Loop Gain in Feedback Circuits: A Unified Theory Using Driving Point Impedance

Agustin Ochoa
Cypress Semiconductor
San Diego

Abstract—The analysis of analog circuits containing feedback has long been confusing due to incomplete definitions of fundamental terms such as loop gain, open loop gain, and port impedance with feedback zeroed, $Z_0'$. In this paper, following a driving point impedance perspective, a unified model is developed that can be translated into the Black’s block diagram to properly identify the feedback analysis parameters and definitions. With this approach there is no ambiguity of how to terminate the sub-circuits used and can be done anywhere in the outer loop. This approach fully accounts for bi-directional properties in the amplifier and in the feedback net as well as loading effects. The approximation inherent in the properly loaded open-loop method is shown.

I. INTRODUCTION

Feedback is a key design tool in analog circuits—with it we can design stable amplifying blocks accurate in gain and with reduced processing and environmental effects. It requires a discipline in the designer: that he understand how to properly apply it. This discipline has been difficult to acquire due to a lack of consistent definitions in the many intuitive based presentations and articles.

In the analysis of feedback-circuits we use concepts such as ‘loop gain’ (LG), ‘Open Loop Gain ($A_{ol}$)’ and ‘Output Impedance with ‘open loop’ conditions, $Z_0$’. We have an intuitive understanding of what these terms mean and usually do not question them. When our designs containing feedback behave as expected we ‘verify by silicon’ that we know what we are doing or at least that we are ‘close enough’. But what is $Z_0$? A general belief is that it is the output impedance of the amplifier [1]. And what does it mean to ‘open the loop”? To do this we usually find a ‘good’ spot in the circuit loop and cut a wire. But is that proper? What is a ‘good’ spot? Other questions arise. What do we believe when we use two different approaches and get different results [2]?

Section II of this paper develops a unified approach to feedback analysis based on the principal of port impedance [3] in a feedback system, the Z-method. From a general perspective I develop the analysis methodology and generate consistent Black’s [4] block diagram variables. The point where we open the loop in the popular ‘open-loop’ method is not always clear and, while we ‘replica load’ the cut we do not always properly terminate this replica load. The open-loop results are shown to be a subset of the Z-results. Further, proper loading of the open loop is shown and the restriction of opening the loop at a high-impedance point or at the non-reachable controlled source are remove. In section III the Z-method is applied to an inverting amplifier/feedback configuration and section IV discusses findings and section V summarizes the paper.

II. MODELING FEEDBACK

Figure 1 shows a general block diagram for a feedback system. The transfer function $H=V_{out}(s)/V_{s}(s)$ using the Driving Point Impedance/Signal Flow Graph approach [5] is given as:

$$H(s) = G_{mos} \cdot DPI_{out}$$

(1)

$G_{mos}$ in (1) is the effective transconductance at the output node due to excitation at the input. The Driving Point Impedance, DPI$_{out}$, requires a little more discussion. The normal approach would be to apply an ac current of magnitude 1 and perform a frequency sweep over the bandwidth of interest. The resulting $V_{out}$ (divided by 1Amp ac) is DPI$_{out}$. Instead, as shown in Figure 2, we split the output node at a point in the external feedback path (here where the feedback net is attached to the amplifier output) and apply two sources, one driving the feedback net, $V_1$, and the other driving the load and amplifier output, $V_2$. Using superposition, we set one source to magnitude 1 with the other zeroed and ac sweep the

\[\begin{align*}
\text{AMP} & \quad \text{Fbk Net} \\
\text{Vs} \uparrow & \quad \text{Vs} \uparrow \\
\text{rS} & \quad \text{rS} \\
\text{Vout} & \quad \text{LOE} \\
\end{align*}\]

Figure 1. General Block Diagram of a Feedback System
frequency. We then switch swept and zeroed source and repeat the ac simulation. In this manner we generate the 4 current components making the net input current \( i_1 + i_2 = i_{11} + i_{22} + i_{12} + i_{21} \) where \( i_{12} = i_{21} \) due to \( V_{s1} = 0 \). The \( i_{xy} \) currents represent cross-currents, currents that appear at one port due to excitation at the other, and are represented as transconductances \( g_{mxy} \) in the equivalent circuit in Figure 3. There are two cross-currents due to the bi-lateral nature of the amplifier and of the feedback net. One cross-current is in the direction through the amplifier in the forward direction while the other traverses the loop in the reverse direction. The \( i_{x} \) currents are ‘self-currents’, currents into node ‘x’ due to excitation at node ‘x’ and are currents from the excitation source to ground. The dashed line indicates that the two nodes are really the same node, \( v_{out} \).

