Earthquake Induced Sloshing in Tanks with Insufficient Freeboard

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Summary

Earthquake induced sloshing in tanks is caused by long-period ground motions which attenuate slowly with distance. A minimum freeboard is needed to accommodate the sloshing waves. Since freeboard results in unused storage capacity, many tanks lack the required freeboard. As a result, sloshing waves impact the roof, generating additional forces on the roof and tank wall. Tanks have suffered extensive damage due to sloshing waves, but the effect of sloshing waves is usually ignored in seismic design of tanks. This paper presents a simple method of estimating sloshing loads in cone and dome roof tanks.

Introduction

The response of cylindrical liquid-storage tanks to earthquake ground motions is reasonably well understood [1–7]. The liquid mass is divided into two parts: (1) the impulsive mass near the base of the tank moves with the tank wall, and (2) the convective mass near the top experiences free-surface sloshing. The impulsive mass experiences high accelerations, therefore, it controls the seismic loads (base shear and overturning moment) in the tank. The convective mass experiences very low accelerations, therefore, it contributes negligibly to the seismic loads in the tank. However, the convective mass needs room to slosh freely in the tank.

It is desirable to provide sufficient freeboard so that the sloshing waves do not impact the roof during earthquakes. For large diameter tanks, the required freeboard can be quite high. For tanks on deep/soft soil deposits or those subjected to near-field motions [8-9], low-frequency ground motions increase freeboard requirement. Elevated tanks on towers or roofs of buildings also require high freeboard. Freeboard means unused storage capacity, which can be quite expensive. Sometimes, there is restriction on the overall height of the tank. Therefore, many tanks lack sufficient freeboard.

Insufficient freeboard causes: (1) upward load on the roof due to impacts from the sloshing wave, and (2) increase in impulsive mass due to constraining action of the roof. The upward force on the roof can damage the roof, break the roof-shell connection (*Fig. 1*) or tear the shell (*Fig. 2*).

Fig. 1: Sloshing damage to an oil tank. The roof suffered damage and roof-shell junction leaked during the 1952 Kern County, California Earthquake (photo by: K. V. Steinbrugge)

Fig. 2: Sloshing damage to a water storage tank. The tank lost its roof and a portion of upper shell during the 1933 Long Beach, California Earthquake (photo by: Harold M. Engle)

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The tank shell could also buckle or tear at the base if not designed for the increased loads resulting from additional impulsive mass. The objective of this paper is to estimate the roof, shell and foundation loads arising from insufficient freeboard in tanks with cone and dome roofs. For flat roof tanks, an approximate solution with engineering accuracy was presented by the author [10]. This paper extends that solution to cone and dome roof tanks.

Model of Tank-Liquid System

A sufficiently accurate model of tank of radius R filled with liquid to height H is shown in *Fig. 3*. The model parameters are: (1) impulsive and convective masses m_i and m_c, (2) impulsive and convective heights h_i and h_c, and (3) impulsive and convective periods T_i and T_c. These can be estimated by the methods presented in previous studies, e.g. [7]. The impulsive damping is assumed to be 2 percent of critical for steel and pre-stressed concrete tanks and 5 percent of critical for reinforced concrete tanks. The convective damping is assumed to be 0,5 percent of critical.

Impulsive and Convective Responses

The impulsive spectral acceleration $SA(T_i)$ is read from the 2% or 5% damping site response spectrum and the convective spectral acceleration $SA(T_c)$ is read from the 0,5% damping site response spectrum (*Fig. 4*). Usually, $SA(T_c) << SA(T_i)$, therefore, nearly half of the liquid mass, moving in convective mode, contributes very little to the seismic loads. However, this can change if the convective mass does not have enough room to move freely in the tank.

