

Effective Math Instruction

DAY 2



Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin



srpowell@utexas.edu



[@sarahpowellphd](https://twitter.com/sarahpowellphd)





Say hello.

Describe two things that resonated with you from yesterday.

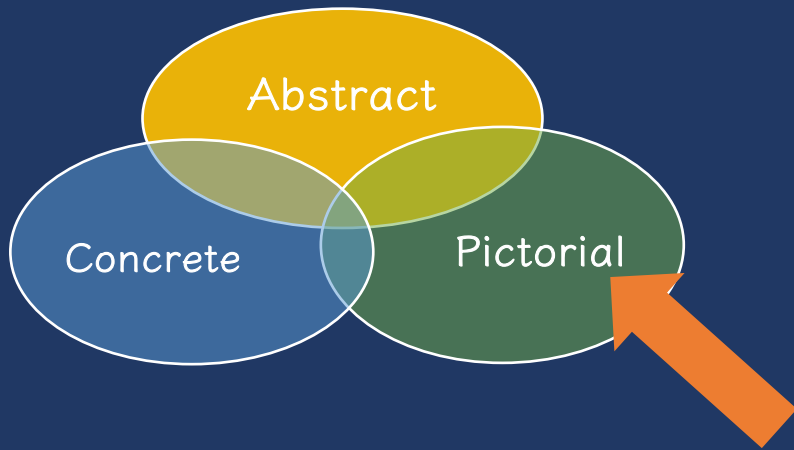




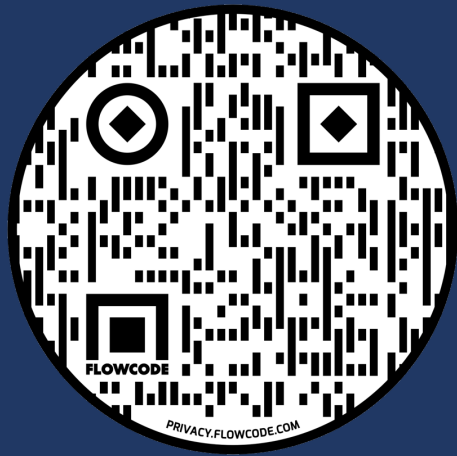
Schedule for Today

8:00-9:30	- Language of mathematics
9:30-9:40	BREAK
9:40-10:15	- Evidence-based practice: Word-problem solving
10:15-10:30	- Word-problem solving: Ineffective strategies
10:30-11:00	- Word-problem solving: Attack strategies
11:00-11:30	- Word-problem solving: Total problems
11:30-1:00	LUNCH
1:00-2:15	<ul style="list-style-type: none">- Word-problem solving: Difference problems- Word-problem solving: Change problems- Word-problem solving: Equal groups problems- Word-problem solving: Comparison problems
2:15-2:25	BREAK
2:25-4:00	<ul style="list-style-type: none">- Word-problem solving: Ratios/Proportions problems- Word-problem solving: Multi-step problems- Wrap-up





Two-dimensional images



Virtual Manipulatives

Help students see and learn math using different tools!

Number & Operations	Place Value
Fractions & Decimals	Integers & Algebra
Geometry	Time & Money
Data & Probability	Extras

Sarah R. Powell, Ph.D.
srpowell@utexas.edu
www.sarahpowellphd.com
@sarahpowellphd

Fractions & Decimals				
	fraction strips	fraction strips	fraction strips	Cuisenaire rods
fraction circles	geoboard	geoboard	geoboard	
two-color counters				
two-color counters	decimal strips	place value disks	percentage strips	



Mathematical Language



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

INSTRUCTIONAL STRATEGIES



Effective Mathematics Practices

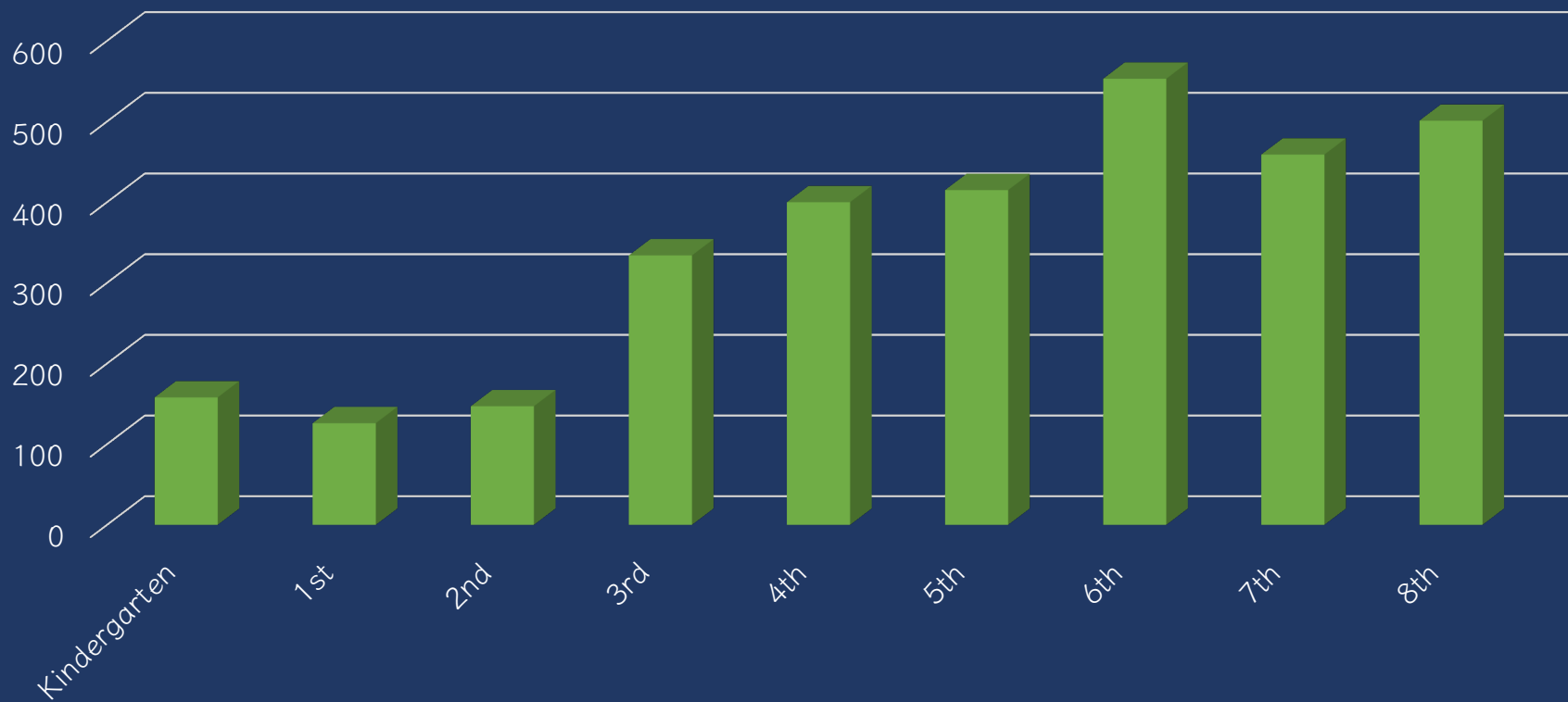
DAY 2

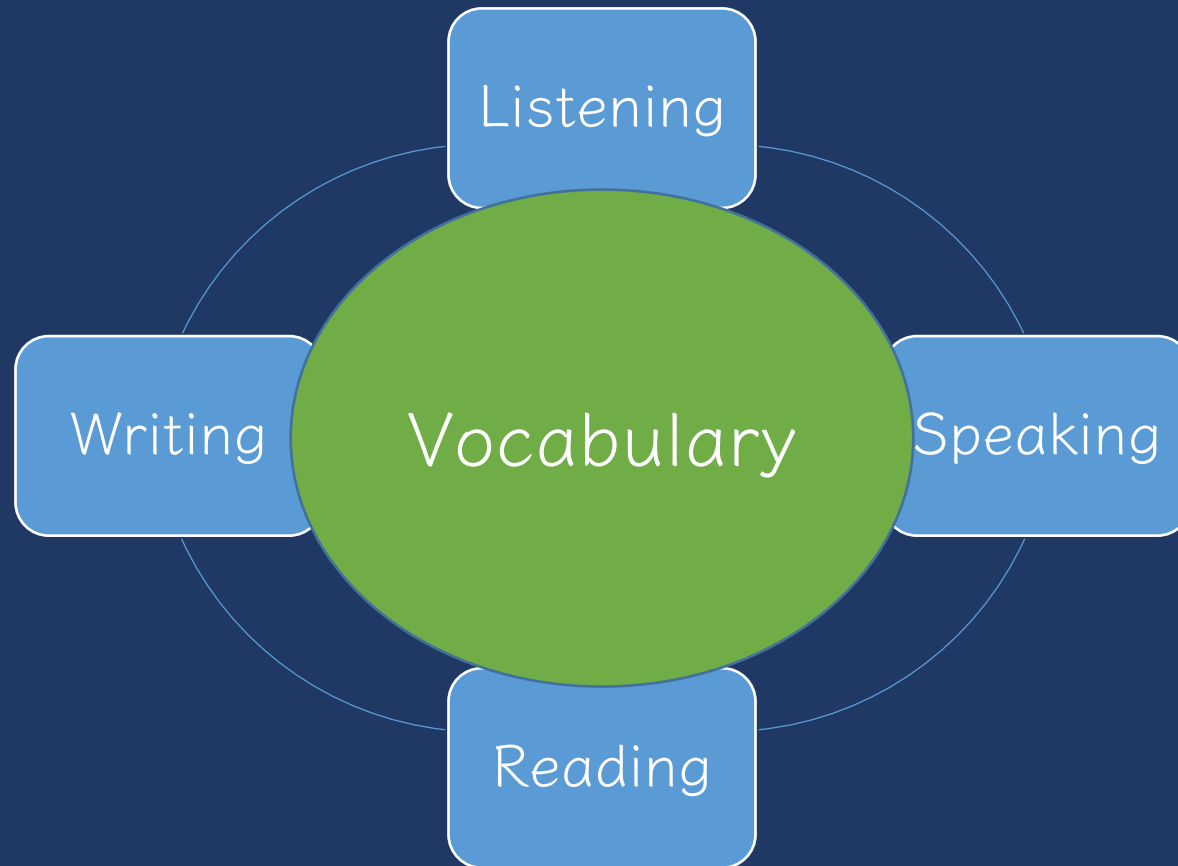
Sarah R. Powell, Ph.D.
srpowell@utexas.edu
@sarahpowellphd
www.sarahpowellphd.com

Mathematical Language

Instead of that...	Say this...







1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
(but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings

divide vs.
Continental
Divide

variable vs.
variably
cloudy

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings

factor vs.
multiple

hundreds vs.
hundredths

numerators
vs.
denominator

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings

mesa vs.
tabla

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way

skip count
vs. multiples

one-fourth
vs. one
quarter



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way
11. Informal terms may be used for formal math terms

rhombus vs.
diamond

vertex vs.
corner



Use formal math language

Use terms precisely



Mathematics

Regroup
Value
Hundreds
Less
Ones
Fewer
Greater
Tens
Balance
Digit
Place



Supporting Clear and Concise Mathematics Language

Instead of That, Say This

Elizabeth M. Hughes, Sarah R. Powell, and Elizabeth A. Stevens

TEACHING Exceptional Children, Vol. 46, No. 1, pp. 2-12, Copyright 2018 The Authors. DOI: 10.1177/0048794618769029

Middle-School Mathematics



8-15-
Right Triangle Relationships
Triangles are similar

Math Language in Middle School


Be More Specific

Sarah R. Powell, Elizabeth A. Stevens, and Elizabeth M. Hughes


TEACHING Exceptional Children, Vol. 51, No. 4, pp. 286-295, Copyright 2018 The Authors. DOI: 10.1177/0048794618769029

286 COUNCIL FOR EXCEPTIONAL CHILDREN





What number is in the tens place?




What digit is in the tens place?
What is the value of the digit in the tens place?

135


Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.





The alligator eats the
bigger number



is less than
OR
is greater than

Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because $<$ and $>$ are two distinct symbols.





carry OR borrow





regroup OR
trade OR
exchange

$$\begin{array}{r} 167 \\ + 294 \\ \hline \end{array}$$

Why this is important...

- “Carry” or “borrow” is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



top number and
bottom number



numerator and
denominator


Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.





reduce the fraction




rename OR
find equivalent OR
simplify


Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.





Four point seven
Four point oh seven



Four and seven tenths
Four and seven hundredths

4.7
4.07

Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.

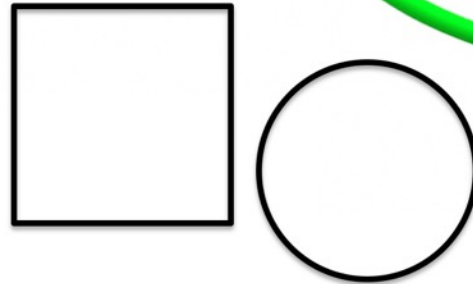




box OR ball



square OR
circle

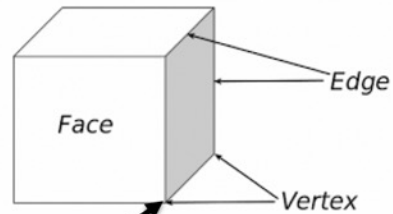


Why this is important...

- Use the formal language of shapes to confirm informal language.

point

vertex



Why this is important...

- This is the endpoint where two or more line segments or rays meet.





flips, slides, turns




reflections,
translations,
rotations

Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.



long hand and
short hand



minute hand and
hour hand

Why this is important...

- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.

Effective Mathematics Practices DAY 2

Sarah R. Powell, Ph.D.
srpowell@utexas.edu
@sarahpowellphd
www.sarahpowellphd.com

Mathematical Language

Instead of that...	Say this...



Identify examples of
“Instead of _____, say
_____.”



