some the the conclusions about the effects of 3D geometry in the literature are incorrect. 3D geometry generally increases the calculated factor of safety, but in some case can reduce it. See Pyke (2019a).

The following slide shows a photograph, taken from the air, of Treasure Island, a man-made island within San Francisco Bay, that consists of hydraulic fill placed behind dikes constructed by the upstream method, and susceptible to liquefaction in earthquakes. As part of the redevelopment of the site the hydraulic fill will be compacted but there are still questions about the possibility of slope failures through the underlying young Bay Mud.

This slide shows the result of a 2D analysis. Note that the critical failure surface passes under the rock dikes so that they do not participate in the calculation.



However, the next slide shows three potential slip surfaces with different aspect ratios. Each of these slip surfaces has to cut through the rockfill dikes twice!



114
42
-30
Z
400
0
Y
-400
0
300
600
X

And the following slide shows the increases in the computed factors of safety using either the Ordinary Method of Columns or Spencer's Method …

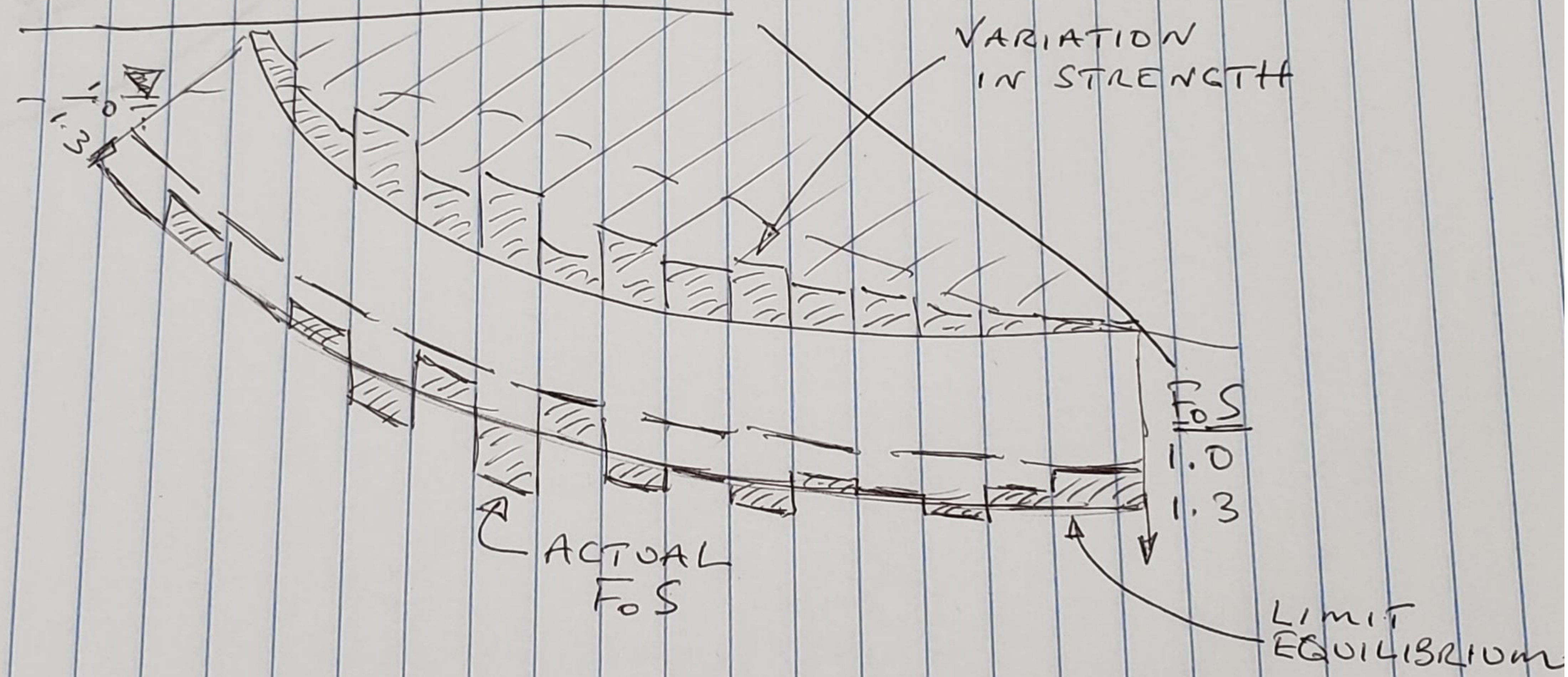
OMC	Spencer	
2.23	2.51	2D FoS
2.59	2.95	3D FoS aspect ratio = 2.0
2.44	2.97	3D FoS aspect ratio = 1.0
2.57	3.52	3D FoS aspect ratio = 0.5