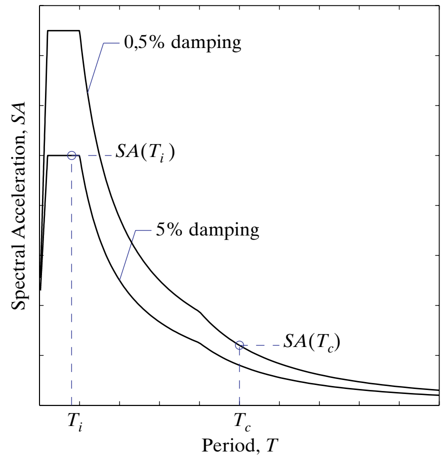

Fig. 4: Site-specific response spectra for 5% and 0,5% damping

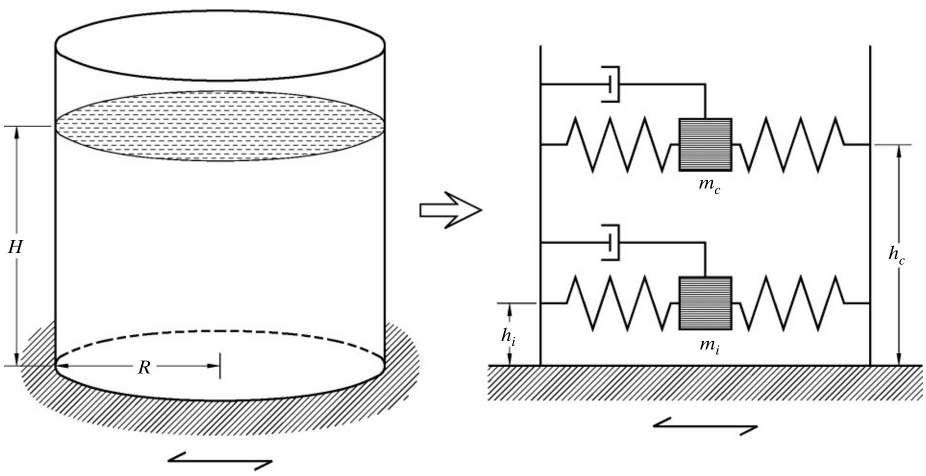

Fig. 3: Simple model of liquid-filled tank

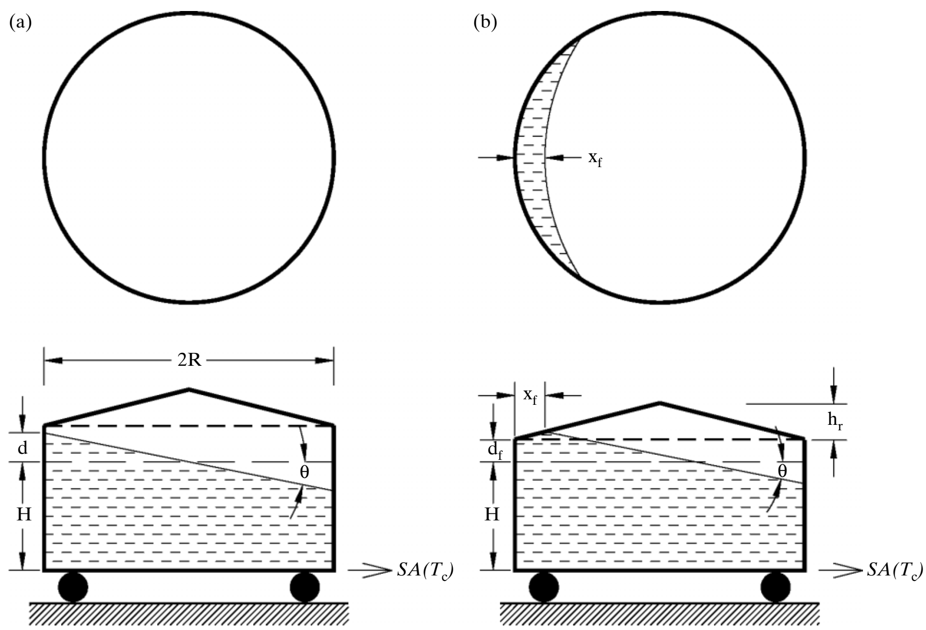

Fig. 5: Liquid-filled tank translating with an acceleration $SA(T_c)$: (a) sufficient freeboard; and (b) insufficient freeboard

Free-Surface Wave Height

The vertical displacement of the liquid surface due to sloshing is:

$$d = R \cdot \frac{SA(T_c)}{g} \tag{1}$$

where, g = acceleration due to gravity. Equation (1) can be understood by visualizing that the liquid-filled tank moves horizontally with an acceleration $SA(T_c)$, as shown in *Fig. 5a*. Under equilibrium, the free-surface would be at an angle θ from the horizontal, where:

$$\theta = \tan^{-1}\left(\frac{SA(T_c)}{g}\right) \tag{2}$$

This gives the height of the sloshing wave to be $d = R \cdot \tan\theta = R \cdot SA(T_c)/g$, thus, the proof of Equation (1). It is assumed in Equation (1) that the entire convective liquid moves in unison, thus giving a somewhat conservative estimate of the sloshing wave height.

