Effective Math Instruction

DAY 2

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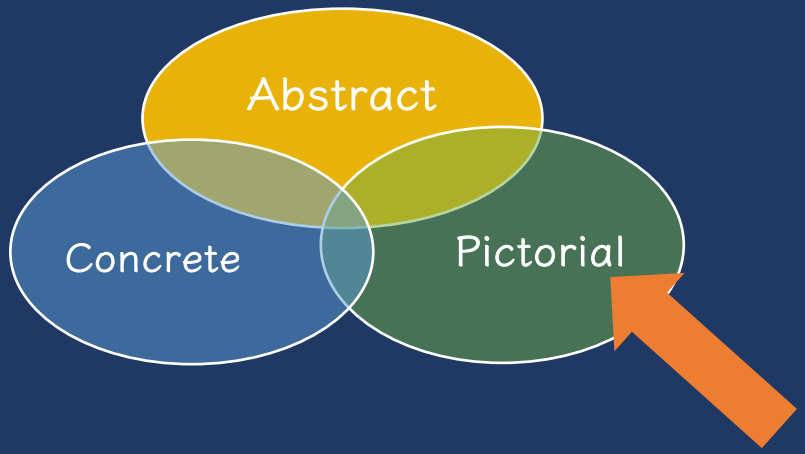
Say hello.

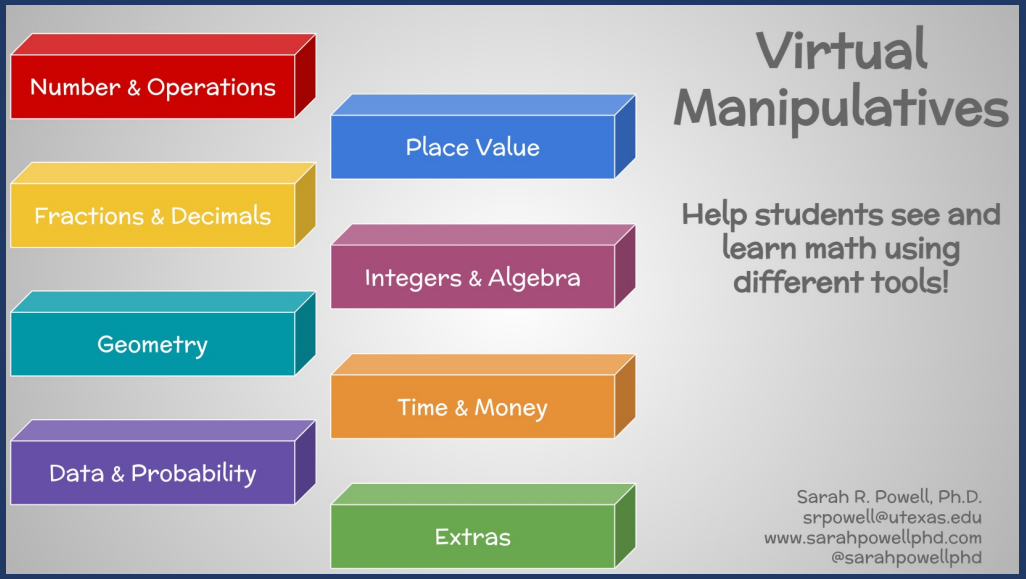
Describe two things that resonated with you from yesterday.

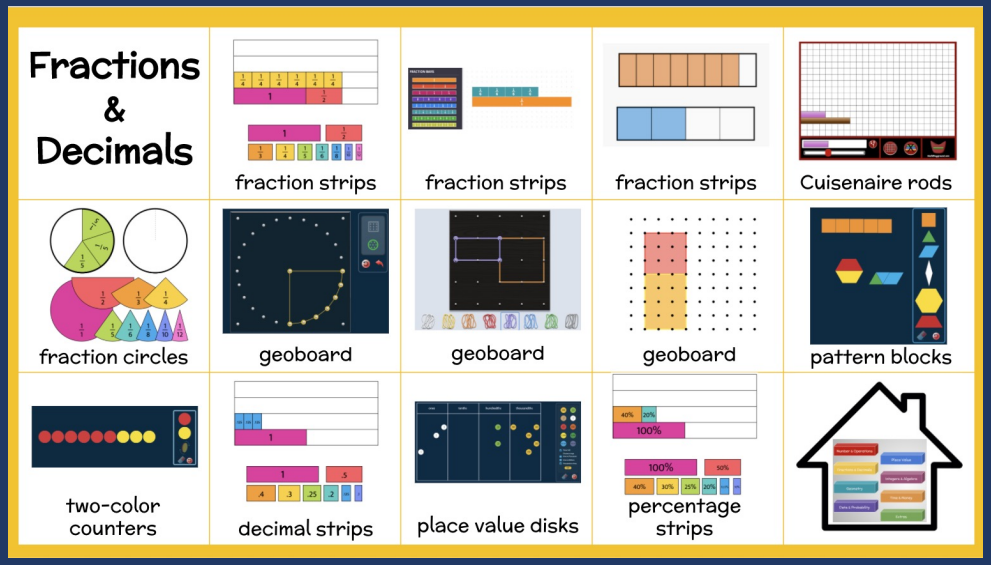
Schedule for Today

8:00-9:30	- Language of mathematics
9:30-9:40	BREAK
9:40-10:15	- Evidence-based practice: Word-problem solving
10:15-10:30	- Word-problem solving: Ineffective strategies
10:30-11:00	- Word-problem solving: Attack strategies
11:00-11:30	- Word-problem solving: Total problems
11:30-1:00	LUNCH
1:00-2:15	- Word-problem solving: Difference problems - Word-problem solving: Change problems - Word-problem solving: Equal groups problems - Word-problem solving: Comparison problems
2:15-2:25	BREAK
2:25-4:00	- Word-problem solving: Ratios/Proportions problems - Word-problem solving: Multi-step problems - Wrap-up



Two-dimensional images

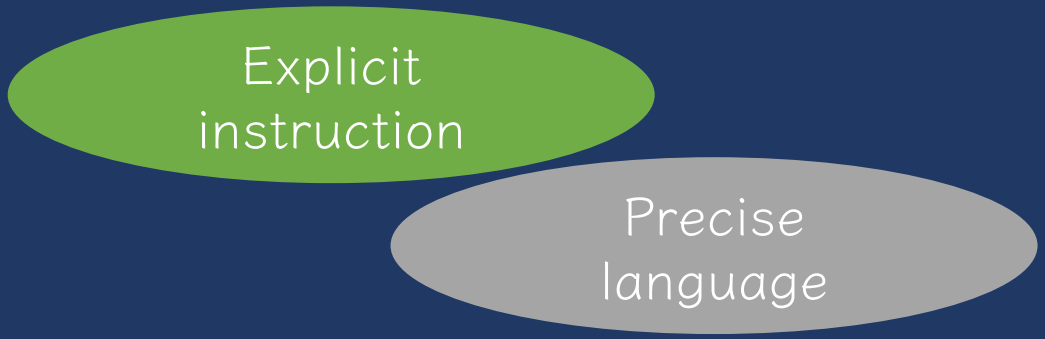




Mathematical Language

Instructional Platform

INSTRUCTIONAL DELIVERY



INSTRUCTIONAL STRATEGIES

Effective Mathematics Practices
DAY 2

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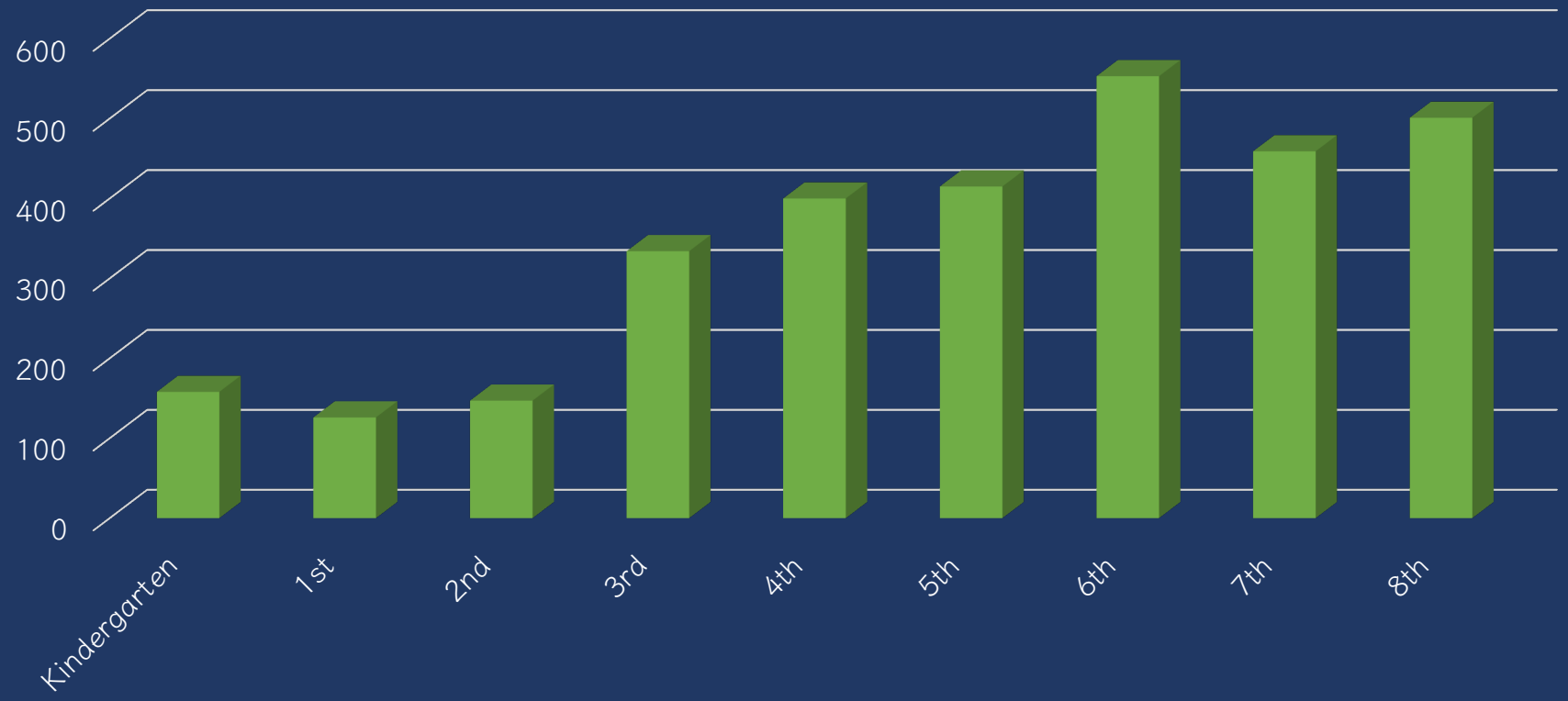
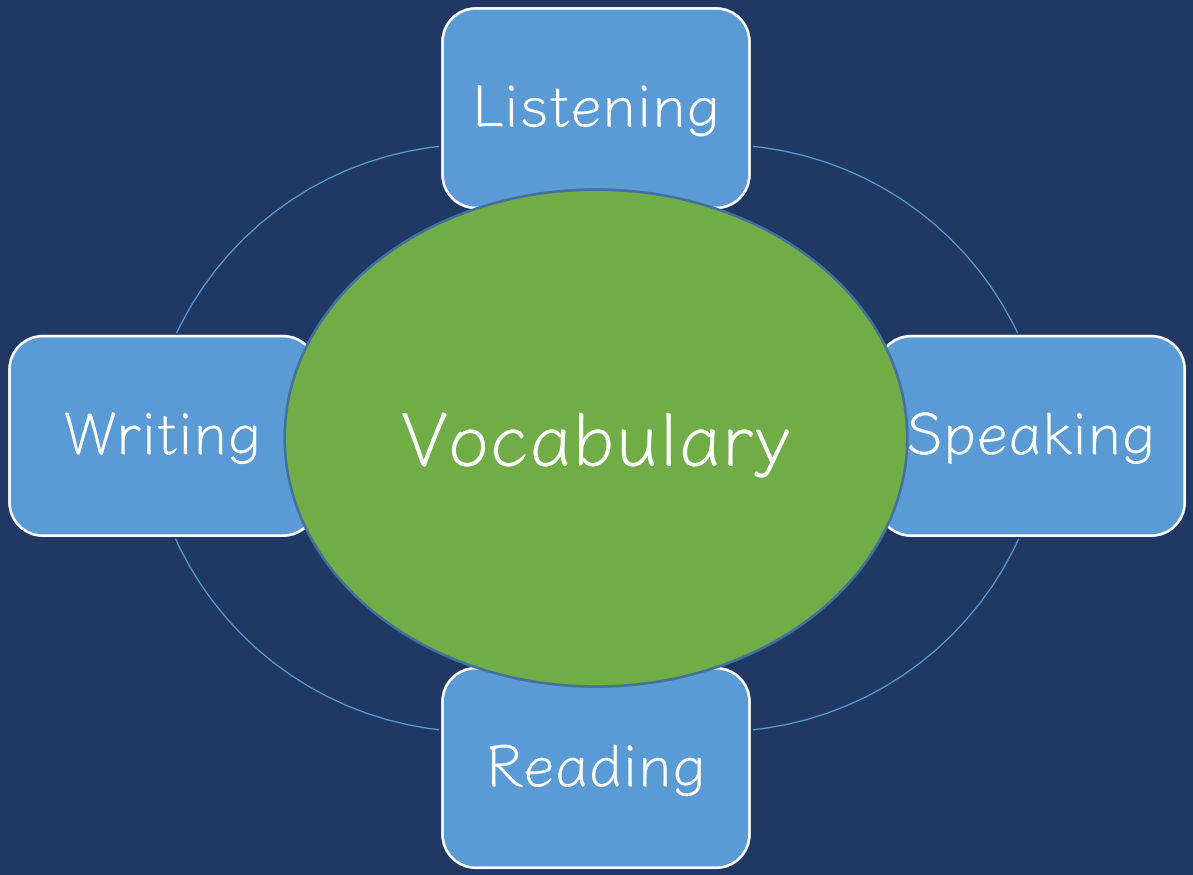
Mathematical Language

Instead of that…	Say this…



Powell, Bos, & Lin (2019)



1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

divide vs. Continental Divide

variable vs. variably cloudy

Rubenstein & Thompson (2002)

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3. Some math terms are only used in math

4. Some math terms have more than one meaning

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6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)

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7. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

Rubenstein & Thompson (2002)

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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

mesa vs. tabla

Rubenstein & Thompson (2002)

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8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)

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6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

skip count vs. multiples

9. English spelling and usage may have irregularities

10. Some math concepts are verbalized in more than one way

one-fourth vs. one quarter

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

rhombus vs. diamond

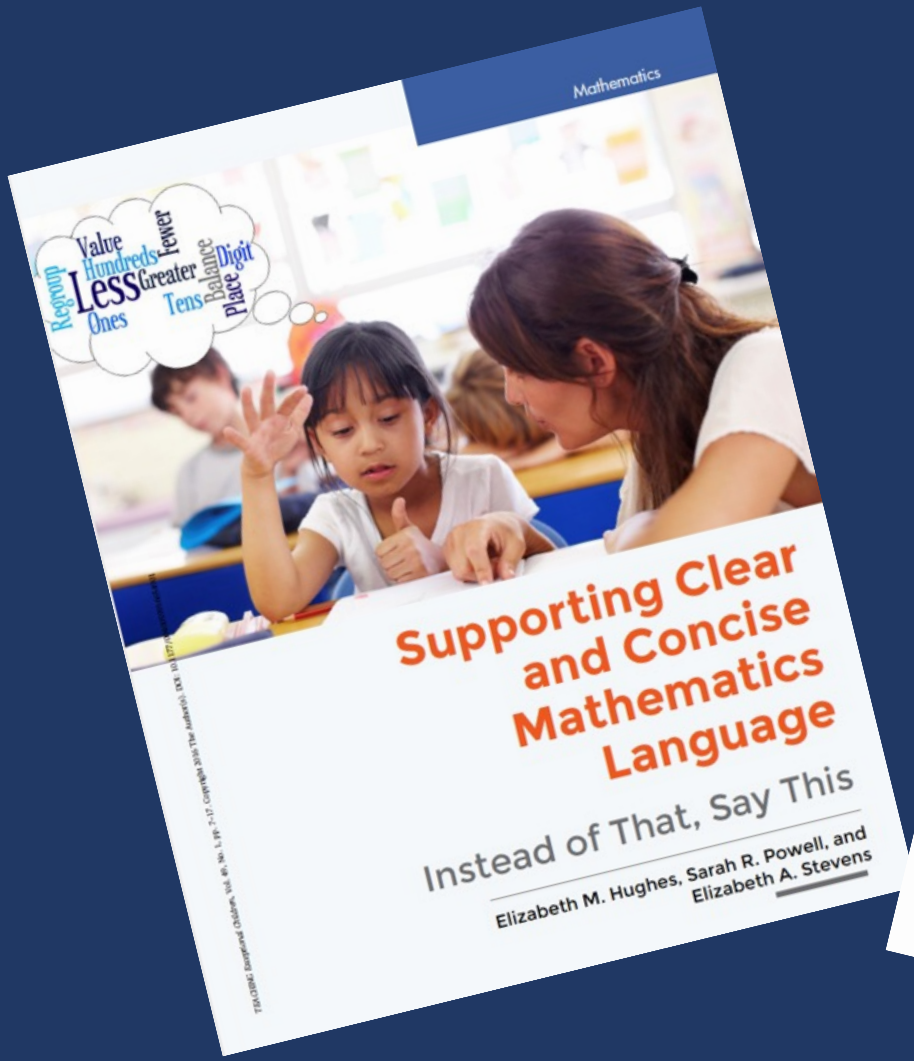
10. Some math concepts are verbalized in more than one way

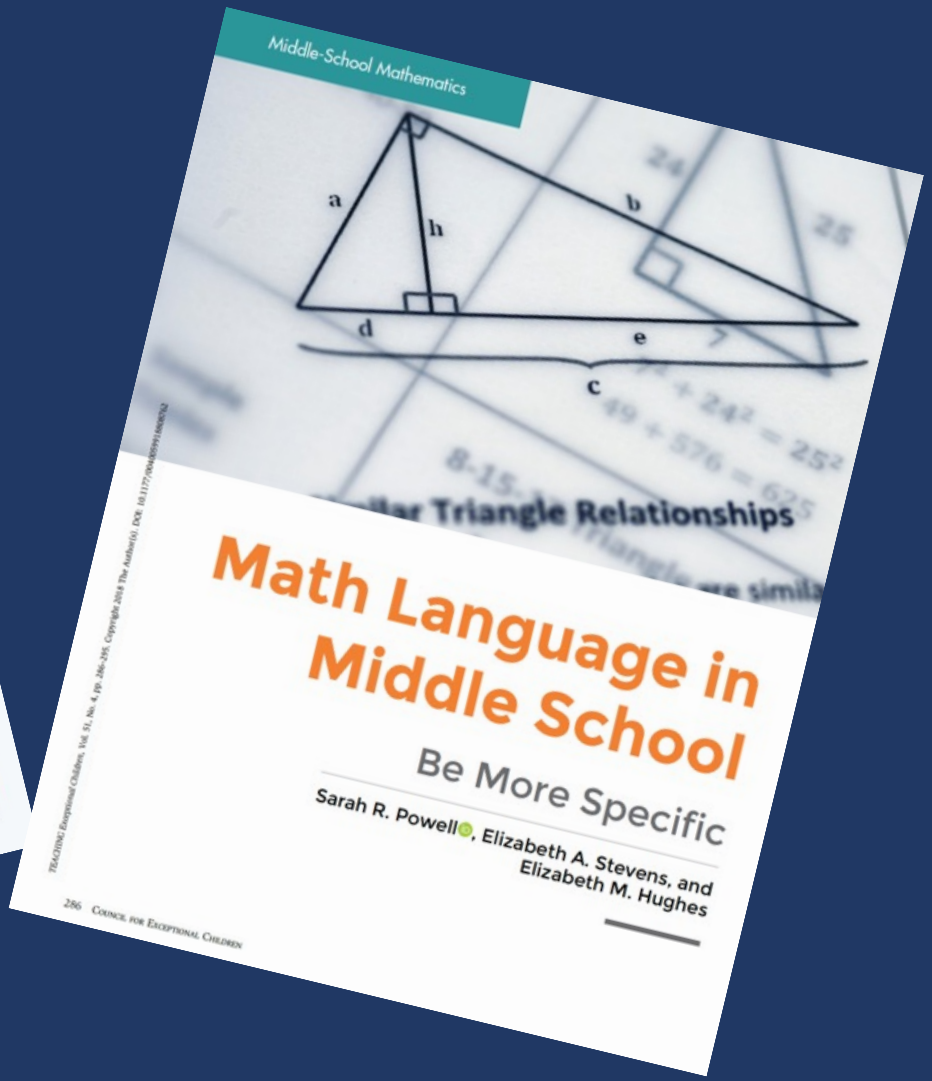
vertex vs. corner

11. Informal terms may be used for formal math terms

Use formal math language

Use terms precisely





What number is in the tens place?

What digit is in the tens place?
What is the value of the digit in the tens place?

135

Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.

The alligator eats the bigger number

is less than
OR
is greater than

Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.

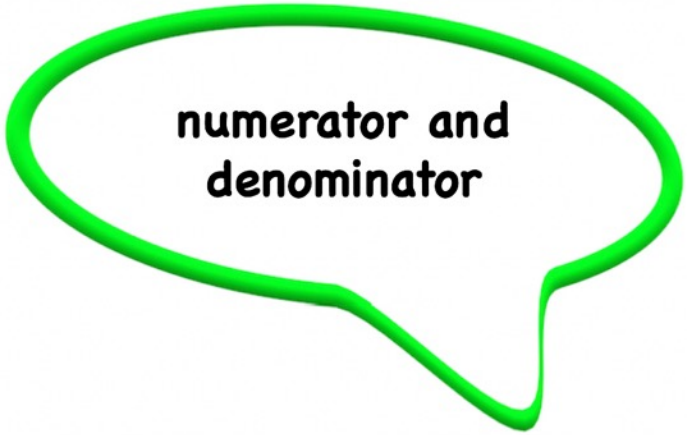
carry OR borrow

regroup OR trade OR exchange

$$\begin{array}{r} 167 \\ +\ 294 \\ \hline \end{array}$$

Why this is important...

- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.

reduce the fraction

rename OR
find equivalent OR
simplify

Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.

Four point seven
Four point oh seven

Four and seven tenths
Four and seven hundredths

4.7

4.07

Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.

box OR ball

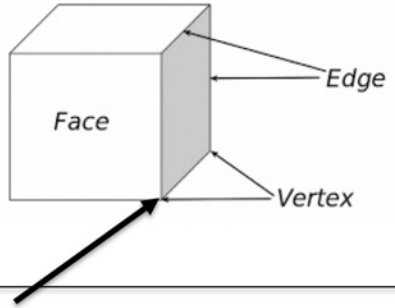
square OR circle

Why this is important...

- Use the formal language of shapes to confirm informal language.

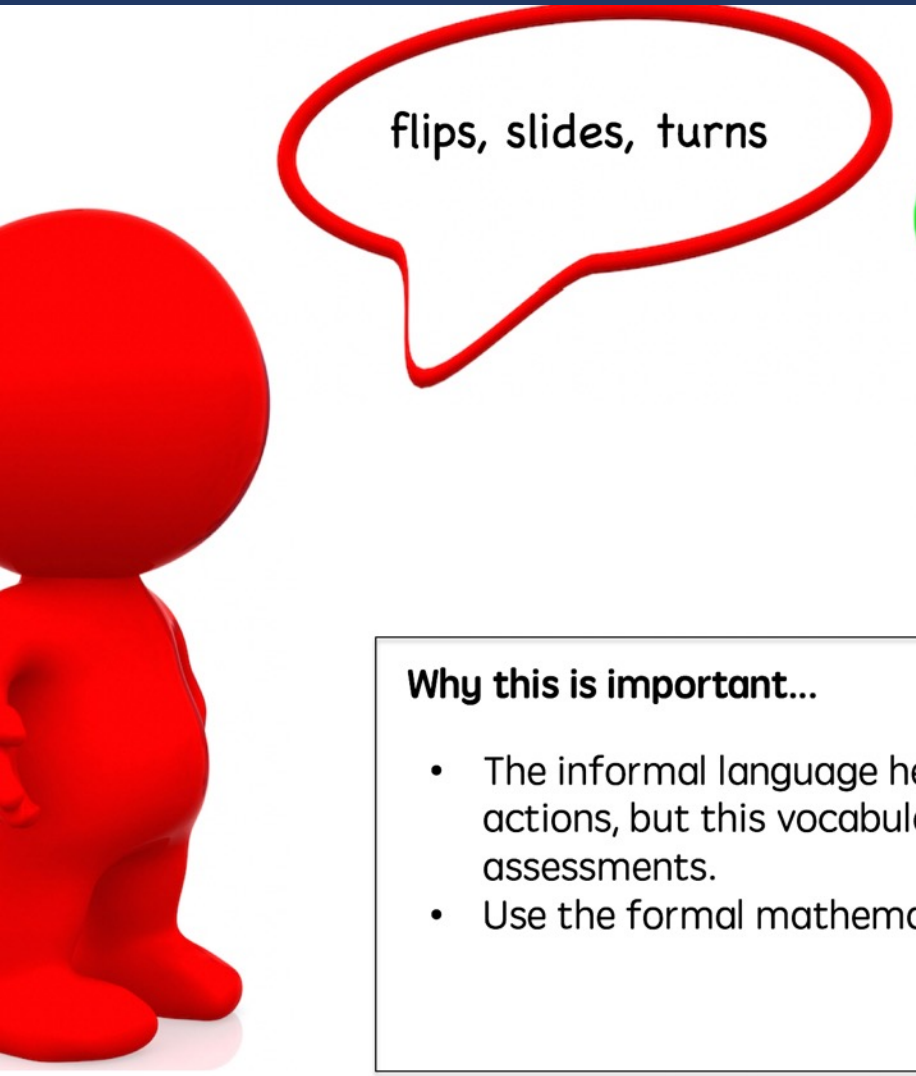
point

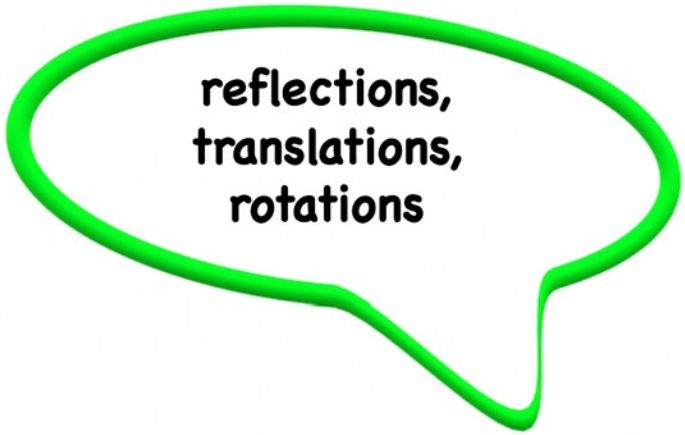
vertex



Why this is important...