Injecting a current \( v_{s}G_{mos} \) into this node as stated in (1) produces the equivalent circuit for the transfer function of the feedback system, Figure 3.

The system flow graph can be drawn directly from Figure 3 by inspection, Figure 4. The transfer function from the flow graph is given in (2). In this form we identify the output impedance with feedback zeroed as the parallel combination of the admittances to ground at each side of the ‘loop cut’ in the feedback path, (3). These admittances are due to the ‘self-currents’. The ‘cross-currents’ represent transconductance currents, currents that traverse the loop. Feedback is identified with these transconductor ‘cross-currents’. There are two transconductors, one for each direction around the loop.

The proper condition of ‘opening the loop’ is a graph (math) property and not a circuit one. Opening the graph removes the effect of the transconductances on the graph algebra effectively zeroing the feedback signal. This cannot be done on the circuit where opening a wire in the feedback path changes load and transmission conditions. The ‘Open Loop Feedback System’, \( A_{ol} \) is the numerator in (2). The factor \( G_{mos} \) at the right side of the summer in Figure 4 can be moved into the loop by combining it with the two block multipliers as shown in Figure 5. In this form we recognize Black’s representation and complete the definition of the parameters and condition of opening the loop enumerated below.

Summary of loop gain results:

1. \( Z_0 = 1/(y_{11} + y_{22}) \), the output impedance with the loop opened in the graph thereby removing the effects of the effective transconductors.
2. \( A_{ol} = G_{mos}Z_0 \), the fully loaded open loop gain with the transconductor elements zeroed.
3. \( \beta = (g_{m12} + g_{m21})/G_{mos} \), Black’s feedback factor
4. Opening the loop is done in the flow graph algebra domain by removing one of the arrows in the feedback path OR in the two-port equivalent circuit by removing the two transconductance elements providing the cross currents in the output impedance circuit.
5. Loop Gain is found as minus the sum of the

\[
H(s) = \frac{G_{mos} \cdot \frac{1}{y_{11} + y_{22}}}{1 - \frac{(g_{m21} + g_{m12})}{y_{11} + y_{22}}}
\]

(2)

\[
Z_0 = \frac{1}{y_{11} + y_{22}}
\]

(3)
transconductances divided by the sum of the self-admittances, or equivalently, the sum of the cross-currents divided by the sum of the self-currents.

\[
LG = - \frac{g_{m12} + g_{m21}}{y_{11} + y_{22}} = \frac{i_{12} + i_{21}}{i_{11} + i_{22}} \tag{4}
\]

III. APPLICATION OF ZO’ LOOP GAIN METHOD

Consider the inverting gain stage in Figure 7. The amplifier will be modeled using its two-port y-parameters \((g_{m,amp}, g_{mr,amp}, y_{11,amp}, y_{22,amp})\) including reverse transmission \(g_{mr,amp}\). The system loop gain parameters, \(y_{11}, y_{22}, g_{m21}, g_{m21}\) are given in (5) through (8) in terms of the amplifier y-parameters and source, load, and feedback admittance/impedance.

To find \(g_{m21}\) in (5), set \(v_2\) to ac ground (Refer to Fig. 2) and solve for the current into \(v_2\) due to excitation at \(v_1\). The amplifier parameter \(g_{m,amp}\) is usually the negative of the amplifier’s forward transconductance, \(-g_m\). This is not always the case especially in high frequency applications where a parasitic capacitance between input and output must be included in the analysis, or in a design with an internal feedback path intentionally included. Here it is fully general. In the high frequency case for a single MOS amplifier, the amplifier transconductance becomes \(g_{m21,amp} = g_m + c_{gd}g_m\).

