Math 2471 – Calc 3

Limits $\displaystyle\lim_{(x,y)\to(a,b)} f(x,y) \overset{?}{=} L$

If following different paths gives different limits, then the limit DNE

Ex $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \begin{cases} 0 & \text{along } x=0 \text{ a } y=0 \\ \frac{1}{2} & \text{along } y=x \end{cases}$

So limit DNE

consider $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2}$

along every path the limit $=0$ so maybe the limit is actually zero

Squeeze Th^m if $f < g < h$

& $\displaystyle\lim_{x\to a} f = \lim_{x\to a} h = L$ then $\displaystyle\lim_{x\to a} g = L$

this actually works for our limits

Now $\quad 0 \le x^2 \le x^2+y^2$

$\quad\quad 0 \le y^2 \le x^2+y^2$

so $\quad 0 \le x^2y^2 \le (x^2+y^2)^2$

so $\quad 0 \le \dfrac{x^2y^2}{x^2+y^2} \le \dfrac{(x^2+y^2)^2}{x^2+y^2} = x^2+y^2$

$$0 \le \lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} \le \lim_{(x,y)\to(0,0)} x^2+y^2 = 0$$

So $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} = 0$

Ex 2 $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^4+2y^4}{x^2+y^2}$

$0 \le x^2 \le x^2+y^2 \Rightarrow \quad 0 \le x^4 \le (x^2+y^2)^2$

$0 \le y^2 \le x^2+y^2 \quad\quad 0 \le y^4 \le (x^2+y^2)$

$$0 \le \frac{x^4+2y^4}{x^2+y^2} \le \frac{(x^2+y^2)^2+2(x^2+y^2)^2}{x^2+y^2} = 3(x^2+y^2)$$

$$0 \le \lim_{(x,y)\to(0,0)} \frac{x^4+2y^4}{x^2+y^2} \le \lim_{(x,y)\to(0,0)} 3(x^2+y^2) = 0$$

so $$\lim_{(x,y)\to(0,0)} \frac{x^4+2y^4}{x^2+y^2} = 0$$

<u>Ex 3</u> $$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^2+y^2}$$

Now $$0 \le x^2 \le x^2+y^2$$
$$0 \le y^2 \le x^2+y^2$$

So $$0 \le y \le \sqrt{x^2+y^2}$$

↑ this could be negative

so $$\sqrt{y^2} \le \sqrt{x^2+y^2} \Rightarrow |y| \le \sqrt{x^2+y^2}$$

correct

so $$-\sqrt{x^2+y^2} \le y \le \sqrt{x^2+y^2}$$

also $-\sqrt{x^2+y^2} \le x \le \sqrt{x^2+y^2}$

$$-(x^2+y^2)^{3/2} \le x^3 \le (x^2+y^2)^{3/2}$$

$$-(x^2+y^2)^2 \le x^3 y \le (x^2+y^2)^2$$

$$-(x^2+y^2) \le \frac{x^3 y}{x^2+y^2} \le \frac{(x^2+y^2)^2}{x^2+y^2} = x^2+y^2$$

$\because \lim_{(x,y)\to(0,0)} x^2+y^2 = 0$

by sq^z th^m $\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^2+y^2} = 0$

so $-\sqrt{x^2+y^2} \le x, y \le \sqrt{x^2+y^2}$

will work pretty much all the time

but we could also ~~have~~ other inequalities

Consider $\lim_{(x,y)\to(0,0)} \frac{x^4+y^2}{x^2+y^2}$

so consider splitting this up

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^2+y^2} + \lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$$

$\uparrow$ this limit $= 0$ $\qquad$ however $\uparrow$ this limit DNE

so the entire limit DNE.

we could also go along

$x=0$ & $y=0$ and get different limits.

Also $\lim_{(x,y)\to 0} \frac{x^2y^2}{x^2+2y^2}$ $\qquad$ $0 \le x^2 \le x^2+y^2 \le x^2+2y^2$

modify a little bit