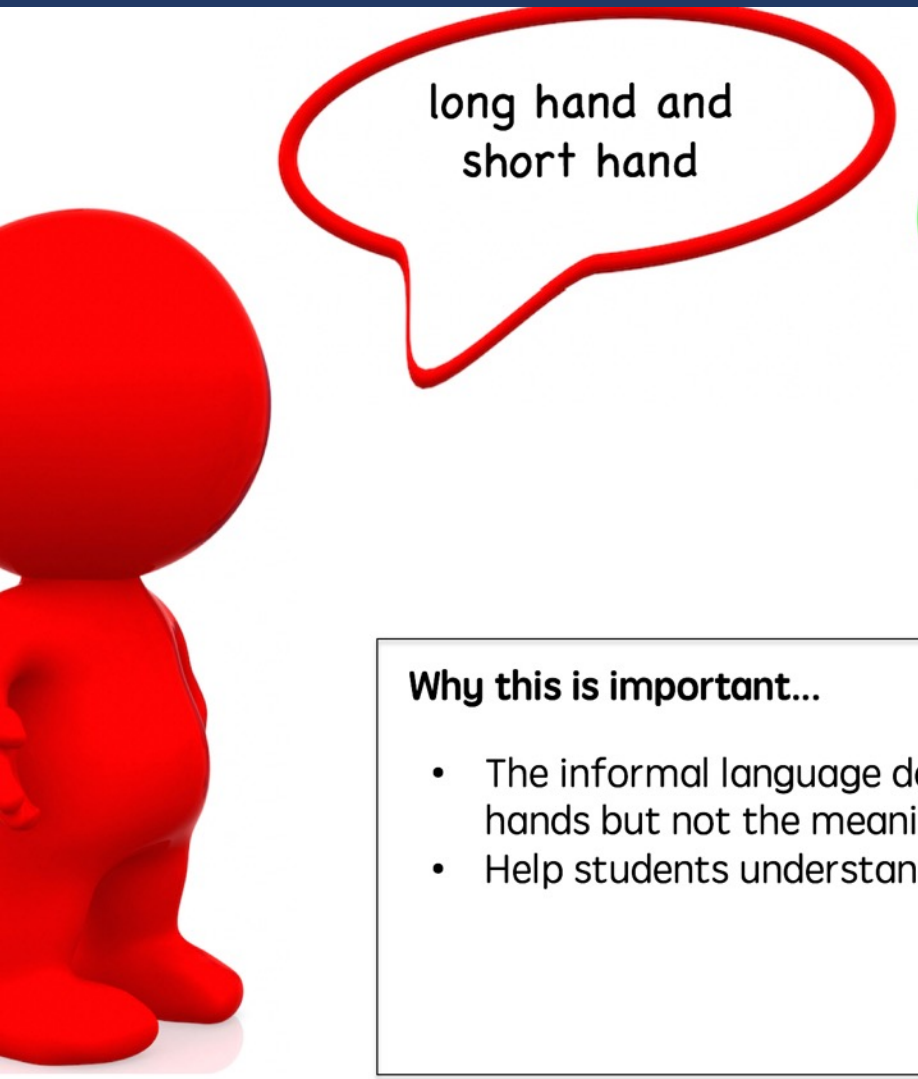
- This is the endpoint where two or more line segments or rays meet.





Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.



Why this is important...

- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.

Effective Mathematics Practices
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Mathematical Language

Instead of that…	Say this…

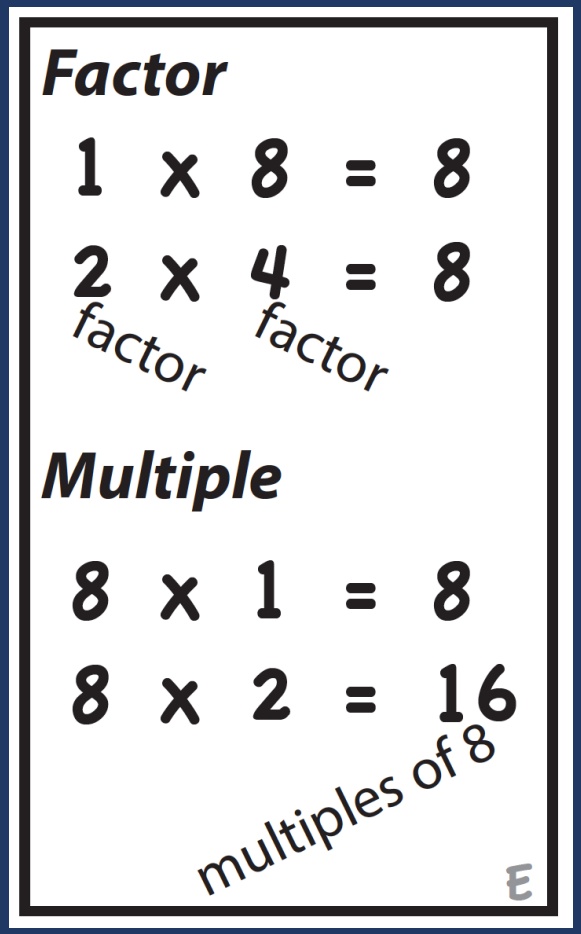
Identify examples of "Instead of ____, say ____."

Use formal math language

Use terms precisely

Use Terms With Precision

Strategies for Teaching Mathematics Language



Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

4:3

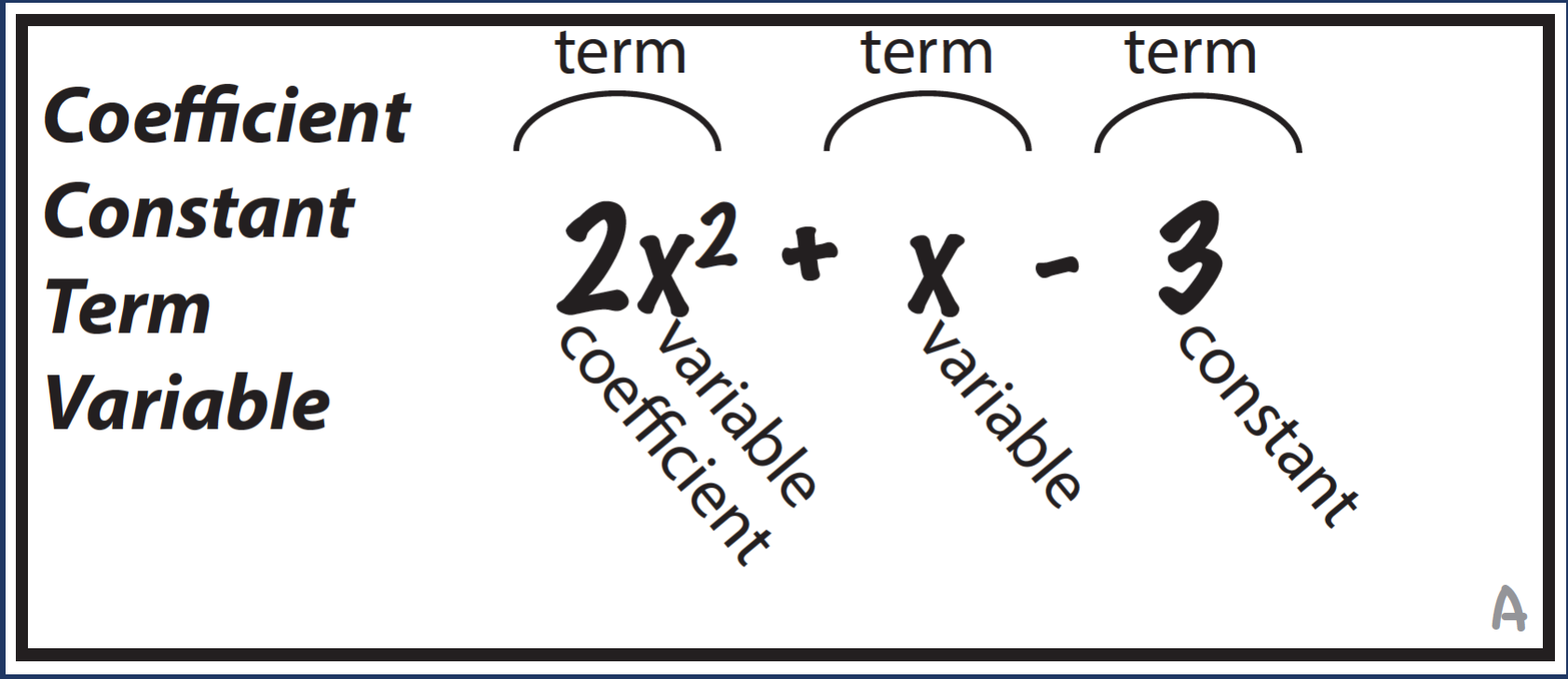
Proper fraction

$$\frac{2}{9}$$

Unit fraction

$$\frac{1}{6}$$

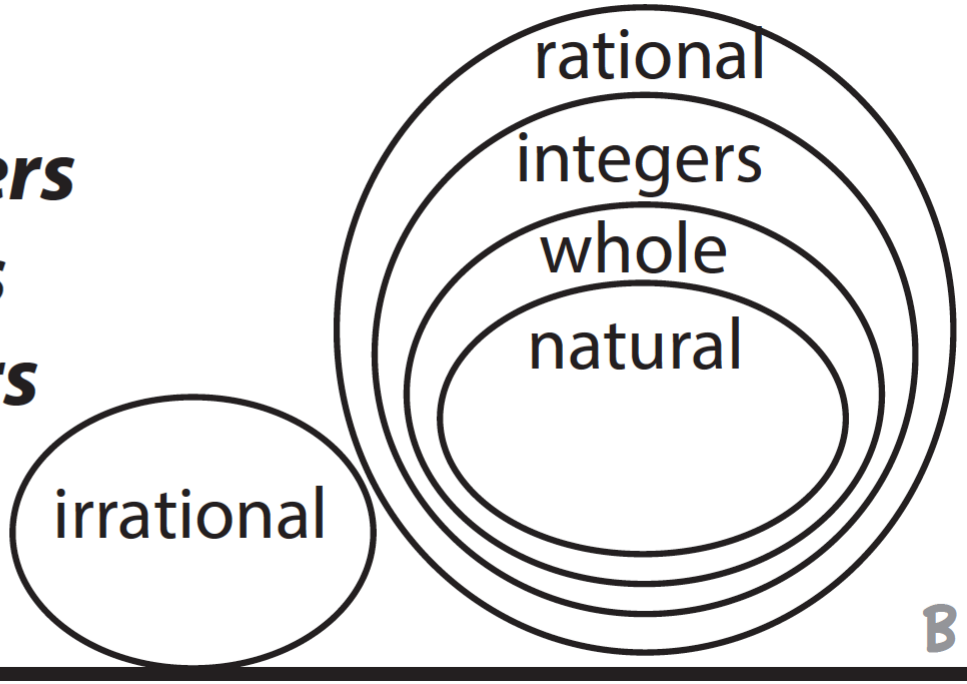
D



Equation	$9x - 4 = 7x$
Expression	$9x - 4$
Formula	$a^2 + b^2 = c^2$
Function	$f(x)$
Inequality	$9x - 4 > 6x$

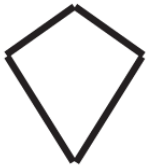
C

Integers
Irrational numbers
Natural numbers
Rational numbers
Whole numbers



Quadrilaterals

Kite



Rhombus

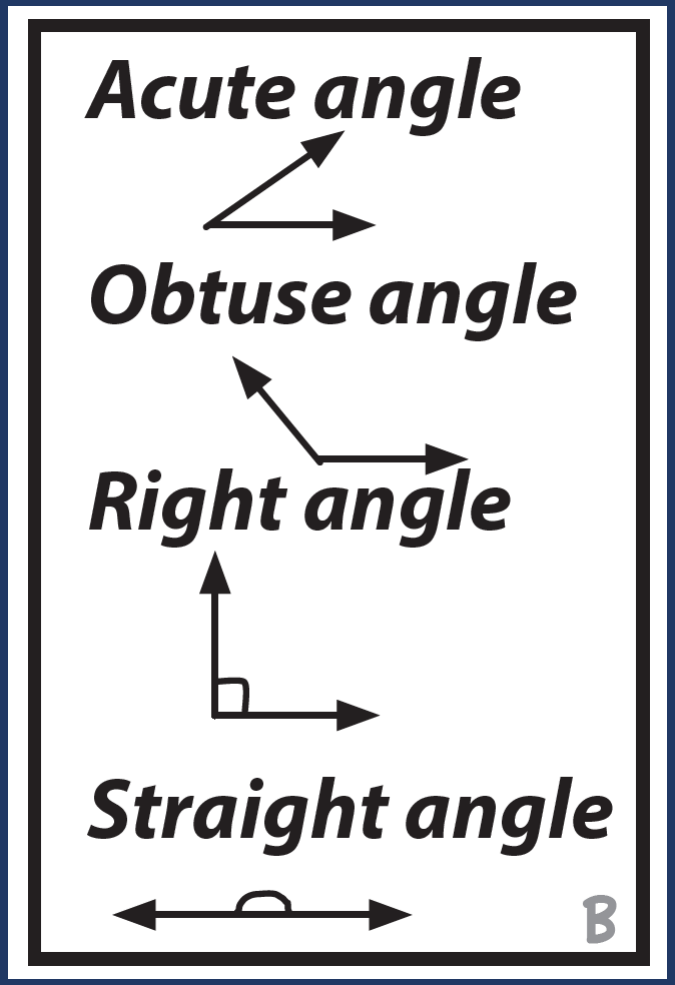
Parallelogram

Square

Rectangle

Trapezoid

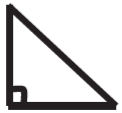
A



Acute triangle

Obtuse triangle

Right triangle



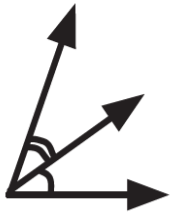
Equilateral triangle

Isosceles triangle

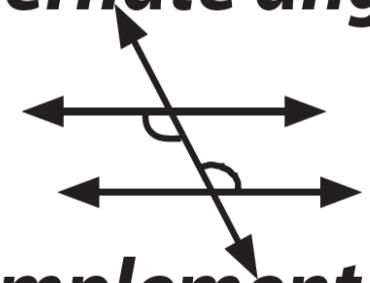
Scalene triangle

C

Adjacent angles

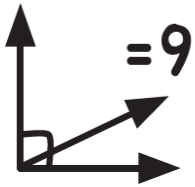


Alternate angles

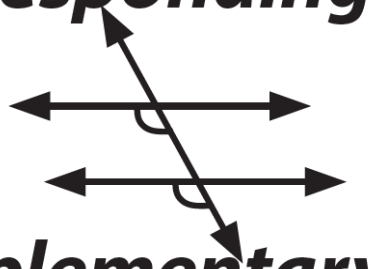


Complementary angles

=90°



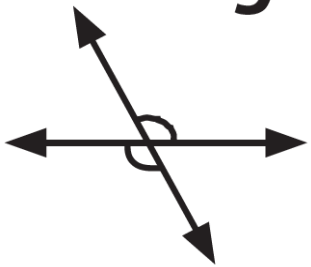
Corresponding angles



Supplementary angles

=180°

Vertical angles

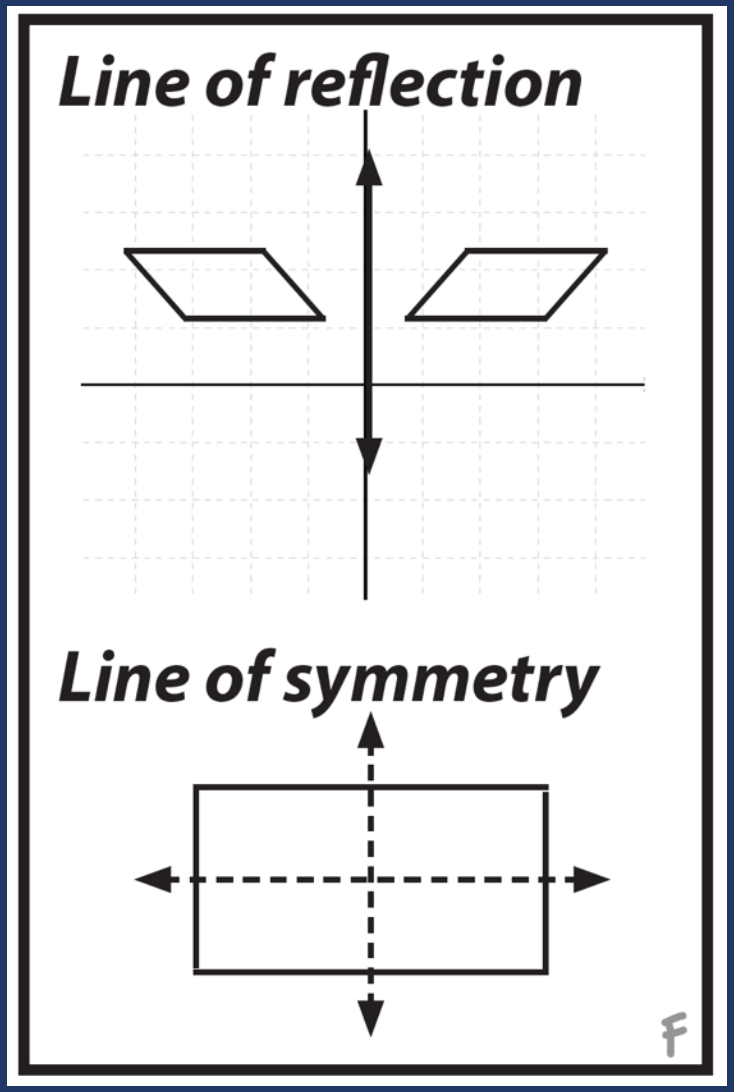


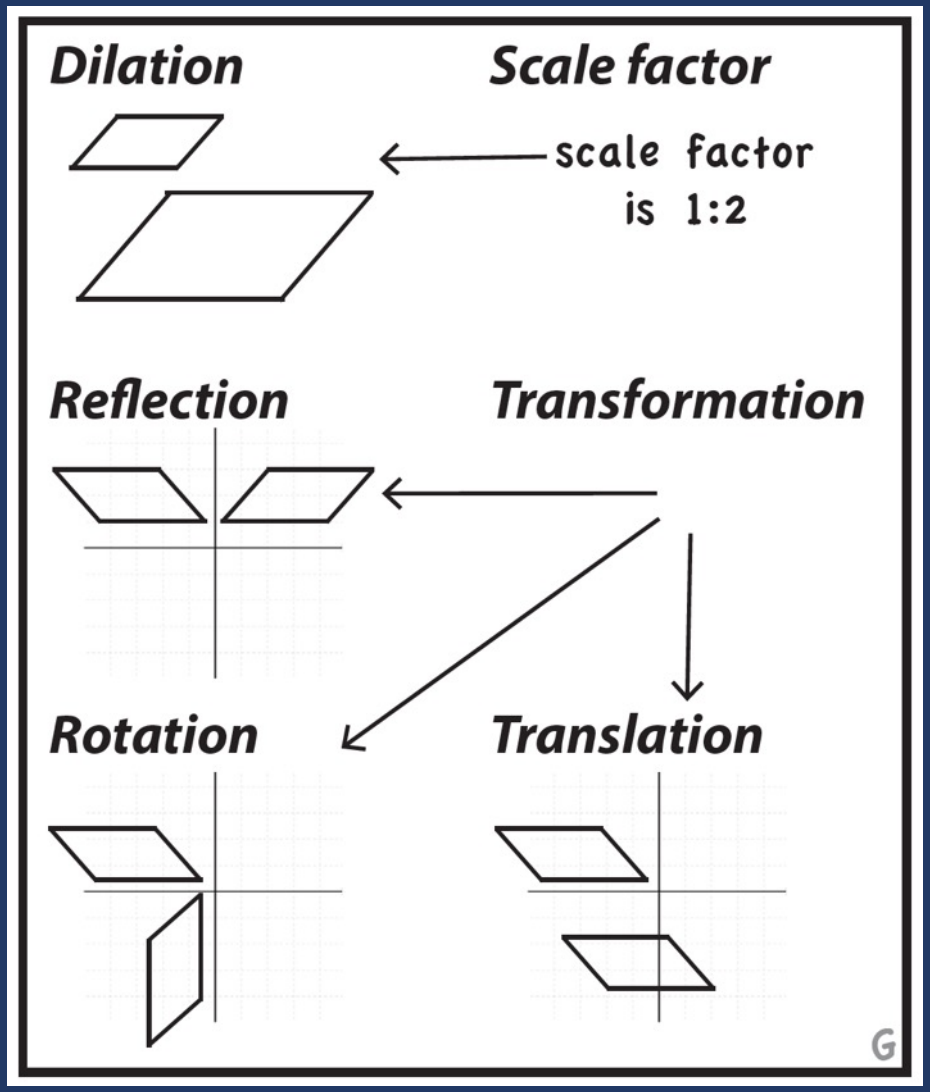
D

Congruent figures

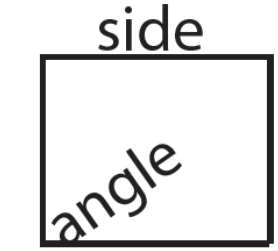
Similar figures

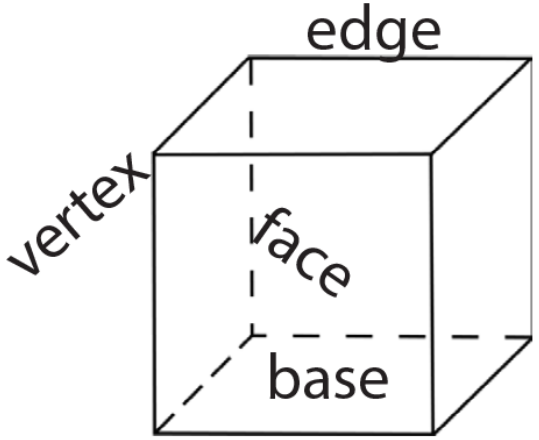
E

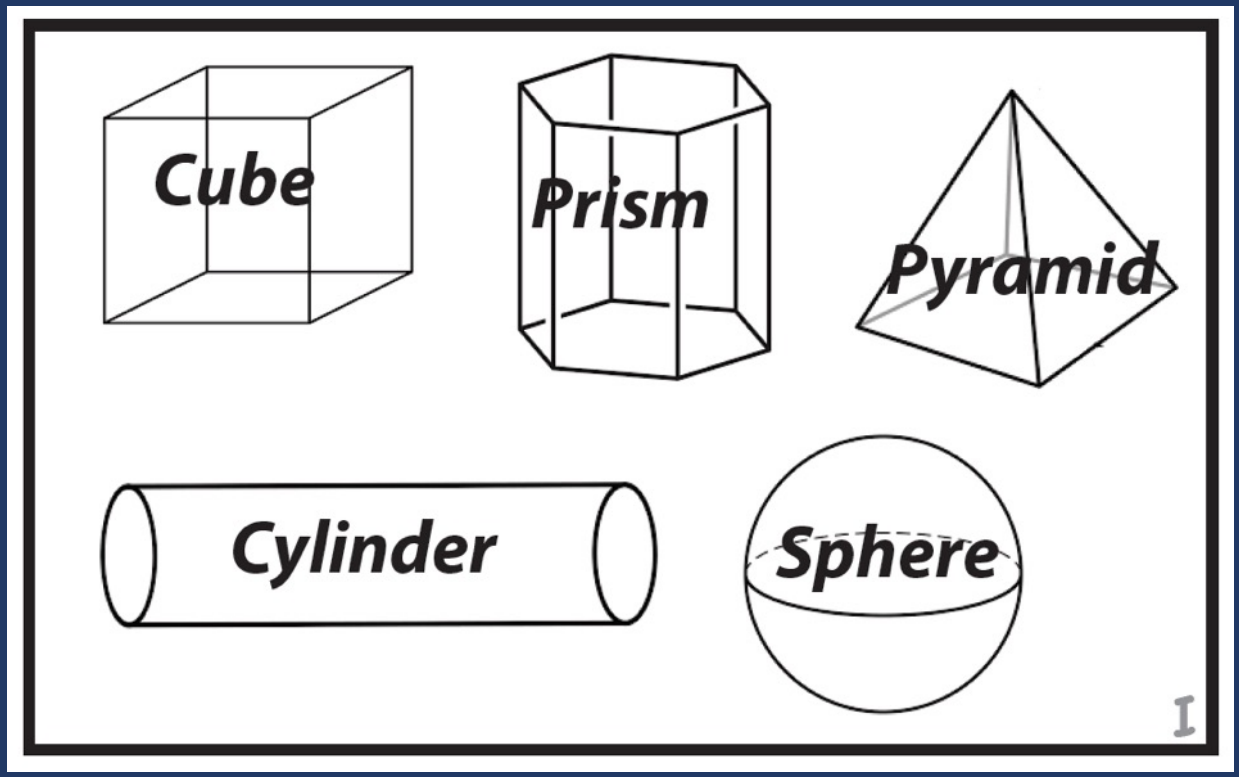




Angle
Base
Edge
Face
Side
Vertex







Coordinate plane

Ordered pair

Quadrants

x

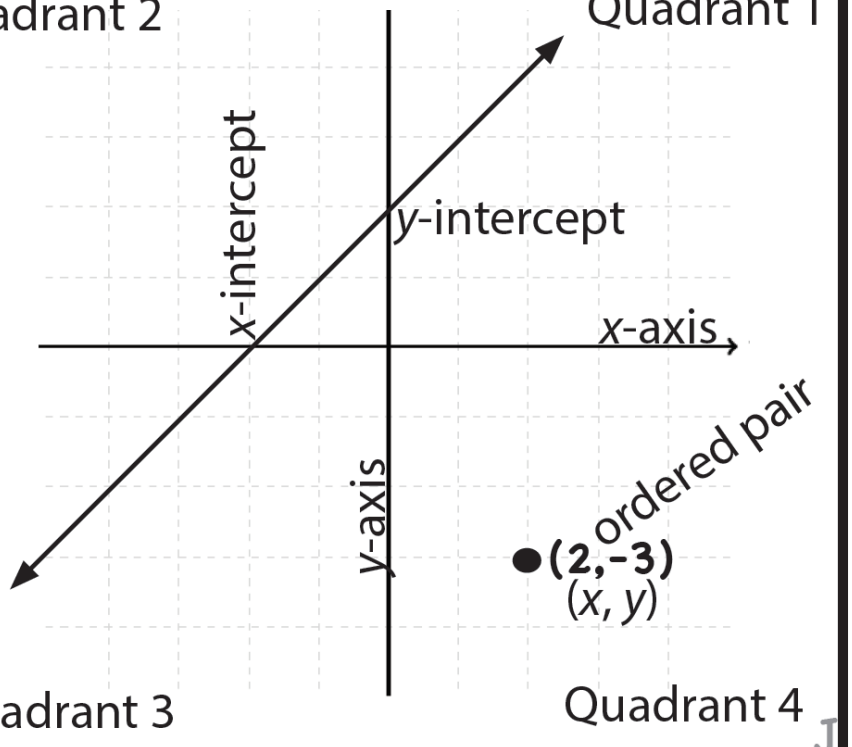
x-axis

x-intercept

y

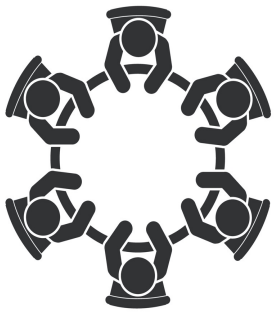
y-axis

y-intercept



Use Terms With Precision

Strategies for Teaching Mathematics Language



Discuss terms you want your students to use with precision.