\[
g_{m21} = \frac{y_{fbk}}{y_s + y_{11,amp} + y_{fbk}} \cdot g_{m,amp} \tag{5}
\]

\[
g_{m12} = \frac{g_{mr,amp}}{y_s + y_{11,amp} + y_{fbk}} \cdot y_{fbk} \tag{6}
\]

\[
y_{11} = \frac{1}{y_{fbk}} + \frac{1}{z_{fbk} + \frac{1}{y_{11,amp} + y_s}} \tag{7}
\]

\[
y_{22} = y_{load} + y_{22,amp} + \frac{g_{mr,amp}}{y_{fbk} + y_{11,amp} + y_s} \cdot g_{m,amp} \tag{8}
\]

By combining (5) through (8) as specified in (4) we obtain the loop gain function. The last term in (8) accounts for internal feedback due to the reverse transconductance \(g_{mr,amp}\) in the model for the amplifier.

An interesting result of this formulation provides the basis for the use of the open-loop method approximation for finding loop gain. To account for the full nodal impedance at the break we replicate the full cell terminated as defined by the driving sources as indicated in Figure 2. With this requirement the proper replica bias is as shown in Figure 8—breaking the loop at the amplifier input and source/feedback node, we add the replica load from the amplifier input with the replica load source/feedback node at its dc operating voltage. When properly loaded we may actually break the loop anywhere and not be limited to a ‘high impedance’ point. The extent of the approximation of this open loop method once properly loaded is in the relative magnitude of the reverse transmission due to the amplifier’s reverse transconductance which is not accounted for in this method. If the amplifier is uni-directional then the open loop method is complete. The Z-method, in contrast, naturally includes both forward and reverse directions of feedback.

IV. DISCUSSION

The Z-method is now applied to the inverting gain stage using the single transistor amplifier as shown in Figure 7. The example is designed to stress the analysis by including an internal feedback capacitor between the gate and drain to increase the amplifier’s reverse transmission. In addition a resonance load is added to the output to further add frequency response and modify the load impedance. While the analysis is shown in closed form suitable for hand calculations, the full analysis is best done using a circuit simulator with full device modeling capability to do the computations. The simulation top schematic uses multiple instances of the gain stage to generate the cross- and self-currents in one simulation run. These signals are algebraically combined performing the algebra specified in (4) and plotted in Figures 9 and 10. The low frequency gain is set to 4 using \(R_{fbk}=100M\) and \(R_s=5M\), \(C_{fbk}=100fF\), \(C_{load}=1pF\), and \(C_{int,fbk}=0\).
Figures 9 and 10 show the results for both the Z and the Open Loop methods. The top traces show the loop gain magnitudes, bottom trace the phases. At the higher frequency end the two methods begin showing differences and the curves split. The lower magnitude trace is paired with the higher phase one and corresponds to the loop gain function using the Open Loop method. Here the differences are seen only at the high frequencies when the reverse transmission through the amplifier can no longer be ignored. This difference can be enhanced by increasing the gate-to-drain capacitance. Figure 10 shows the responses with $C_{fbk_int}=20 \text{fF}$. The increased reverse transmission through the amplifier affects the Open Loop method at lower frequencies especially in the phase. The resonance effects of the L-C branch in the load are clearly seen at around 4MHz.

V. SUMMARY

A direct and consistent development of key feedback parameters has been derived using driving point impedance and flow graph equivalent transformations. The process fully accounts for all loading effects due to the feedback net and system load, and for bidirectional nature of elements—of the amplifier itself and of the feedback network. A consistent definition of what it means to open the loop, how to find parameters $Z_0'$, $A_0$, and loop gain has been shown. The process transformed the problem systematically from the original circuit to the equivalent Black’s form resulting in functional definitions to Black feedback parameters for Open Loop Gain and Feedback Factor. In addition, the Open Loop Method has been shown to be a subset of the Z-method in that it gives results only for the primary direction of loop gain. The loop that traverses the amplifier in the reverse direction is neglected. The Open Loop method requires proper termination to fully account for loading at the cut. Generally this is done by replica loading at the loop break. Terminating the replica properly is shown to be done by introducing a dc voltage source at the break, of the quiescent value before the break. The Z-method does this by definition and includes loop transmission in the reverse direction automatically—this signal becomes significant in high frequency designs and in designs where the amplifier is bi-lateral.

ACKNOWLEDGMENT

I have had many discussions over a long period of time with Don Patterson, Roger Gill and Howard Tang and gratefully acknowledge their contributions to this work.

VI. REFERENCES