Use formal math language

Use terms precisely



Use Terms With Precision



Strategies for Teaching Mathematics Language



Factor

$$1 \times 8 = 8$$

$$2 \times 4 = 8$$

factor

factor

Multiple

$$8 \times 1 = 8$$

$$8 \times 2 = 16$$

multiples of 8

E



Improper fraction

$$\frac{8}{5}$$

Mixed number

$$1\frac{3}{5}$$

Proper fraction

$$\frac{2}{9}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Ratio

$$4:3$$

Unit fraction

$$\frac{1}{6}$$

D



Coefficient

Constant

Term

Variable

term term term

$2x^2 + x - 3$

coefficient variable variable constant

A



Equation $9x - 4 = 7x$

Expression $9x - 4$

Formula $a^2 + b^2 = c^2$

Function $f(x)$

Inequality $9x - 4 > 6x$

c



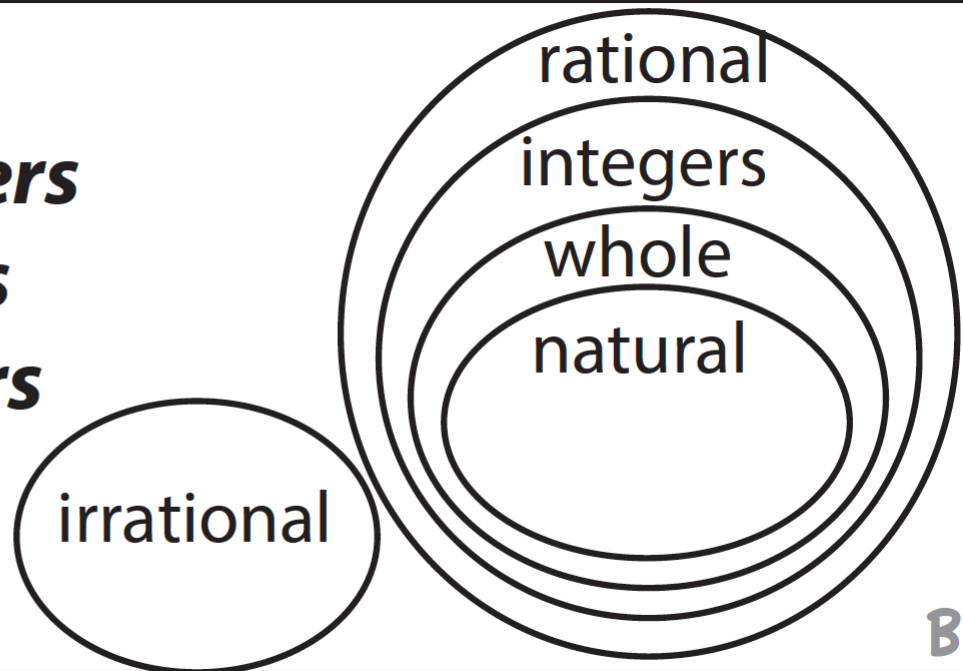
Integers

Irrational numbers

Natural numbers

Rational numbers

Whole numbers



Quadrilaterals

Kite



Rhombus



Parallelogram



Square



Rectangle

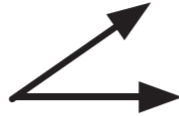


Trapezoid

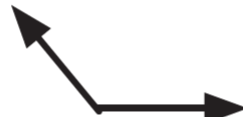


A

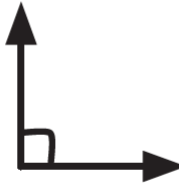
Acute angle



Obtuse angle



Right angle



Straight angle



B

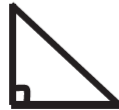
Acute triangle



Obtuse triangle



Right triangle



Equilateral triangle



Isosceles triangle

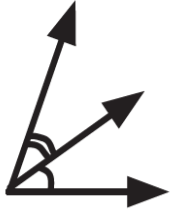


Scalene triangle

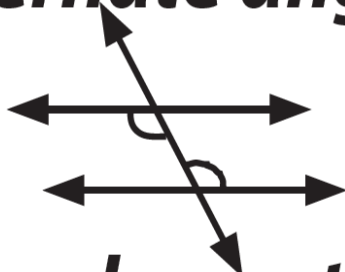


C

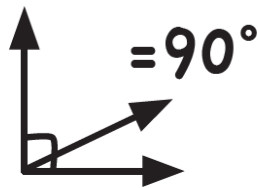
Adjacent angles



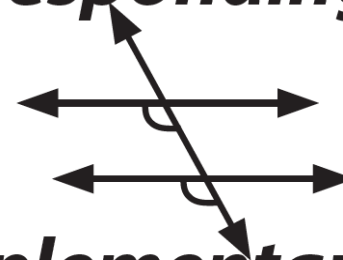
Alternate angles



Complementary angles



Corresponding angles

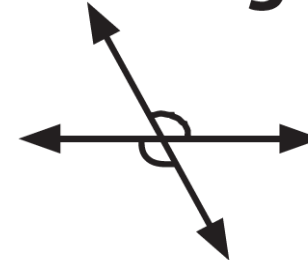


Supplementary angles

= 180°



Vertical angles

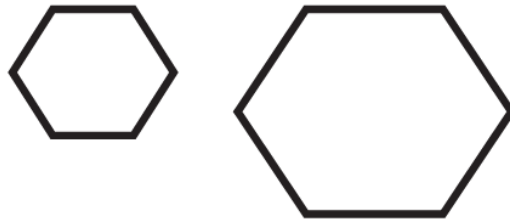


D

Congruent figures

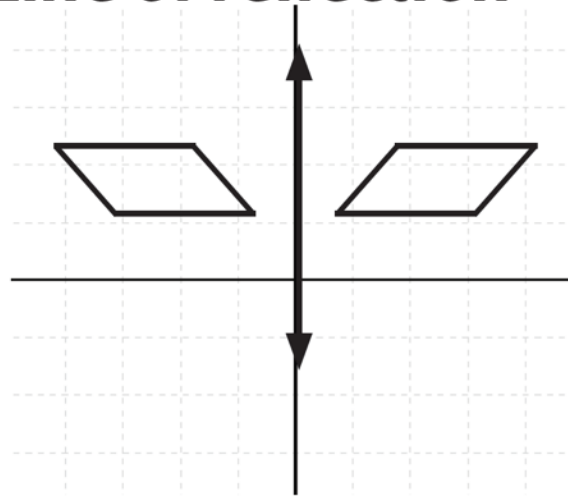


Similar figures

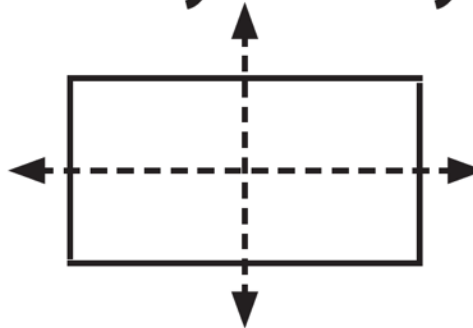


E

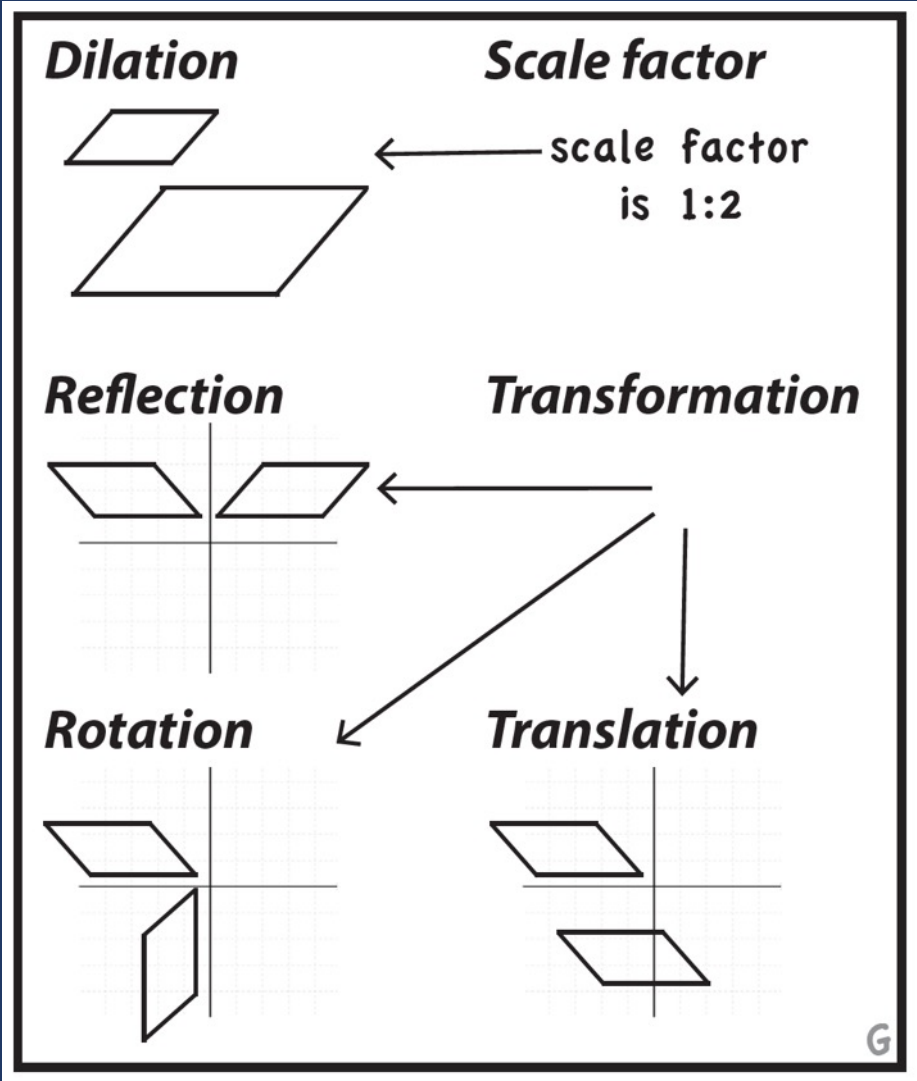
Line of reflection



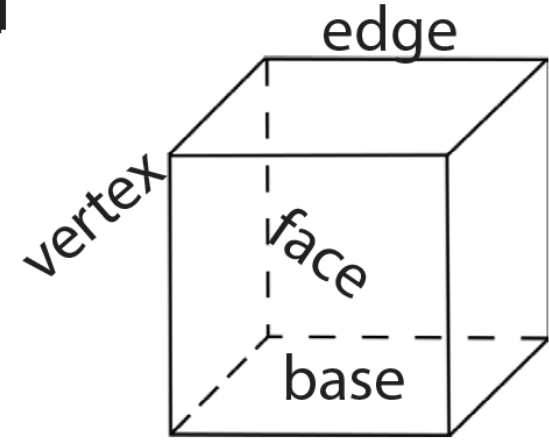
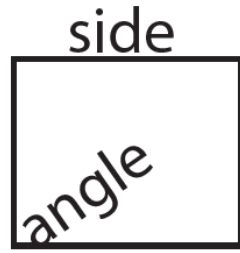
Line of symmetry



F

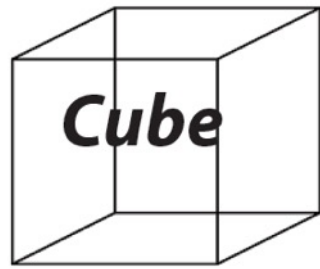


Angle
Base
Edge
Face
Side
Vertex



#





I

Coordinate plane

Ordered pair Quadrant 2

Quadrants

x

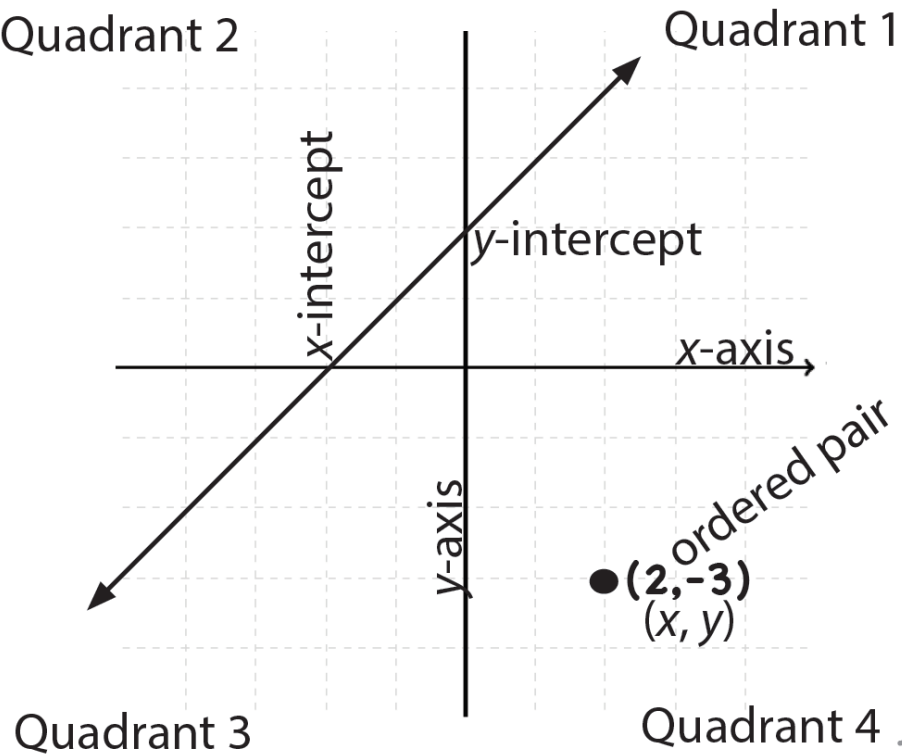
x-axis

x-intercept

y

y-axis

y-intercept



Use Terms With Precision

Strategies for Teaching Mathematics Language



Discuss terms you want your students to use with precision.



Use formal math language

Use terms precisely

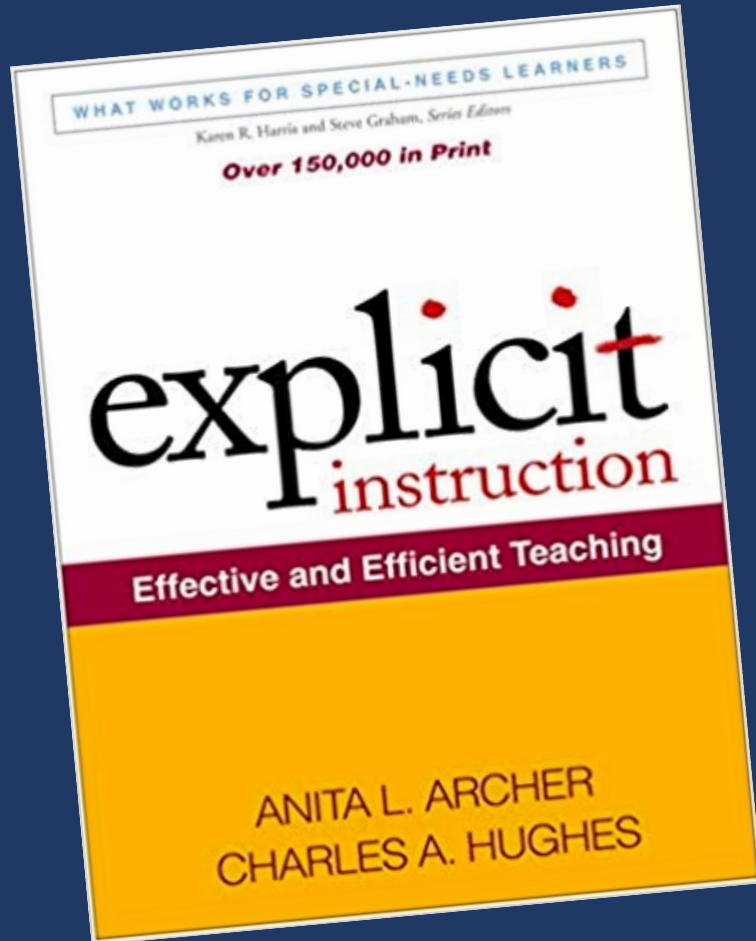


Use Terms With Precision

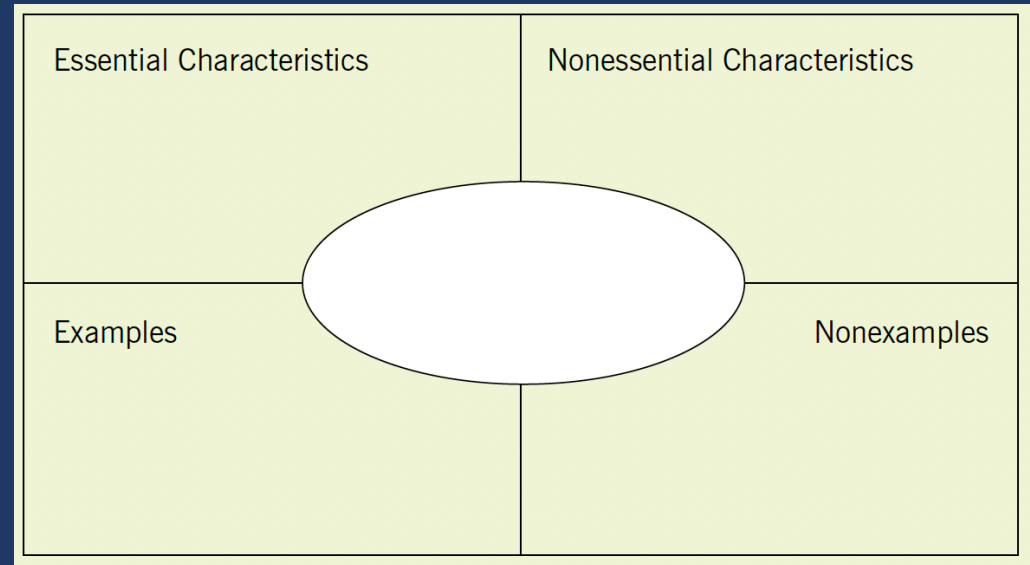
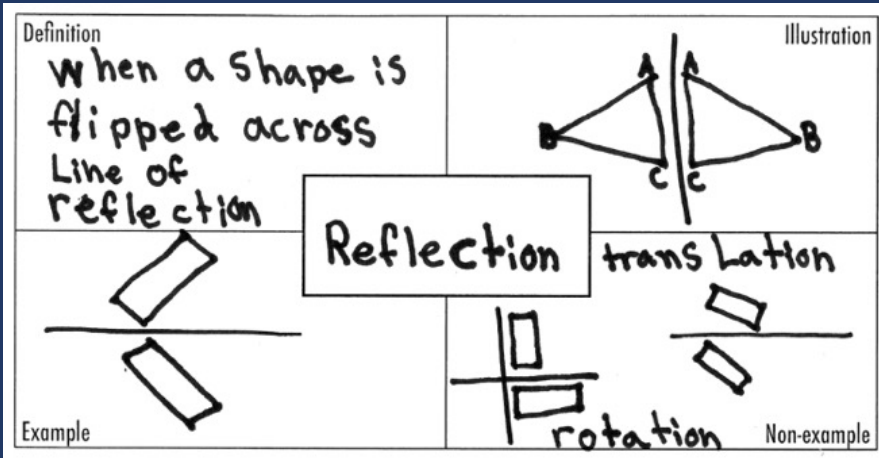
Strategies for Teaching Mathematics Language



1. Use explicit instruction



2. Use graphic organizers



Dunston & Tyminski (2013)



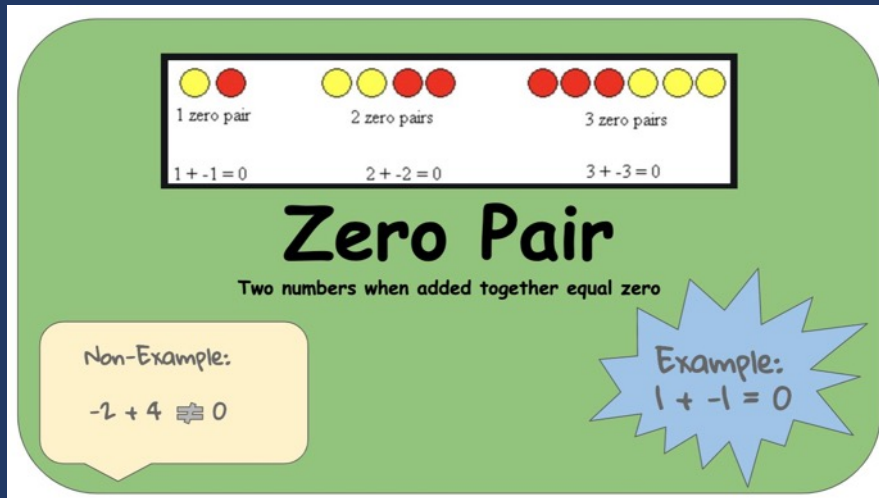
2. Use graphic organizers

Word	Lightbulb Word
Definition	Picture

Dunston & Tyminski (2013)

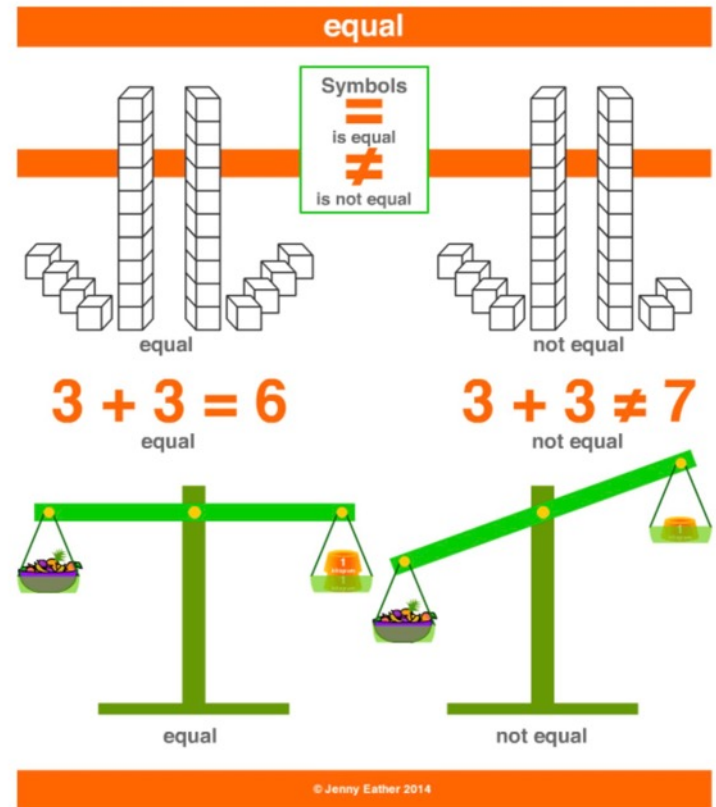


3. Have students create vocabulary cards



A green-bordered card titled "Zero Pair" with the subtitle "Two numbers when added together equal zero". At the top, three pairs of colored circles (yellow and red) illustrate 1, 2, and 3 zero pairs, each with a corresponding equation: $1 + -1 = 0$, $2 + -2 = 0$, and $3 + -3 = 0$. The main title "Zero Pair" is in large black font. Below it, a yellow box labeled "Non-Example:" contains the equation $-2 + 4 \neq 0$. A blue starburst labeled "Example:" contains the equation $1 + -1 = 0$.

6. Equal: having the same amount or value.




A white card with orange accents illustrating "equal" and "not equal". At the top, an orange bar contains the word "equal". Below it, a green box lists symbols: $=$ is equal and \neq is not equal. Two sets of blocks are shown: one with two stacks of 3 blocks each, labeled "equal" with the equation $3 + 3 = 6$; and another with two stacks of 3 blocks each and one extra block, labeled "not equal" with the equation $3 + 3 \neq 7$. Below these, two balance scales are shown: one balanced and labeled "equal", and one tilted and labeled "not equal". At the bottom, an orange bar contains the copyright notice "© Jenny Esther 2014".