Effects of Insufficient Freeboard

Wetted Width of Roof

Next, consider the effect of insufficient freeboard, i.e., actual freeboard d_f is less than the required freeboard d given by Equation (1). For a horizontal acceleration of $SA(T_c)$, the slope of the free-surface θ is still given by Equation (2). However, a portion of the tank roof is wetted, as shown in *Fig. 5b*. For a conical roof of height h_r (measured from the top of the tank shell), the

wetted width x_f (*Fig. 5b*) can be calculated as follows:

– From $SA(T_c)$, calculate θ using Equation (2).
– Find x_f such that the empty volume above the inclined water surface is equal to the empty volume in the tank before the earthquake, that is, $\pi R^2 d_f + \pi R^2 h_r / 3$.

For practical applications, relationships between d_f/d and x_f/R are generated for tanks with different normalized roof height h_r/d. Following steps are taken to generate these relationships:

Step 1: Certain values of h_r/R, x_f/R and d/R are assumed. These are the roof height, wetted width and required freeboard normalized by the tank radius.

Step 2: From d/R, the angle of the free-surface θ is calculated by using Equations (1) and (2), that is, $\theta = \tan^{-1}(d/R)$.

Step 3: The empty volume above the liquid surface in the tank (normalized by πR^3) is calculated by numerical integration using MATLAB routine *dblquad* [11]. Let us call this $V_{empty}/\pi R^3$.

Step 4: Since the empty volume in the tank is same before and during the earthquake (Equation 3),

$$\frac{V_{empty}}{\pi R^3} = \frac{\pi R^2 \cdot d_f + \pi R^2 \cdot h_r/3}{\pi R^3} = \frac{d_f}{R} + \frac{h_r}{3R} \tag{3}$$

or,

$$\frac{d_f}{R} = \frac{V_{empty}}{\pi R^3} - \frac{h_r}{3R} \tag{4}$$

d_f/R is calculated from Equation (4). It corresponds to the assumed values of h_r/R, x_f/R and d/R in Step 1.

Step 5: Different values of h_r/R, x_f/R and d/R are assumed in Step 1. Steps 1 through 4 are repeated to calculate the corresponding values of d_f/R and finally, $d_f/d = (d_f/R)/(d/R)$.

For cone roof tanks, it is found that the relationship between d_f/d and x_f/R depends only on h_r/d (instead of both h_r/R and d/R). *Fig. 6* shows the relationship between d_f/d and x_f/R for tanks with different h_r/d. As expected, a decrease in d_f/d increases x_f/R. For the same d_f/d, an increase in h_r/d reduces x_f/R. Note that $h_r/d = 0$ for a flat roof tank. This was the only result presented in [10]. The wetted roof width is significantly shorter for cone roof tanks than for flat roof tank.

Similar calculations were also performed for dome roof tanks. For dome roof tanks, the relationship between d_f/d and x_f/R depends strongly on h_r/d but it also depends weakly on d/R. The weak dependence on d/R is ignored and the relationship between d_f/d and x_f/R for different h_r/d is presented in *Fig. 7*. As expected, a decrease in d_f/d increases x_f/R. For the same d_f/d, an increase in h_r/d decreases x_f/R. For the same d_f/d and h_r/d, the normalized wetted width x_f/R is shorter for a dome roof tank than for a cone roof tank. This is because a dome roof has slightly larger empty volume to accommodate the sloshing wave than a cone roof of the same height. Because the results for cone roof are conservative, the rest of this paper deals only with cone roofs.

Roof, Shell and Foundation Loads

The maximum upward pressure on the tank roof due to sloshing wave is at the base of roof (top of shell). The vertical distance of the base of roof from the extension of the free surface of liquid is $x_f \tan\theta + x_f \cdot h_r / R$ (*Fig. 8*). Therefore, the maximum upward pressure on the roof is given by Equation (5).

$$P_{\max} = \rho \cdot g \cdot x_f \left(\tan\theta + \frac{h_r}{R} \right) \tag{5}$$

where, ρ = mass density of liquid. The roof pressure linearly reduces to zero at the point where the free surface of liquid meets the tank roof (*Fig. 9*).