And now, a note on the limitations of limit equilibrium analyses

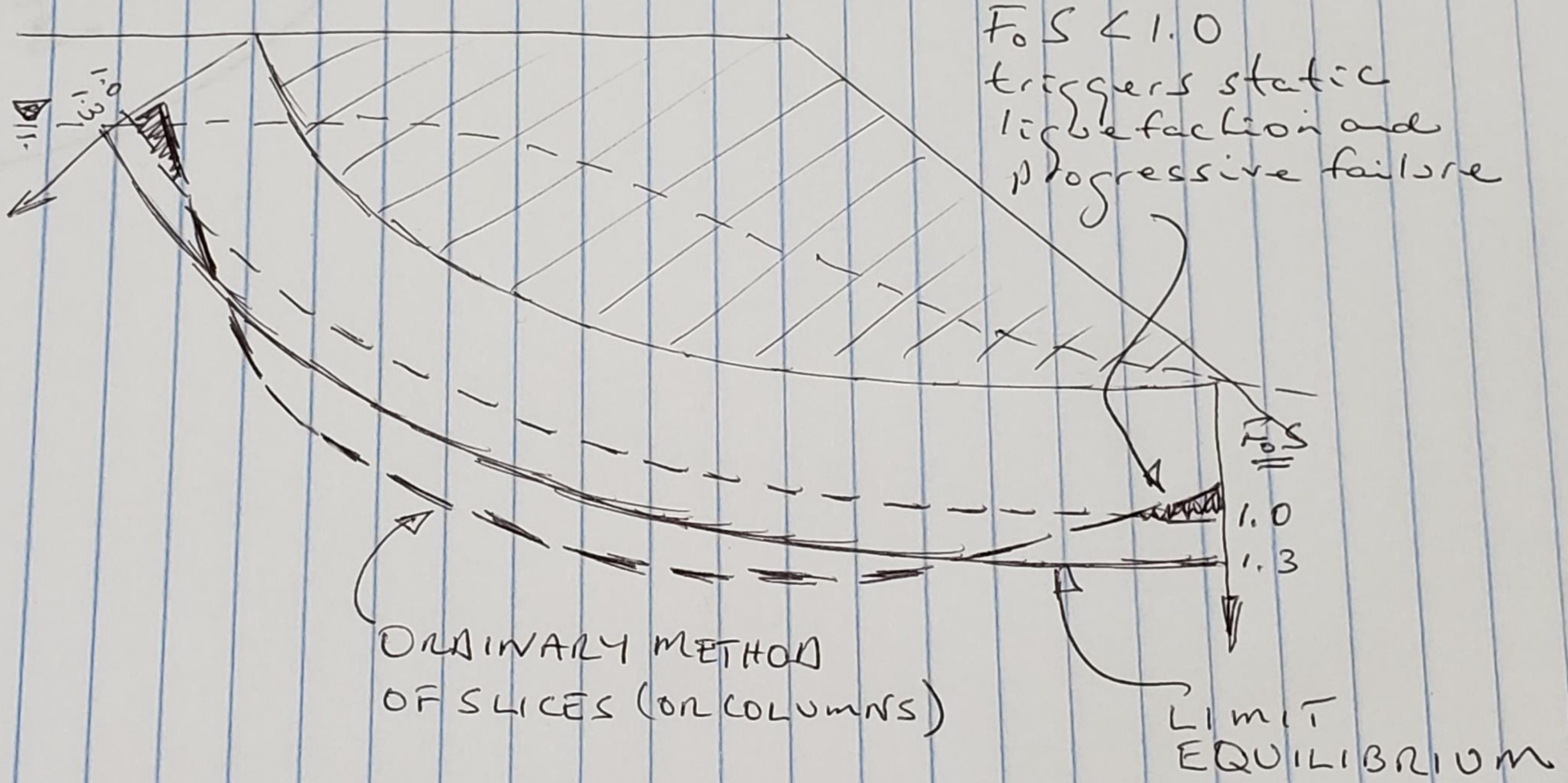
- See Pyke (2019b) and other articles on LinkedIn
- Methods that “fully satisfy equilibrium” imply a rigid sliding mass; force the local FoS to be the same at the base of each slice and hence an artificial stress distribution; they also do not include seepage forces
- The Ordinary Method of Slices or Columns offers some advantages, including the ability to include seepage forces, but the stress distribution around the potential failure surface may still not be totally correct
- Intervention and iteration are required to model progressive failure. No method of slices analysis provides displacements and therefore this class of analysis cannot follow the decline from peak strength to residual strength, even in a progressive analysis. They also do not account for excess pore pressure redistribution and dissipation