4. Have students create glossaries

Integer Definitions

Zero Pairs
A positive and negative cancel one another;

Positive
A number that is greater than zero.



Absolute Value
The distance of a number from zero on a number line; shown as $||$

Negative
A number that is less than zero. Identified by a minus sign.

Numerator: how many parts of the whole

- Ex. $\frac{4}{10}$

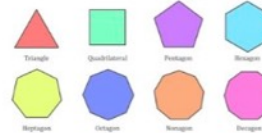
Odd number: a number not divided evenly by 2

- Ex. 1, 3, 5, 7, 9....

Percent: a specific number in comparison to 100

- 74%

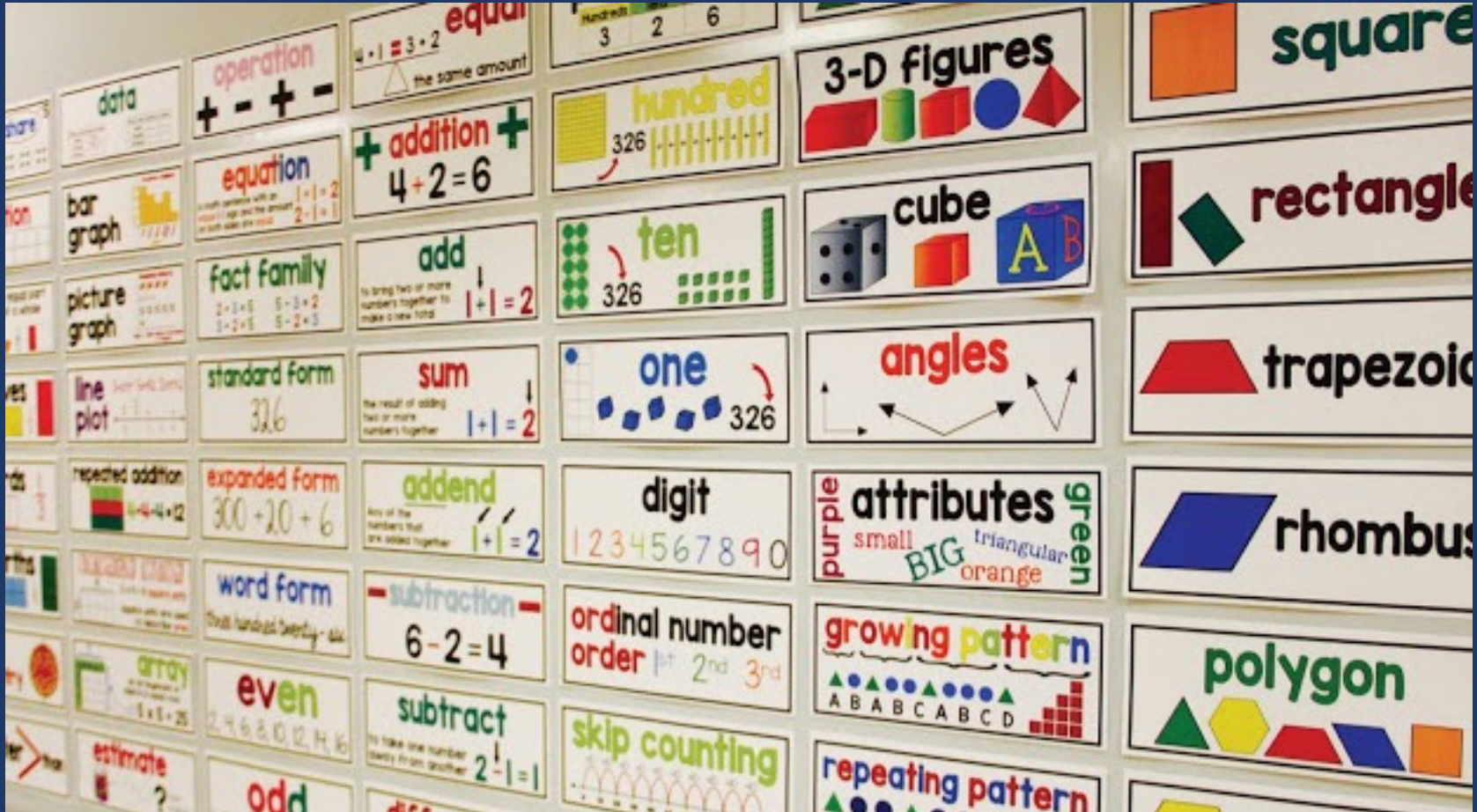
Polygon: any enclosed shape that is made up of 3 or more straight lines



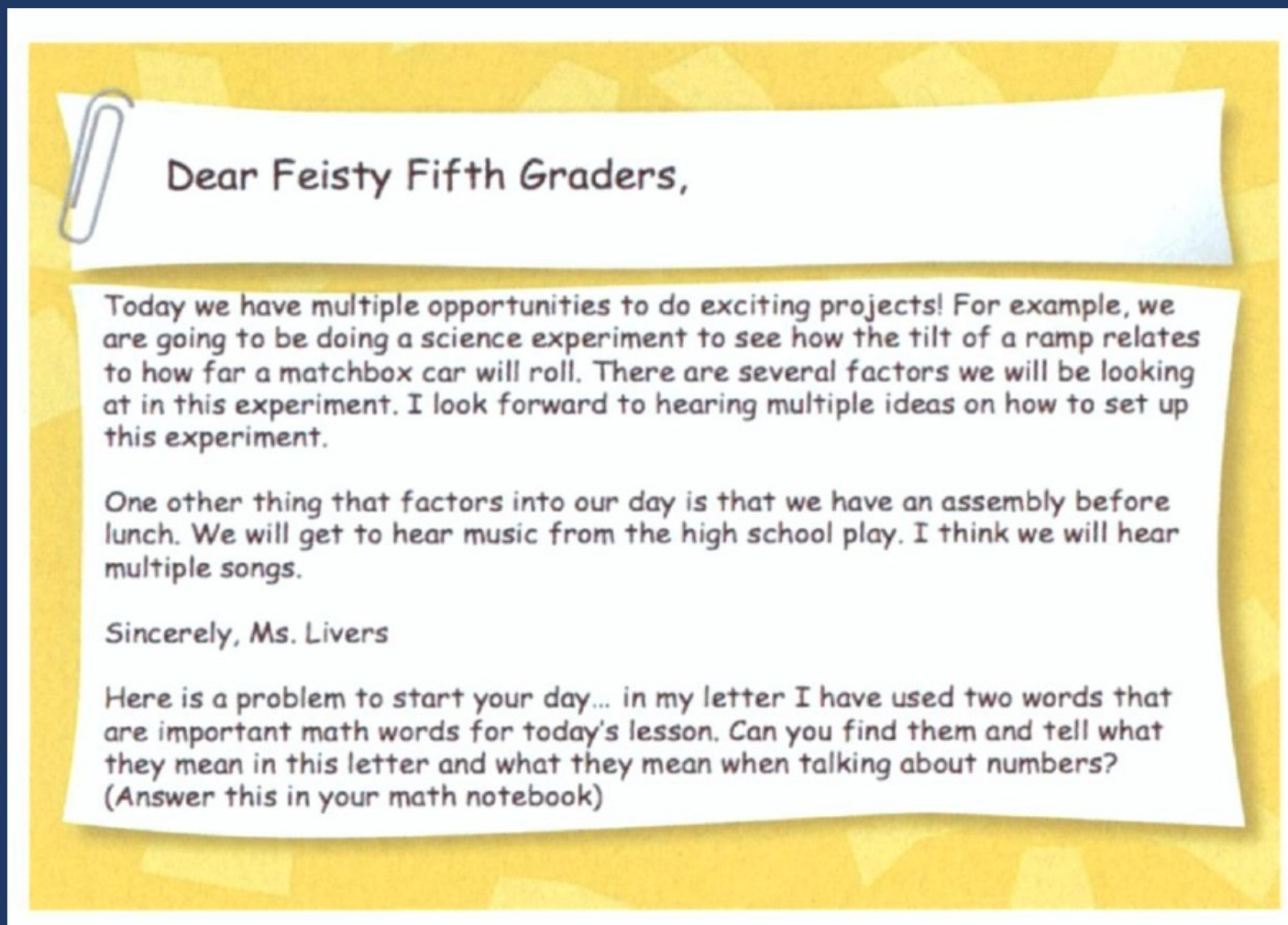
- Ex.



5. Create a word wall



6. Preview vocabulary



Dear Feisty Fifth Graders,

Today we have multiple opportunities to do exciting projects! For example, we are going to be doing a science experiment to see how the tilt of a ramp relates to how far a matchbox car will roll. There are several factors we will be looking at in this experiment. I look forward to hearing multiple ideas on how to set up this experiment.

One other thing that factors into our day is that we have an assembly before lunch. We will get to hear music from the high school play. I think we will hear multiple songs.

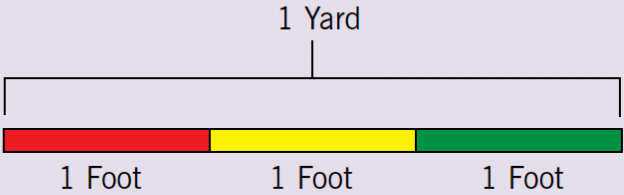
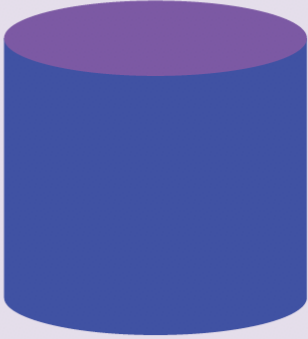
Sincerely, Ms. Livers

Here is a problem to start your day... in my letter I have used two words that are important math words for today's lesson. Can you find them and tell what they mean in this letter and what they mean when talking about numbers?
(Answer this in your math notebook)

Bay-Williams & Livers (2009)



7. Cluster vocabulary

	Length	Weight
Meaning	How long something is	How heavy something is
Visual	 <p>1 Yard</p> <p>1 Foot 1 Foot 1 Foot</p>	<p>2000 pounds = 1 ton</p> 

Livers & Bay-Williams (2014)



7. Cluster vocabulary

Rating	Word	Definition	Synonym(s)	Example	Sample Problem
2	expression	a mathematical phrase combining operations, numbers and/or variables.	phrase algebraic expression	6 6n 6+n <i>no equal sign</i>	Lucia earns \$8 per hour for babysitting and gets a \$5 tip. Write an <u>expression</u> to represent the amount she would earn if she worked for x hours.
2	variable	a quantity that can change or take many values. (refers to the letter or symbol representing the quantity)	unknown	x D y T	The <u>variable</u> x represents the number of hours Charlie works in a week. Write an expression to represent his earnings if he earns \$9 per hour.
1	product	the result when two or more numbers are multiplied	total answer	$3 \times 2 = 6$ ↑ product	The <u>product</u> of 6 and a number is 24. What is the number?
3	quotient	the result of a division (refers to the number of times the divisor divides the dividend)	answer	$18 \div 2 = 9$ $2 \overline{)18}$ ↑ quotient	Estimate the <u>quotient</u> when 365 is divided by 12.

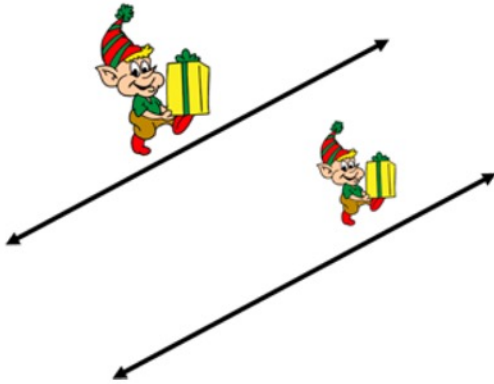
Marin (2018)



8. Use mnemonics

Parallel Lines (Pair of Elves)

Lines that are the same distance apart and will never intersect



The Pair of Elves are the same distance apart and will never intersect.

The Pair of Elves are on Parallel Lines

Ray (Run Away)

A line that has a starting point but no endpoint

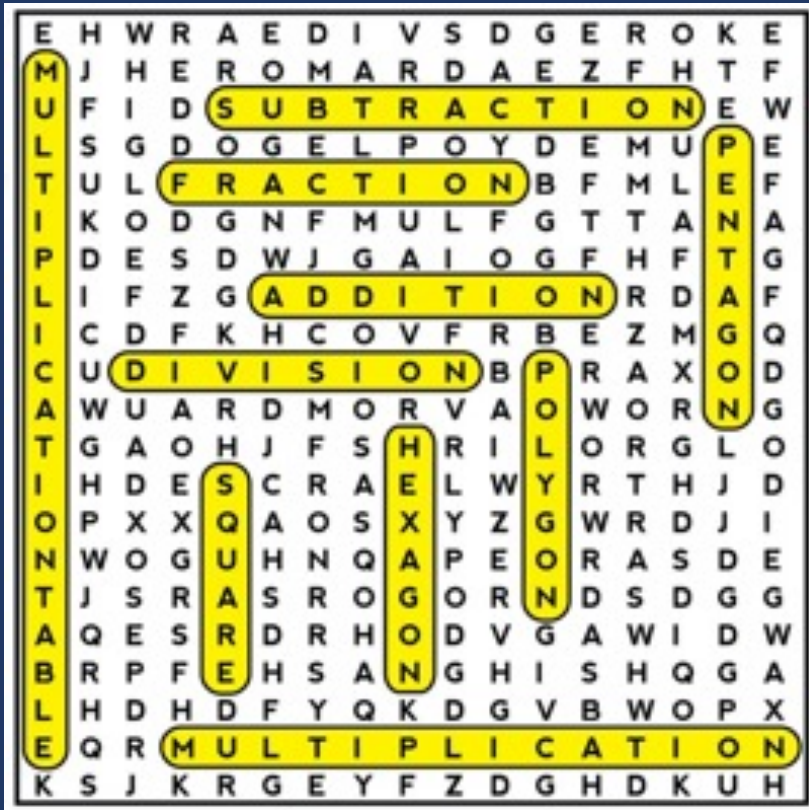


Start here!! Run away and never stop running Ray.

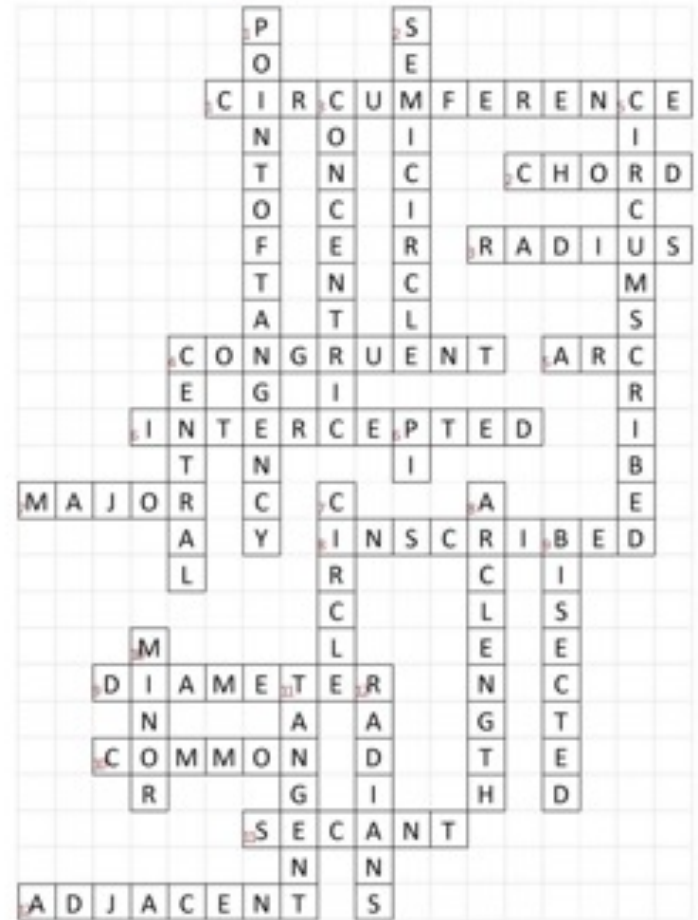
Riccomini et al. (2015)



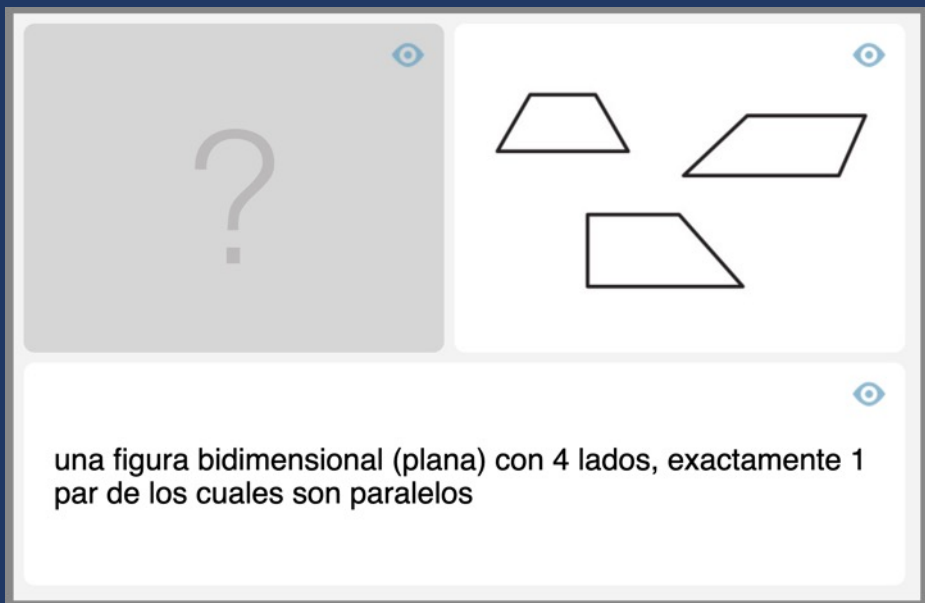
9. Do word games



CIRCLES VOCABULARY CROSSWORD ANSWER KEY



10. Use technology



una figura bidimensional (plana) con 4 lados, exactamente 1 par de los cuales son paralelos

Math Learning Center



Houghton Mifflin Math eGames **Math Lingo** Grade 2

How to Play New Game

60 minutes = 1 ____

change	minute hand	hour hand
hour	equal amounts	second
quarter-hour	half-hour	minute

Math Lingo



Use Terms With Precision

Strategies for Teaching Mathematics Language



Discuss your strategy for focusing on mathematical language in your teaching.



