Today

- Introduce debt and default.
- Previously:
  - Erick had showed how introducing borrowing constraints affects consumption
  - Induces a precautionary motive, generates saving

Borrowing Constraints

- Ad-hoc borrowing constraints:
  - Total amount

$$a' \geq -b$$

  - Share of income

$$a'(\epsilon) \geq -\varphi w \epsilon$$

Borrowing Constraints

- Natural borrowing constraints:
  - Inada conditions

$$\lim_{c \to 0} u(c) = -\infty$$

    ensure households never borrow and amount that could lead to negative consumption in any possible state.

  - Gives rise to natural, endogenous constraint:

$$a'(\epsilon) \geq \frac{w\epsilon_{min}}{r}$$

    where ϵ_{min} is the lowest possible realization of individual productivity.

Borrowing Constraints

- All borrowing constraints considered above have similar unstated assumption:
    - Enforcement technology that forces individuals to repay. e.g., infinite pain

$$v(a, \epsilon) = -\infty$$
$$if \;\; a \leq -b$$

    - Means that no scope for default.
    - In contrast, default is common/important feature in US.

No-default Borrowing Constraints

- Suppose households can default.
- How can we ensure that debts are settled?
- Zhang 1997 proposed mechanism and characterized equilibrium with endogenous borrowing constraints:

No-default Borrowing Constraints

- Suppose households can default.
- How can we ensure that debts are settled?
- Zhang 1997 proposed mechanism and characterized equilibrium with incomplete markets and endogenous borrowing constraints

No-default Borrowing Constraints

- How do we know that $b^\star(\epsilon)$ exists?
    - $b^\star(\epsilon)$ is constant if $\pi(\epsilon', \epsilon) > 0 \forall \epsilon, \epsilon\prime$
    - $v(-b, \epsilon)$ is decreasing in b since it is decreasing in a. Also continuous.
    - $v(0, \epsilon) \geq v^{aut}(\epsilon)$.
    - Need to find a point b such that $v(-b, \epsilon) \leq v^{aut}(\epsilon)$.
        - Natural borrowing constraint, b^{nat}
        - b^{nat} - $\{c^{aut}(\epsilon)\}_{t=0}^{\infty} \geq \{c(-b_{nat}, \epsilon)\}_{t=0}^{\infty} \forall t$
    - Then $b^\star(\epsilon)$ exists by the Mean Value Theorem

No-default Solution

1. Assume R.
2. Calculate $v^{aut}(\epsilon)$ - Simple
3. Guess $b^{\star} \in (0, b^{nat})$.
4. Solve household problem for given borrowing constraint (EGM)
5. Using solved policy functions, simulate long history of households starting at states $(-b^{\star}(\epsilon), \epsilon) \forall \epsilon$. Use simulations to compute $v(-b^{\star}(\epsilon), \epsilon)$.
6. Compare $v(-b^{\star}(\epsilon), \epsilon)$ with $v^{aut}(\epsilon)$. If $v(-b^{\star}(\epsilon), \epsilon) > v^{aut}(\epsilon)$, increase b, etc. Iterate until $v(-b^{\star}(\epsilon), \epsilon) = v^{aut}(\epsilon)$ within desired tolerance.
7. Check market clearing and update guess of R.

Default in Equilibrium

- Above solution does not permit equilibrium default.
- Arbitrage opportunities:
  - Suppose that financial intermediary (no productive technology) entered market and offered to lend more than no-default limit at a higher interest rate. Some households might find such a contract profitable.
- Present Livshits McGee Tertilt (2007)

Consumers

- Unit continuum of infinitely-lived ex-ante identical agents
- Preferences: $u' > 0, u'' < 0$, in DARA class (e.g., CRRA)
- Discount rate β

Labor Supply and Idiosyncratic Risk

- Unit mass of agents inelastically supply unit endowment of labor effort:
- Receive income y that follows a Markov chain π.

Default

- Agents can always default on their debt. If the do:
  1. Debts ($a < 0$) are set to zero next period.
  2. They are excluded from borrowing from financial markets for one period.
  3. A fraction of income γ is seized by the intermediary as partial repayment.
- Default, as a function of assets a and income y, is denoted by indicator $D(a, y)$.
  - $D(a, y) = 1$ if defaulted last period

Production

- Production not stated in this economy
- However, could be. There exists firm with production technology F that gives rise to labor demand with price w such that $y = \epsilon w$.

Financial markets

- Financial intermediaries act competitively as price takers.
- Take in savings and pay out next period at interest rate $1/\bar{q}$. Thus, if an agent deposits $\bar{q}$ units of consumption good they get 1 unit back next period.
- Issue loans
    - Price of loan depends on amount of debt issued and income y, as both predict the default probability.
    - Let $\theta(a', y)$ denote the default probability.
    - Let $q(a', y)$ denote the price of the loan.