But, if you use the Ordinary Method of Slices, you can obtain the local factors of safety and thus conduct a progressive failure analysis.

The following slide illustrates a single run in which the user randomizes the peak undrained shear strengths around the potential failure surface. In this example, a limit equilibrium analysis shows a constant factor of safety of 1.3, but the local factors of safety should approach 1.0 for some slices simply because of the variation in the undrained shear strength.

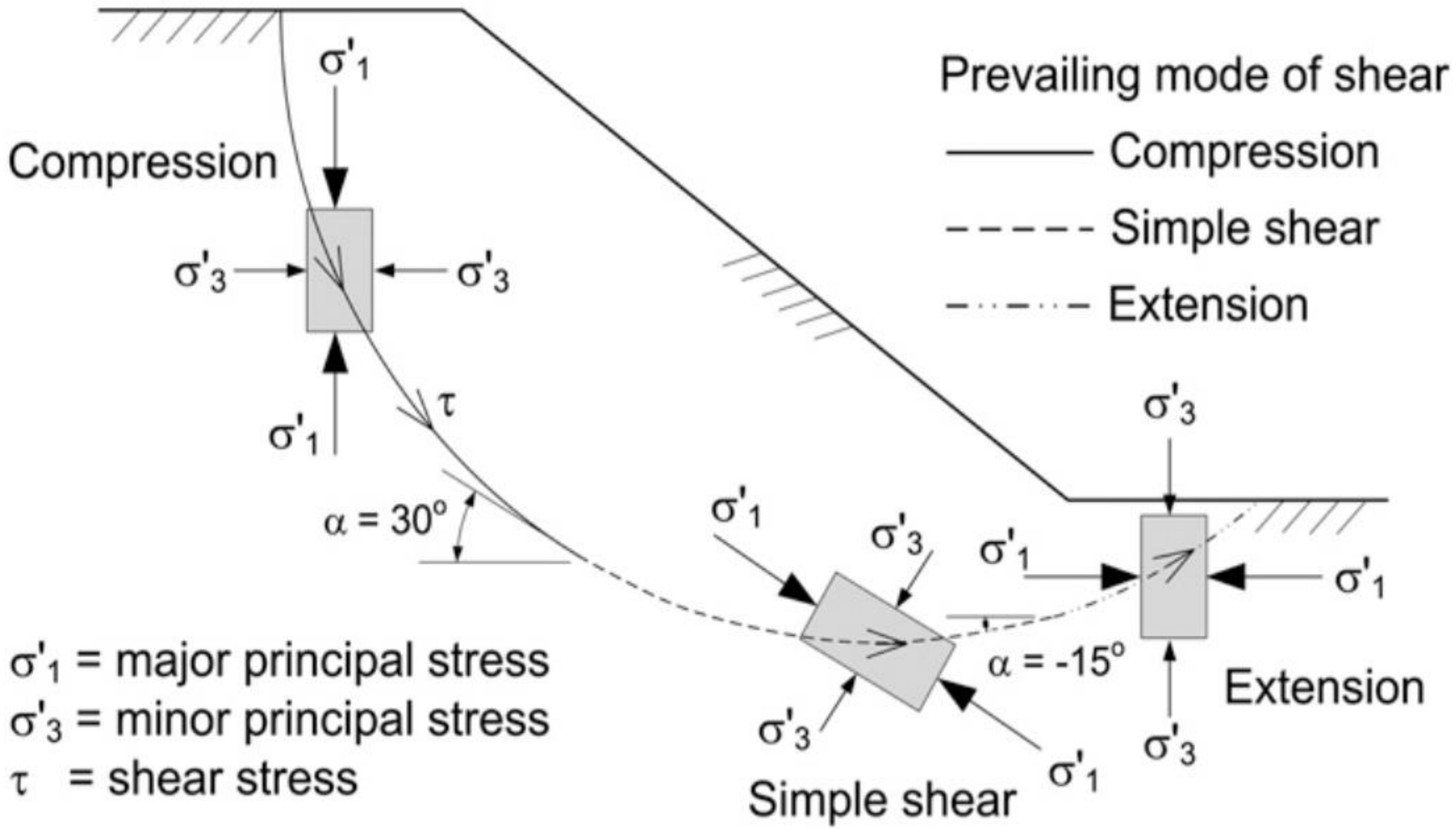

VARIATION IN STRENGTH
1.0
1.3
FoS
1.0
1.3
ACTUAL FoS
LIMIT EQUILIBRIUM

Even with a constant undrained shear strength, the Ordinary Method of Slices will typically produce a pattern of local factors of safety as shown in the following slide. Especially at the ends of the potential slip surface, the local factor of safety may drop below 1.0. This means that the undrained shear strength should be reset to the residual strength and the analysis repeated as necessary until stability or failure is realized.



FoS < 1.0
triggers static
liquefaction and
progressive failure
1.0
1.3
FoS
1.0
1.3
ORDINARY METHOD
OF SLICES (OR COLUMNS)
LIMIT
EQUILIBRIUM

The variation in the undrained strength with the style of loading should also be taken into account. The conventional wisdom is that the stress states around a potential failure surface vary as shown in the following slide, taken from Sadrekarimi (2014), but obviously this is approximate. In critical situations, the site-specific variation in the existing stress staes and styles of loading to failure might be obtained from a companion continuum analysis.



Prevailing mode of shear
Compression
Simple shear
Extension
Compression
σ'1
σ'3
σ'3
σ'1
τ
α = 30°
σ'1
σ'3
σ'1
σ'1
σ'3
σ'3
σ'1
α = -15°
Extension
σ'3
Simple shear
σ'1 = major principal stress
σ'3 = minor principal stress
τ = shear stress

Common slope stability programs do not at present automate modelling of progressive failure, but they do offer the ordinary method of slices as an alternative, so that progressive analyses can be run manually.

Ideally all three of these factors, material variability, variation of the undrained shear strength with the style of loading, and progressive failure should be modelled in one set of analyses. Because in this class of analysis the shear strength has to drop from the peak strength to the residual strength in one iteration, the analysis may end up being conservative and a factor of safety in the order of 1.1 might normally be deemed sufficient.