Word-Problem Solving



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



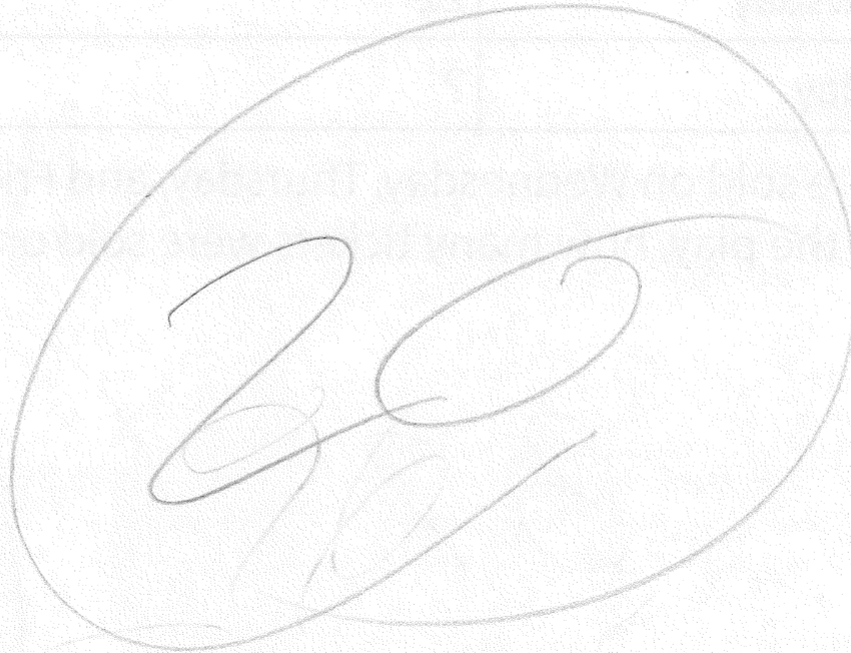
How Students Solve Word Problems



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?



71

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

71

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

Donna



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

96 paper fold



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ + 25 \\ \hline 121 \end{array}$$



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ + 25 \\ \hline 121 \end{array}$$

✓

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ \times 25 \\ \hline 121 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ + 25 \\ \hline 121 \end{array}$$



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ - 25 \\ \hline 71 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} + 1 \\ + 96 \\ + 25 \\ \hline 121 \end{array}$$



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ + 25 \\ \hline 121 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 1 \\ 96 \\ + 25 \\ \hline 121 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ - 25 \\ \hline 71 \end{array}$$



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ - 25 \\ \hline \end{array}$$

Correct



Undetermined



Repeated information



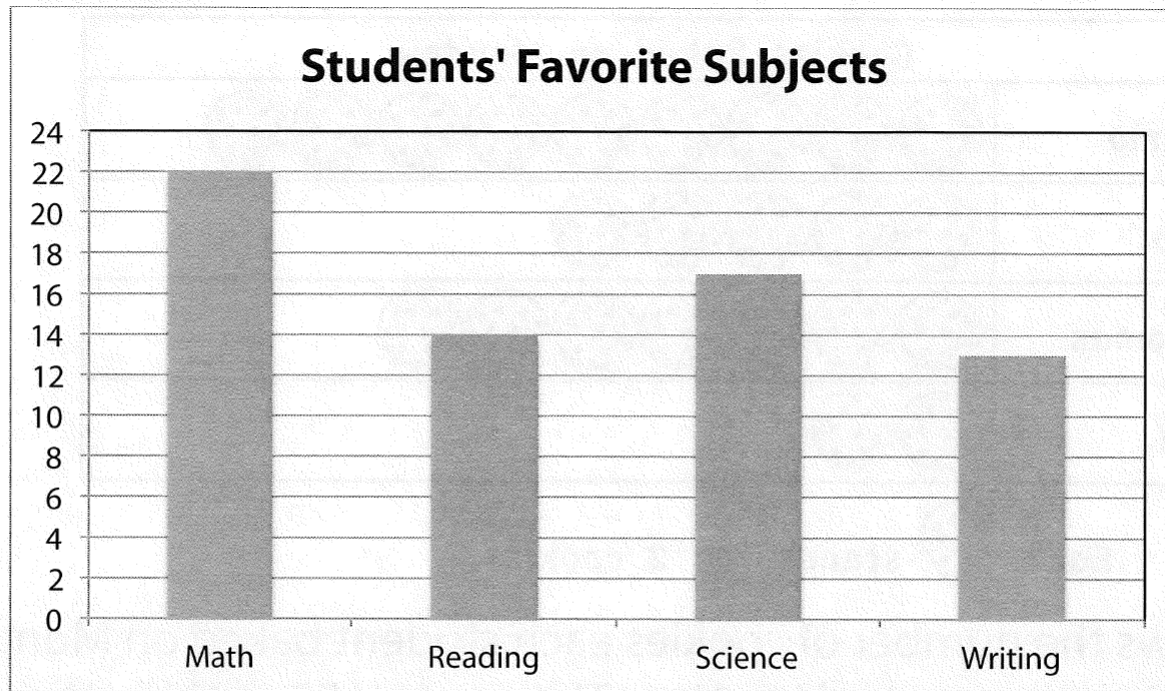
Wrong information



Wrong (misapplied) operation



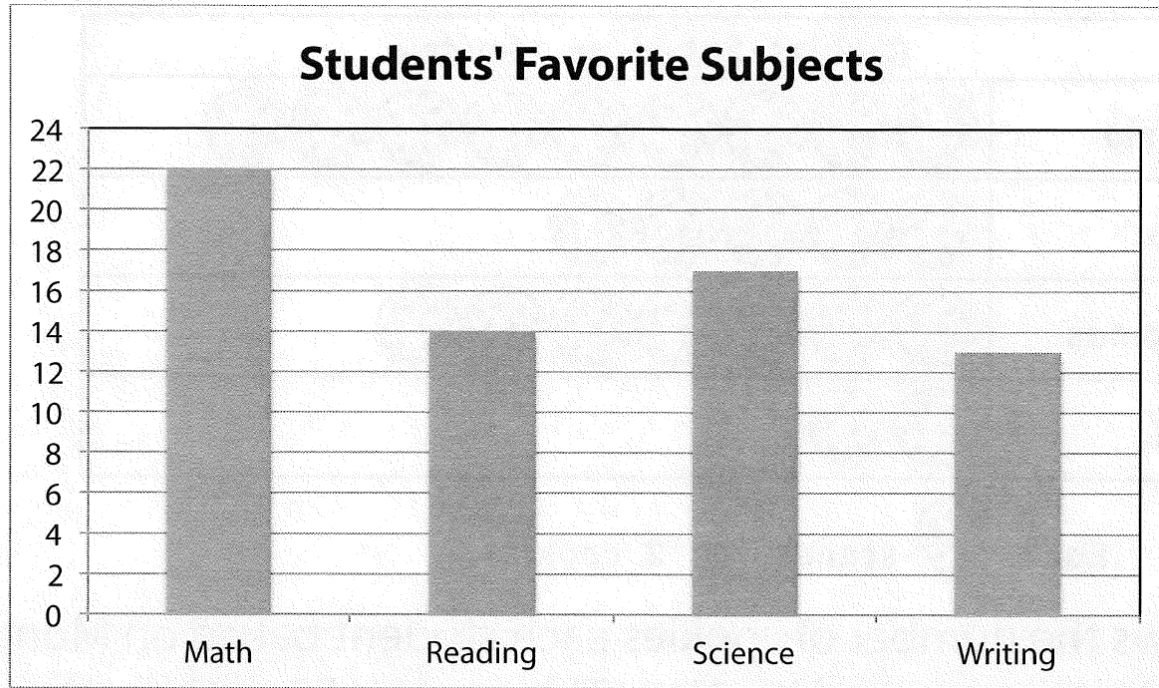
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?



J.

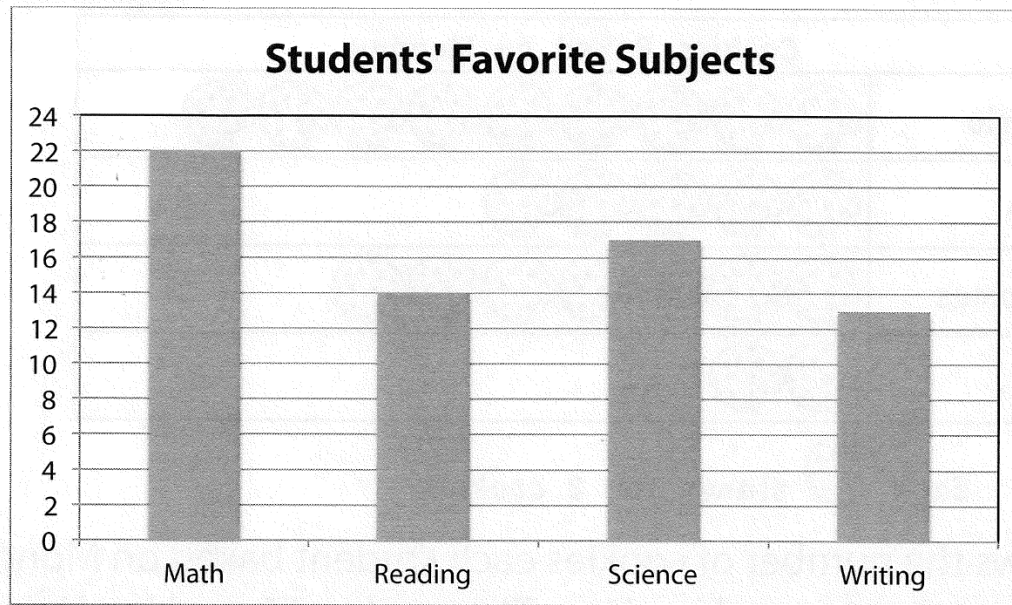


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

60

|||||

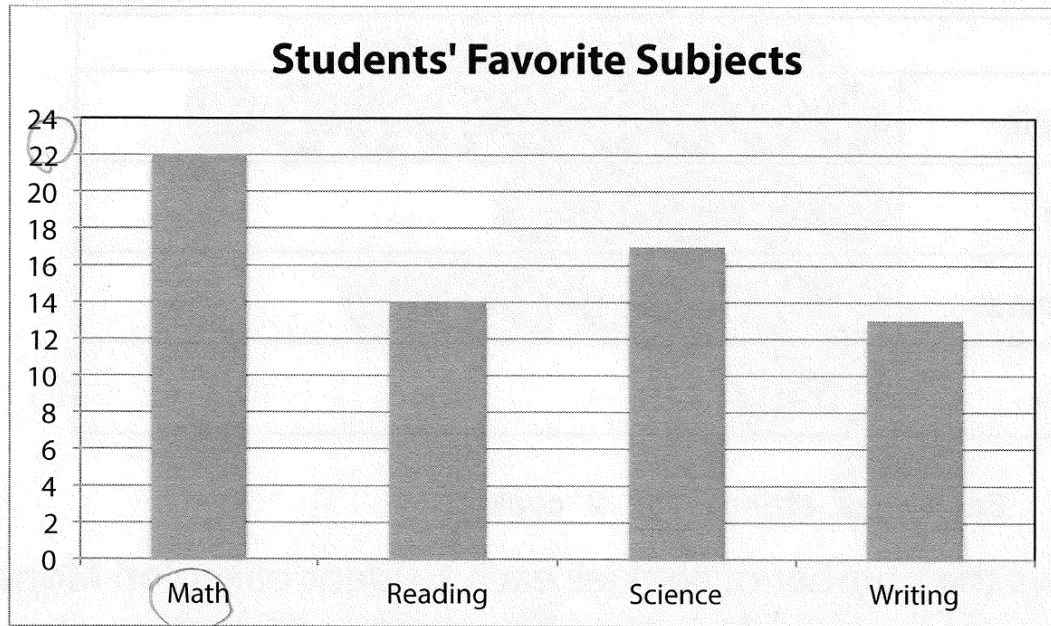
|||||



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

They choose 8% more than reading

J.

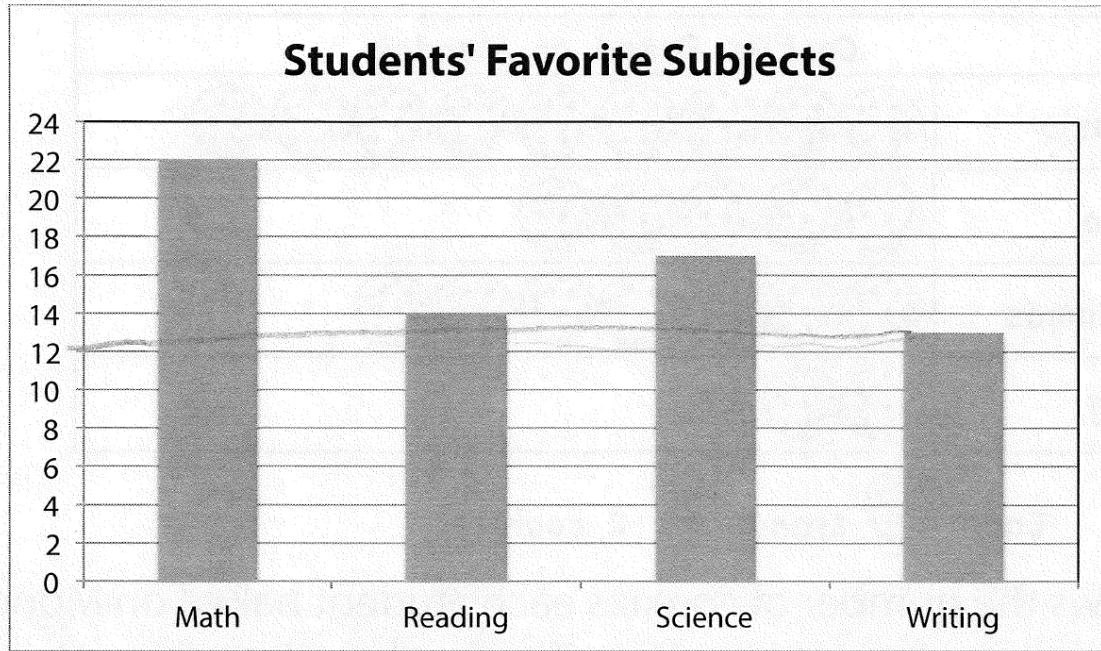


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

22
Math



J.

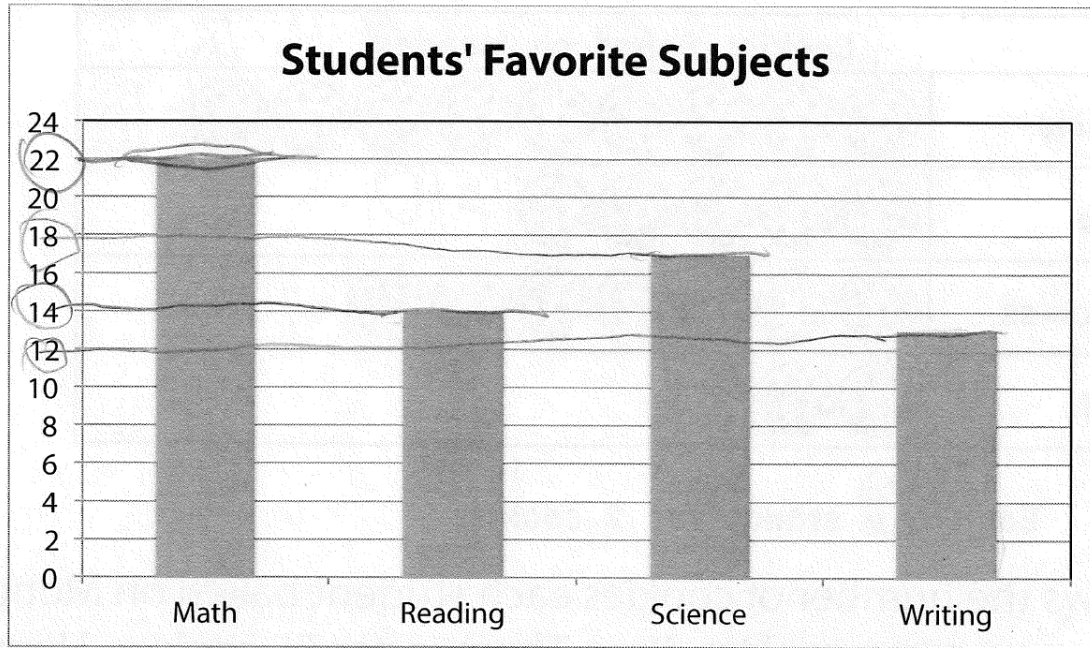


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

(22) math (13) writing



J.

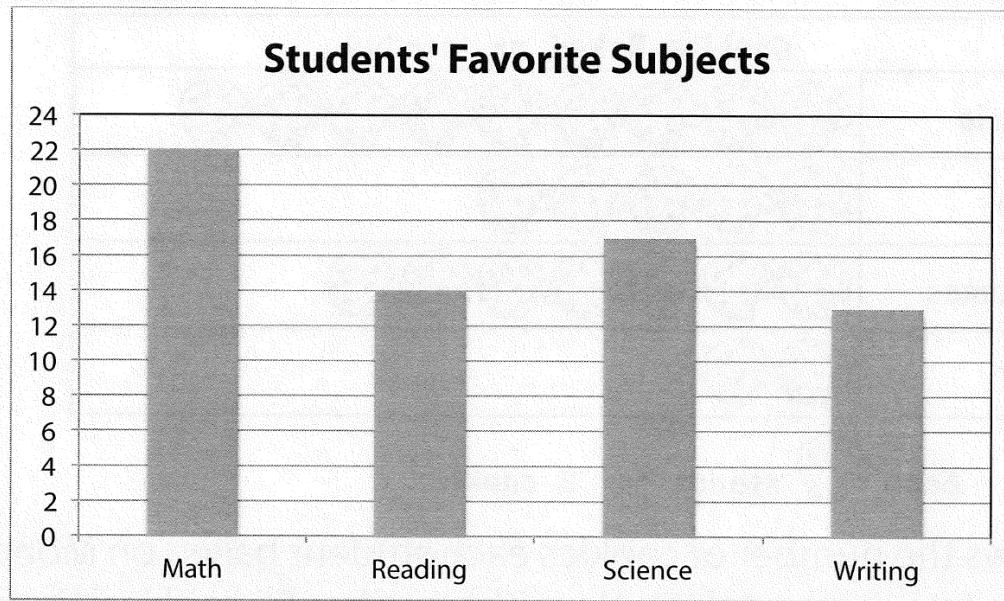


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

22
Math

14
writing

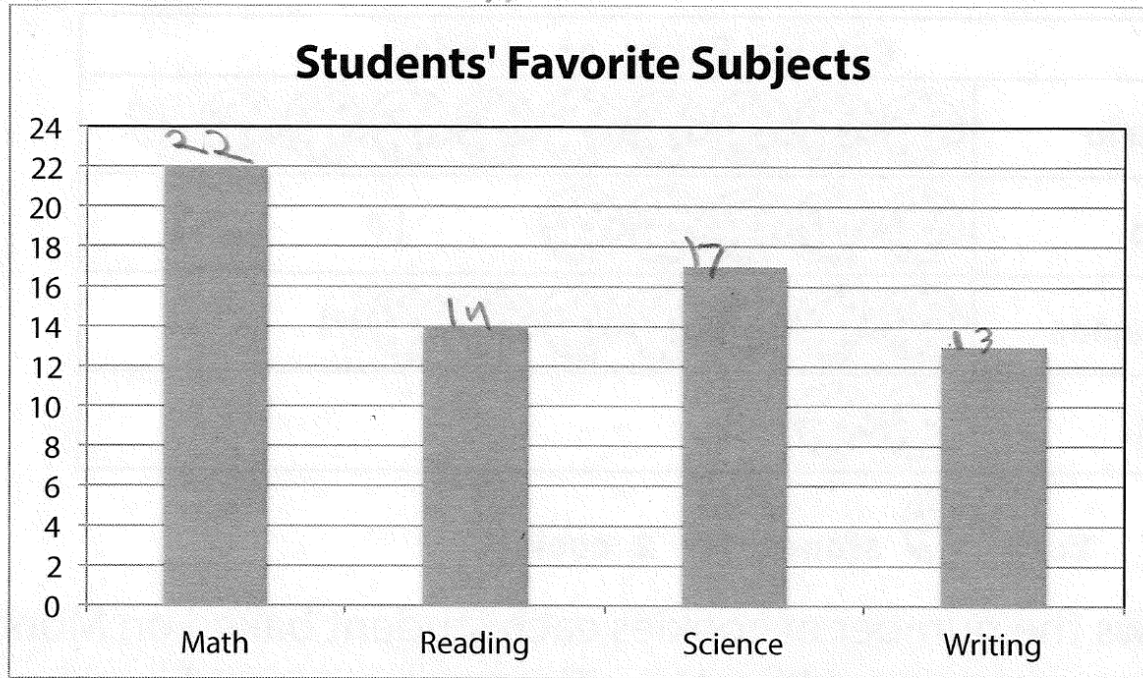
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

the students chooses they
love more math dicar es
so to 22 math

J.