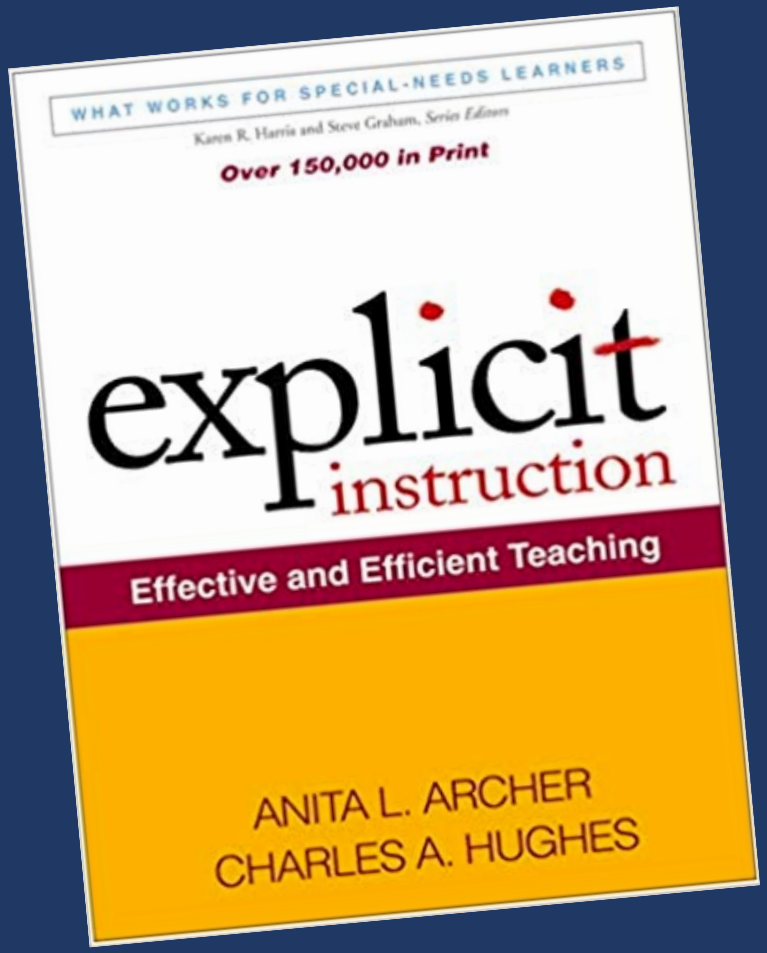
Use formal math language

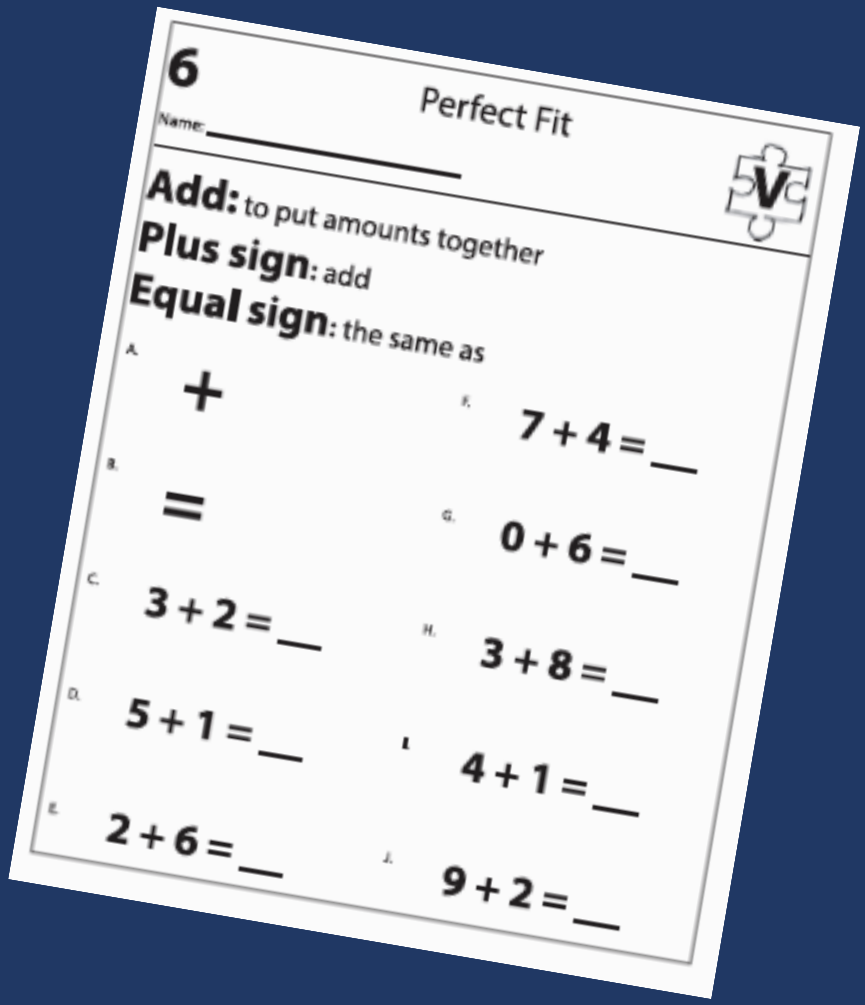
Use terms precisely

Use Terms With Precision

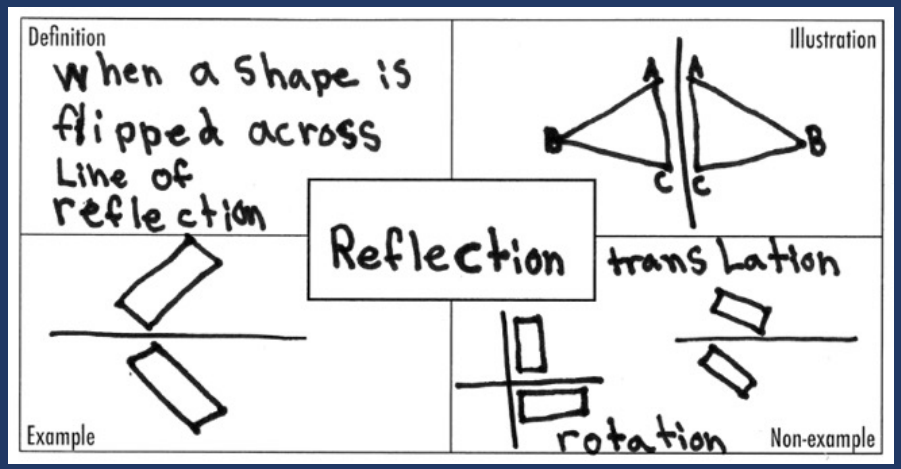
Strategies for Teaching Mathematics Language

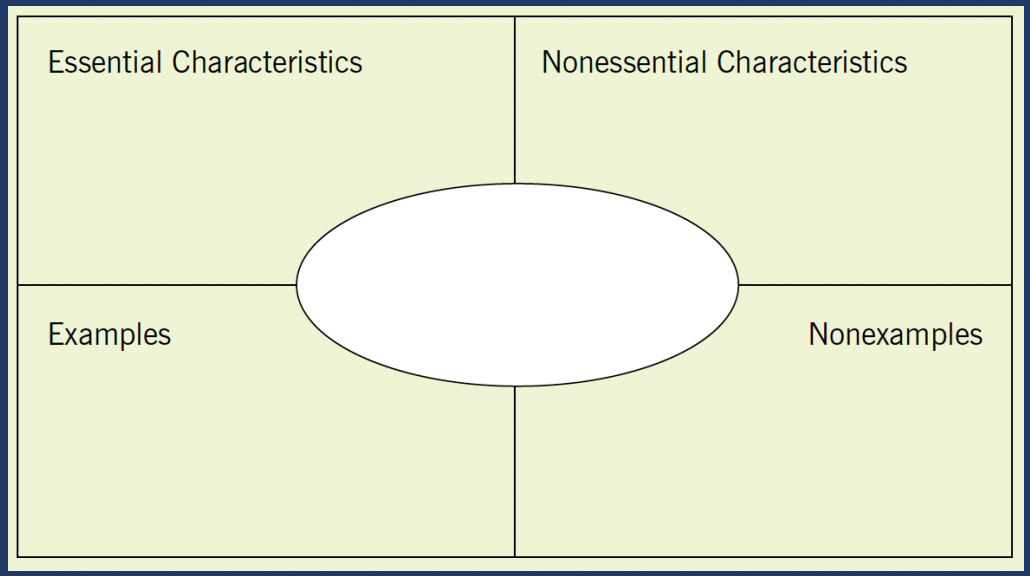
1. Use explicit instruction





2. Use graphic organizers





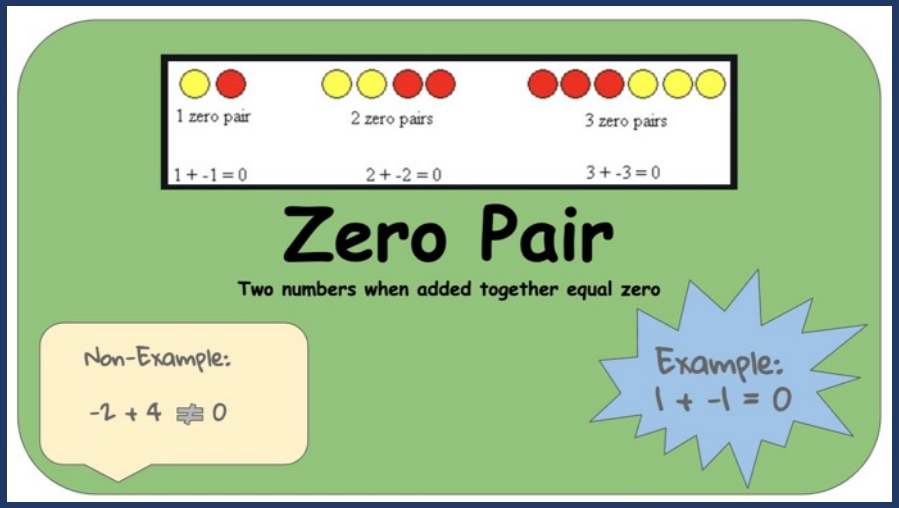
Dunston & Tyminski (2013)

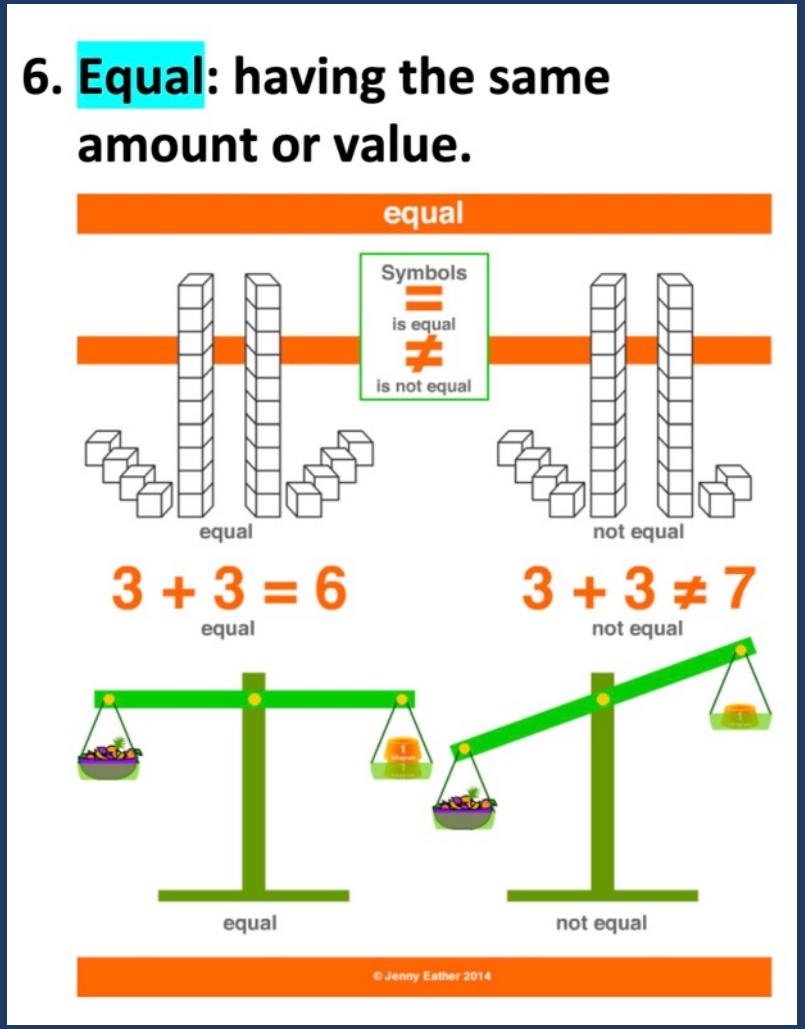
2. Use graphic organizers

Word	Lightbulb Word
Definition	Picture

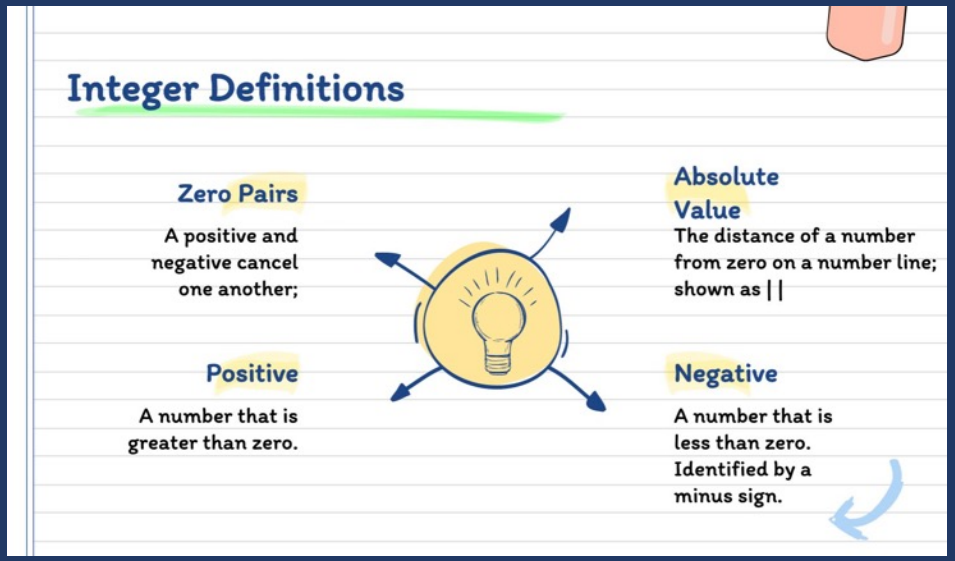
Dunston & Tyminski (2013)

3. Have students create vocabulary cards





4. Have students create glossaries



Numerator: how many parts of the whole

- Ex. $\frac{4}{10}$

Odd number: a number not divided evenly by 2

- Ex. 1, 3, 5, 7, 9....

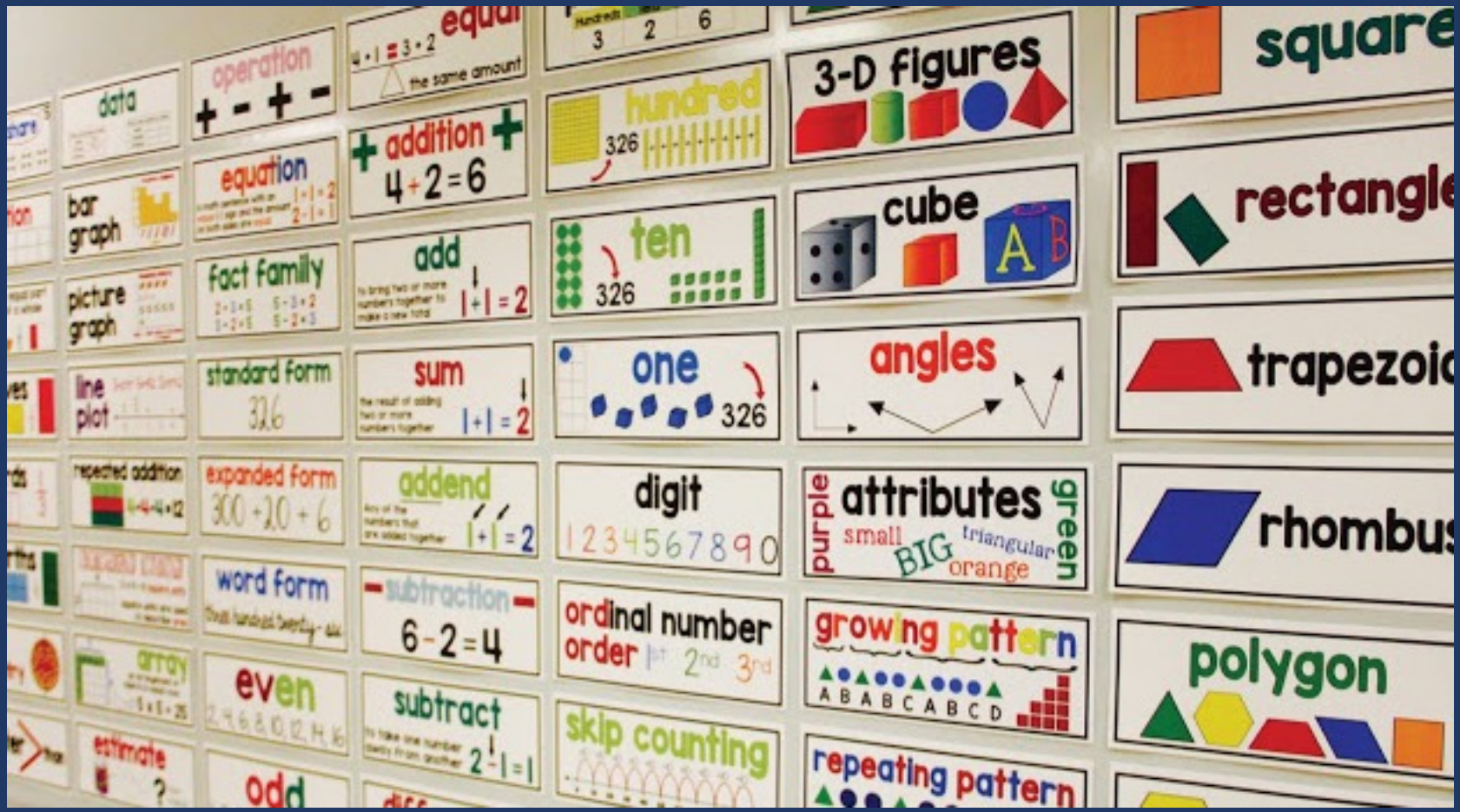
Percent: a specific number in comparison to 100

- 74%

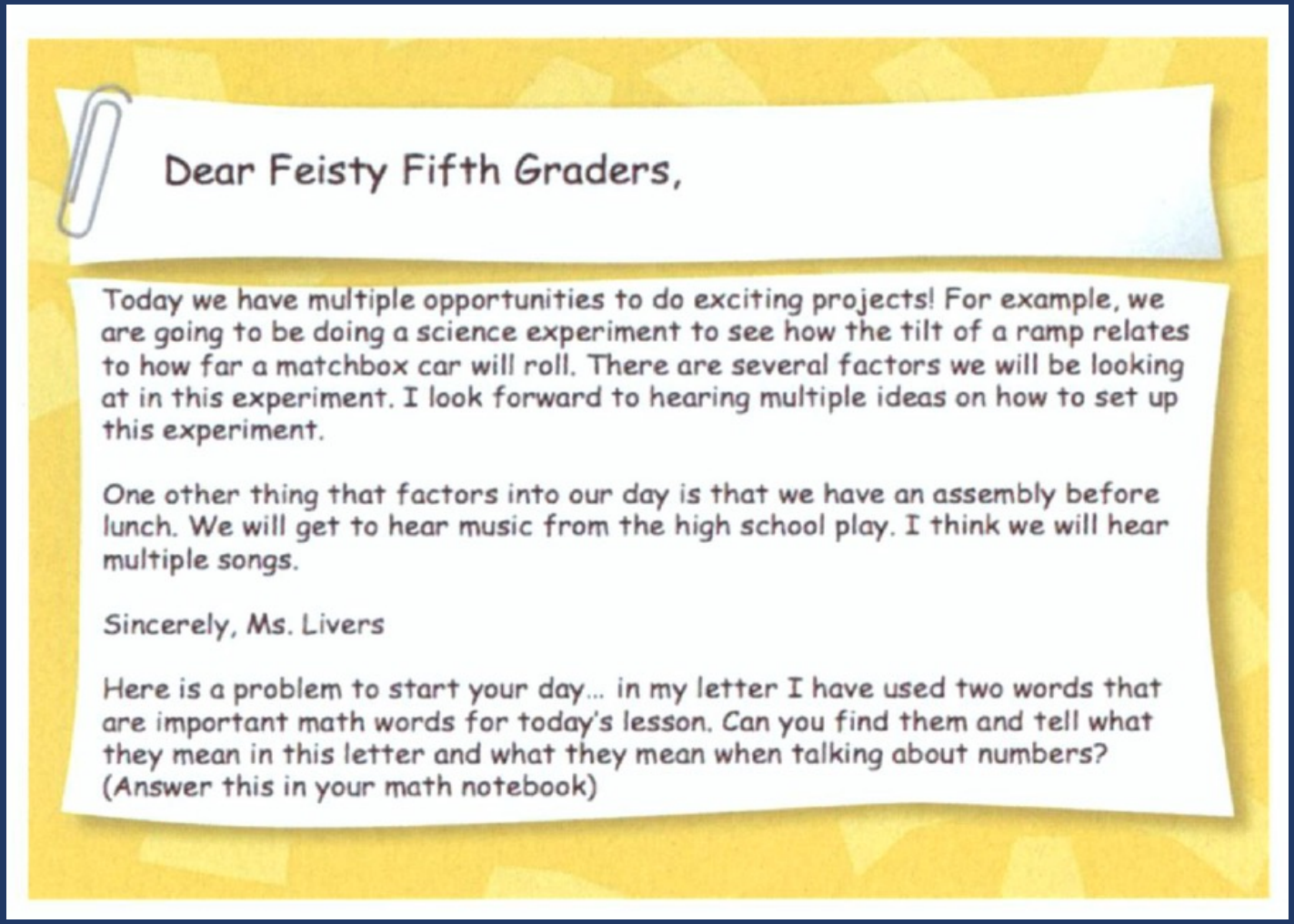
Polygon: any enclosed shape that is made up of 3 or more straight lines

- Ex. 