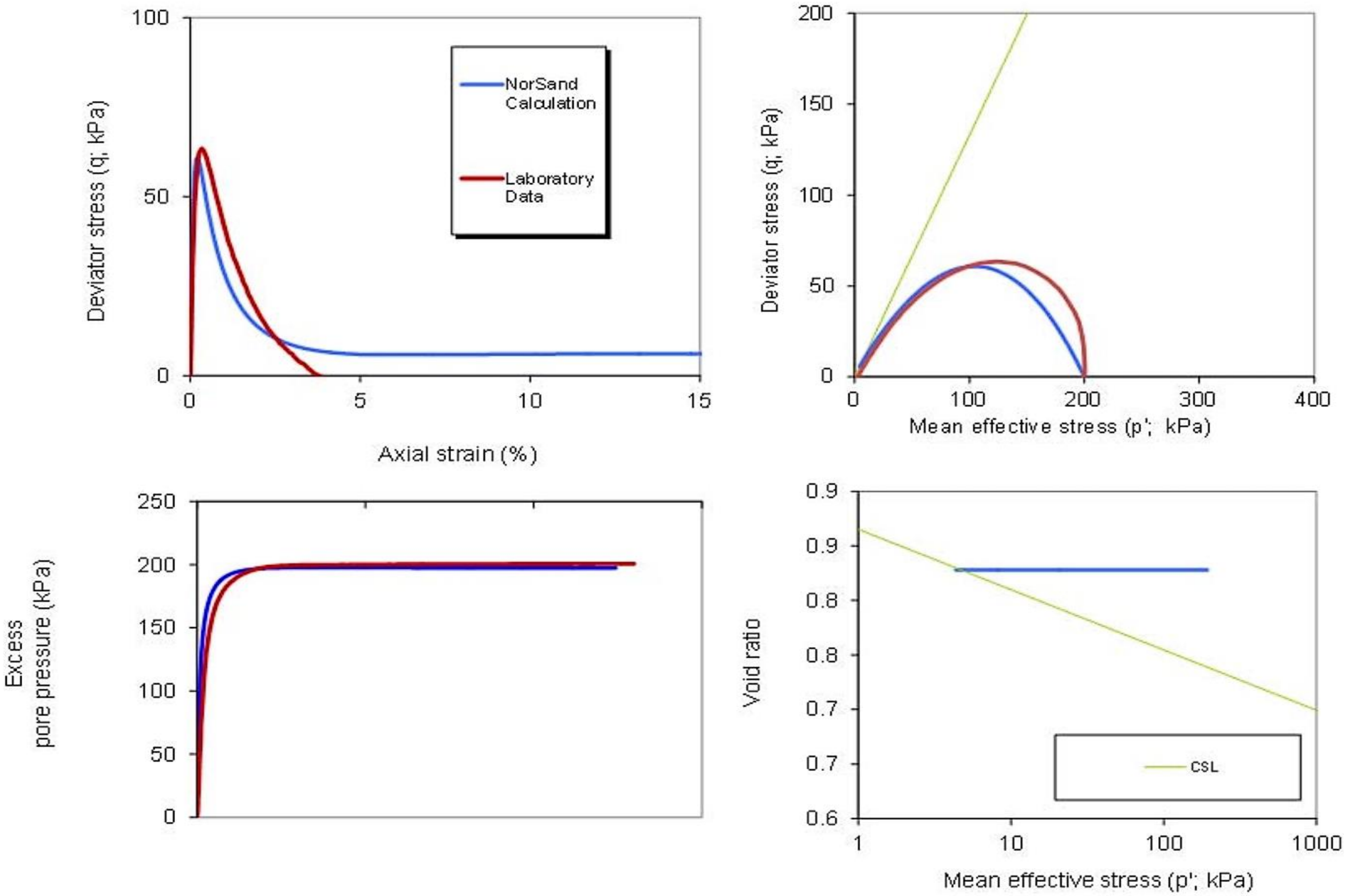
The more conventional factors of safety of 1.3 or 1.5 are only required when material variability, progressive failure and the variation of the undrained strength with the style of loading are not taken into account.

Limitations of continuum analyses

- Require moduli (or stress strain relationship) as well as strength. But the stress distribution is not so sensitive to moduli, therefore continuum analyses might be of value in determining the stress state around potential failure surfaces in a side analysis.
- In theory can model deformations all the way to failure, but that requires soils models that can model the drop to residual strengths and that everyone can understand and agree on
- Effective stress continuum analyses can account for excess pore pressure redistribution and dissipation but this requires accurate, and likely anisotropic, hydraulic conductivities
- May give a false impression of accuracy

But …

The next slide shows a nice example of modeling the drop from peak to residual strength from the Fundao Investigations report, using the Norsand model of Jefferies and Been (2016). But the investigators only adopted this model in their third round of analyses, suggesting that although there may be a role for continuum analyses in special situations such as investigating failures, they are not necessarily suitable for use as a tool for routine analysis and design.



Deviator stress (q; kPa)
Axial strain (%)
NorSand Calculation
Laboratory Data
Deviator stress (q; kPa)
Mean effective stress (p'; kPa)
Excess pore pressure (kPa)
Void ratio
Mean effective stress (p'; kPa)
CSL

And now, a caution before we proceed to talk about uncertainty and probabilities of failure – even the best soil models and laboratory tests are approximations of the behavior in the field. As Terzaghi is reported to have said (by Davies et al, 2002):

"Nature has no contract with mathematics – she has even less of an obligation to laboratory test procedures and results".