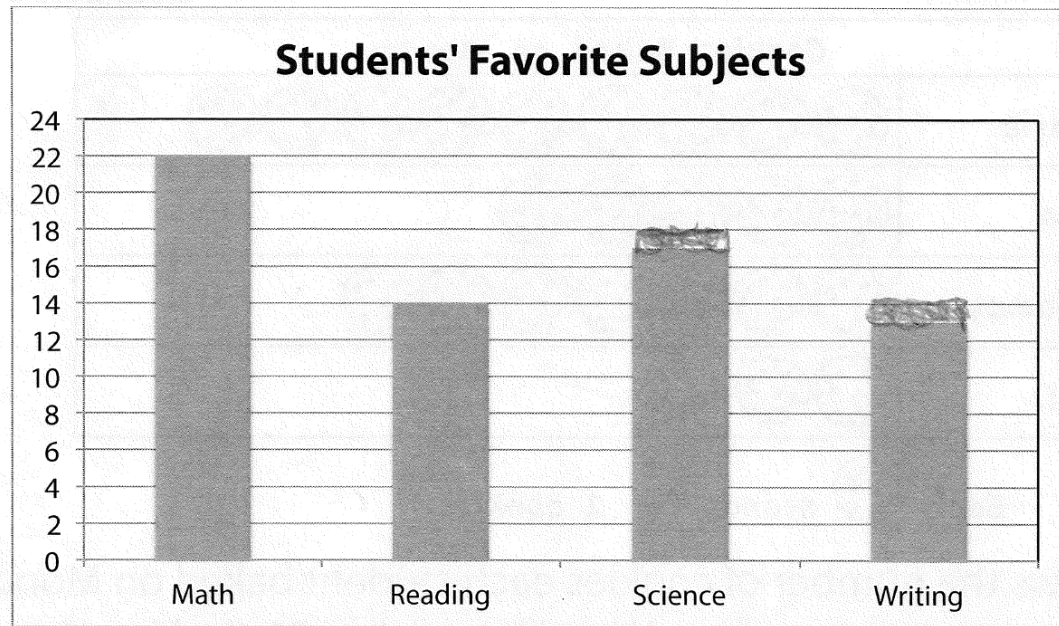


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 14 \\ + 13 \\ \hline 27 \end{array}$$



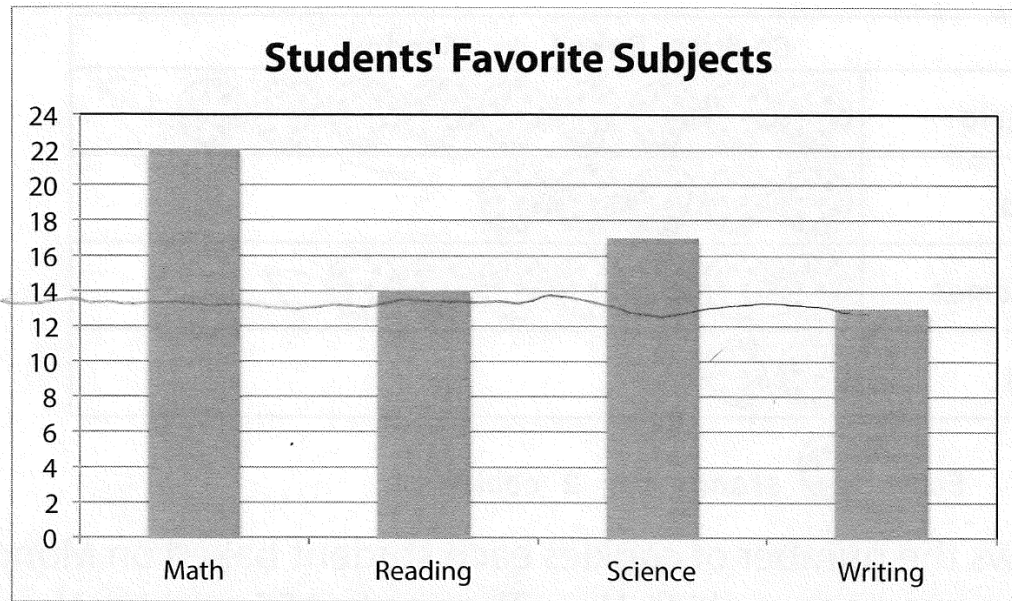
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ + 14 \\ \hline 36 \end{array}$$

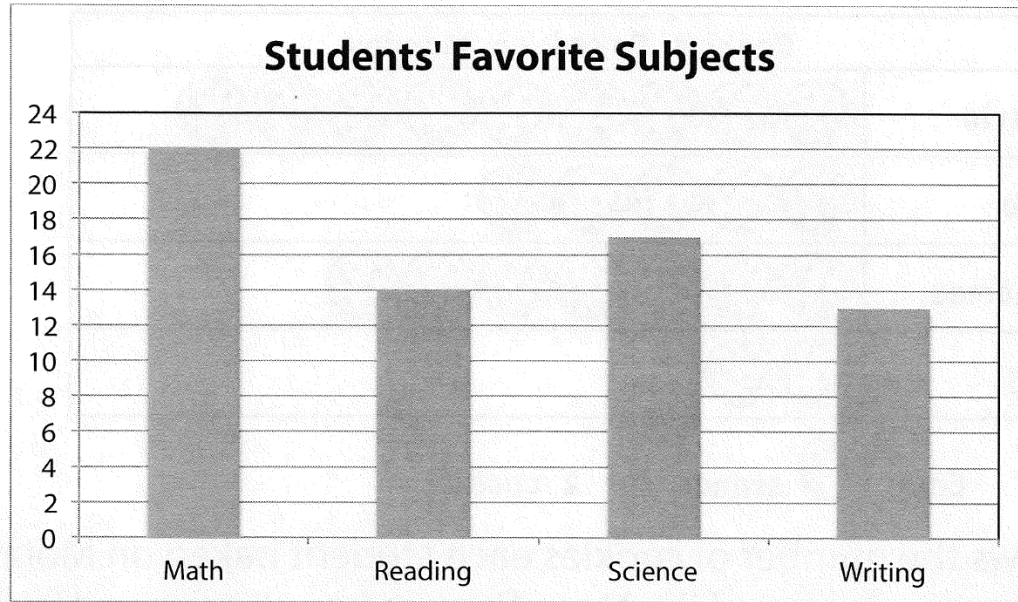
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 24 \\ + 13 \\ \hline 37 \end{array}$$

J.

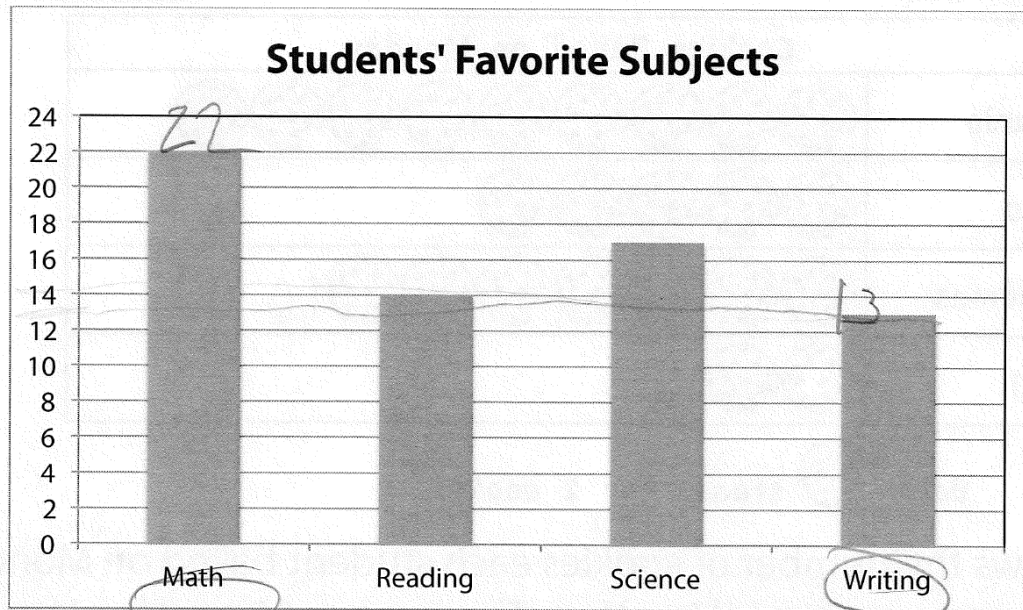


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ + 12 \\ \hline 34 \end{array}$$

34

J.



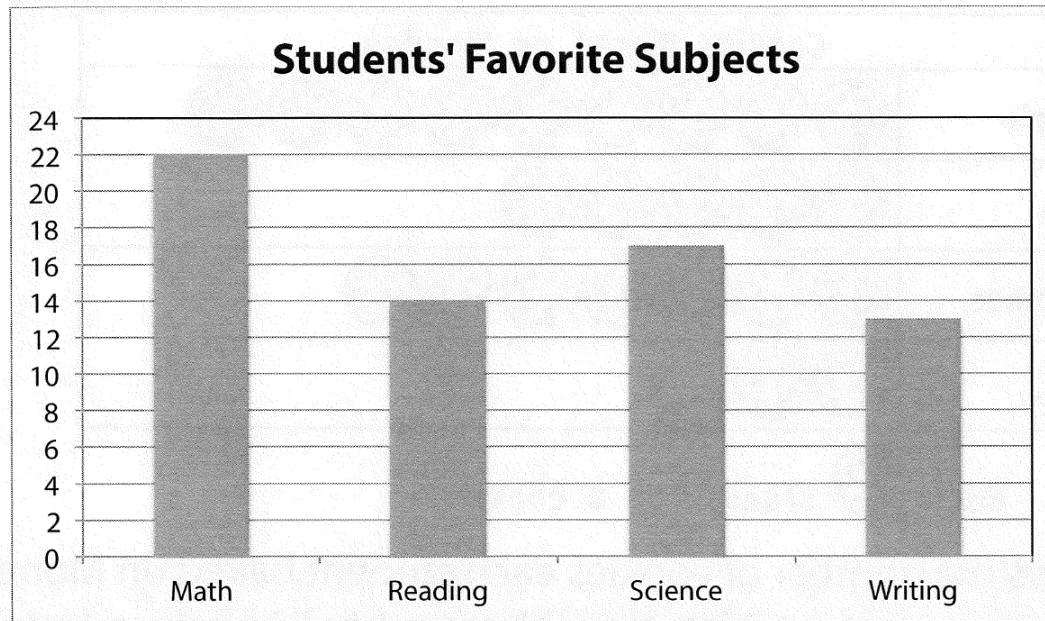
The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$22$$
$$+ 13$$

$$35$$

14

J.

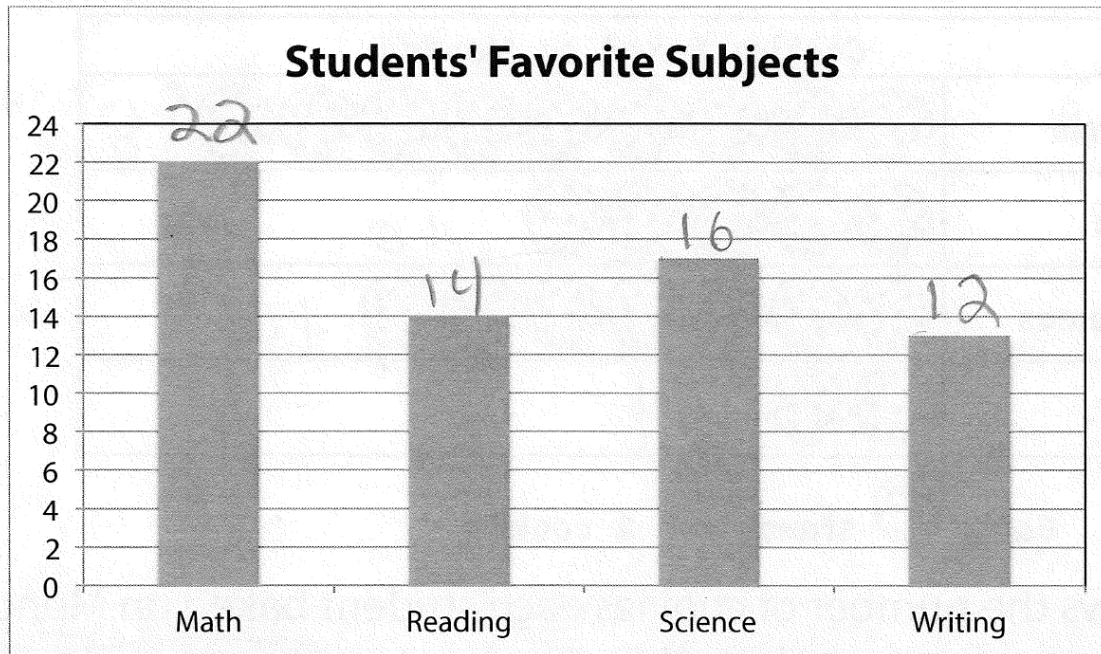


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 13 \\ \hline 9 \end{array}$$



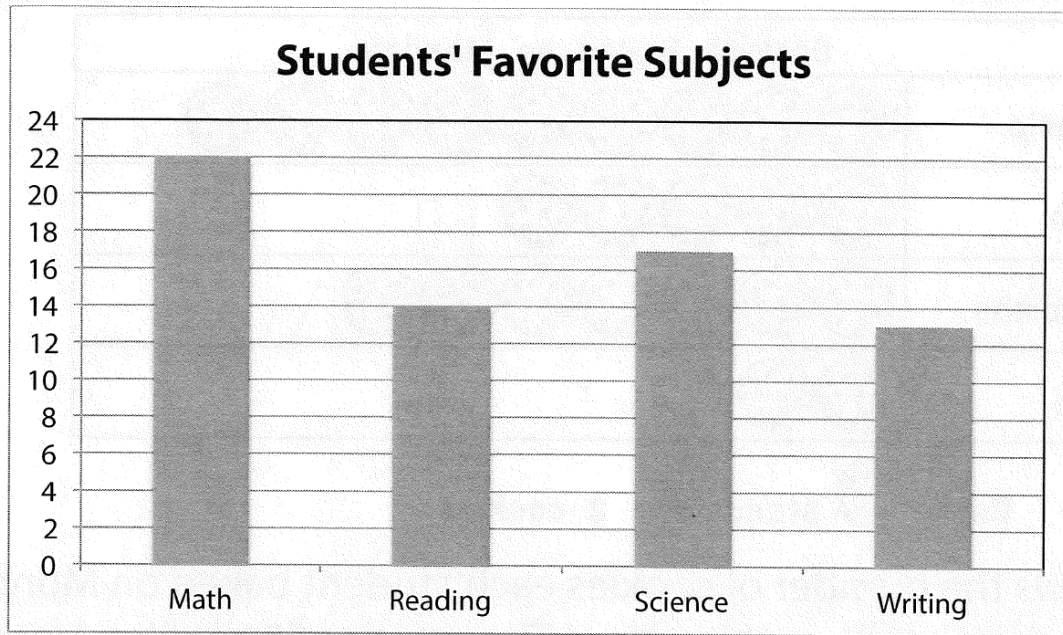
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 12 \\ \hline 10 \end{array}$$

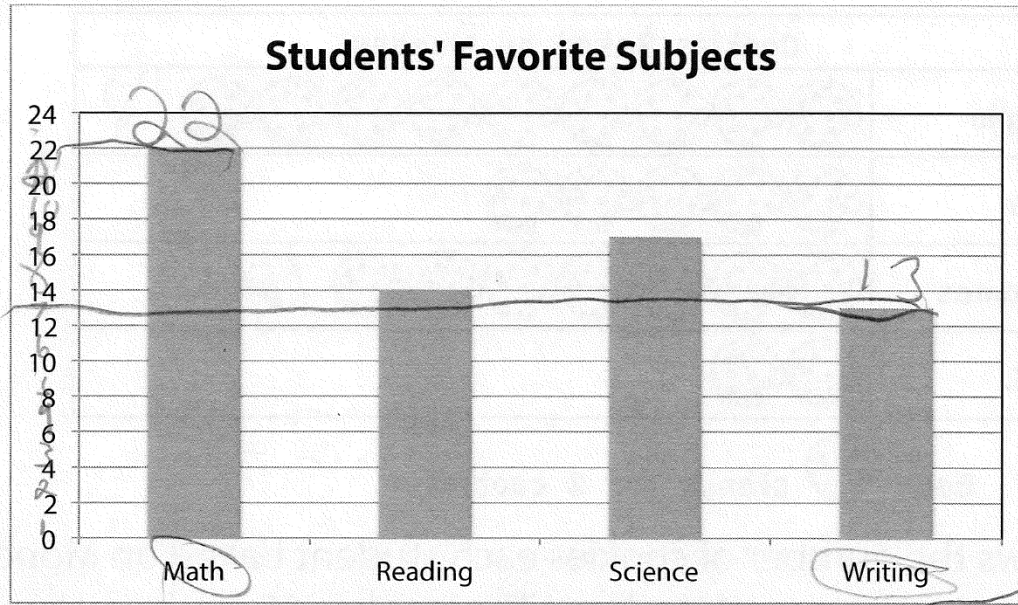
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 13 \\ \hline 9 \end{array}$$

J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 13 \\ \hline 9 \end{array}$$



Correct



Undetermined



Didn't provide
answer



Wrong
(misapplied)
operation



Wrong
information



Subtraction error



COMMON

Undetermined

Repeated
information from
problem

Didn't provide
answer

Wrong
information

Wrong
(misapplied)
operation

UNCOMMON

Addition error

Subtraction error



Word-Problem Solving

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20

Solve the problem

What skills are necessary to solve this problem?



Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20



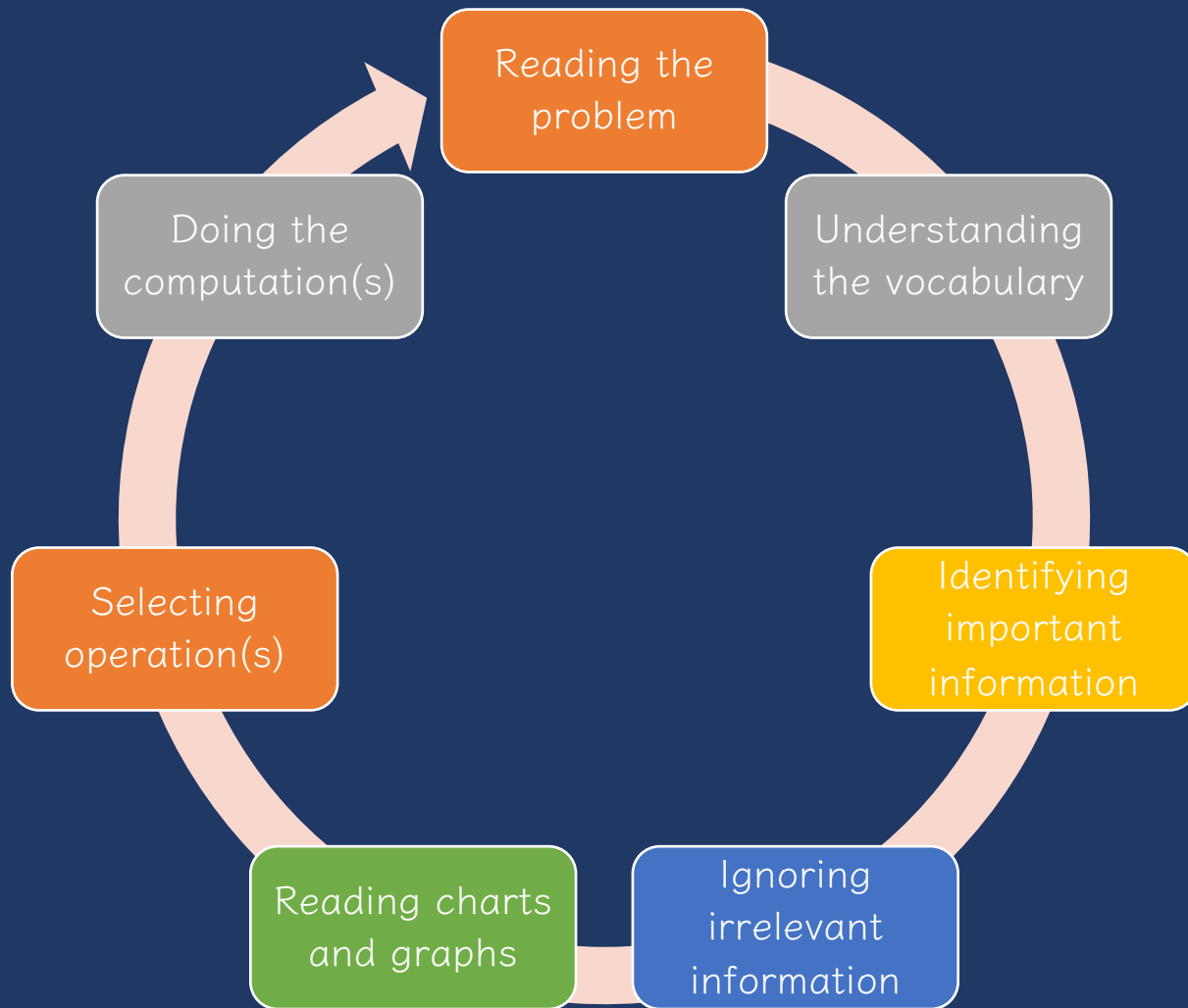
How would you solve this problem?
What skills are necessary to solve this problem?



Word-Problem Solving

Teaching Problem Solving





Ineffective Strategies





1. Keywords tied to operations



Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?

Key Words Used in Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> add all together or altogether and both combined how many in all how much in all increased by plus sum together total 	<ul style="list-style-type: none"> change decreased by difference fewer or fewer than how many are left (or have left) how many did not have how many how much taller, heavier, less or less lost minus need to reduce remain subtract take away

+

OPERATION cue words

ADDITION	SUBTRACTION
<ul style="list-style-type: none"> and total join more than in all sum altogether increased 	<ul style="list-style-type: none"> less than decreased remaining left fewer take away difference minus
MULTIPLICATION	DIVISION
<ul style="list-style-type: none"> product times as many as of by equal groups 	<ul style="list-style-type: none"> quotient each broken into distributed evenly parts

Math POSTER

ADDITION	MULTIPLICATION
<ul style="list-style-type: none"> -sum -total -more than -plus 	<ul style="list-style-type: none"> -both -combined -increased by -perimeter -product -per -double -every -each -by
SUBTRACTION	DIVISION
<ul style="list-style-type: none"> -difference -remain -left -less than -minus -how many more 	<ul style="list-style-type: none"> -fewer than -decrease -give away -reduce -discount -how many more -quotient -divide by -into -split -out of -shared -per -every -each -evenly -equal groups -half

Problem Solving Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add together 	<ul style="list-style-type: none"> are not decrease difference fewer, larger, shorter left less than minus remain take away

key words

combined

addition: sum, both

in all: together, perimeter

total: plus, add

more than

triple

factor

product

multiply

each

per

in all

multiple

area

double

times

average

division: equal groups, half

split

quotient

divide

shared

equally

each

distribute

Math Operation - Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add altogether and both in all sum total increase 	<ul style="list-style-type: none"> difference fewer than gave/take away decreased by how many more show much longer/smaller/shorter minus remaining
Multiplication	Division
<ul style="list-style-type: none"> area product Each by - of - per Times double, twice, triple total increase 	<ul style="list-style-type: none"> quotient divide into equal parts/share equally per amount of each

Math Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> plus sum add total all together increase more combine 	<ul style="list-style-type: none"> subtract minus difference left left over decrease take away fewer 	<ul style="list-style-type: none"> times product factor double groups each area rows 	<ul style="list-style-type: none"> quotient split share divide separate each average equal groups



Math Words Poster Set

★★★★★ (4.1)

75%

[7 Reviews](#)
[Write a Review](#)

Dependents would recommend this to a friend

Do you want to know about this product?

Answer in about 48 hours

with 3 monthly payments of just \$4.33. No fees.

[Learn more](#)

1 post

Quantity

-

1

Ready to buy

Add to Wish list





Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords ^a		Multiple keywords ^a		Keyword(s) led to correct solution ^a	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

^aWhen a problem featured a keyword.





Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema ^a		Any keyword		Keyword(s) led to correct solution ^b	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

^aSum across schemas does not equal 100 because each word problem featured more than one schema.

^bWhen a problem featured a keyword.



Mr. Rivera's taxable income is \$20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?



Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords
("What does *more than* tell you about?")



But, do not tie a keyword to a specific operation!





2. Presenting problems by operation



Name: _____

Date: _____

Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. How many books did Noah have in all?
2. Bonnie found 8 rocks on her front yard and 7 rocks in her backyard. How many rocks did she find in all?
3. Edward had 5 toy cars. He got 3 more toy cars. How many toy cars did Edward have in all?
4. Mariela collected 11 feathers. She found 3 more feathers. How many feathers did she have in all?
5. LaMonte made 14 cookies. He made 7 more cookies. How many cookies did LaMonte have in all?

Division Word Problems

1. Zookeeper Al had 567 bananas. He gave an equal number of bananas to 9 monkeys in the zoo and 567 bananas. How many bananas did each monkey get? And how many are left over?
2. Betty has 427 oranges. She wants to pack them up equally in 23 boxes. How many oranges will she have in each box and how much does she have left over?
3. Mr. King has 1376 pages of paper. He wants to give 32 pages to each student. How many students can he give paper to? How many extra pages will he have left over?
4. Mr. King has 1376 pages of paper. He wants to give 32 pages to each student. He instead gives 33 pages to each student. Will there be enough paper for all the students. How much more scrap paper does he need?



Effective Strategies



Teach an attack strategy

Teach about schemas

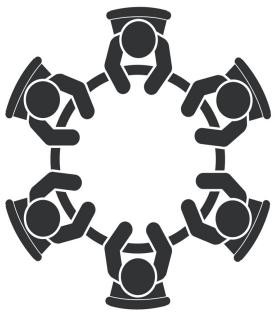


Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20



What was your process for working through this problem?



Attack Strategy

SOLVE

- Study the problem.
- Organize the facts.
- Line up the plan.
- Verify the plan with computation.
- Examine the answer.

R-CUBES

- Read the problem.
- Circle key numbers.
- Underline the question.
- Box action words.
- Evaluate steps.
- Solve and check.

UPS✓
UNDERSTAND
Read and explain.

PLAN
How will you solve the problem?

SOLVE
Set up and do the math!

✓CHECK
Does your answer make sense?



RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.



STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve the problem.

Review your answer.

RICE

Read and record the problem.

Illustrate your thinking.

Compute.

Explain your thinking.



SUPER

Slowly read the story problem twice.

Underline the question and circle the numbers you need.

Picture it. Draw the scenario to show what is happening.

Explain the problem with a number sentence.

Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.

Highlight or underline key information.

Identify the question by drawing a circle around it.

Now solve the problem. Show your work.

Examine your work for precision, accuracy, and clarity.

Share your answer by writing a sentence.



SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

Box action words.

Evaluate steps.

Solve and check.



UPS✓

UNDERSTAND

Read and explain.

PLAN

How will you solve the problem?

SOLVE

Set up and do the math!

✓CHECK

Does your answer make sense?

Created by: Sarah Powell (srpowell@austin.utexas.edu)





Share your favorite attack strategy.



Teach an attack strategy

Teach about schemas



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Additive Word Problems

Meanings of Addition

Meanings of Subtraction



Total

Count one set, count another set, put sets together, count sum



$$2 + 3 = 5$$

Total

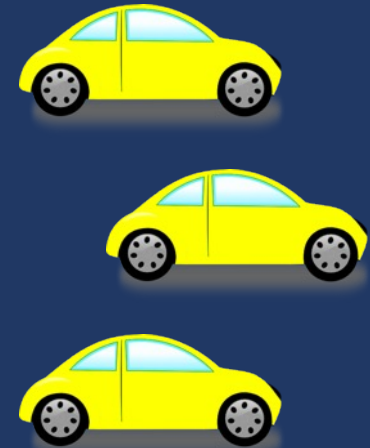
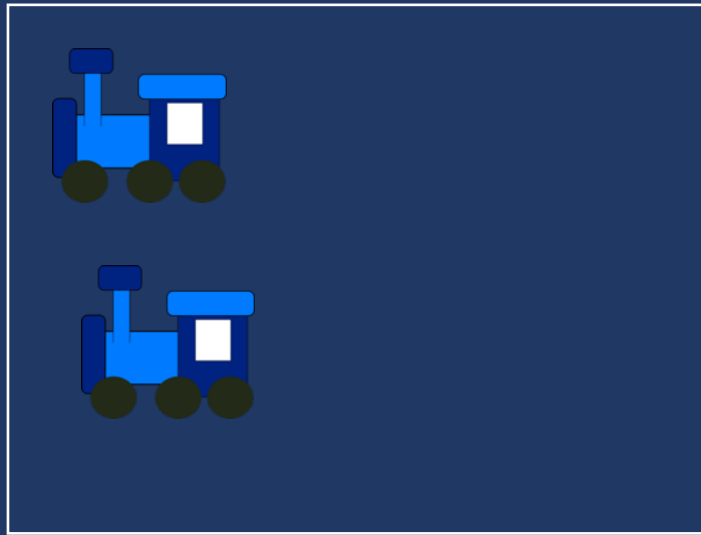
Parts put together into a **total**

Karly saw **4** cardinals and **5** blue jays. How many birds did Karly see?



Change

Start with a set, add the other set, count sum



$$2 + 3 = 5$$

Change

An amount that **increases** or **decreases**

Silas had \$4. Then they earned \$5 for cleaning their room. How much money does Silas have now?



$$6 + 7 = \underline{\quad}$$



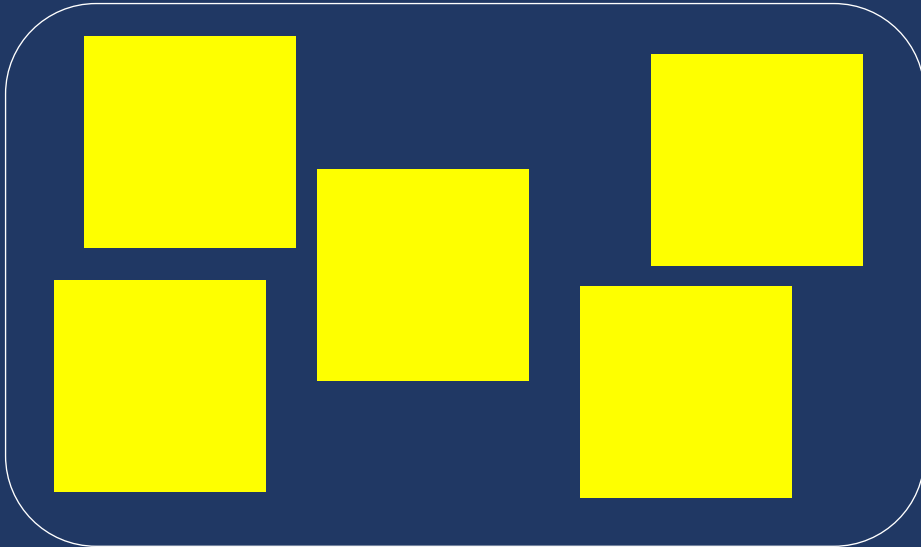
Share a Total story.

Share a Change/Join story.



Change

Start with a set, take away from that set, count difference



$$5 - 3 = 2$$



Change

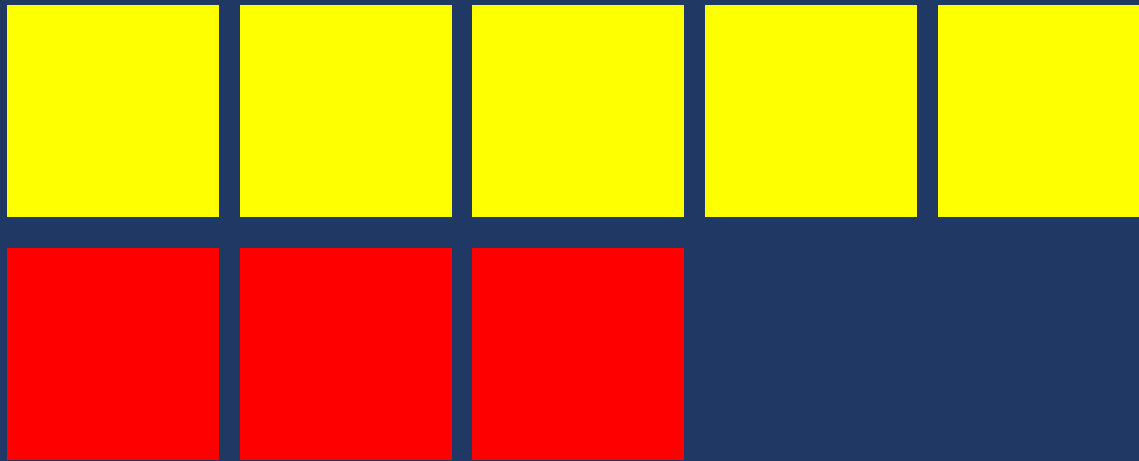
An amount that increases or decreases

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?



Difference

Compare two sets, count difference



$$5 - 3 = 2$$



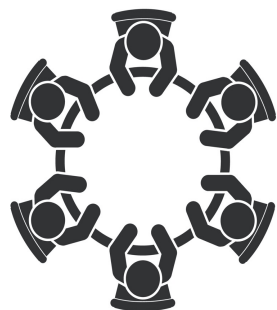
Difference

Greater and **less** amounts compared for a **difference**

Rachel has **9** pencils. Jodie has **4** pencils. How many more pencils does Rachel have? (How many fewer does Jodie have? What's the difference between Rachel's and Jodie's pencils?)







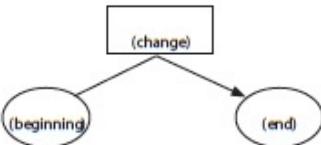
$$14 - 8 = \underline{\quad}$$



Share a Change/Separate story.

Share a Difference story.



Schema and Definition	Equations and Graphic Organizers	Examples	Variations
<p>Total (Combine; Part-part-whole) Parts combined for a sum</p>	<p>$P1 + P2 = T$ (part + part = total)</p> 	<p>Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?</p> <p>Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?</p>	<p>More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?</p>
<p>Difference (Compare) Sets compared for a difference</p>	<p>$B - s = D$ (bigger - smaller = difference)</p>  <p>$G - L = D$ (greater - less = difference)</p> 	<p>Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?</p> <p>Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?</p> <p>Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?</p>	<p>(None)</p>
<p>Change (Join; Separate) An amount that increases or decreases</p>	<p>$ST \pm C = E$ (start +/- change = end)</p>  	<p>End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?</p> <p>Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?</p>	<p>Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?</p>
		<p>End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?</p> <p>Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?</p>	



Total

Additive Word Problems

A.
Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.
In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.
Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

NOTES ABOUT TOTAL PROBLEMS:



Total

Part-part-whole
Combine

Parts put together into a **total**

Daniela saw **3** canoes and **8** kayaks.
How many boats did Daniela see?

Total

Daniela saw **11** boats. If **3** of the boats
were canoes, how many were kayaks?

Part

Daniela saw **11** boats. **8** of the boats
were kayaks, how many were canoes?