The upward force on the roof is resisted by the vertical tensile force in the shell. The connection between the shell and the roof should be designed to transfer this force. If $x_f << R$, the force per unit

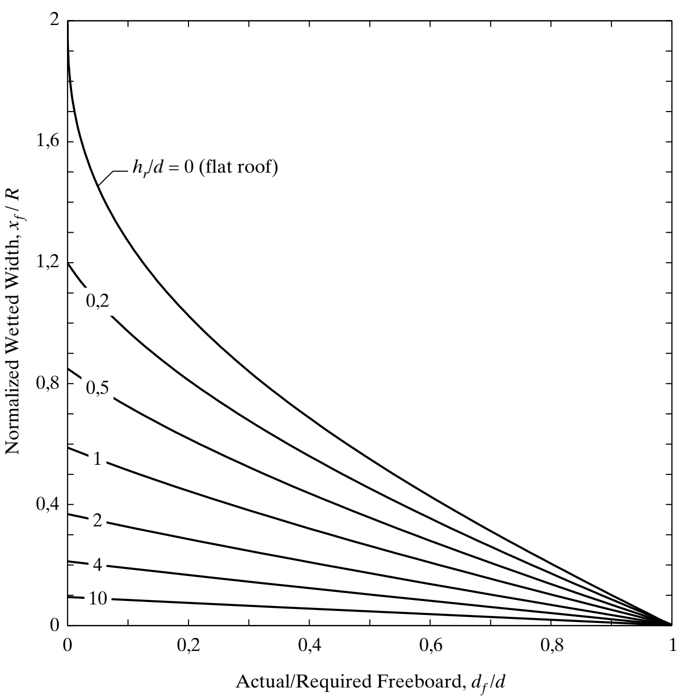

Fig. 6: Cone roof tank. Normalized wetted width of tank roof x_f/R as a function of actual/required freeboard d_f/d and normalized roof height h_r/d

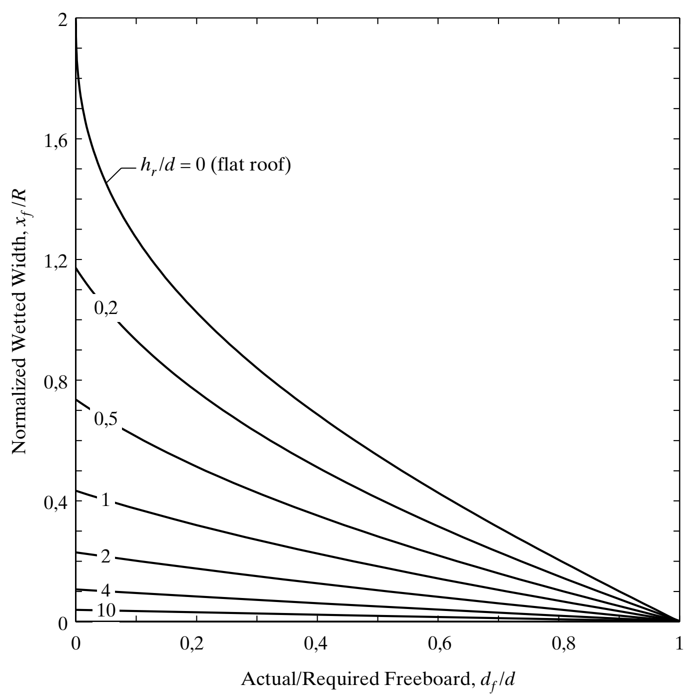

Fig. 7: Dome roof tank. Normalized wetted width of tank roof x_f/R as a function of actual/required freeboard d_f/d and normalized roof height h_r/d

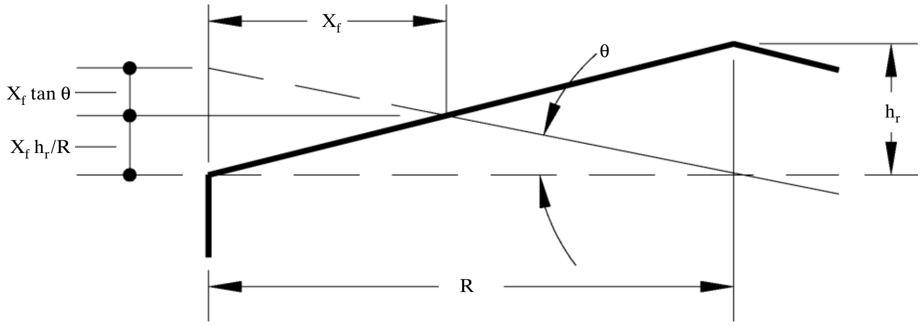

Fig. 8: Vertical distance of base of roof (top of shell) from the extension of free-surface of liquid