And, this caution can also be extended to field tests. No single field test represents the actual conditions at failure in a real case history.

Hence the need for informed judgement.

6. Regarding the probability of failure

- Several of the standard slope stability computer programs allow the user to specify normal or log-normal distributions around the estimate of the mean value of one or more parameters and the probability that the computed factor of safety falls below 1.0 is then computed using Monte Carlo simulations. For instance, the best estimate of the factor of safety might be 1.4 and the probability of the factor of safety falling below 1.0 might be said to be 0.05 percent.
- However, aside from the limitations of such calculations –how are the answers impacted by differences between the analytical model and the field conditions? - are all the uncertainties properly included? – such calculations are much more an indication of the uncertainty in the analysis of the safety of the existing condition, rather than an evaluation of the probability of failure.
- The probability of failure is much more a function of the probability of various triggering mechanisms being activated, taking into account all the uncertainties involved in each possible mechanism.
- But, note that the more experts that are involved and the more uncertainties that are modelled, the more the mean likelihood of failure goes up – a paradox!

Possible triggering mechanisms

Applied loadings (and unloadings)

These are under the control of the operator so that the probability of the occurrence of the event is either 0 or 1. The question then becomes, given a certain decision by the operator, what is the probability that decision could trigger a failure. That part is quantifiable as the uncertainty in the calculation, just as for assessing the existing condition, but with the same limitations.

Mechanical undercutting

This is a special case of an applied loading or unloading but is worth listing separately because of its likely adverse consequences.

Continued on the following slides …

Sudden changes in the water level in the pond

This refers principally to sudden rises in the pond level, but sudden lowering can cause rapid drawdown failures in the upstream face of a dam. Sudden changes are both a function of operator decisions and natural events. These can be related in the sense that an operator's decision to improve the diversion of surface water around an impoundment reduces the probability of a natural event triggering a failure. Given a specified set of operator decisions, the probability of occurrence of a failure than becomes the product of the probability of a certain rise in the water level and the probability of failure for that new water level.

Internal stress re-distribution, lateral spreading (Fundao?)

Difficult to quantify but again the product of the probability of an adverse condition developing and the probability of that condition causing failure.

Small earthquakes, mine blasts, magazine explosions

My current hypothesis is that these are unlikely to affects the stresses and strains in the overall structure to a sufficient degree that they could constitute a trigger, but that they might cause the rearrangement or collapse of a small assembly of grains, particularly if there is another process such as lateral spreading or piping and erosion under way. The rearrangement and collapse can be looked at by either or both of: (a) experimental studies at 1g or in a centrifuge, or (b) analytical studies using a discrete element program such as UDEC. Subsequent flow can be studied using newer MPM analyses.

Other vibratory loads

Even less likely to be significant but can be studies as for small earthquakes and mine blasts

Seepage, piping and erosion

Likely to be one of the more significant triggers but difficult to quantify probability of occurrence.

Scour of the toe

Bad news if it happens. Operator should minimize probability of occurrence.

Failure of an upstream dam

Also likely to be bad news. Probability of failure the product of probability of such and event times probability of it triggering failure.

7. Conclusions

- Define the geometry, phreatic surface and state of consolidation in sufficient detail
- Quantify material properties as well as you can and note uncertainties
- Only use a level of analysis that you fully understand
- Conduct "instability" analyses and model progressive failure
- Try multiple sets of assumptions on geometry and material properties as opposed to doing a formal probabilistic calculation
- Recognize that analyses provide insight, not precise answers, and only for the assumed model, not necessarily the field condition

More Conclusions

- Ideally, given sufficient time and money, the engineer will conduct both field and laboratory tests, and will conduct more than one kind of analysis
- There is no one correct approach to evaluating the potential for triggering of a flow slide. It depends on the particulars of the situation
- Don’t rely any one paper or lecture (even my own). Study multiple sources and make your own best judgment

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The End!

Please send any comments or questions to me at bobpyke@attglobal.net