Part



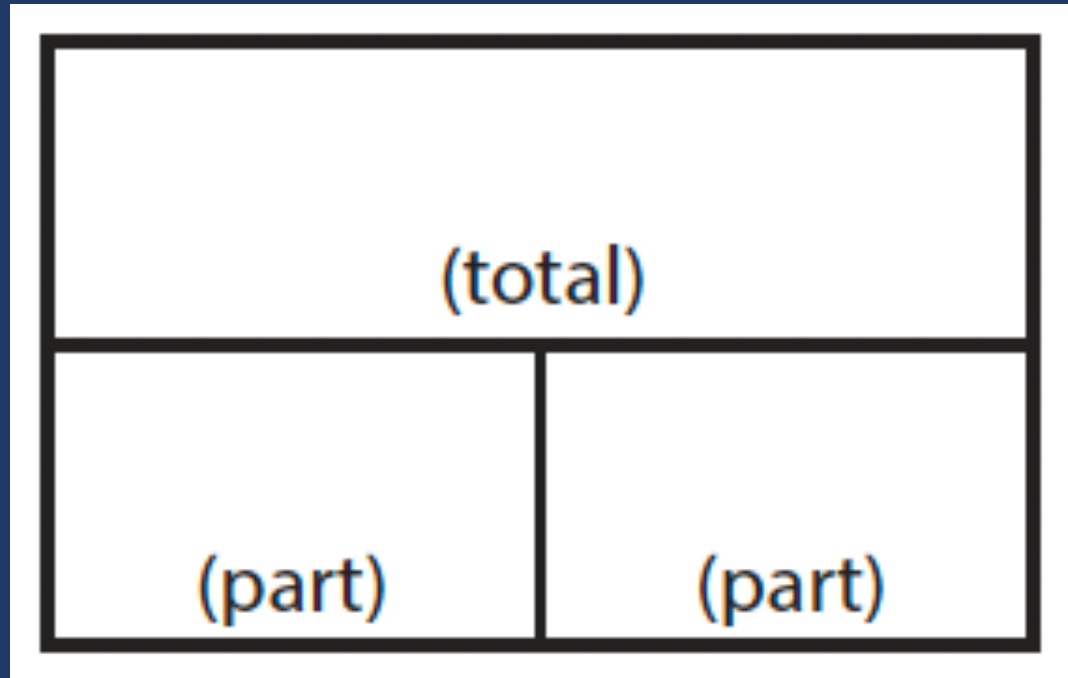
Total

“Are parts put together for a total?”



Total

$$P1 + P2 = T$$



Total

Additive Word Problems

A.
Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.
In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.
Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

NOTES ABOUT TOTAL PROBLEMS:



Total



Write a Total problem.

(Label the type on the back of the card.)



Difference Compare

Greater and **lesser** amounts compared for a **difference**

Adrianna has **10** pencils. Tracy has **4** pencils. How many more pencils does Adrianna have?

Difference

Adrianna has **6** more pencils than Tracy. If Tracy has **4** pencils, how many does Adrianna have?

Greater amount

Tracy has **6** fewer pencils than Adrianna. Adrianna has **10** pencils. How many pencils does Tracy have?

Lesser amount



Total

“Are parts put together for a total?”

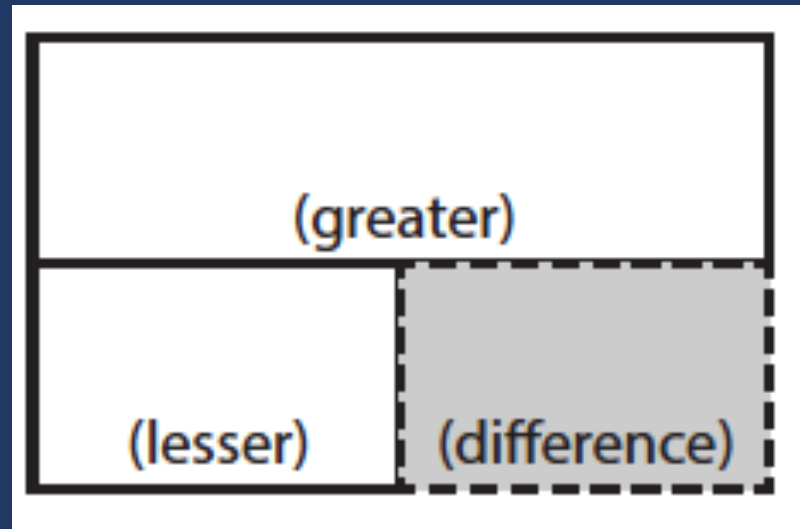
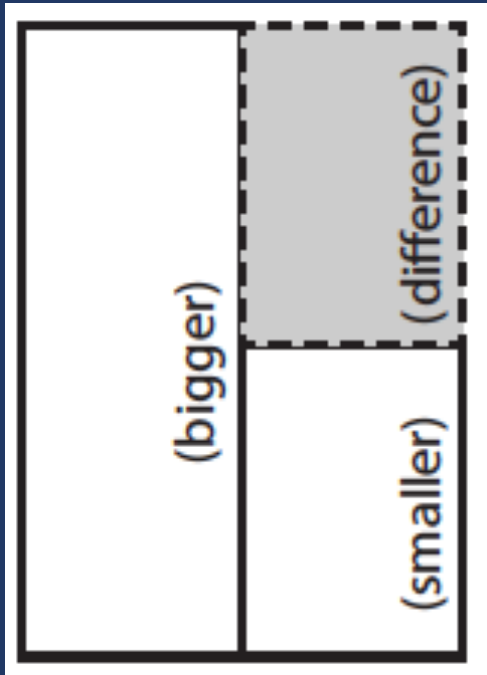
Difference

“Are amounts compared for a difference?”



Difference

$$G - L = D$$



Difference

Additive Word Problems

D.

Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?

E.

Damian's dog eats $5\frac{1}{2}$ cups of dog food each week. Monte's dog eats $4\frac{1}{2}$ cups more each week than Damian's dog. How much does Monte's dog eat in a week?

F.

The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?

NOTES ABOUT DIFFERENCE PROBLEMS:



Difference



Write a Difference problem.

(Label the type on the back of the card.)



Change

Join

An amount that **increases** or **decreases**

Nickole had **6** notebooks. Then, she bought **3** notebooks. How many notebooks does Nickole have now?

End amount

Nickole had **6** notebooks. Then, she bought a few more notebooks. Now, Nickole has **9** notebooks. How many notebooks did she buy?

Change amount

Nickole had some notebooks. Then, she bought **3** notebooks. Now, Nickole has **9** notebooks. How many notebooks did she have to start with?

Start amount



Change

Separate

An amount that increases or **decreases**

Samantha baked **20** cookies. Then, she ate **3** of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked **20** cookies. Then, she ate some of the cookies. Now, she has **17** cookies. How many cookies did Samantha eat?

Change amount

Samantha baked some cookies. She ate **3** of the cookies and has **17** cookies left. How many cookies did Samantha bake?

Start amount



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

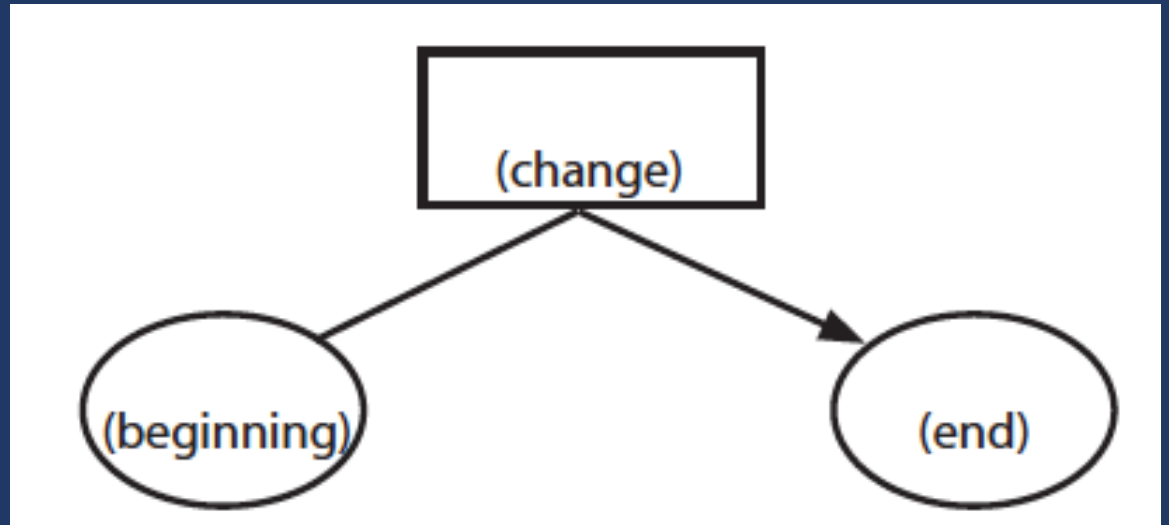
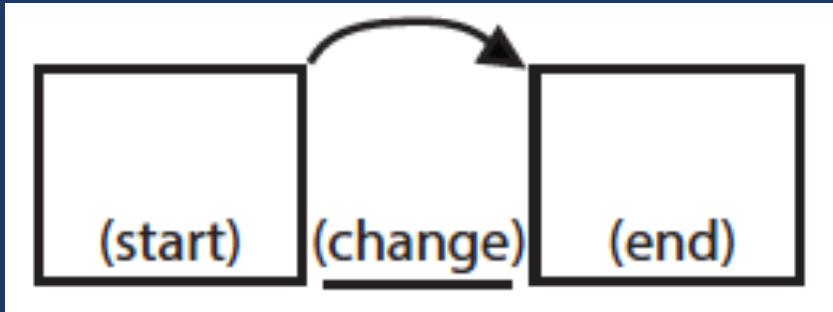
Change

“Does an amount increase or decrease?”



Change

$$ST \quad + / - \quad C \quad = \quad E$$



Change

Additive Word Problems

G.

A plant was $3\frac{3}{4}$ inches tall at the beginning of June. By the end of July, the plant was $9\frac{1}{8}$ inches tall. How many inches did the plant grow in 2 months?

H.

Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?

I.

Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?

NOTES ABOUT CHANGE PROBLEMS:



Change



Write a Change problem.

(Label the type on the back of the card.)





Schema Check!



Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

26. Part A

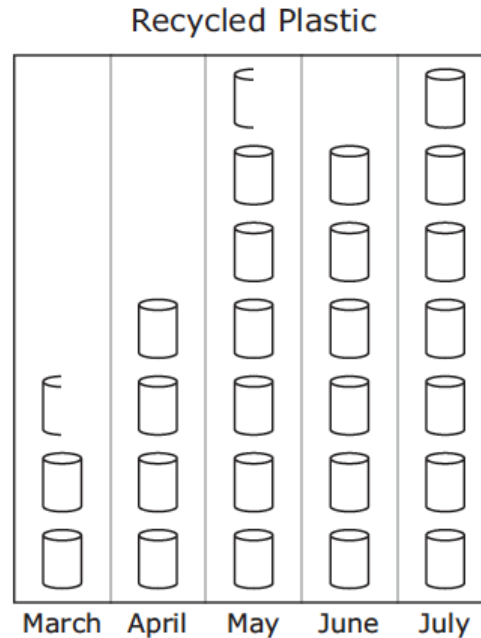
The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.


What is the total number of stamps that Pablo has after the first day of the stamp show?



Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Each  means 20 pounds.

Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Total


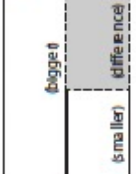
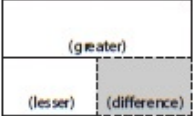

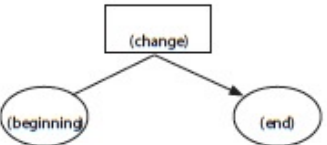
Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.



10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

- Ⓐ 300
- Ⓑ 340
- Ⓒ 350
- Ⓓ 360

Schema and Definition	Equations and Graphic Organizers	Examples	Variations		
<p>Total (Combine; Part-part-whole) Parts combined for a sum</p>	<p>$P1 + P2 = T$ (part + part = total)</p> 	<p>Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?</p> <p>Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?</p>	<p>More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?</p>		
<p>Difference (Compare) Sets compared for a difference</p>	<p>$B - s = D$ (bigger - smaller = difference)</p>  <p>$G - L = D$ (greater - less = difference)</p> 	<p>Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?</p> <p>Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?</p> <p>Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?</p>	<p>(None)</p>		
<p>Change (Join; Separate) An amount that increases or decreases</p>	<p>$ST +/- C = E$ (start +/- change = end)</p>  	<p>End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?</p> <p>End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?</p>	<p>Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?</p> <p>Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?</p>	<p>Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?</p> <p>Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?</p>	<p>Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?</p>



Multiplicative Word Problems

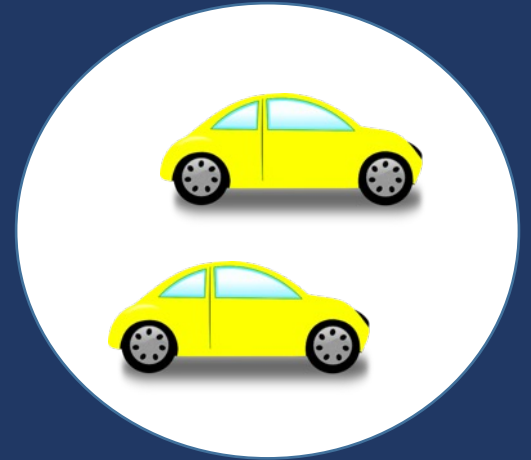
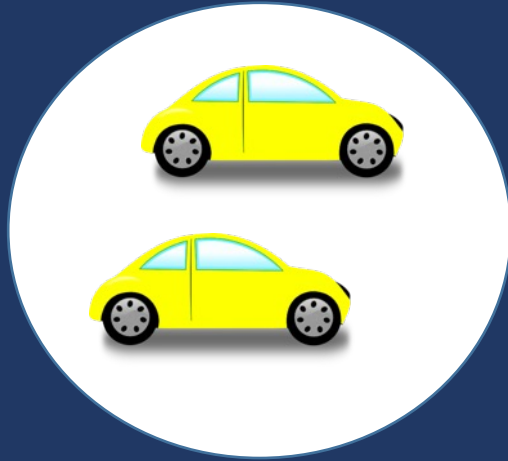
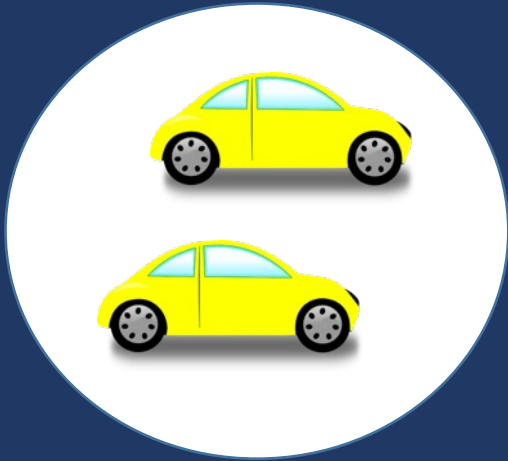
Meanings of Multiplication

Meanings of Division



Equal Groups

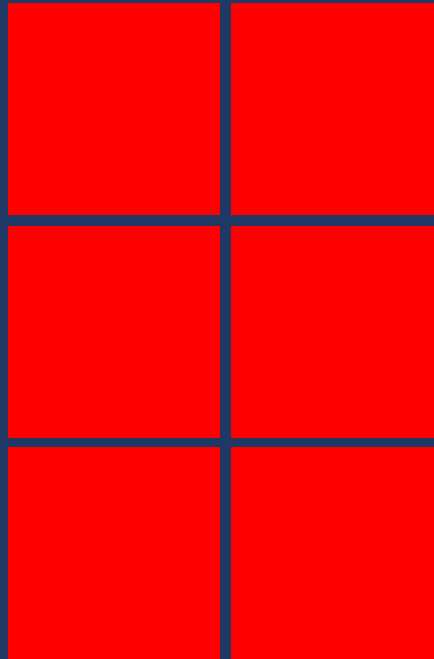
Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

Equal Groups

Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$



Equal Groups

Groups multiplied by **number in each group** for a **product**

Rhiannon has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Rhiannon have altogether?



Comparison

Show a set, then multiply the set



$$3 \times 2 = 6$$

Comparison

Set multiplied by a number of **times** for a **product**

Vivienne picked **6** apples. Jessica picked **2** times as many apples as Vivienne. How many apples did Jessica pick?



$$4 \times 3 = \underline{\quad}$$



Share an Equal Groups story.

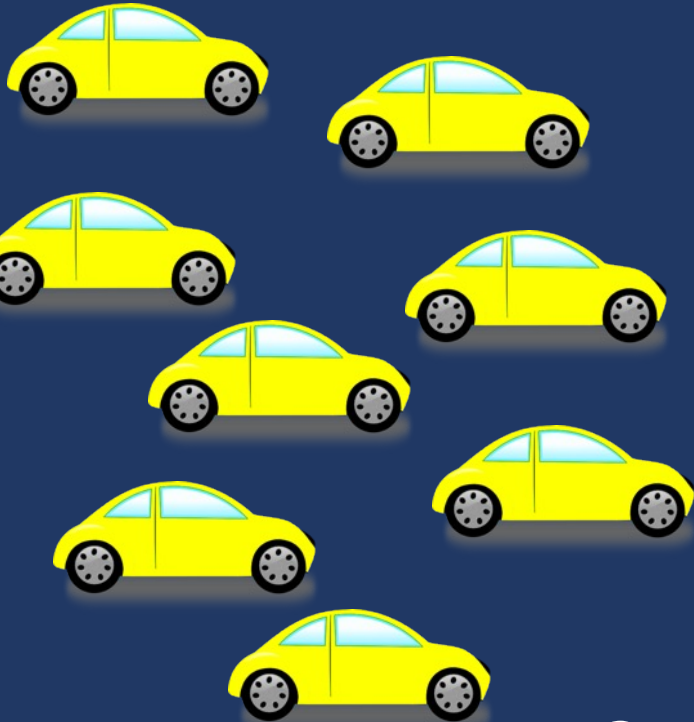
Share a Comparison story.



Equal Groups

(Partitive Division)

Show the dividend, divide equally among divisor, count quotient

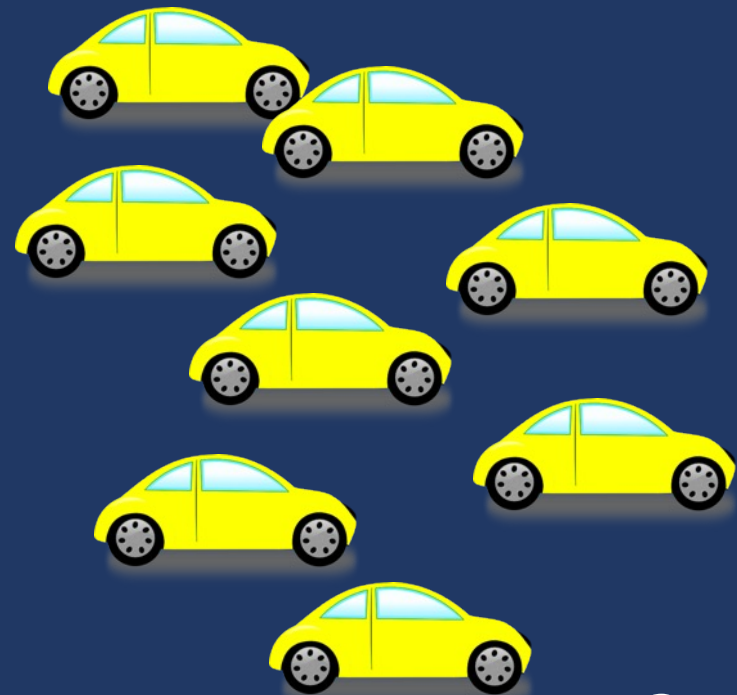


$$8 \div 2 = 4$$

Equal Groups

(Quotative Division)

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$



Equal Groups

Groups multiplied by **number in each group** for a **product**

Stefanie has **12** apples. She wants to share them equally among her **2** friends. How many apples will each friend receive?