5. Create a word wall



6. Preview vocabulary



Dear Feisty Fifth Graders,

Today we have multiple opportunities to do exciting projects! For example, we are going to be doing a science experiment to see how the tilt of a ramp relates to how far a matchbox car will roll. There are several factors we will be looking at in this experiment. I look forward to hearing multiple ideas on how to set up this experiment.

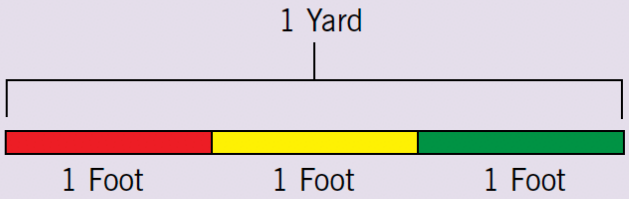
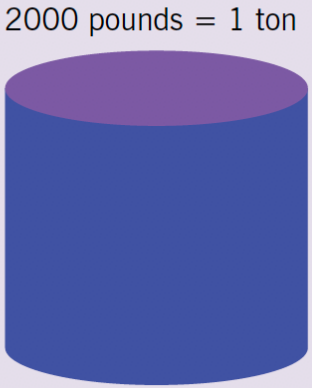
One other thing that factors into our day is that we have an assembly before lunch. We will get to hear music from the high school play. I think we will hear multiple songs.

Sincerely, Ms. Livers

Here is a problem to start your day... in my letter I have used two words that are important math words for today's lesson. Can you find them and tell what they mean in this letter and what they mean when talking about numbers? (Answer this in your math notebook)

Bay-Williams & Livers (2009)

7. Cluster vocabulary

	Length	Weight
Meaning	How long something is	How heavy something is
Visual		

Livers & Bay-Williams (2014)

7. Cluster vocabulary

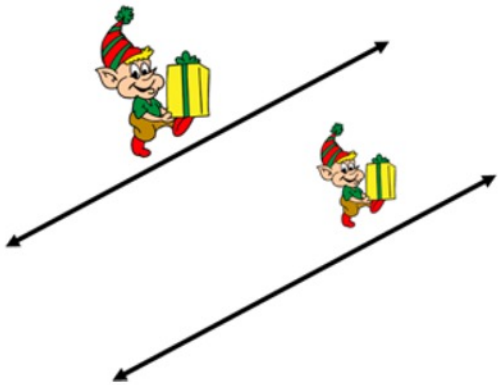
Rating	Word	Definition	Synonym(s)	Example	Sample Problem
2	expression	a mathematical phrase combining operations, numbers and/or variables.	phrase algebraic expression	6 6n 6+n (no equal sign)	Lucia earns $8 per hour for babysitting and gets a $5 tip. Write an expression to represent the amount she would earn if she worked for x hours.
2	variable	a quantity that can change or take many values. (refers to the letter or symbol representing the quantity)	unknown	x D y T	The variable x represents the number of hours Charlie works in a week. Write an expression to represent his earnings if he earns $9 per hour.
1	product	the result when two or more numbers are multiplied	total answer	3 x 2 = 6 (product)	The product of 6 and a number is 24. What is the number?
3	quotient	the result of a division (refers to the number of times the divisor divides the dividend)	answer	18 ÷ 2 = 9 (quotient) 9 ← 2⟌18	Estimate the quotient when 365 is divided by 12.

Marin (2018)

8. Use mnemonics

Parallel Lines (Pair of Elves)

Lines that are the same distance apart and will never intersect



The Pair of Elves are the same distance apart and will never intersect.

The Pair of Elves are on Parallel Lines

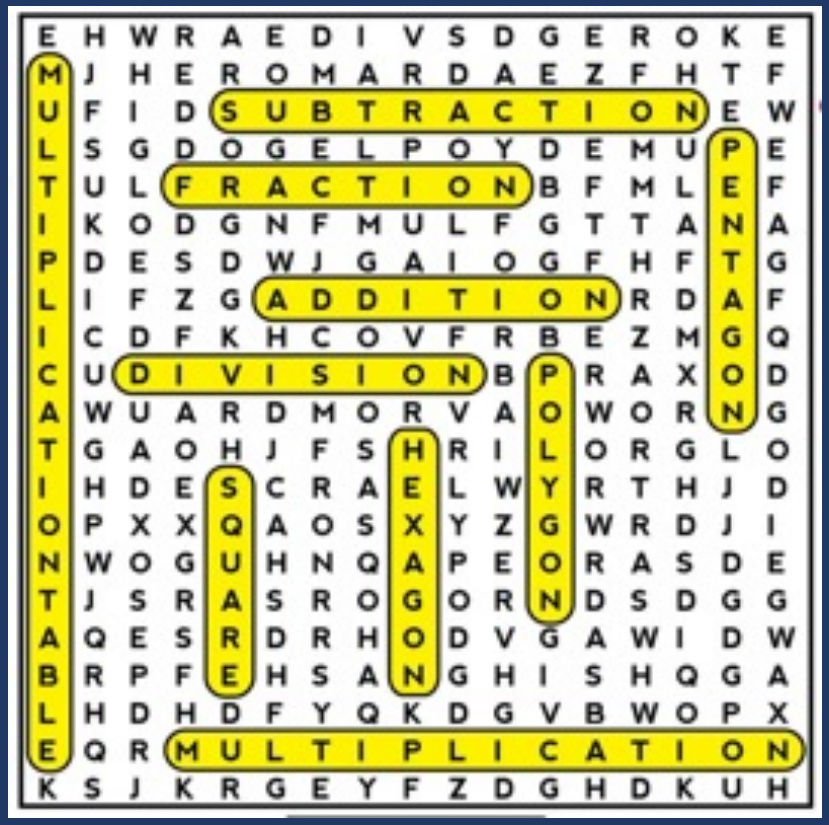
Ray (Run Away)

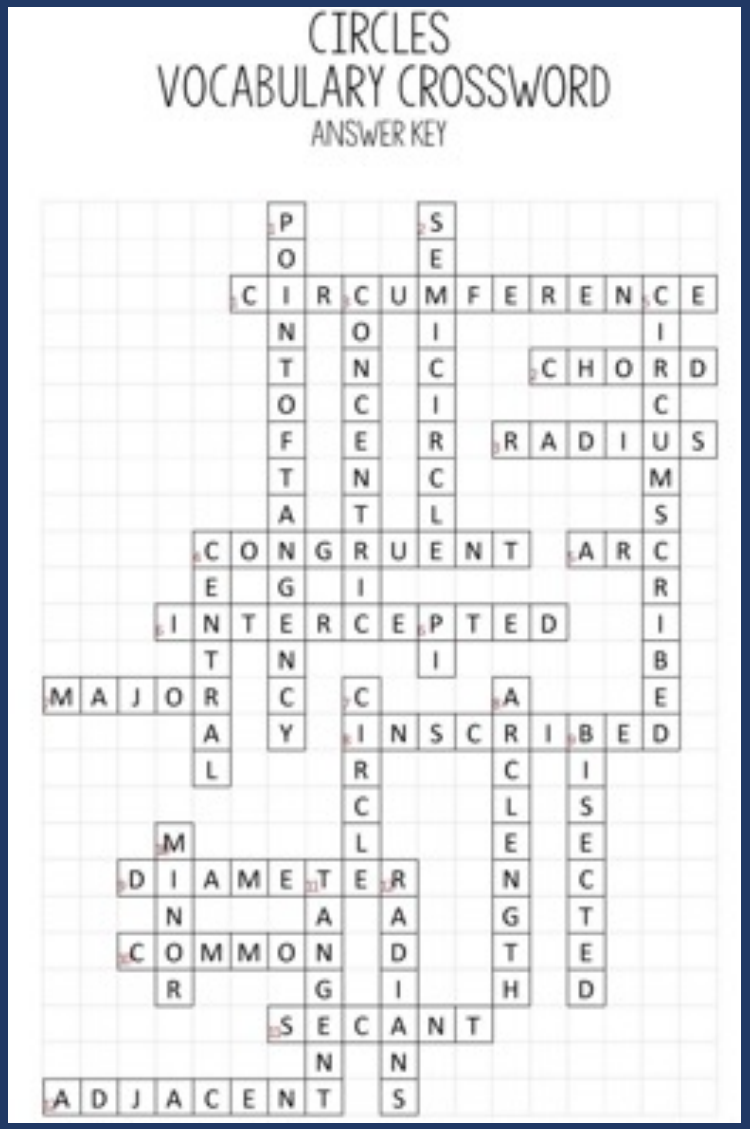
A line that has a starting point but no endpoint

Start here!! Run away and never stop running Ray.

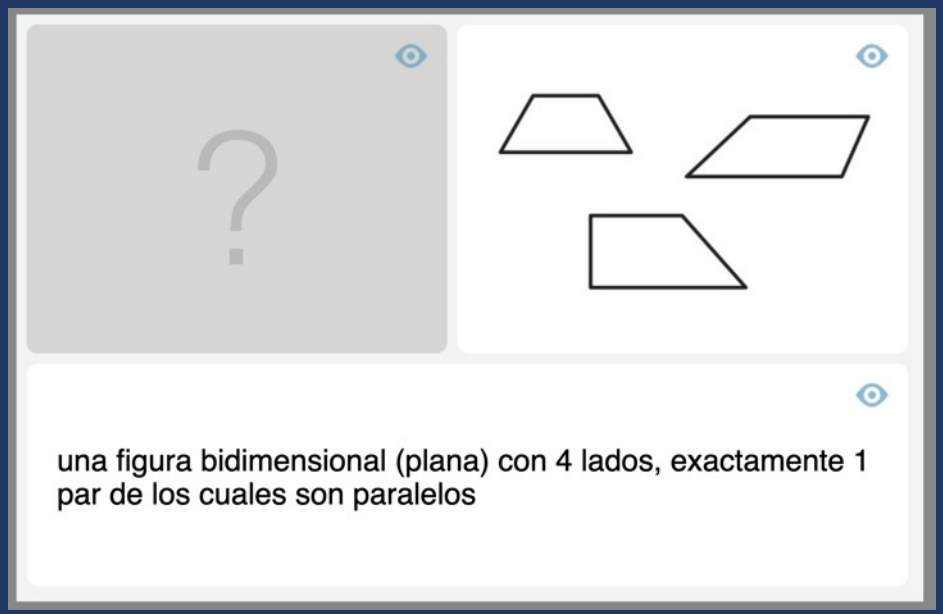
Riccomini et al. (2015)

9. Do word games





10. Use technology



Math Learning Center

Math Lingo

Use Terms With Precision

Strategies for Teaching Mathematics Language

Discuss your strategy for focusing on mathematical language in your teaching.

Word-Problem Solving

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

How Students Solve Word Problems

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

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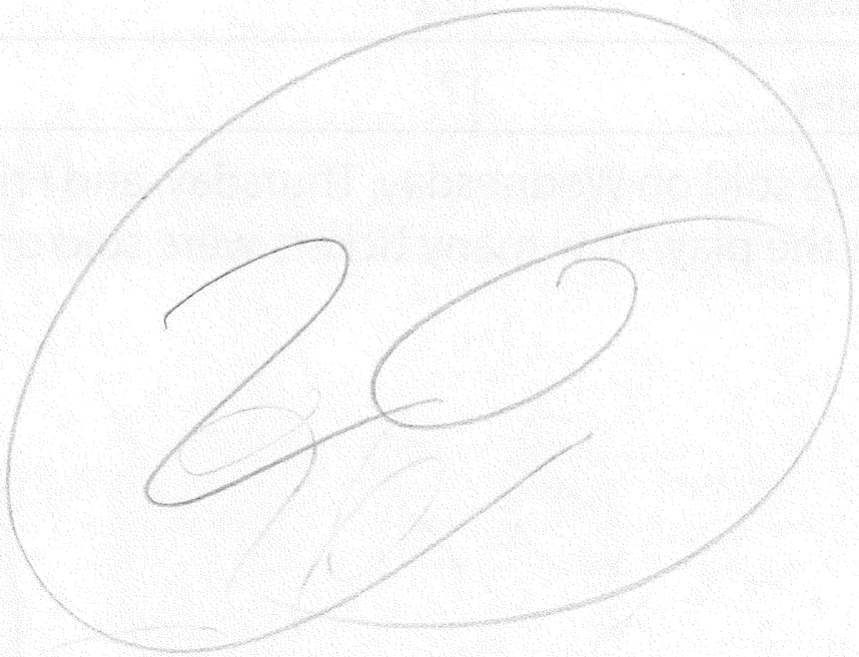

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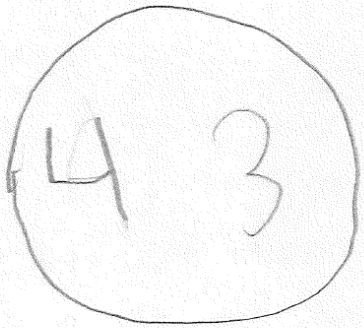

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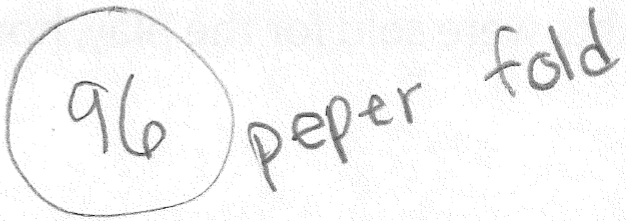

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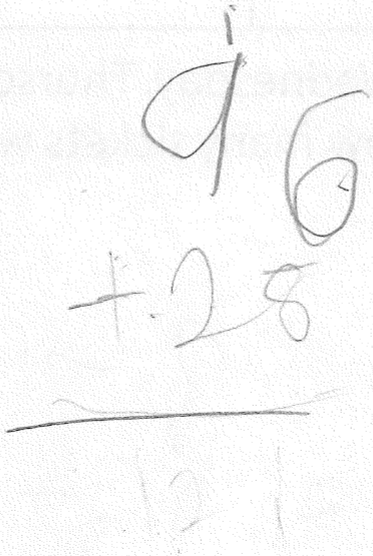

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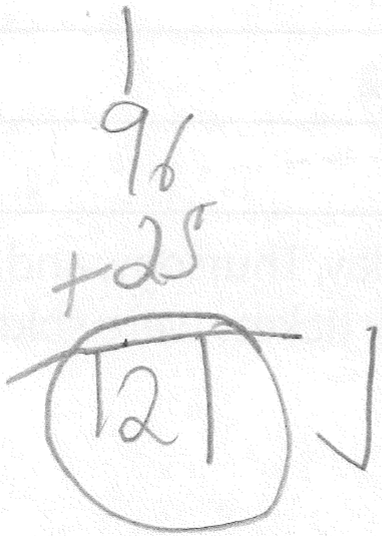

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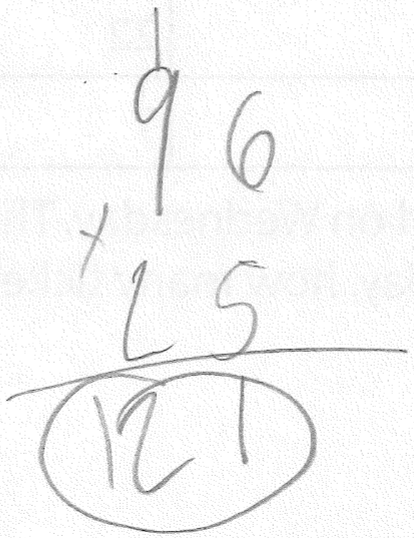

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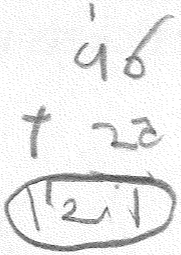

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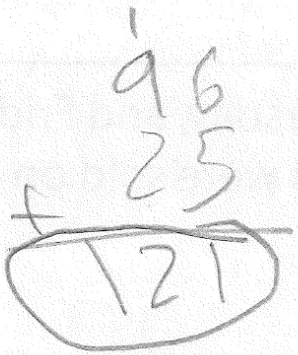

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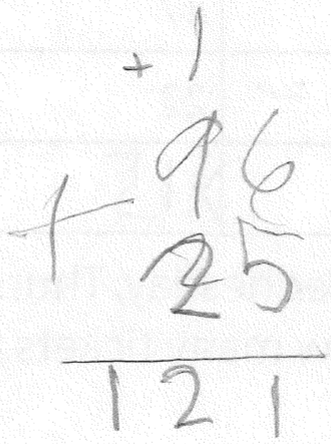

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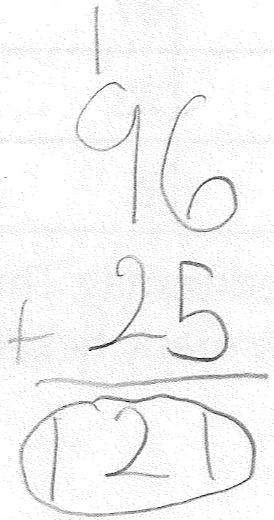

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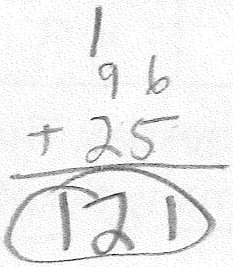

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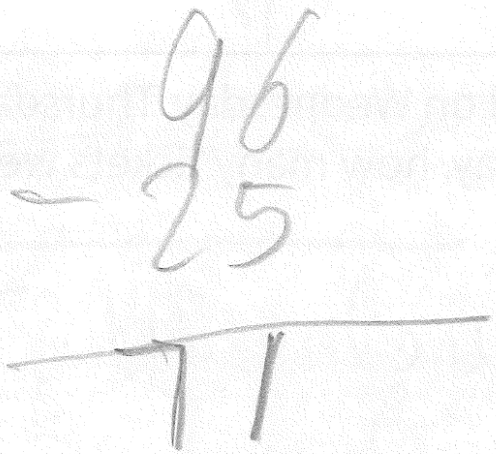


Correct

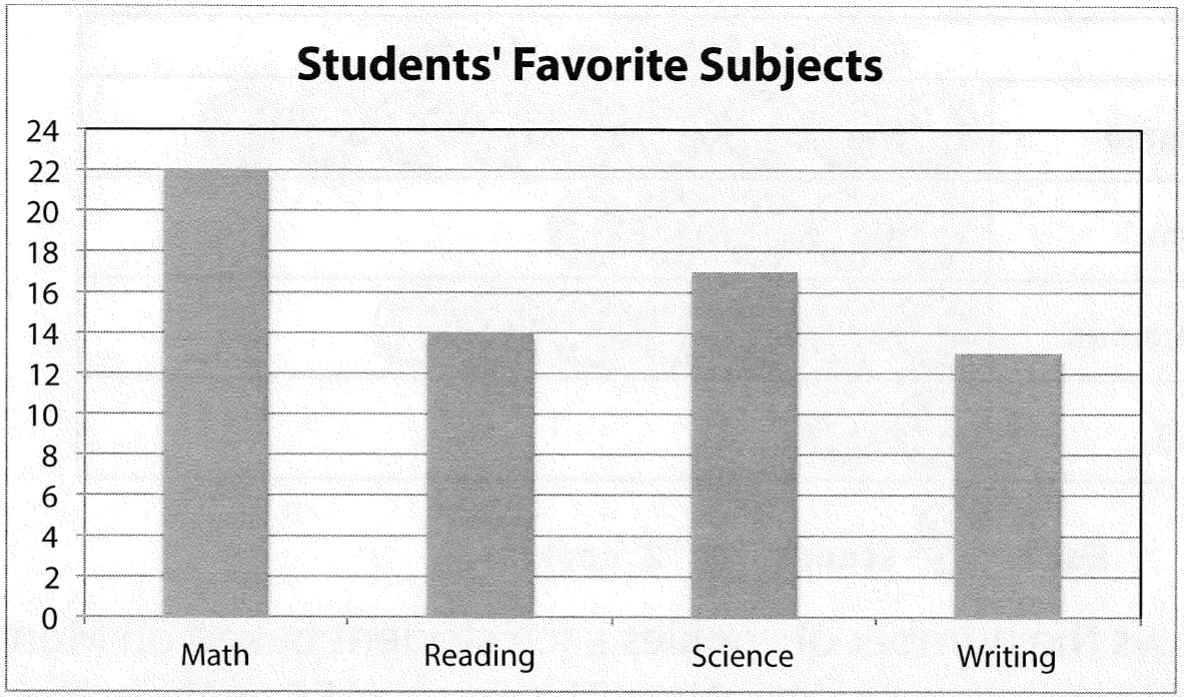
Undetermined

Repeated information

Wrong information

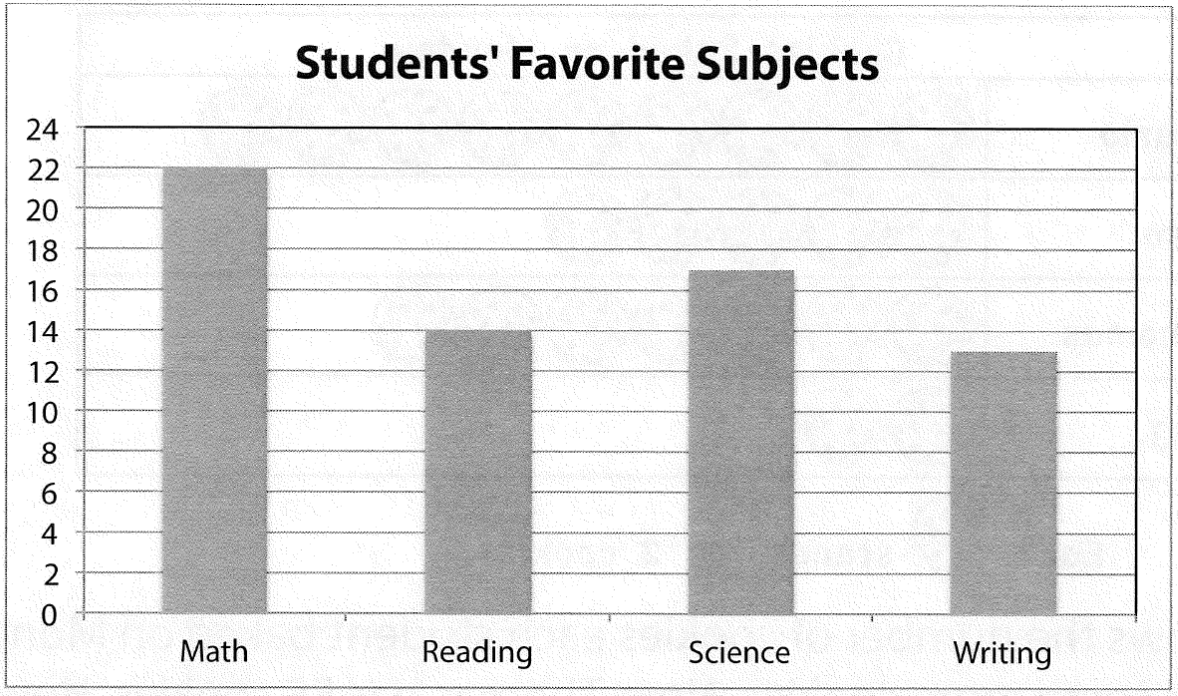
Wrong (misapplied) operation

J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

J.



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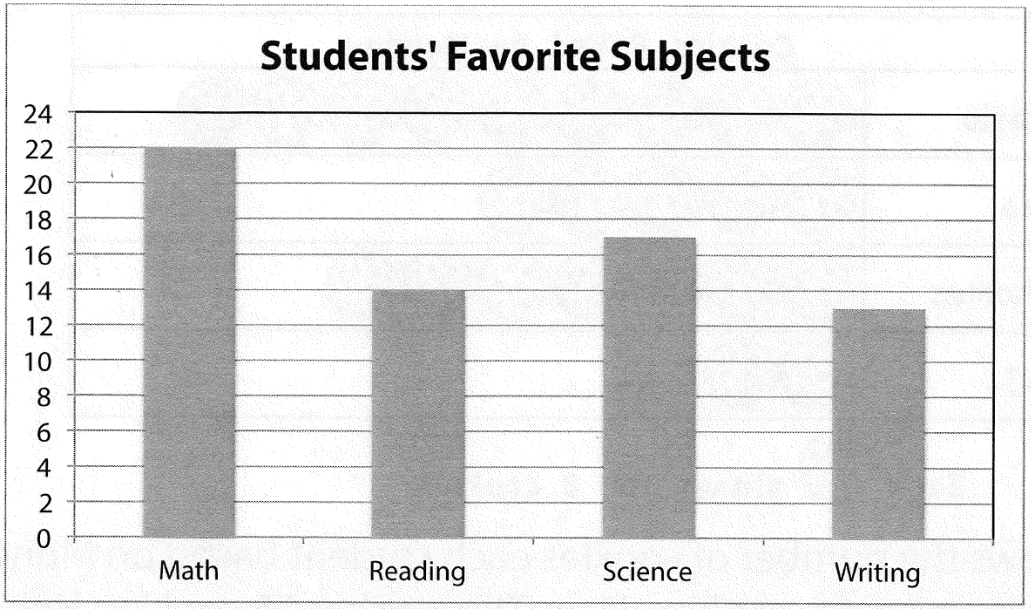
60

J.