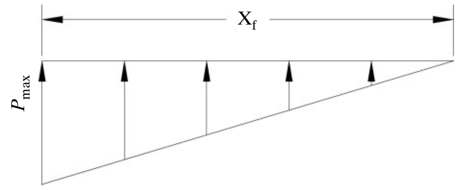

Fig. 9: Radial variation of pressure on tank roof

circumference of the tank shell may be approximated as given in Equation (6).

$$F_{\max} \approx \frac{1}{2} P_{\max} \cdot x_f = \\ = \frac{1}{2} \rho \cdot g \cdot x_f^2 \cdot \left(\tan\theta + \frac{h_r}{R} \right) \qquad (6)$$

Substituting, $\tan\theta = d/R$, Equation (7) can be obtained.

$$F_{\max} \approx \frac{1}{2} \rho \cdot g \cdot x_f^2 \cdot \left(\frac{d + h_r}{R} \right) \qquad (7)$$

Equations (6) and (7) assume that the upward force is resisted by the wet side of the tank shell only. This is not a good assumption when x_f/R is greater than say 0,5. $F_{\max}$ should then be calculated from structural analysis of the tank roof. In Equations (5) to (7), the amplification of roof pressure due to dynamic response of the tank roof has not been considered. This is justified by the fact that the sloshing loads on the roof are applied slowly compared to the natural period of vibration of the tank roof. Typically, the period of the sloshing wave is longer than 3 s and because it is applied near the circumference of the roof, it can only excite higher modes of vibration of the roof, which are generally quite stiff (short-period). The constraint on the sloshing motion increases the mass participation in the impulsive mode. In the limiting case, if the empty space above the liquid surface is zero, the entire liquid in the tank is impulsive. The required empty space in the tank to accommodate sloshing action is $\pi R^2 d$ and the actual empty space is $\pi R^2 d_f + \pi R^2 h_r/3$. Smaller the actual/required empty space in the tank, smaller the convective mass and larger the impulsive mass. Assuming that the convective mass reduces linearly from m_c to 0 as the actual/required empty space reduces from 1 to 0, the adjusted values of the impulsive and convective masses are given in Equation (8) and (9).

$$\bar{m}_i = \begin{cases} m_i + m_c \cdot \left(1 - \dfrac{d_f + h_r/3}{d} \right) & \text{for } d_f + h_r/3 < d \\ m_i & \text{for } d_f + h_r/3 \geq d \end{cases} \qquad (8)$$

$$\bar{m}_c = m_l - \bar{m}_i \qquad (9)$$

Where, $m_l = \rho \cdot \pi R^2 H =$ total liquid mass in the tank. The impulsive and convective periods may also be adjusted as follows:

$$\bar{T}_i = T_i \cdot \sqrt{\frac{\bar{m}_i}{m_i}} \qquad (10)$$

$$\bar{T}_c = T_c \cdot \sqrt{\frac{\bar{m}_c}{m_c}} \qquad (11)$$

For tanks with insufficient freeboard, masses $\bar{m}_i$ and $\bar{m}_c$ should be used instead of m_i and m_c to calculate the base shear and moments [10]. The impulsive and convective spectral accelerations should be read from the site response spectra (*Fig. 4*) using the adjusted impulsive and convective periods $\bar{T}_i$ and $\bar{T}_c$.

Conclusion

A simple method has been presented to estimate the additional loads on tank's roof, wall and foundation due to impacts from the sloshing waves. In many cases, it will be economical to design tanks for these additional loads than to build taller tanks with sufficient freeboard. The sloshing loads in cone and dome roof tanks are significantly smaller than those in flat roof tanks of same size. The sloshing loads in dome roof tanks are slightly smaller than those in cone roof tanks. Therefore, the results for cone roof tanks may also be used for dome roof tanks.

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