Household Problem

Non-default Problem

$$v(a, y, D = 0) = \max_{c,a'} u(c) + \beta \sum_{y'} \pi(y', y) \max\left(v(a', y', D' = 0), \omega(0, y', D' = 1)\right)$$

$$s.t.$$

$$a + y - c = \begin{cases} \bar{q}a' \ if \ a' \geq 0 \\ q(a', y) \ if \ a' < 0 \end{cases}$$

Default Problem

$$\omega(0, y, D = 1) = \max_{c,a'} u(c) + \beta \sum_{y'} \pi(y', y) v(a', y', D' = 0)$$

$$s.t.$$

$$c + \bar{q}a' = (1 - \gamma)$$

$$a' > 0$$

Stationary Equilibrium

A recursive competitive equilibrium is

1. A set of value functions v, ω
2. Decision rules c, a', D'
3. Prices q
4. Default probabilities θ
5. Invariant distribution $\lambda(a, y, D)$

Stationary Equilibrium

Such that

1. Given prices, consumers optimize
2. Intermediaries make zero profits

$$\frac{1}{\bar{q}} = \frac{1}{q(a',y)}\{[1-\theta(a',y)] + \sum_{y'\in Y} D(a',y')\left(\frac{\gamma y'}{a'}\right)\pi(y',y)\}\forall(a',y)$$

3. Default probabilities are consistent with agent decisions:

$$\theta(a',y) = \sum_{y'\in Y} D(a',y')\pi(y',y)$$

4. Asset market clears

$$\int_{A\times Y\times D} a d\lambda = 0$$

5. The invariant distribution is consistent with agent decisions and solves the fixed point problem.

Solution

Two differences from standard Aiyagari:

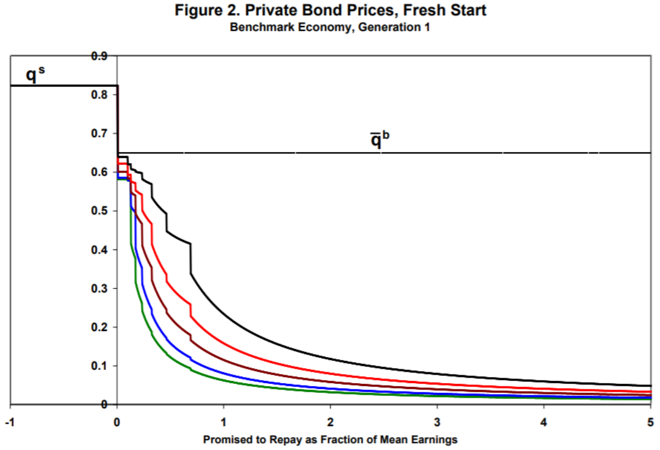
- Use VFI because FOC involves comparison of v, ω
- Price is a function q and a number $\bar{q}$, not a number (or two numbers)

Algorithm:

- Guess q and $\bar{q}$.
- Solve household problem
- Use zero-profit condition of intermediaries to update q until zero profit condition is satisfied.
- Check market clearing. If not satisfied, update guess of $\bar{q}$ (use same q) and start over.

Results

Bond Prices



Results

Table VI
Results

	Results	Rule	Debt/Earnings	Defaults	Avg. r^b	better rule	Cons. Equiv.
1	Benchmark U.S.	FS	8.33%	0.53%	11.66%	FS	0.39%
		NFS	17.04%	0.10%	9.64%		
	U.S. Data	FS	8.8%	0.68%	12.0%	-	-
2	No expense U.S.	FS	11.04%	0.00%	9.01%	NFS	0.68%
		NFS	31.69%	0.22%	13.01%		
3	$r^s \downarrow 1\%$ U.S.	FS	12.02%	0.60%	10.01%	FS	0.03%
		NFS	25.93%	0.17%	9.97%		
4	$g \uparrow 0.10$ U.S.	FS	10.15%	0.35%	10.87%	NFS	0.00%
		NFS	19.35%	0.04%	9.23%		
5	Benchmark Germany	FS	6.11%	0.06%	9.44%	NFS	0.04%
		NFS	9.36%	0.02%	9.20%		

Results

Table VII
Age Profile of Bankrupts:
FS Benchmark Parameter Values and U.S. Data

Age	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	avg.
Model	2.64	1.95	1.31	1.06	0.78	0.84	0.74	0.67	1.00
Data	1.25	1.53	1.44	1.57	1.45	0.84	0.91	0.17	1.00