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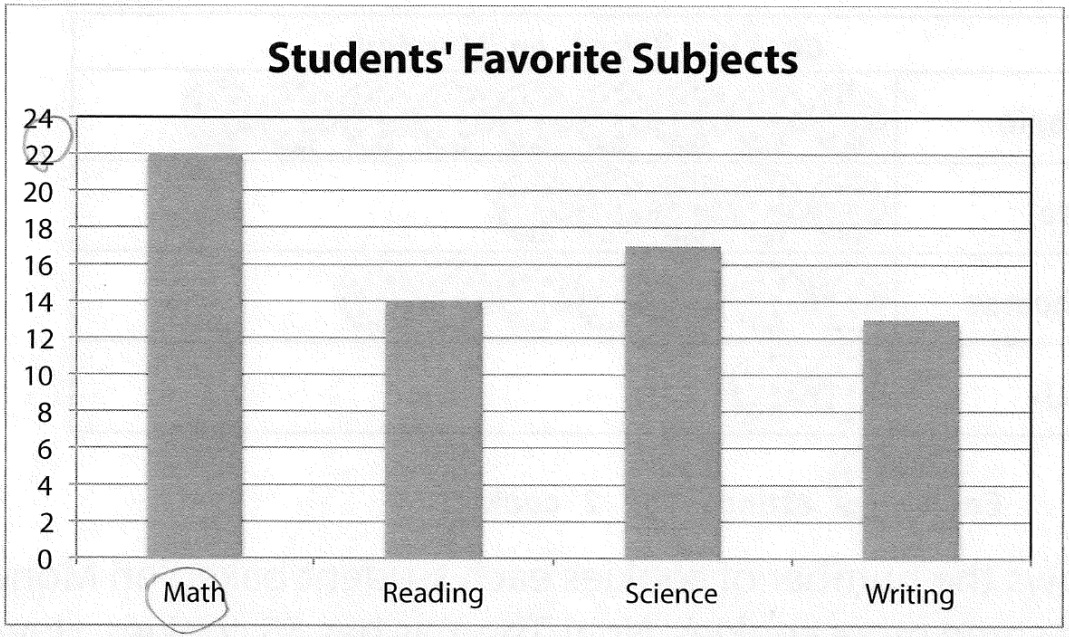
they choose reading 8% more than

J.



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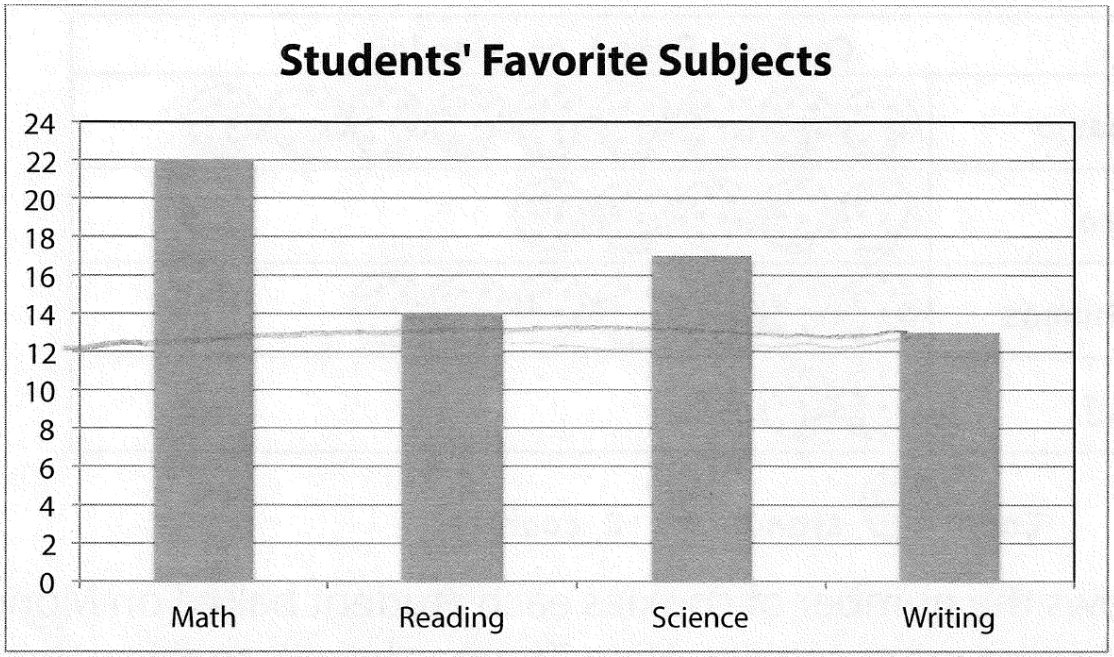
22
Math

J.



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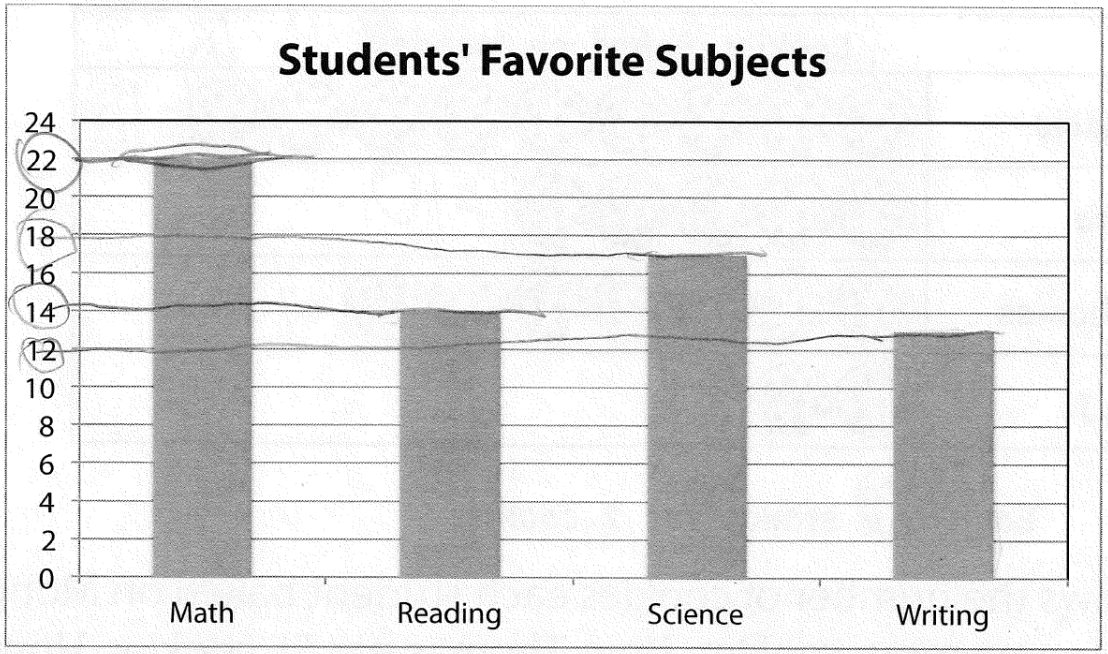
22 math

12 writing

J.



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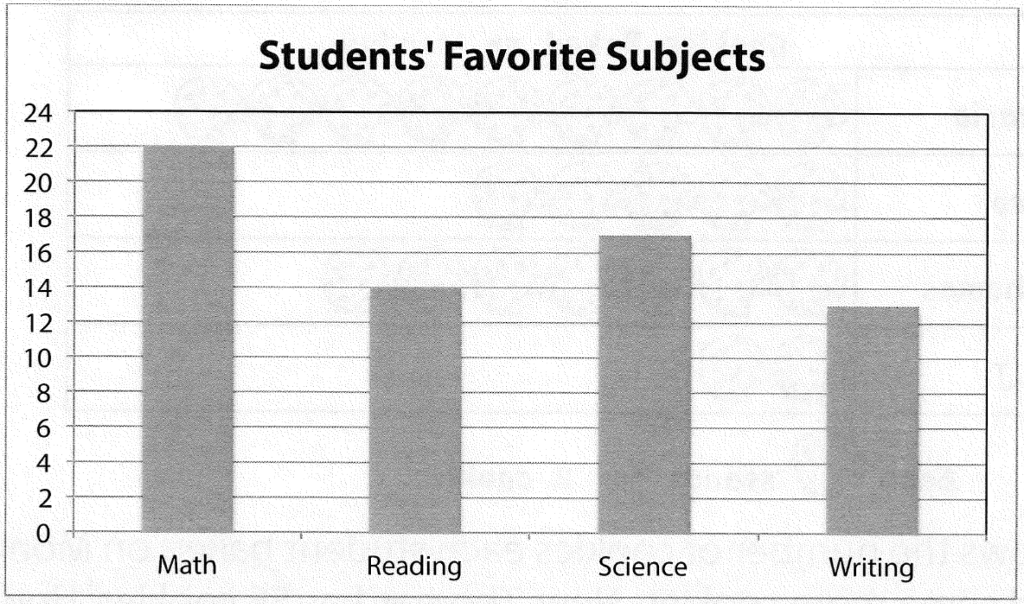
22 Math

14 writing

J.



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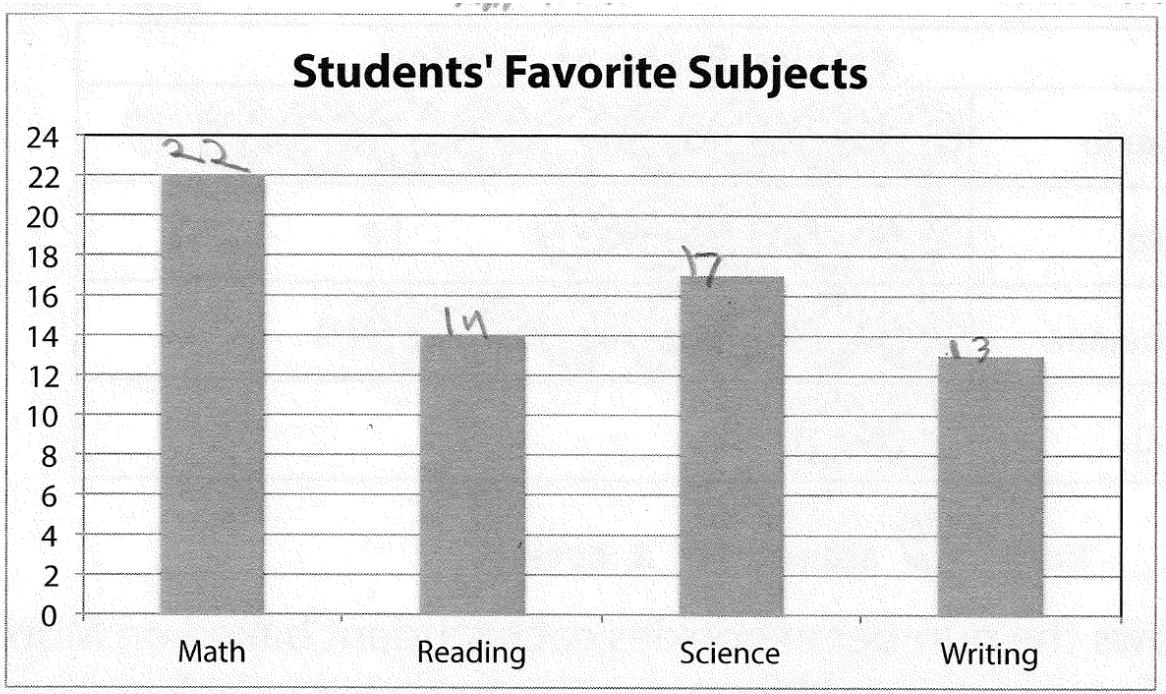
J.



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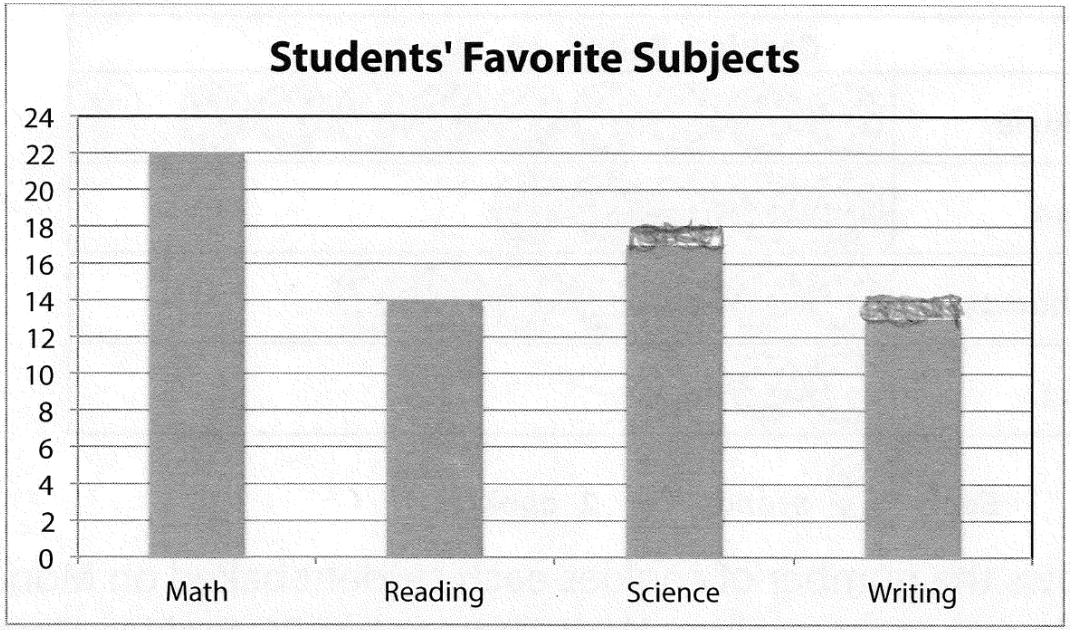
14
+13
27

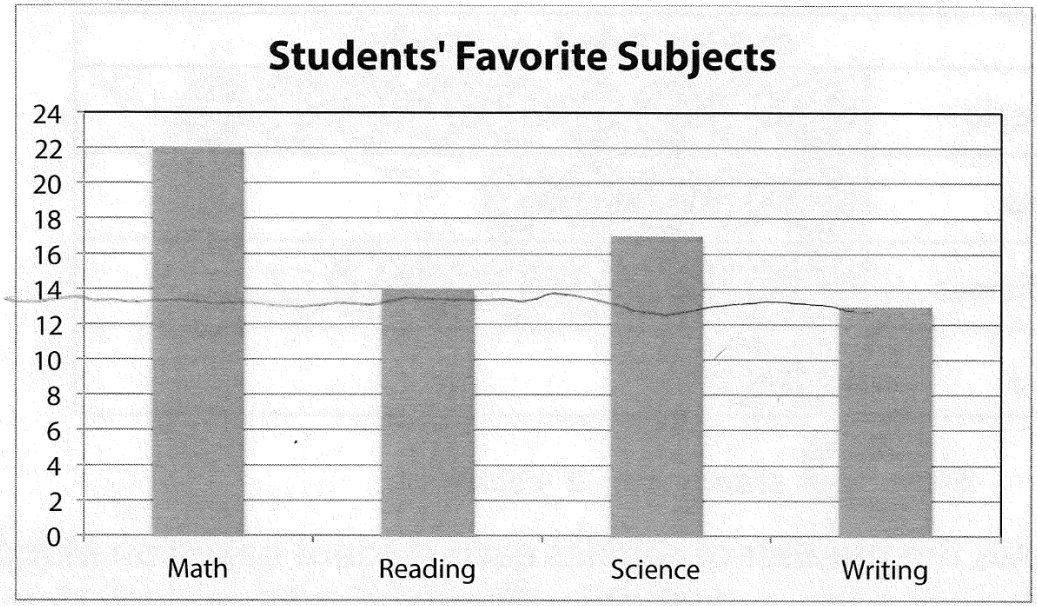
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ +\ 14 \\ \hline 36 \end{array}$$

J.



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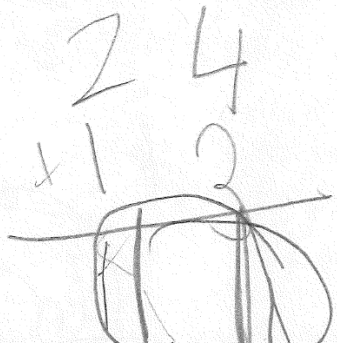

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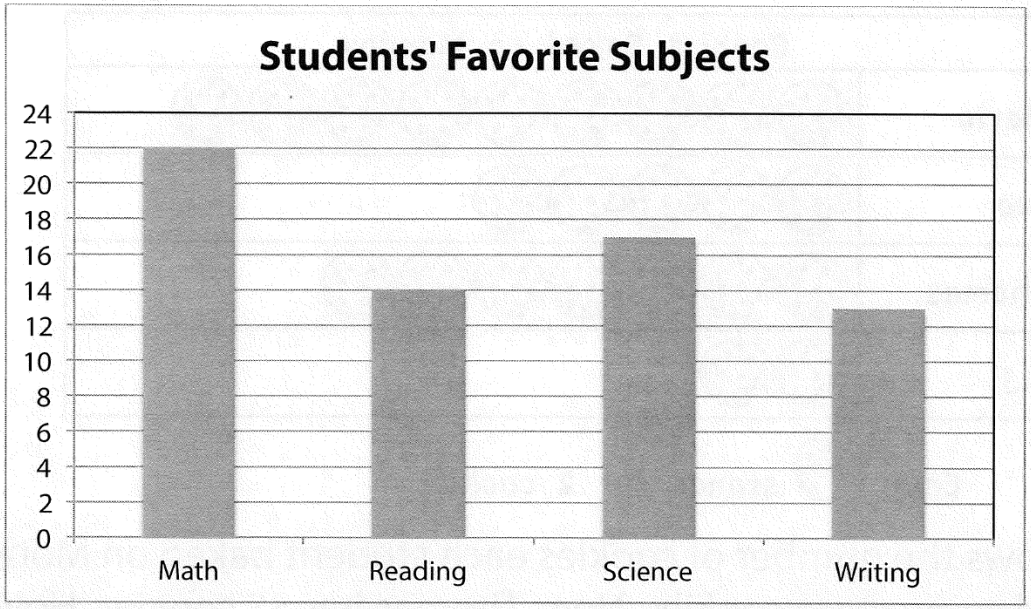


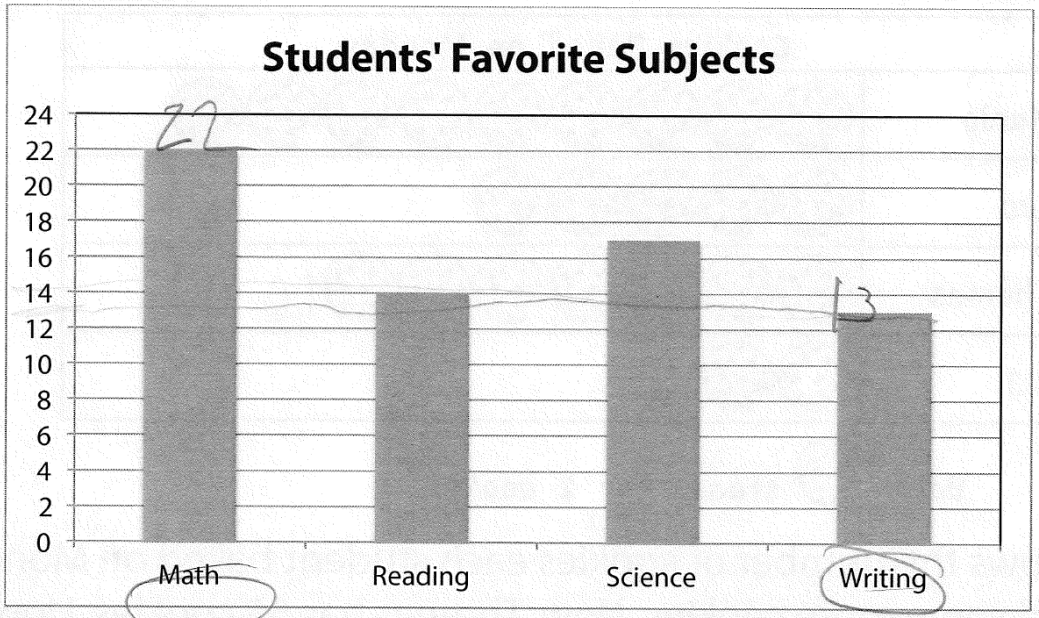
The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

22
+ 12

34

34

J.



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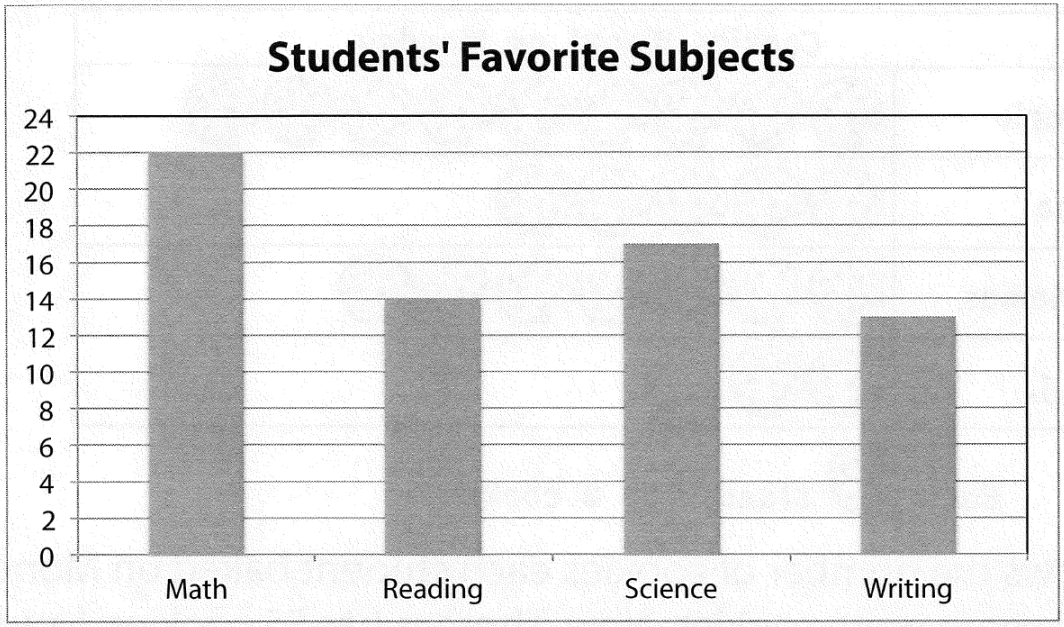
14

$$\begin{array}{r} 22 \\ +\ 13 \\ \hline 35 \end{array}$$

J.



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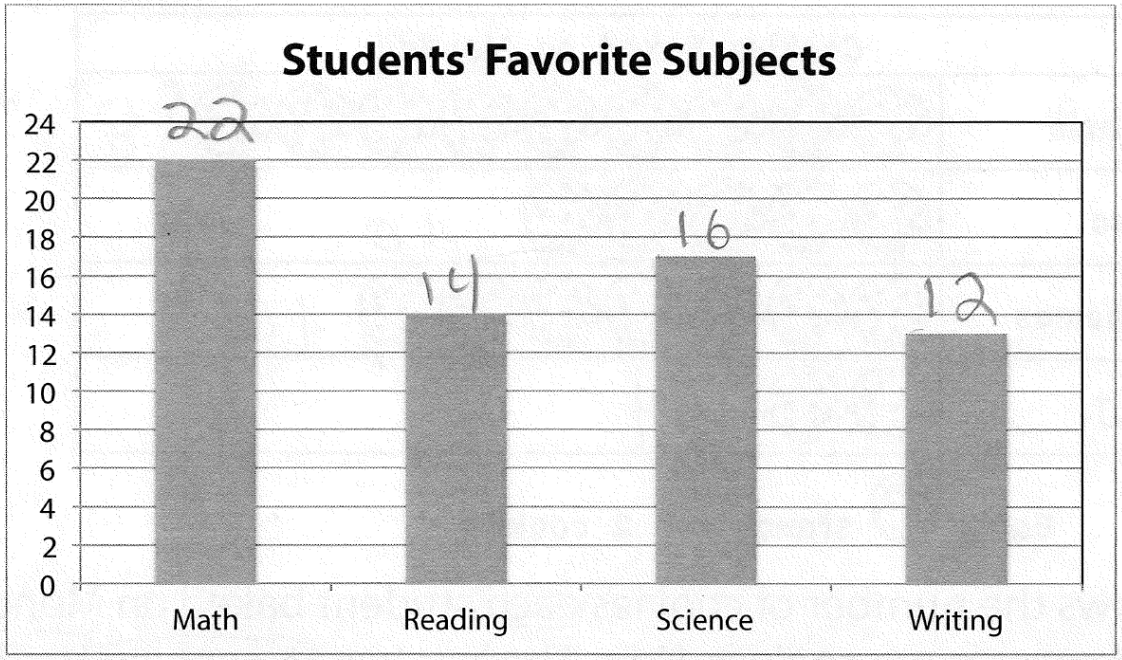
22
-15
7

J.