Nicole has **12** apples. She put them into bags containing **6** apples each. How many bags did Nicole use?



$$15 \div 5 = \underline{\quad}$$



Share a Partitive story.

Share a Quotative story.



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions		Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly? With unit rate: Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?



Equal Groups

Array
Vary

Groups multiplied by **number in each group** for a **product**

Toni has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Toni have altogether?

Product

Toni has **24** crayons. They want to place them equally into **2** boxes. How many crayons will Toni place in each box?

Number in each group

Toni has **24** crayons. They put them into boxes with **12** crayons each. How many boxes did Toni use?

Groups



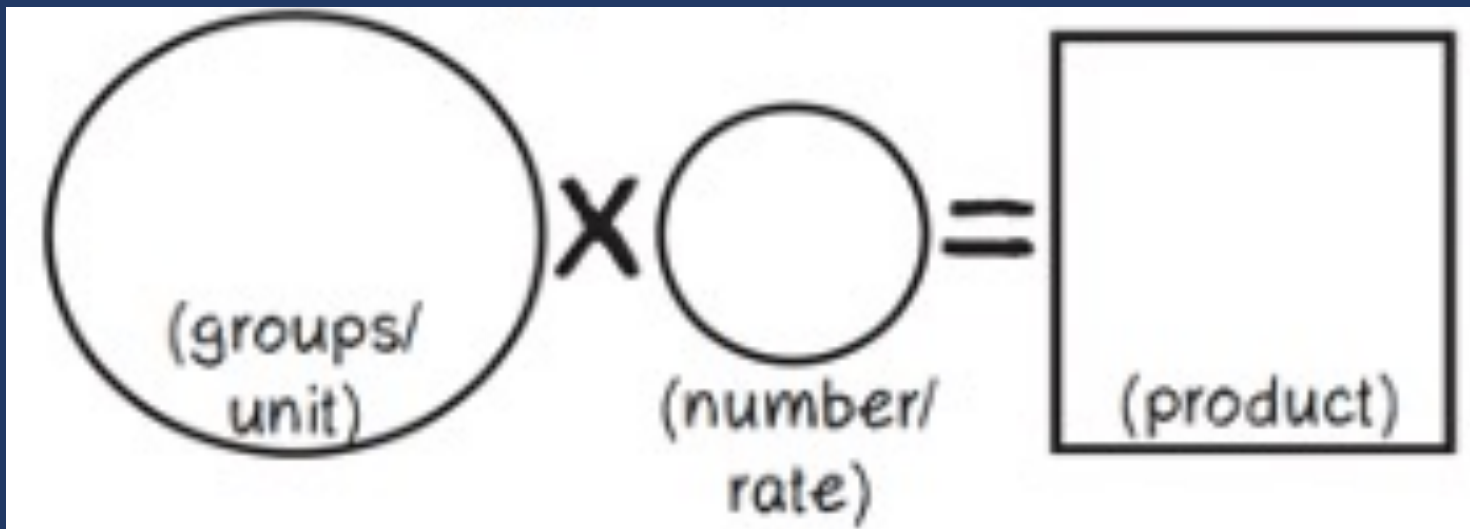
Equal Groups

“Are there groups with an equal number in each group?”



Equal Groups

$$GR \times N(E) = P$$



Equal Groups

Multiplicative Word Problems

A.
Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?

B.
Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light bulbs did Jane buy?

C.
Zachary has 3 feet of string. He makes bracelets, and each bracelet needs $5 \frac{1}{4}$ inches of string. How many bracelets could Zachary make?

NOTES ABOUT EQUAL GROUPS PROBLEMS:



Equal Groups



Write an Equal Groups problem.

(Label the type on the back of the card.)



Comparison

Set multiplied by a number of **times**
for a **product**

Brooke ran **6** minutes. Shaleeni ran **4**
times longer than Brooke. How many
minutes did Shaleeni run?

Set

Number of
times

Product



Equal Groups

“Are there groups with an equal number in each group?”

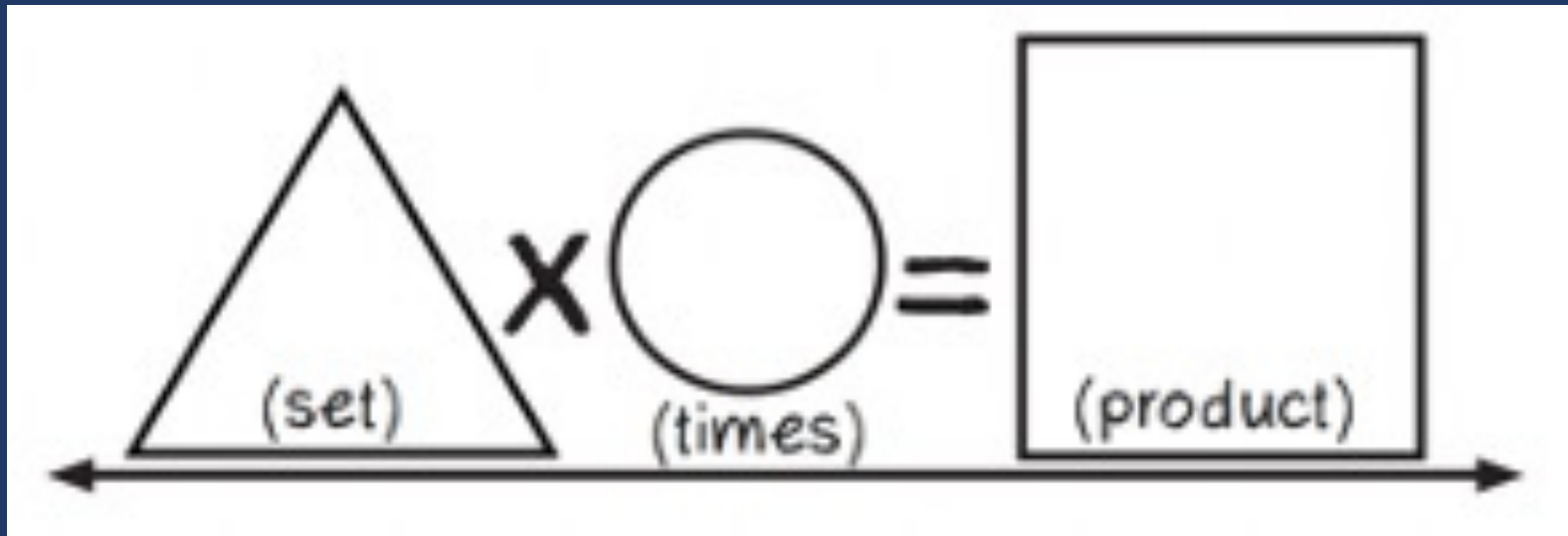
Comparison

“Is a set compared a number of times?”



Comparison

$$S \times T = P$$



Comparison

Multiplicative Word Problems

D.
Enrique has 2 times as many pencils as Ava.
Ava has 6 pencils. How many pencils does
Enrique have?

E.
Susan has 7 times as many books as Mo. Mo
has 18 books. How many books Susan has?

NOTES ABOUT COMPARISON PROBLEMS:

F.
Sally typed 56 words in 2 minutes. At this
rate, how many words could Sally type in 7
minutes?

G.
An airplane's altitude changed -378 feet over
7 minutes. What was the mean change of
altitude in feet per minute?



Comparison



Write a Comparison problem.

(Label the type on the back of the card.)



Ratios/Proportions

Description of **relationships** among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was **3:5**. If she baked **25** brownies, how many cookies did she bake?

Emma typed **56** words in **2** minutes. At this rate, how many words could Emma type in **7** minutes?

Equal Groups

“Are there groups with an equal number in each group?”

Comparison

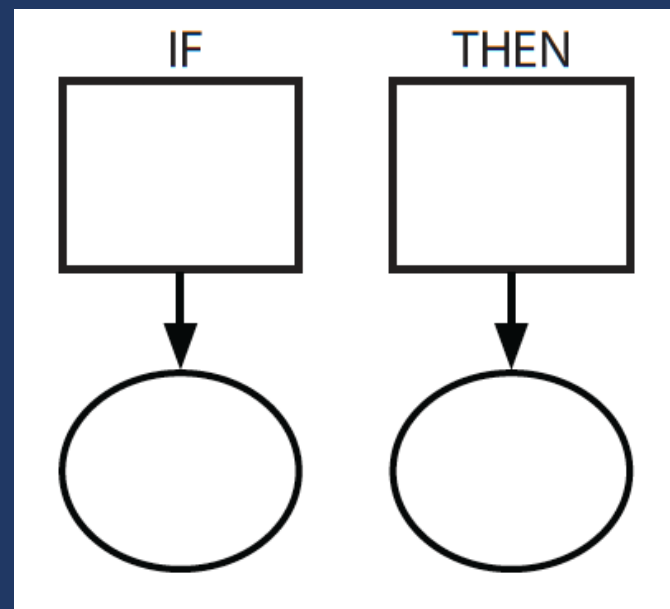
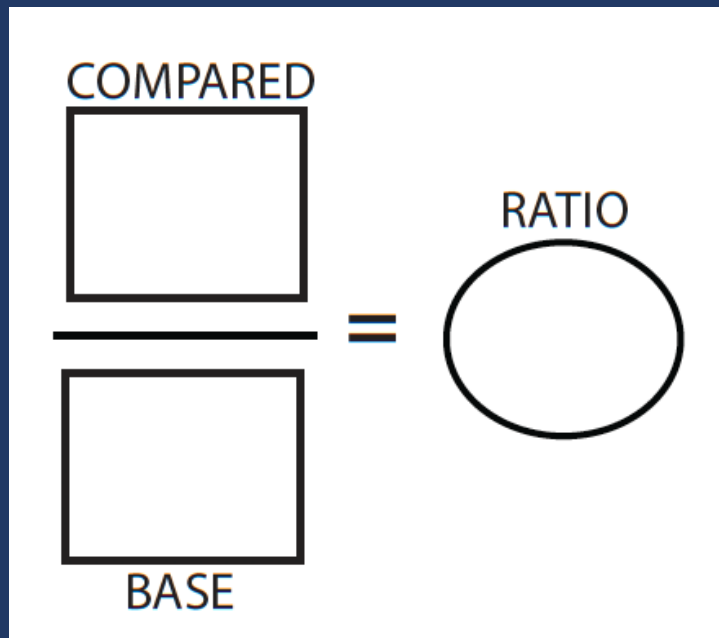
“Is a set compared a number of times?”

Ratios/Proportions

“Are there relationships among quantities - if this, then this?”



Ratios/Proportions



Ratios/Proportions

Multiplicative Word Problems

H.
The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?

I.
Sara buys a sweater at a department store. The sweater costs \$30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.

J.
Margarita baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

NOTES:



Ratios/Proportions



Write a Ratios or Proportions problem.
(Label the type on the back of the card.)





Schema Check!



Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

Ratios/Proportions

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?



Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions		Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
		Base unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	Compared unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?		With unit rate: Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Total

Difference

Change

Equal
Groups

Comparison

Ratio/
Proportion



Teach an attack strategy

Teach about schemas



Word-Problem Solving

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20

Solve the problem

What skills are necessary to solve this problem?



Revisit this problem.
Discuss the schemas in
the problem.



If needed, number the graph.



U

- Understand by reading
- Underline the label

P

- (Parentheses) needed numbers
- Put the numbers in order

S

- Schema(s)
 - Total Equal Groups
 - Difference Equal Shares
 - Change
- Solve

✓²

- Check the number answer
- Check the label answer



Multi-Step Problems

A.
Leslie had 3 pizzas. Each pizza was cut into 8 pieces. Leslie ate 2 pieces. How many pieces were left?

B.
Mr. Kahn gave away 8 blue balloons and 6 red balloons. He gave away 3 times the number of white balloons as red balloons. What was the total number of balloons Mr. Kahn gave away?

C.
An egg farm packages 264 total cartons of eggs each month. The farm has 3 different sizes of cartons.
The small carton hold 8 eggs, and $\frac{1}{6}$ of the total cartons are small.
The medium carton holds 12 eggs, and $\frac{2}{3}$ of the total cartons are medium.
The large carton holds 18 eggs, and the rest of the total cartons are large.

Determine how many each size of carton is needed each month. Then determine how many eggs are needed to fill the 264 cartons.



Teach an attack strategy

Teach about schemas



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction





Pirate Math Equation Quest

About

Research

Individual

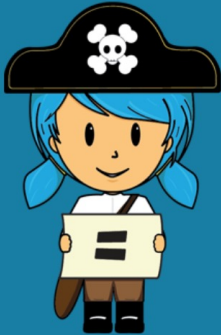
Small Group

STAAR

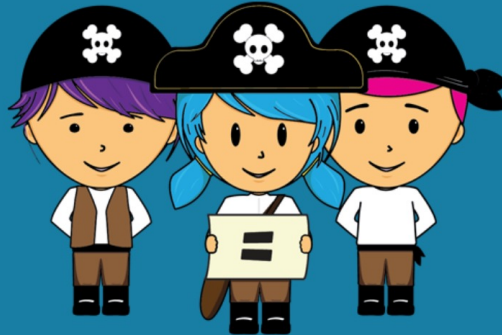
Videos

Welcome to Pirate Math Equation Quest!

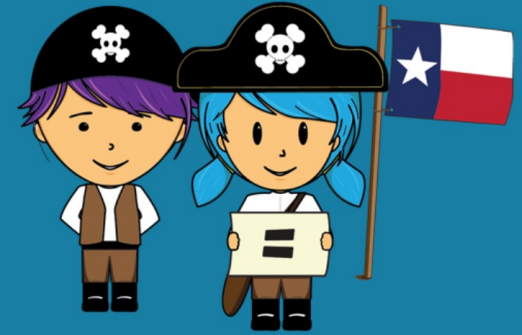
Individual Word-Problem Intervention



Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



Licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License



Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.



MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES



The cover features a central illustration of a blue calculator with the number '195' on its screen, surrounded by colorful mathematical symbols like plus, minus, and multiplication signs. Below the calculator is a blue notepad with a yellow pencil and a red eraser. In the top right corner, there is a logo for 'INCLUSION IN TEXAS' with a green icon of three people. The title 'Instructional Routines for Mathematics Intervention' is written in large white letters on a blue background at the bottom. A paragraph of text describes the purpose of the routines, and the TEA logo is at the bottom right.

**INCLUSION
IN TEXAS**

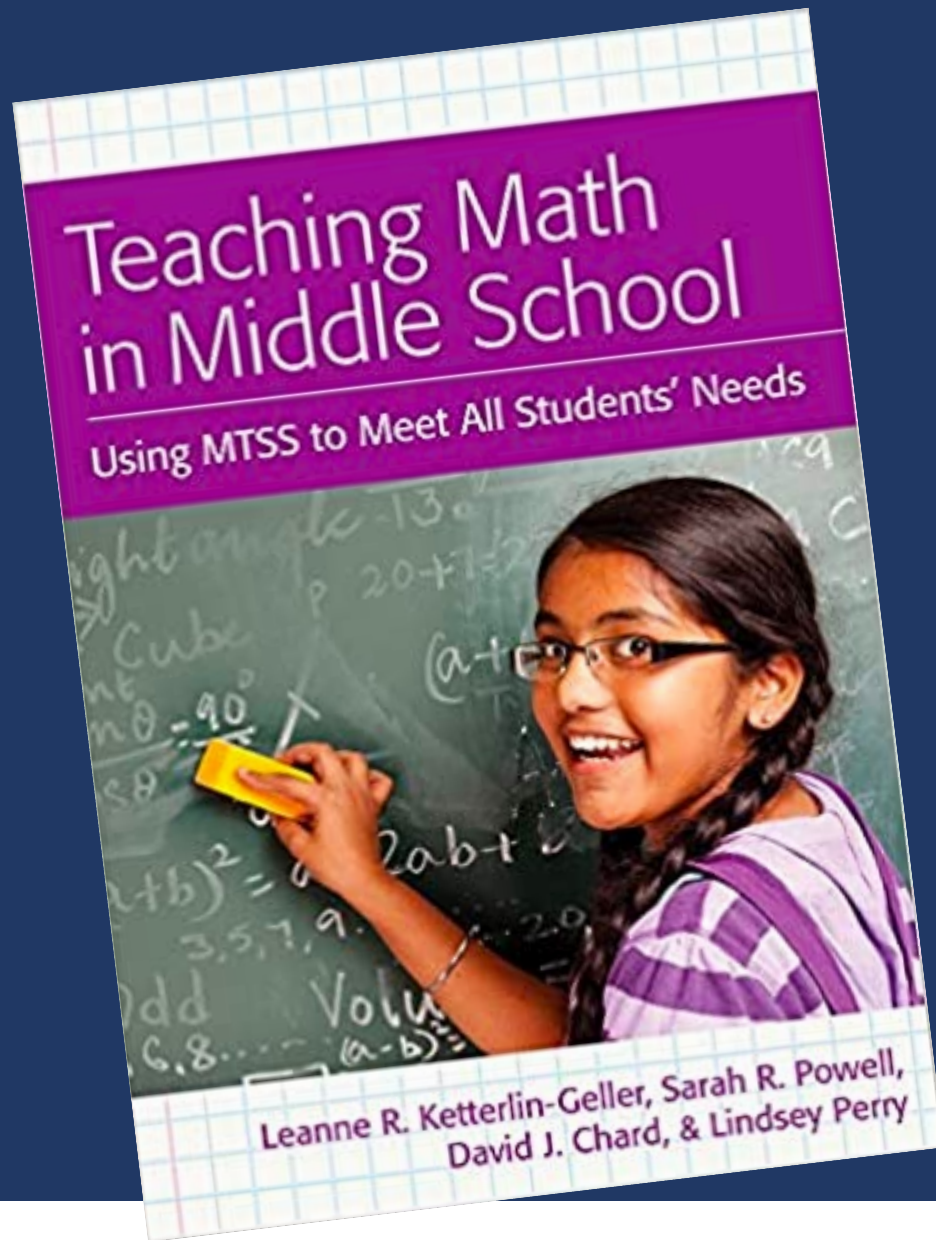
Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859







THE SCIENCE OF MATH

www.thescienceofmath.com



The Science of Math



Science of Math



@4ScienceofMath



We're recruiting Kindergarten teachers for a math program that uses music to teach math. If interested: srpowell@utexas.edu



**young academic
music study**

Information for School Districts



What is the Young Academic Music (YAM) curriculum?

YAM is a supplemental, evidence-based kindergarten curriculum that provides math, music, and computational thinking instruction. It is aligned with both the Common Core Standards in Mathematics and the Texas Essential Knowledge and Skills. This project is being conducted by Touro Graduate School of Education in partnership with the University of Texas at Austin and SRI International. SRI International is the independent evaluator for the project.



Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin



srpowell@utexas.edu



[@sarahpowellphd](https://twitter.com/sarahpowellphd)