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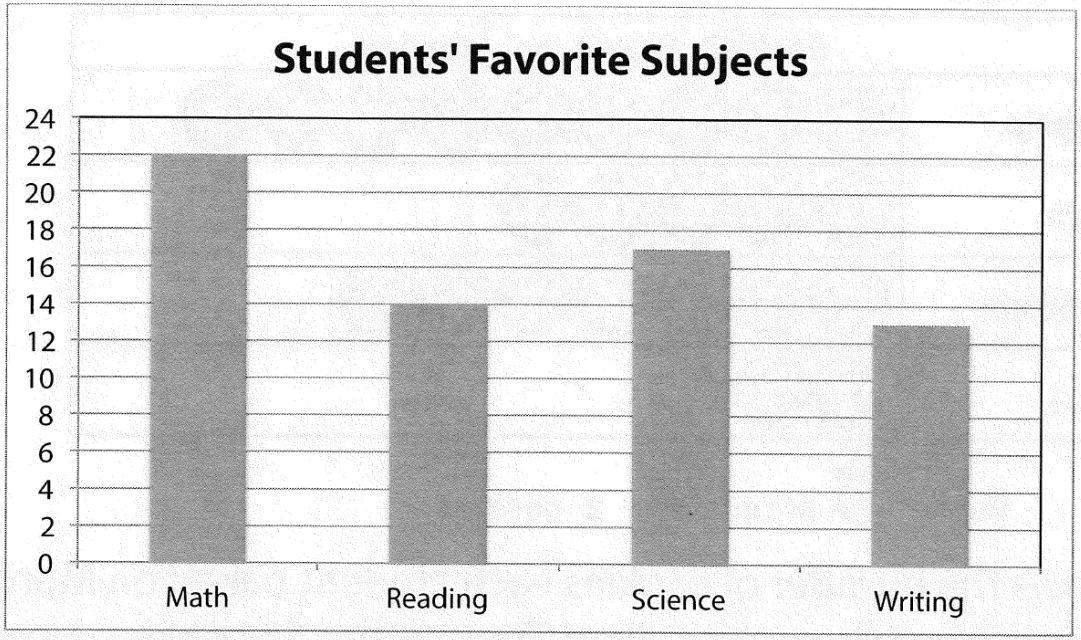
$$\begin{array}{r} 22 \\ -\ 12 \\ \hline 10 \end{array}$$

J.



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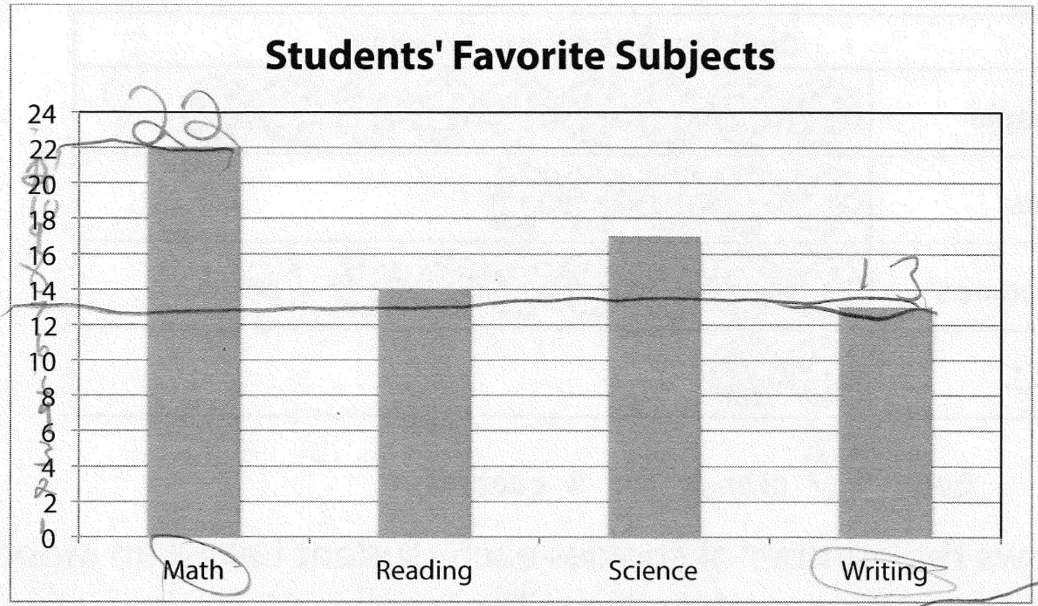

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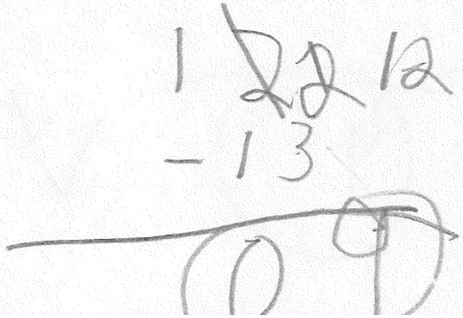

Correct

Undetermined

Didn't provide answer

Wrong (misapplied) operation

Wrong information

Subtraction error

COMMON

- Undetermined
- Repeated information from problem
- Didn't provide answer
- Wrong information
- Wrong (misapplied) operation

UNCOMMON

- Addition error
- Subtraction error

Word-Problem Solving

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

C 25

D 20

Solve the problem

What skills are necessary to solve this problem?

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
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- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

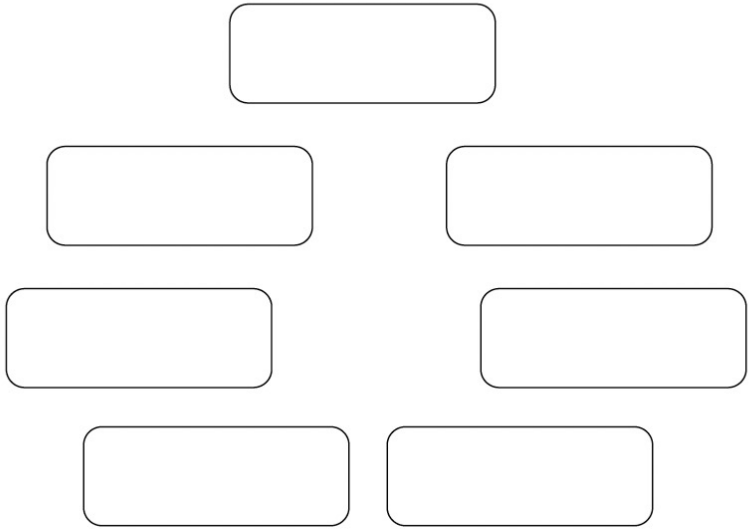
C 25

D 20

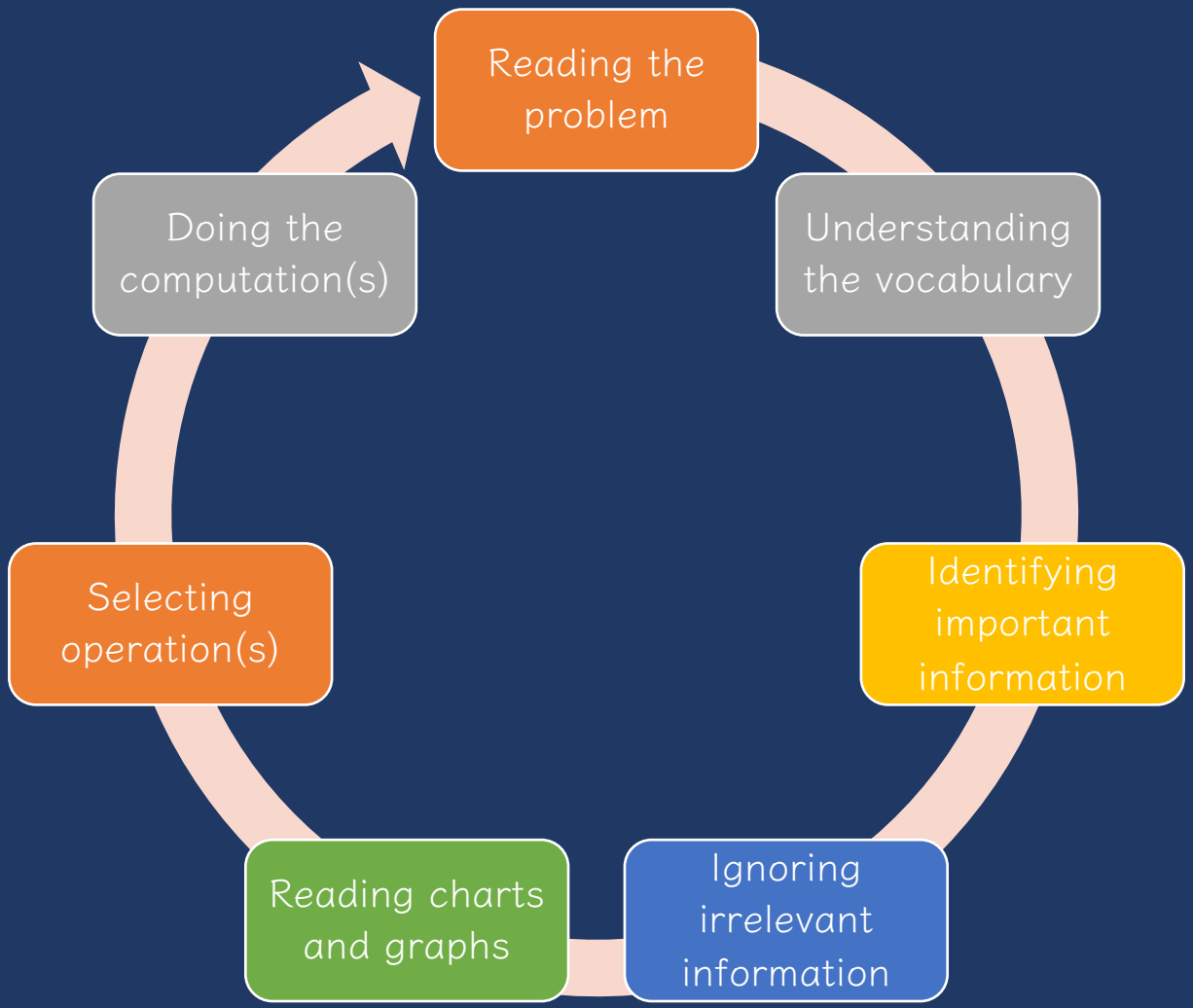
How would you solve this problem?

What skills are necessary to solve this problem?

Word-Problem Solving



Teaching Problem Solving

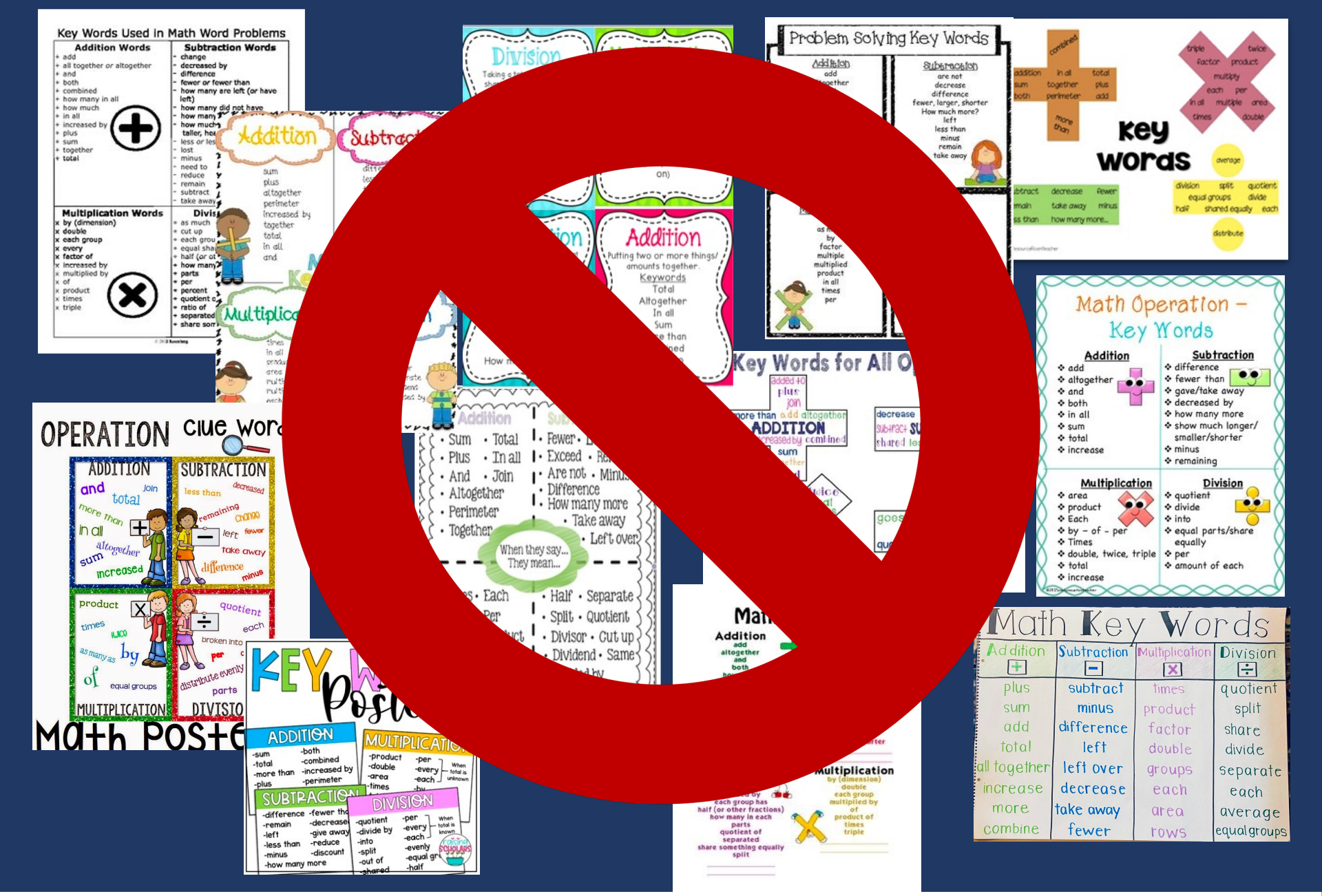


Ineffective Strategies

1. Keywords tied to operations

Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?



Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords[a]		Multiple keywords[a]		Keyword(s) led to correct solution[a]	
	n	%	*n*	%	*n*	%	*n*	%	*n*	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

[a]When a problem featured a keyword.

Description of Multi-Step Word Problems (n = 84)

	Occurrence of schema[a]		Any keyword		Keyword(s) led to correct solution[b]	
Schema	*n*	%	*n*	%	*n*	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

[a]Sum across schemas does not equal 100 because each word problem featured more than one schema.

[b]When a problem featured a keyword.

Mr. Rivera's taxable income is $20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of $11.50.

- The video game cost $4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

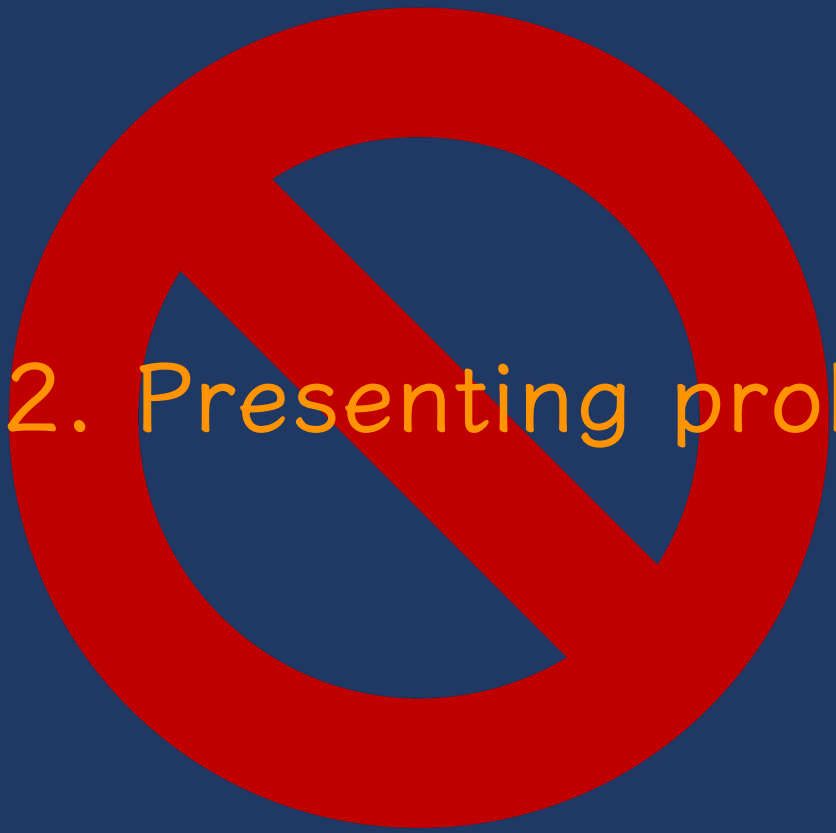
The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?

Keywords are important to identify and understand

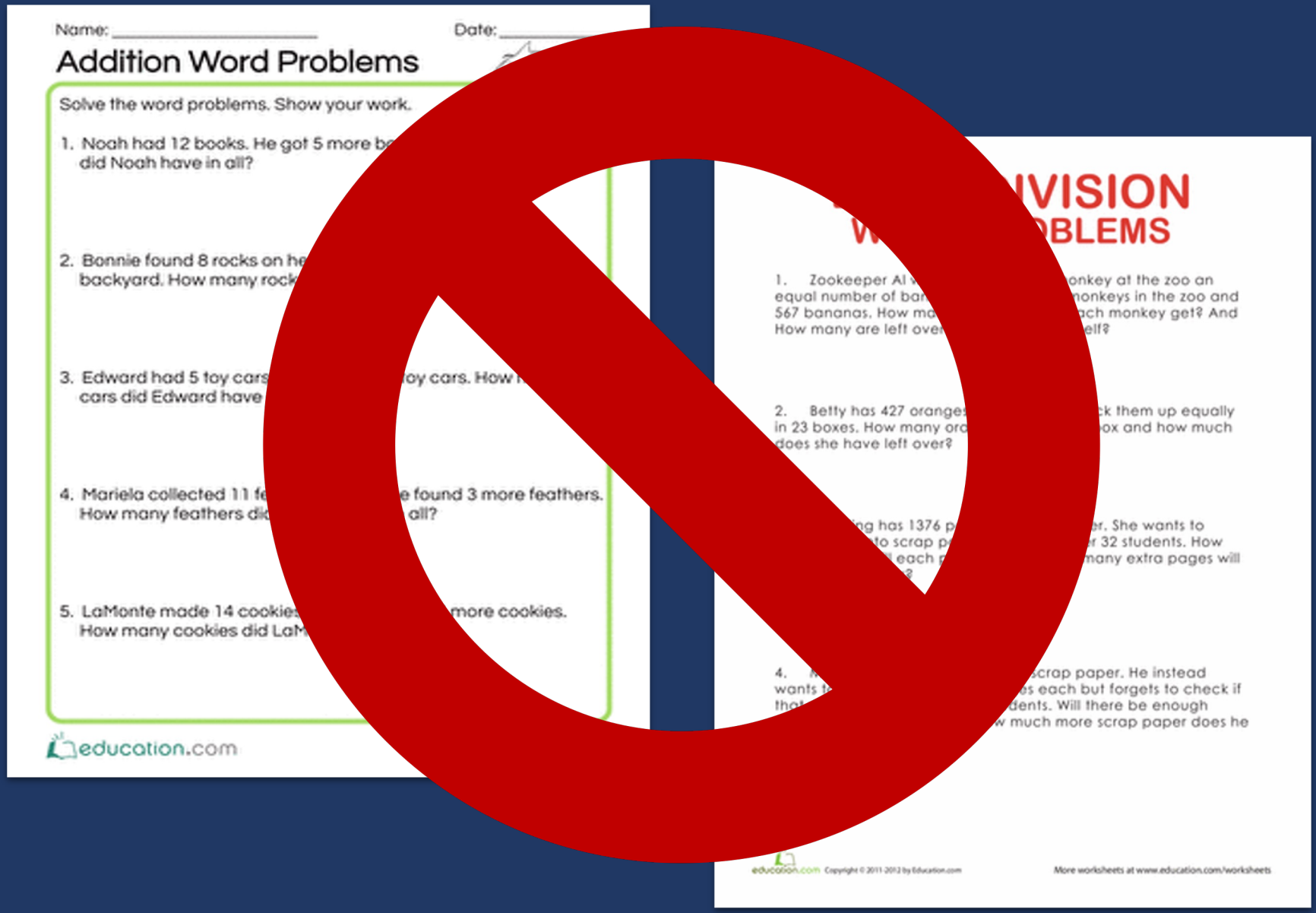
Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")

But, *do not* tie a keyword to a specific operation!



2. Presenting problems by operation



Effective Strategies

Teach an attack strategy

Teach about schemas

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

C 25

D 20

What was your process for working through this problem?

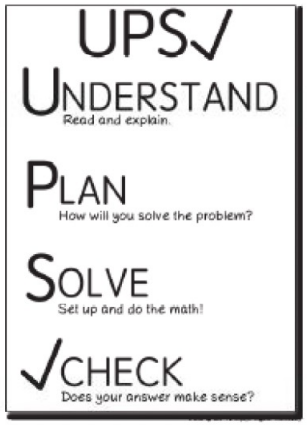
Attack Strategy

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.



RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development.
Solve the equation.

RICE

Read and record the problem.
Illustrate your thinking.
Compute.
Explain your thinking.

STAR

Stop and read the problem carefully.
Think about your plan and the strategy you will use.
Act. Follow your plan and solve the problem.
Review your answer.

SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

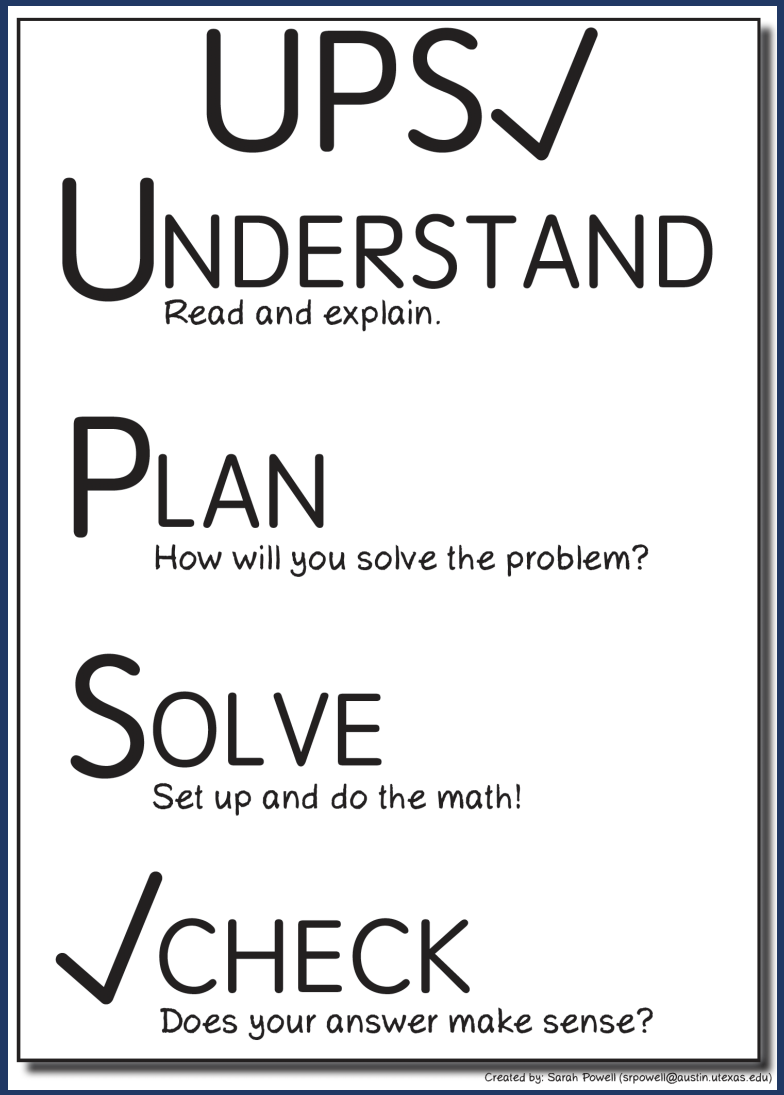
Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.



Share your favorite attack strategy.

Teach an attack strategy

Teach about schemas

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Additive Word Problems

Meanings of Addition
Meanings of Subtraction

Total

Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

Total

Parts put together into a total

Karly saw 4 cardinals and 5 blue jays. How many birds did Karly see?

Change

Start with a set, add the other set, count sum

$$2 + 3 = 5$$

Change

An amount that **increases** or decreases

Silas had $4. Then they earned $5 for cleaning their room. How much money does Silas have now?

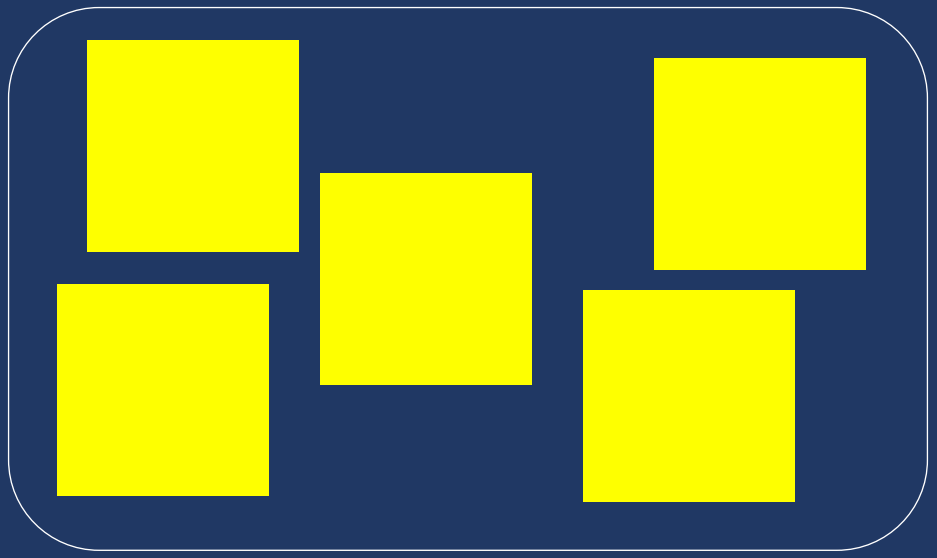
6 + 7 = __

Share a Total story.

Share a Change/Join story.

Change

Start with a set, take away from that set, count difference



$$5 - 3 = 2$$

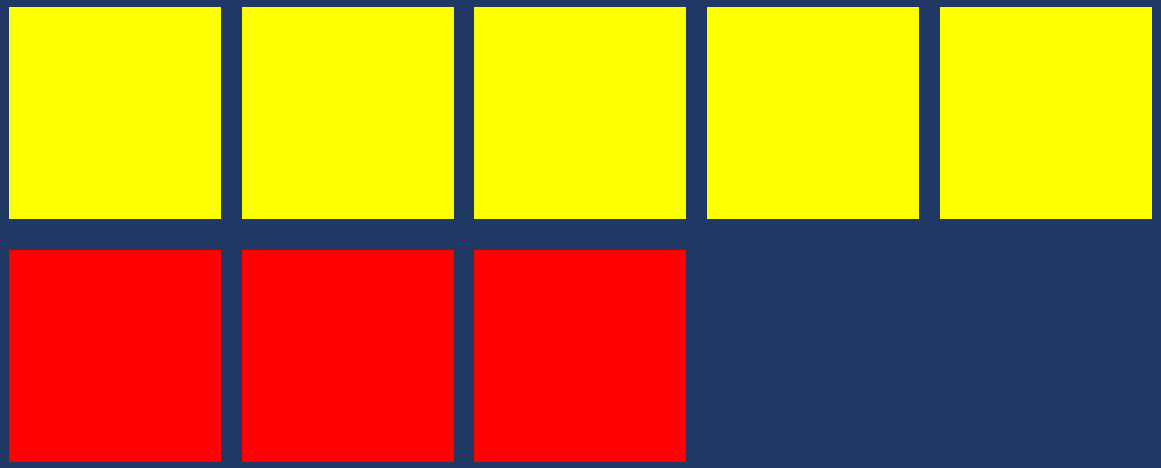
Change

An amount that increases or **decreases**

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

Difference

Compare two sets, count difference



$$5 - 3 = 2$$

Difference

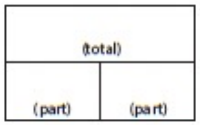
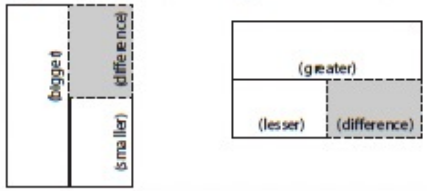
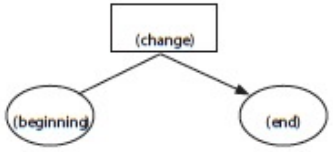
Greater and less amounts compared for a difference

Rachel has 9 pencils. Jodie has 4 pencils. How many more pencils does Rachel have? (How many fewer does Jodie have? What's the difference between Rachel's and Jodie's pencils?)

$14 - 8 = \_\_$

Share a Change/Separate story.

Share a Difference story.

Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total)  	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B – s = D (bigger – smaller = difference) G – L = D (greater – less = difference)  	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/– C = E (start +/– change = end)   	End (increase) unknown: Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now? End (decrease) unknown: Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	Change (increase) unknown: Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting? Change (decrease) unknown: Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	Start (increase) unknown: Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with? Start (decrease) unknown: Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	Multiple changes: Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?

Total

Additive Word Problems

A. Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?	B. In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?
C. Sam mows lawns and made $560 last week. She made $95 on Monday, $135 on Tuesday, and $70 on Wednesday. How much did Sam make on Thursday and Friday?	NOTES ABOUT TOTAL PROBLEMS: 

Total

Part-part-whole
Combine

Parts put together into a total

Daniela saw 3 canoes and 8 kayaks. How many boats did Daniela see?

Total

Daniela saw 11 boats. If 3 of the boats were canoes, how many were kayaks?

Part

Daniela saw 11 boats. 8 of the boats were kayaks, how many were canoes?

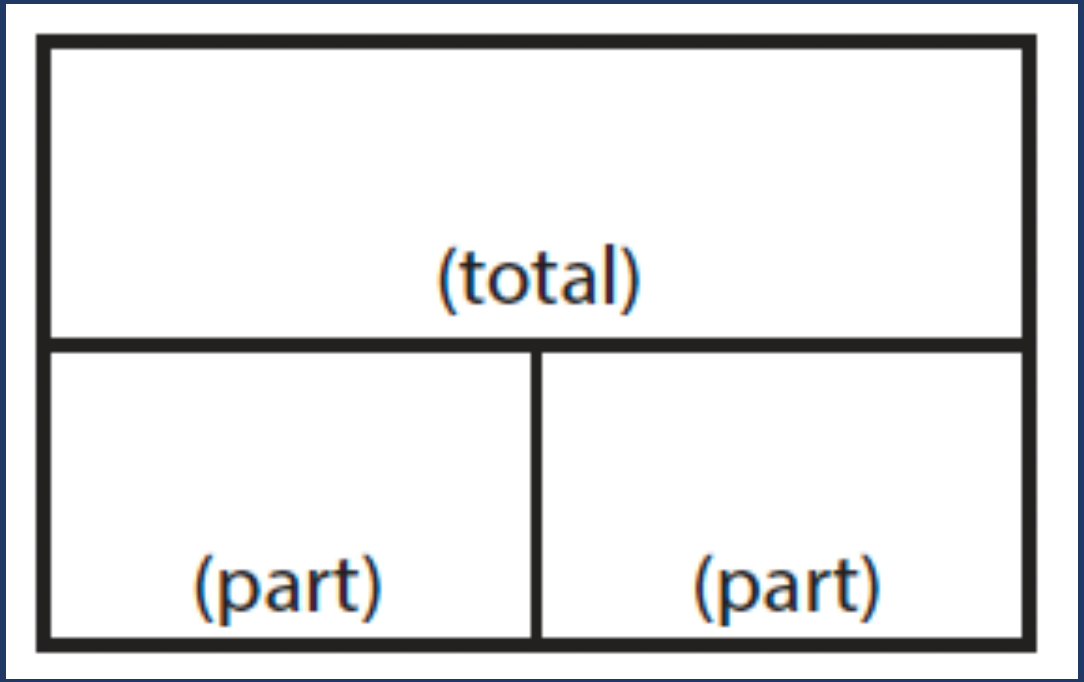
Part

Total

"Are parts put together for a total?"

Total

P1 + P2 = T



Fuchs et al. (2008); Griffin & Jitendra (2009)

Total

Additive Word Problems

A. Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?	B. In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?
C. Sam mows lawns and made $560 last week. She made $95 on Monday, $135 on Tuesday, and $70 on Wednesday. How much did Sam make on Thursday and Friday?	NOTES ABOUT TOTAL PROBLEMS:

Total

Write a Total problem.

(Label the type on the back of the card.)

Difference

Compare

Greater and lesser amounts compared for a difference

Adrianna has 10 pencils. Tracy has 4 pencils. How many more pencils does Adrianna have?

Difference

Adrianna has 6 more pencils than Tracy. If Tracy has 4 pencils, how many does Adrianna have?

Greater amount

Tracy has 6 fewer pencils than Adrianna. Adrianna has 10 pencils. How many pencils does Tracy have?

Lesser amount

Total

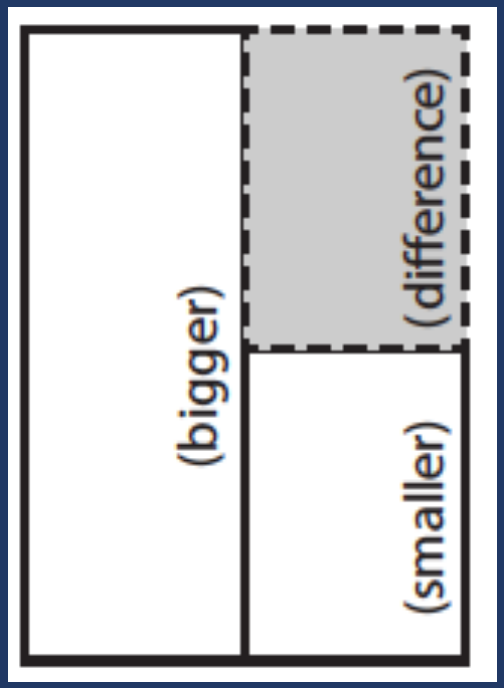
"Are parts put together for a total?"

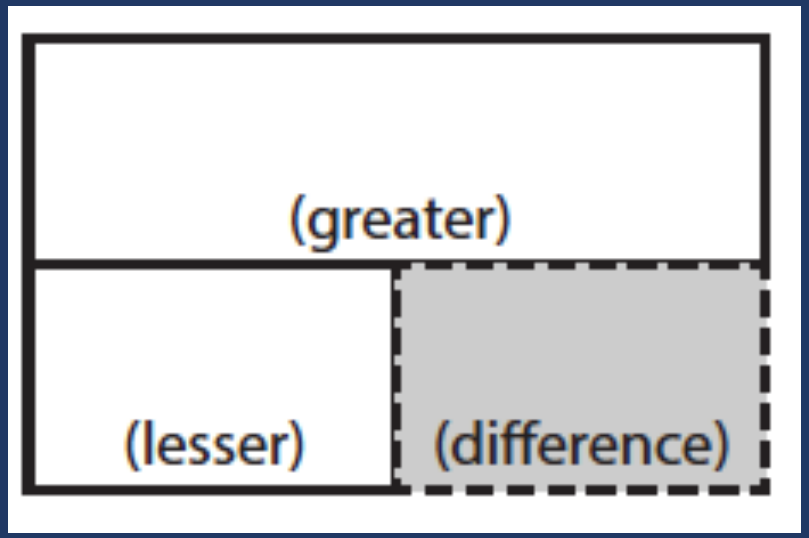
Difference

"Are amounts compared for a difference?"

Difference

G – L = D





Fuchs et al. (2008); Griffin & Jitendra (2009)

Difference

Additive Word Problems

D. Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?	E. Damian's dog eats 5 1/2 cups of dog food each week. Monte's dog eats 4 1/2 cups more each week than Damian's dog. How much does Monte's dog eat in a week?
F. The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?	NOTES ABOUT DIFFERENCE PROBLEMS:

Difference

Write a Difference problem.

(Label the type on the back of the card.)

Change

Join

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

End amount

Change amount

Start amount

Change

Separate

An amount that increases or **decreases**

Samantha baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Samantha eat?

Change amount

Samantha baked some cookies. She ate 3 of the cookies and has 17 cookies left. How many cookies did Samantha bake?

Start amount

Total

"Are parts put together for a total?"

Difference

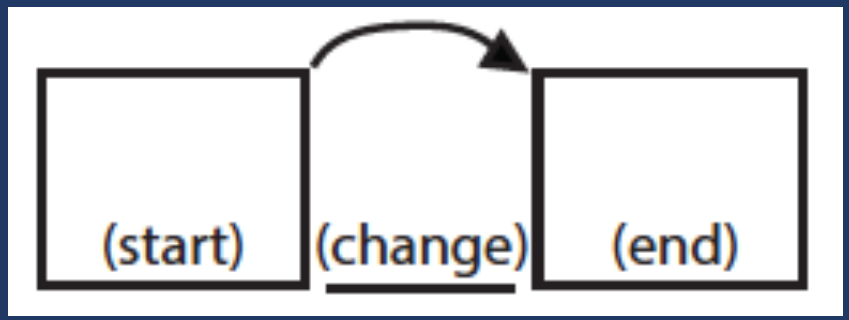
"Are amounts compared for a difference?"

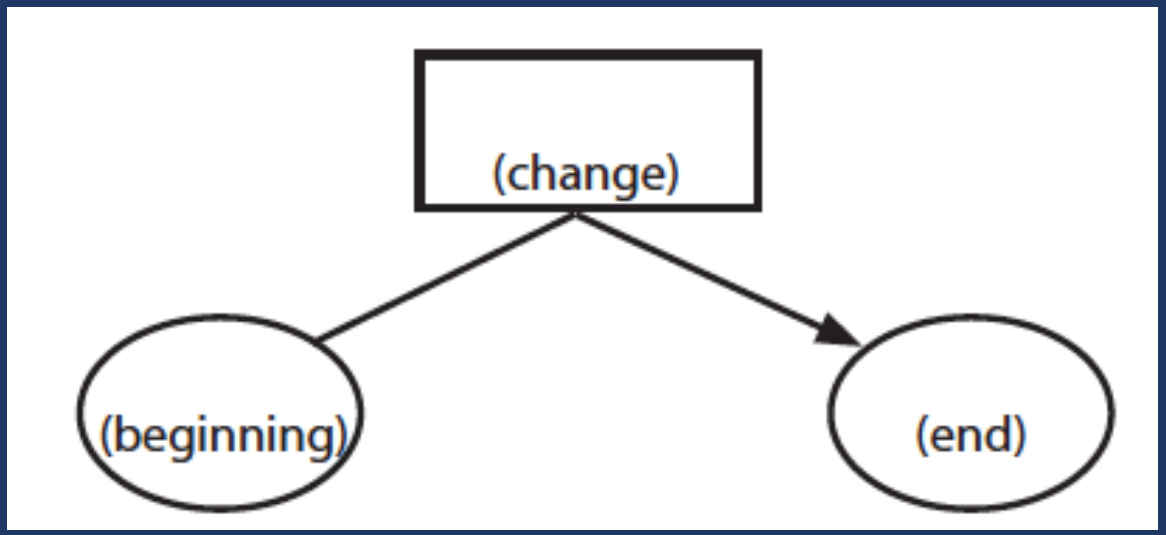
Change

"Does an amount increase or decrease?"

Change

ST +/− C = E





Fuchs et al. (2008); Griffin & Jitendra (2009)

Change

Additive Word Problems

G. A plant was 3 3/4 inches tall at the beginning of June. By the end of July, the plant was 9 1/8 inches tall. How many inches did the plant grow in 2 months?	H. Martina has some money in her bank account. Then, she spent $135.69 and has a balance of -$24.80. How much money did Martina have to begin with?
I. Hui saved $70 in January. In February, she spent $64 of the money she saved. She saved $92 more in March. How much has Hui saved by the end of March?	NOTES ABOUT CHANGE PROBLEMS:

Change

Write a Change problem.

(Label the type on the back of the card.)

Schema Check!

Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

26. Part A

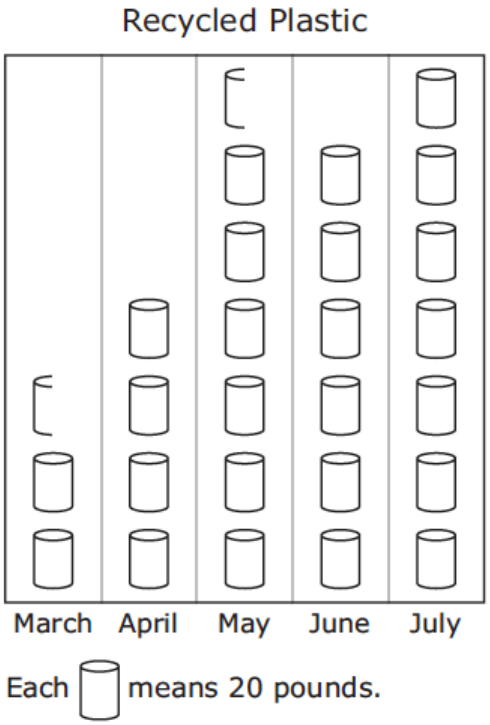
The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

Grade 3 PARCC

Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Grade 3 STAAR

Total

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.

10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

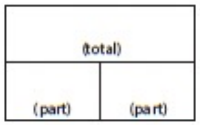
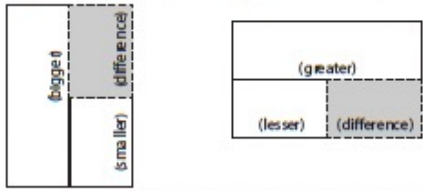
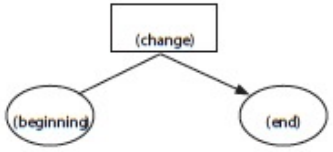
Ⓐ 300

Ⓑ 340

Ⓒ 350

Ⓓ 360

Grade 3 PARCC

Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total)  	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B – s = D (bigger – smaller = difference) G – L = D (greater – less = difference)  	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/– C = E (start +/– change = end)   	End (increase) unknown: Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now? End (decrease) unknown: Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	Change (increase) unknown: Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting? Change (decrease) unknown: Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	Start (increase) unknown: Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with? Start (decrease) unknown: Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	Multiple changes: Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?

Multiplicative Word Problems

Meanings of Multiplication
Meanings of Division

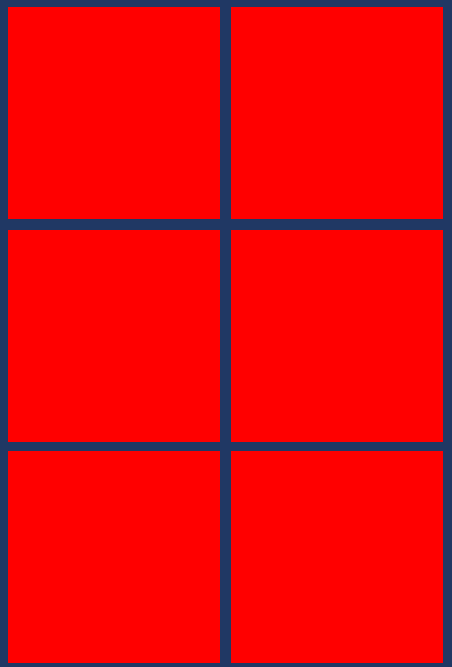
Equal Groups

Show the groups, show the amount for each group, count product

$$3 \times 2 = 6$$

Equal Groups

Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

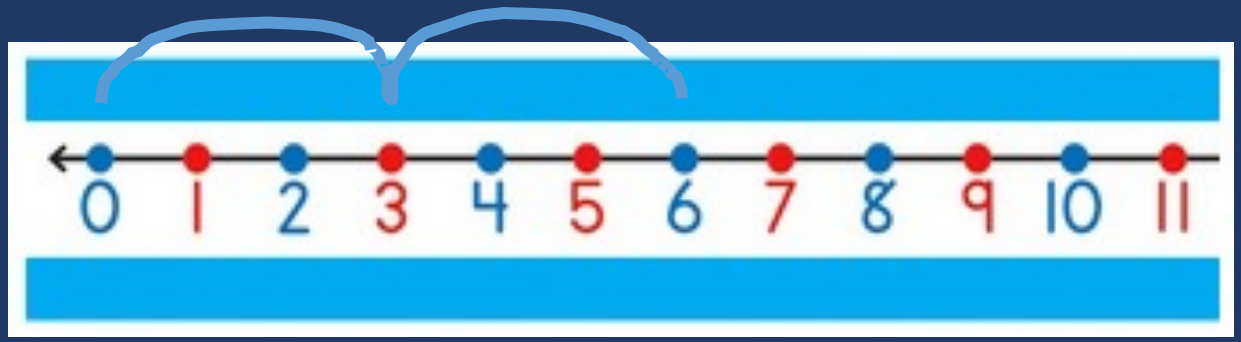
Equal Groups

Groups multiplied by number in each group for a product

Rhiannon has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Rhiannon have altogether?

Comparison

Show a set, then multiply the set



$$3 \times 2 = 6$$

Comparison

Set multiplied by a number of times for a product

Vivienne picked 6 apples. Jessica picked 2 times as many apples as Vivienne. How many apples did Jessica pick?

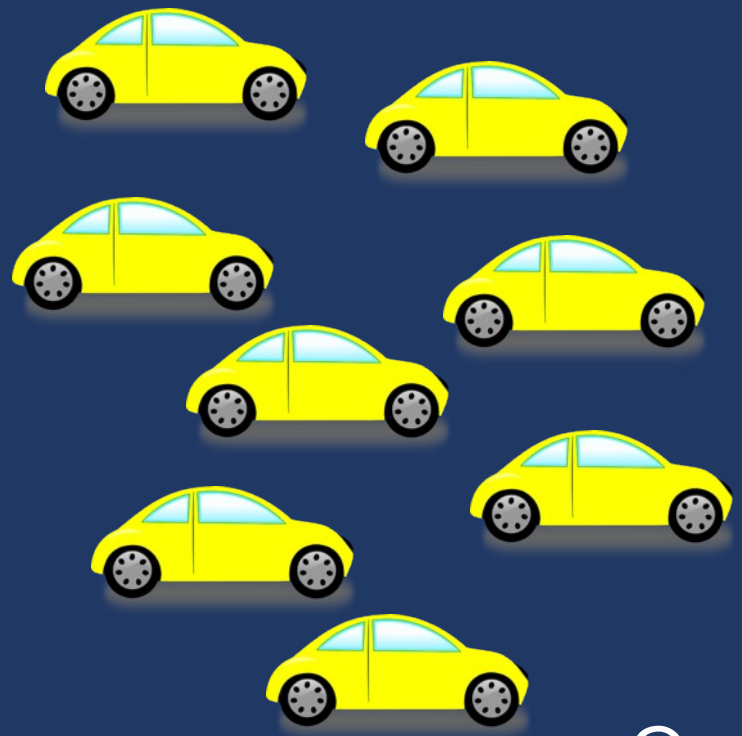
$4 \times 3 = \_\_$

Share an Equal Groups story.

Share a Comparison story.

Equal Groups (Partitive Division)

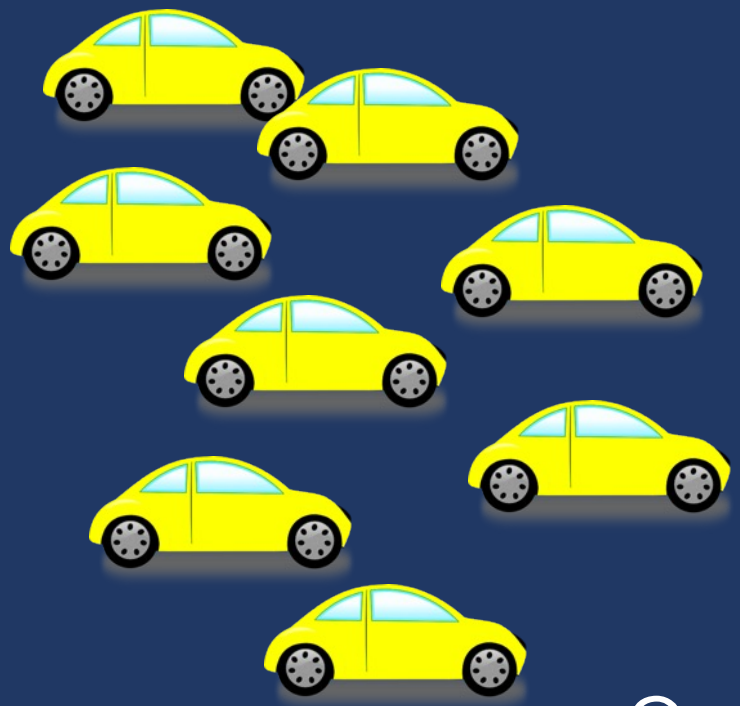
Show the dividend, divide equally among divisor, count quotient



$$8 \div 2 = 4$$

Equal Groups (Quotative Division)

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$

Equal Groups

Groups multiplied by number in each group for a product

Stefanie has 12 apples. She wants to share them equally among her 2 friends. How many apples will each friend receive?

Nicole has 12 apples. She put them into bags containing 6 apples each. How many bags did Nicole use?

$15 \div 5 = \_\_$

Share a Partitive story.

Share a Quotative story.

Total

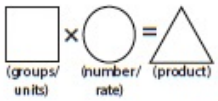
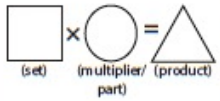
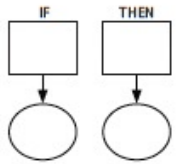
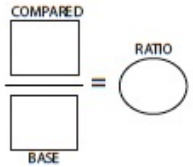
Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions		Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?		With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
		Base unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	Compared unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With unit rate: Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

Equal Groups

Array
Vary

Groups multiplied by number in each group for a product

Toni has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Toni have altogether?

Product

Toni has 24 crayons. They want to place them equally into 2 boxes. How many crayons will Toni place in each box?

Number in each group

Toni has 24 crayons. They put them into boxes with 12 crayons each. How many boxes did Toni use?

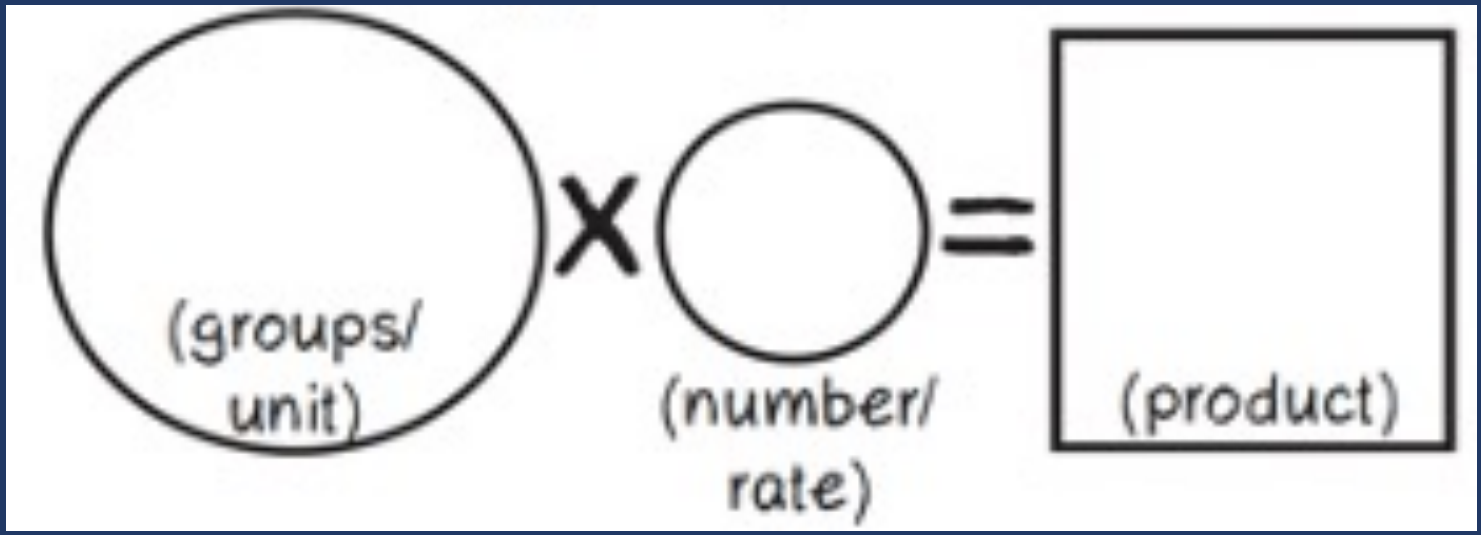
Groups

Equal Groups

"Are there groups with an equal number in each group?"

Equal Groups

GR × N(E) = P



Equal Groups

Multiplicative Word Problems

A. Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?	B. Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light blubs did Jane buy?
C. Zachary has 3 feet of string. He makes braclets, and each bracelet needs 5 1/4 inches of string. How many bracelets could Zachary make?	NOTES ABOUT EQUAL GROUPS PROBLEMS:

Equal Groups

Write an Equal Groups problem.

(Label the type on the back of the card.)

Comparison

Set multiplied by a number of times for a product

Brooke ran 6 minutes. Shaleeni ran 4 times longer than Brooke. How many minutes did Shaleeni run?

Set

Number of times

Product

Equal Groups

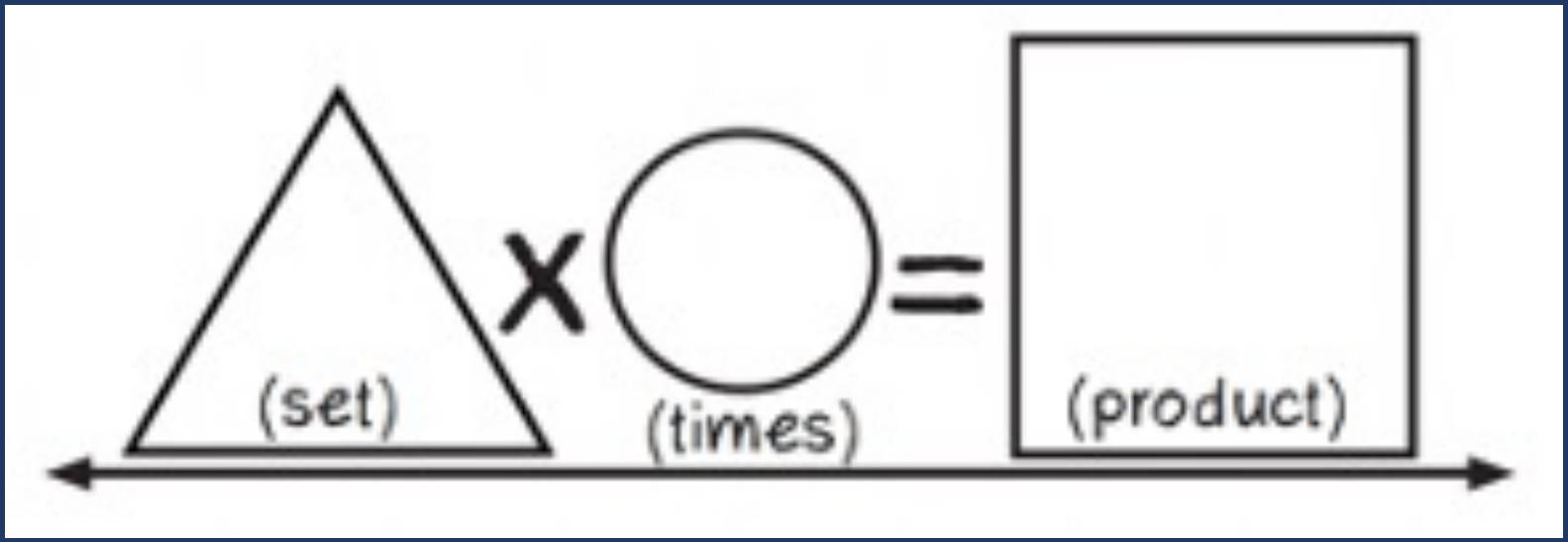
"Are there groups with an equal number in each group?"

Comparison

"Is a set compared a number of times?"

Comparison

S × T = P



Comparison

Multiplicative Word Problems

D. Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?	E. Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has? NOTES ABOUT COMPARISON PROBLEMS:
F. Sally typed 56 words in 2 minutes. At this rate, how many words could Sally type in 7 minutes?	G. An airplane's altitude changed -378 feet over 7 minutes. What was the mean change of altitude in feet per minute?

Comparison

Write a Comparison problem.

(Label the type on the back of the card.)

Ratios/Proportions

Description of relationships among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

Xin et al. (2005)

Equal Groups

"Are there groups with an equal number in each group?"

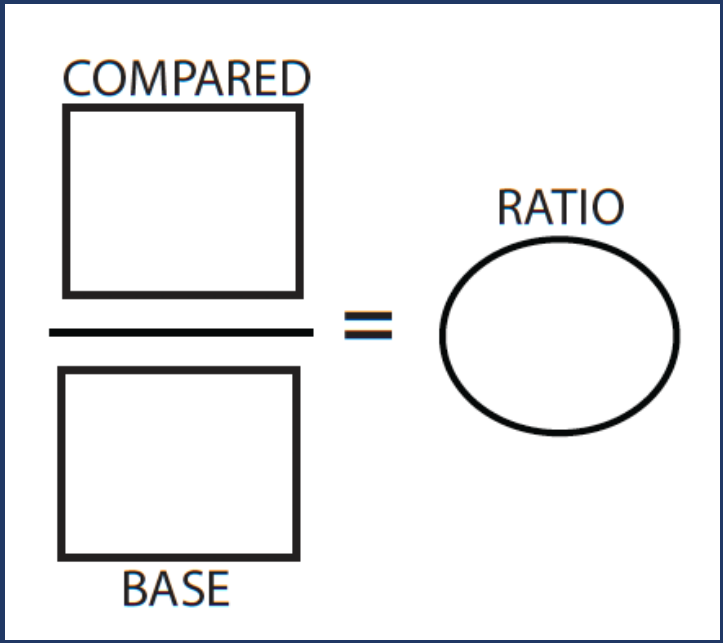
Comparison

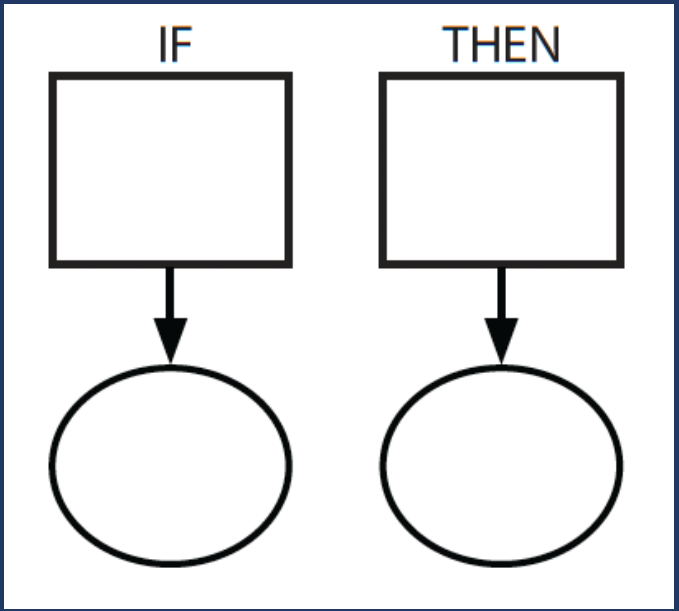
"Is a set compared a number of times?"

Ratios/Proportions

"Are there relationships among quantities - if this, then this?"

Ratios/Proportions





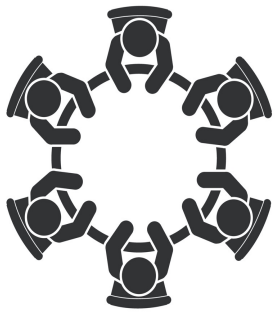
Xin et al. (2005)

Ratios/Proportions

Multiplicative Word Problems

H. The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?	I. Sara buys a sweater at a department store. The sweater costs $30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.
J. Margarita baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?	NOTES:

Ratios/Proportions



Write a Ratios or Proportions problem.

(Label the type on the back of the card.)

Schema Check!

Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

Grade 4 PARCC

Ratios/Proportions

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?

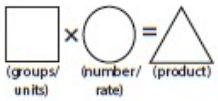
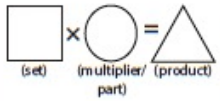
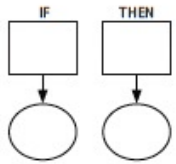
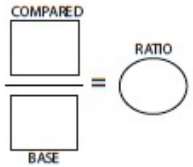
Grade 5 STAAR

Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

Grade 4 Smarter Balanced

Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions		Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?		With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
		Base unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	Compared unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With unit rate: Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Total

Difference

Change

Equal Groups

Comparison

Ratio/ Proportion

Teach an attack strategy

Teach about schemas

Word-Problem Solving

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

C 25

D 20

Solve the problem

What skills are necessary to solve this problem?

Revisit this problem. Discuss the schemas in the problem.

If needed, number the graph.

U

- **U**nderstand by reading
- **U**nderline the label

P

- (**P**arentheses) needed numbers
- **P**ut the numbers in order

S

- **S**chema(s)
  - **T**otal
  - **D**ifference
  - **C**hange
  - **E**qual **G**roups
  - **E**qual **S**hares
- **S**olve

2

- **C**heck the number answer
- **C**heck the label answer

Multi-Step Problems

A.
Leslie had 3 pizzas. Each pizza was cut into 8 pieces. Leslie ate 2 pieces. How many pieces were left?

B.
Mr. Kahn gave away 8 blue balloons and 6 red balloons. He gave away 3 times the number of white balloons as red balloons. What was the total number of balloons Mr. Kahn gave away?

C.
An egg farm packages 264 total cartons of eggs each month. The farm has 3 different sizes of cartons.
The small carton hold 8 eggs, and 1/6 of the total cartons are small.
The medium carton holds 12 eggs, and 2/3 of the total cartons are medium.
The large carton holds 18 eggs, and the rest of the total cartons are large.

Determine how many each size of carton is needed each month. Then determine how many eggs are needed to fill the 264 cartons.

Teach an attack strategy

Teach about schemas

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

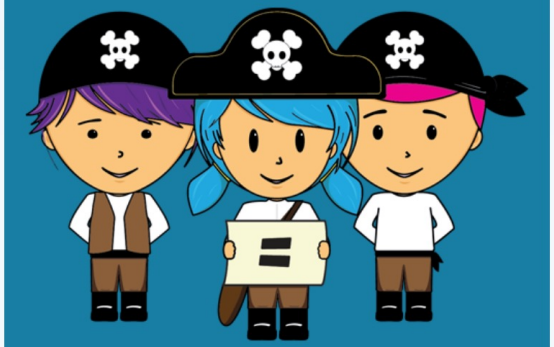
Pirate Math Equation Quest

About | Research | Individual | Small Group | STAAR | Videos

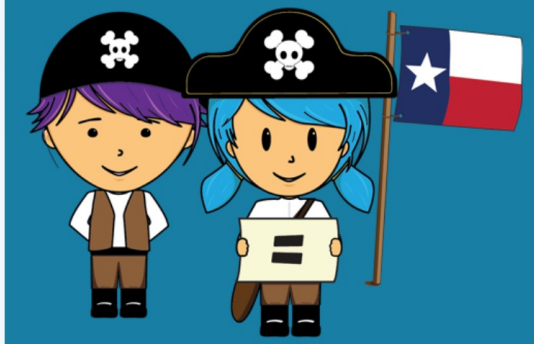
Welcome to Pirate Math Equation Quest!

Individual Word-Problem Intervention

Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



https://intensiveintervention.org/intensive-intervention-math-course

National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

Search

Intensive Intervention | Tools Charts | Implementation Support | Intervention Materials | Information For...

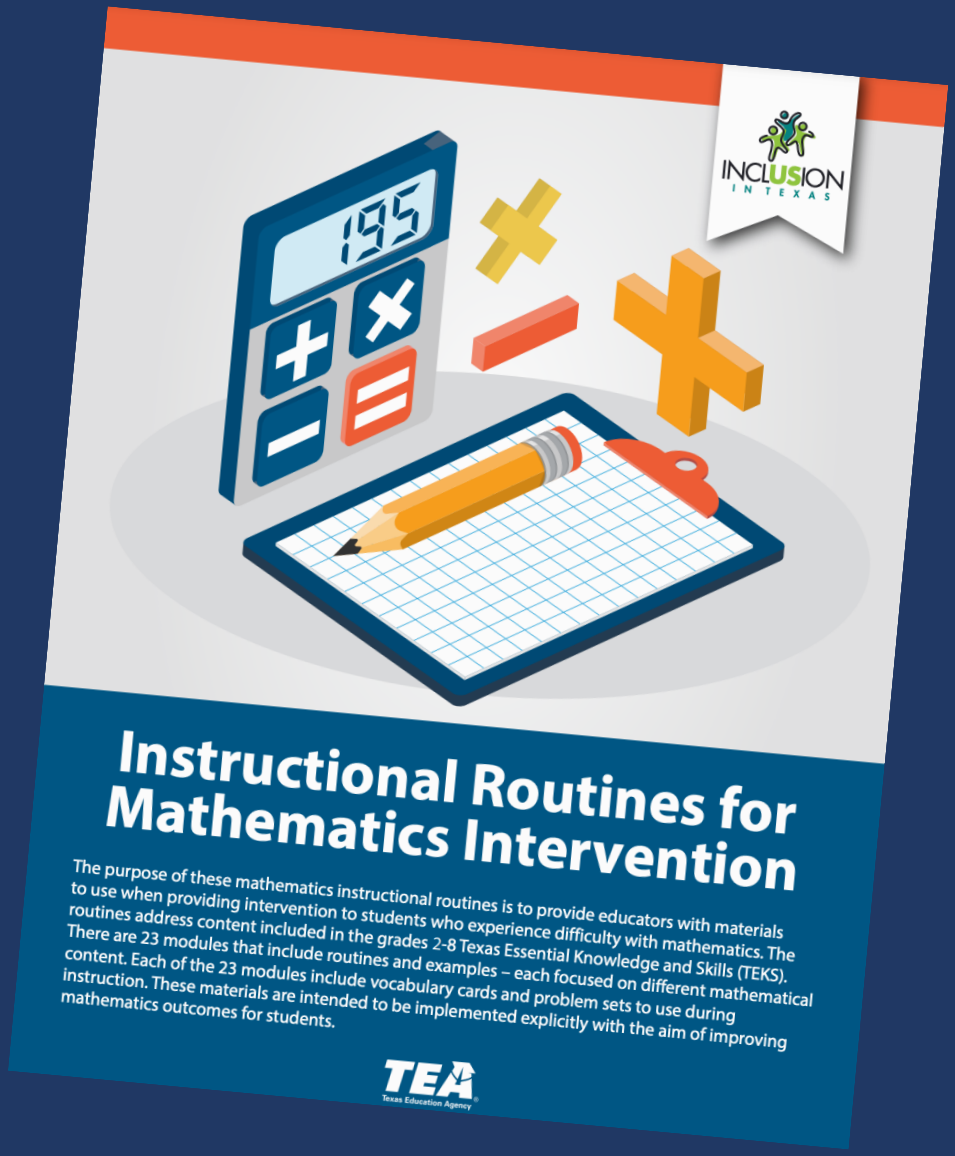
Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a high-leverage practice in special education, and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

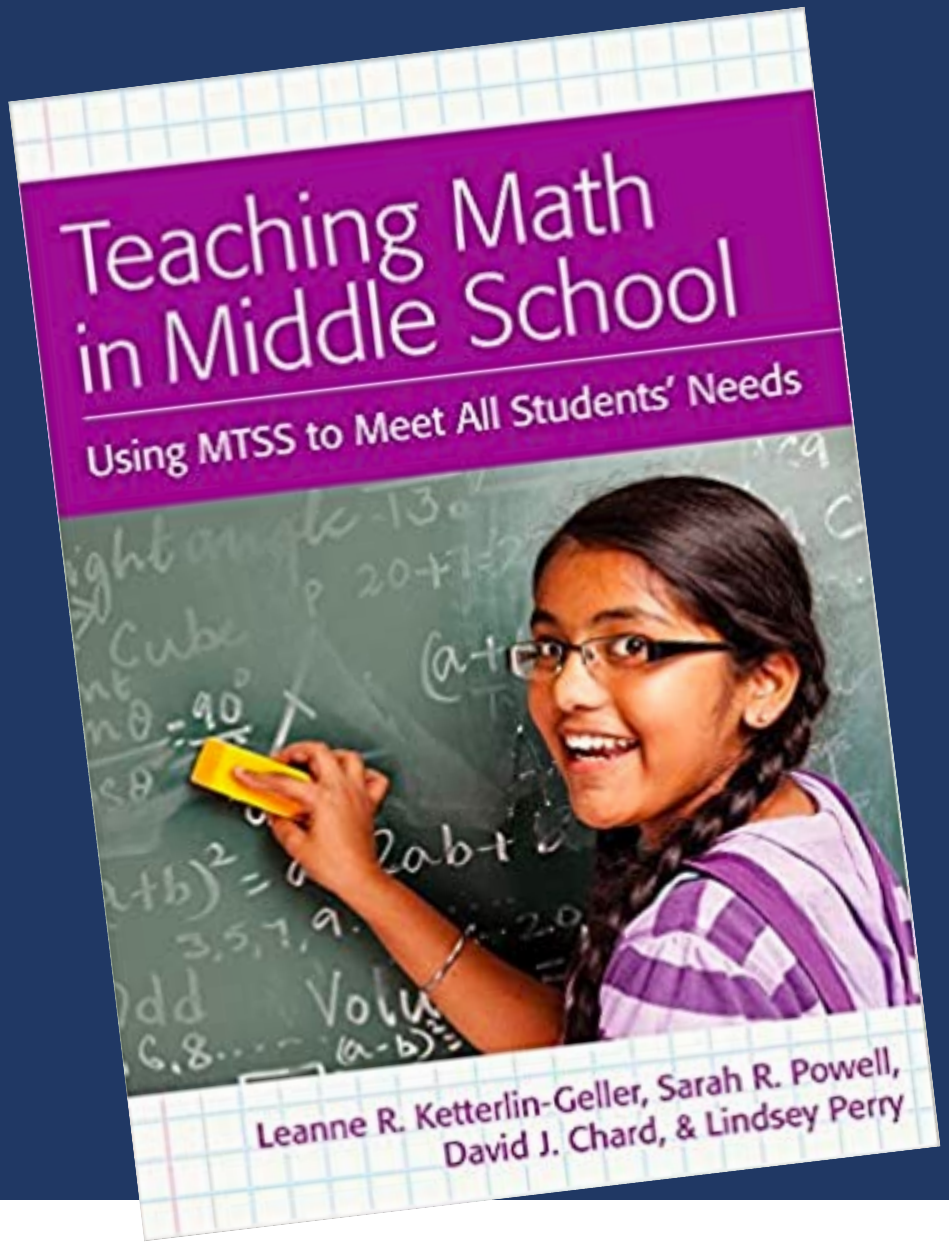
NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEEDAR Center, developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.

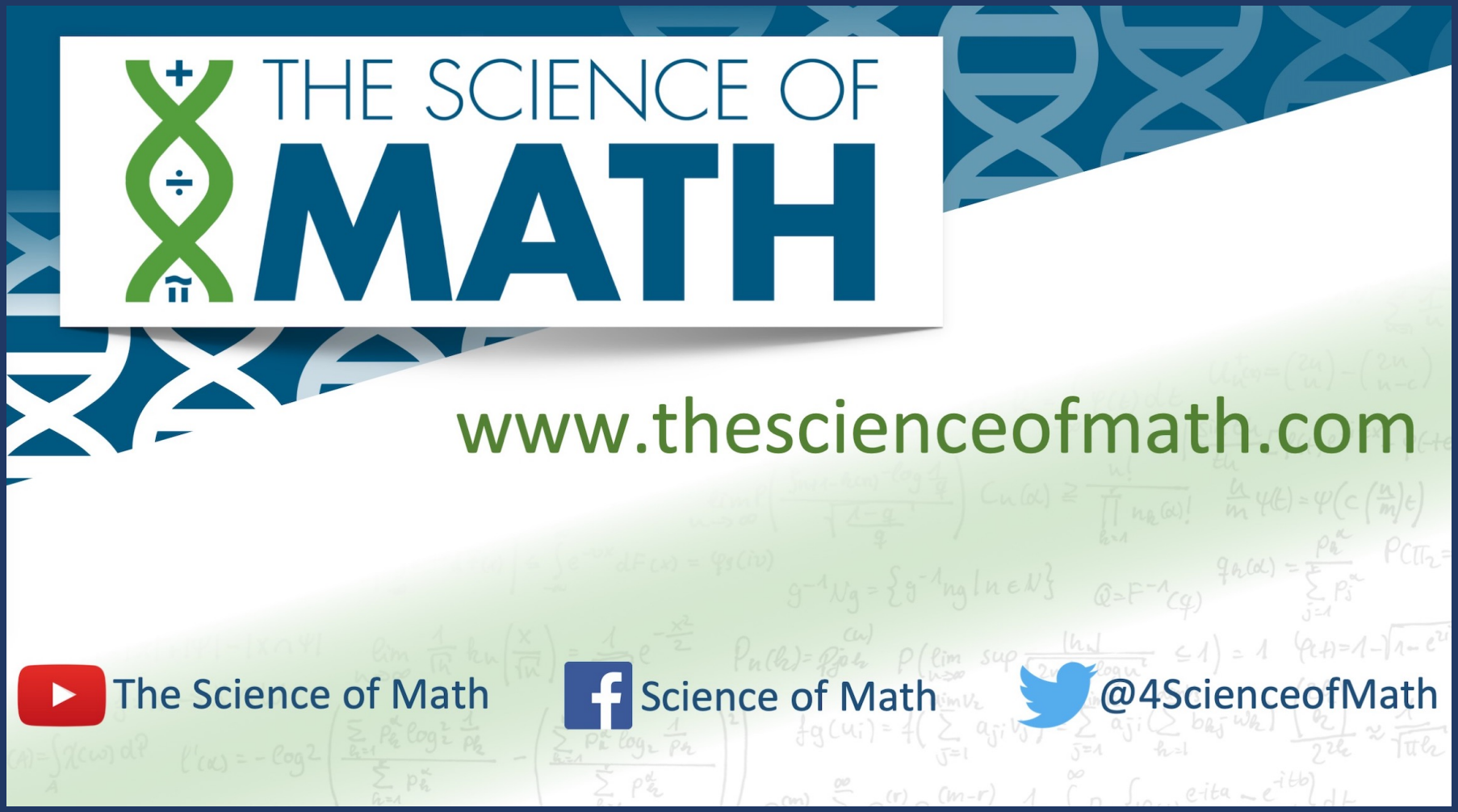
MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES



https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859

https://www.amazon.com/Teaching-Math-Middle-School-Students/dp/1598572741





We're recruiting Kindergarten teachers for a math program that uses music to teach math. If interested: srpowell@utexas.edu

young academic music study

Information for School Districts

What is the Young Academic Music (YAM) curriculum?

YAM is a supplemental, evidence-based kindergarten curriculum that provides math, music, and computational thinking instruction. It is aligned with both the Common Core Standards in Mathematics and the Texas Essential Knowledge and Skills. This project is being conducted by Touro Graduate School of Education in partnership with the University of Texas at Austin and SRI International. SRI International is the independent evaluator for the project.

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