AAAI-17 Tutorial on Computer Poker

Part 3: Local Search and Reinforcement Learning

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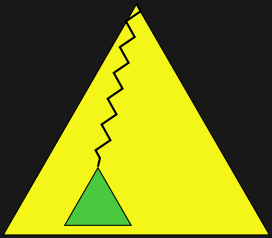
Topics

Scope

- Local Tree Search
  - IS-MCTS
  - Online Outcome Sampling
  - Continuous Resolving
- Reinforcement Learning from Self-Play
  - Extensive-Form Fictitious Play
  - (Neural) Fictitious Self-Play



Local Tree Search

Problems with full-game equilibria

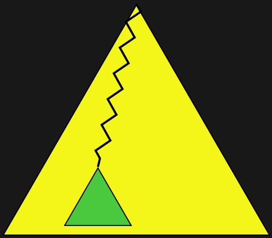
- Memory (Space requirements)
  - How do we obtain an equilibrium if we cannot fit the whole game tree in memory?
- Computation (Time requirements)
  - How do we compute an equilibrium in a reasonable amount of time?
- Locality
  - How do we map a real information state (sequence) to our full-game abstraction?

Local Tree Search

Approach

Compute or refine an approximate equilibrium locally for the situation at hand.

- Standard approach in large-scale perfect-information games
  - e.g. MCTS in Go

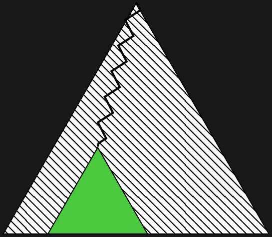


Local Tree Search

IS-MCTS

Refs: Cowling et al. 2012, Heinrich&Silver 2015

- Extension of common MCTS to imperfect-information games:
  - One information-state subtree per player
  - Sampling from local belief state
  - Information-state policy regret minimizing or fictitious play
- Achieved practical successes
  - E.g. Imperfect-information board&card games, including LHE
- Lacking equilibrium guarantees
  - Exploitable in local search
  - May plateau in full-game search



Local Tree Search

Equilibrium recovery

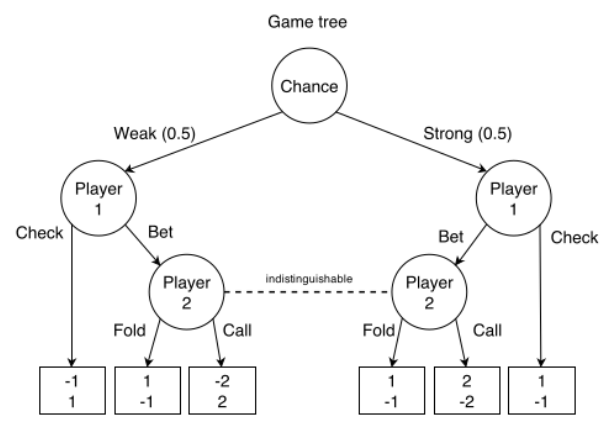


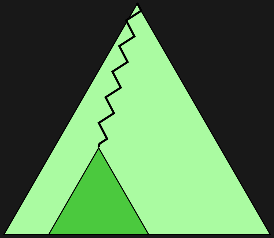
Figure: Bluff catching game

Local Tree Search

Online Outcome Sampling

Refs: Lisy et al. 2015

- Focus Outcome Sampling (MCCFR) on relevant information states
- Achieves equilibrium guarantees
- Does not fully address memory, computation and abstraction-mapping problems

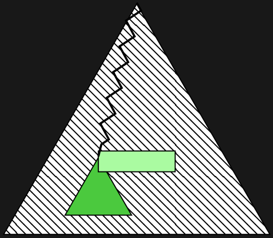


Local Tree Search

Continuous resolving

Refs: Burch et al. 2014, Moravcik et al. 2017

- Solve gadget game that hashes essential information of the global equilibrium in counterfactual values
  - Recovers global equilibrium with strictly local search
  - Continuous resolving carries forward the counterfactual values
- Resolving may be intractable except at the end of the game
  - DeepStack bootstraps a limited-depth search from learned counterfactual values



Topics

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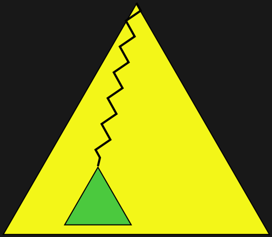
- Reinforcement Learning from Self-Play
  - Extensive-Form Fictitious Play
  - (Neural) Fictitious Self-Play



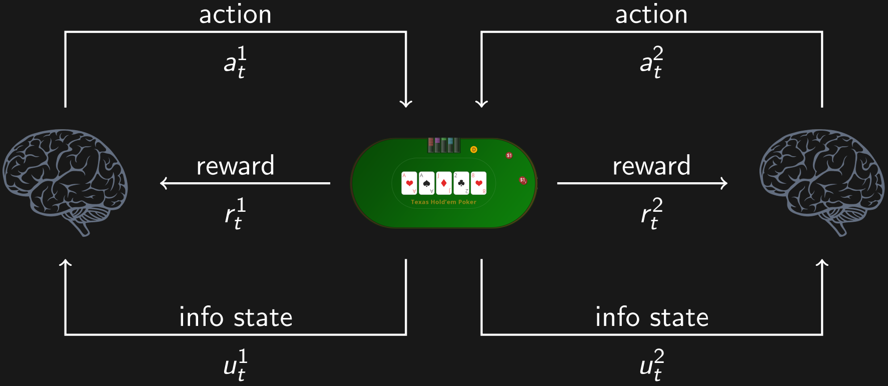
Reinforcement Learning from Self-Play

Problems

- Handcrafted abstractions
  - Do you have the domain expertise and time to handcraft an abstraction?
    - Different variants may require different kinds of abstraction, e.g. limit, no-limit, stud, draw poker variants
  - How do we abstract action sequences for multi-player variants?
- Static abstractions
  - Did your abstracton lose essential information for achieving a desired equilibrium quality?
  - Does it have any deficiencies or result in pathologies?
- Scalability
  - Does your approach scale well with the number of players?

Reinforcement Learning from Self-Play

Approach



Learn strategies, π, that optimize action-values

$$Q_\pi(u, a) = \mathbb{E}_\pi \left[\sum_{k=t}^{T} R_{k+1} \,\middle|\, U_t = u, A_t = a \right]$$

Reinforcement Learning from Self-Play

Generalised weakened fictitious play

Refs: (Brown, 1951; Leslie & Collins, 2006)

At each iteration each player

1. Computes a best response to fellow players' average strategies
2. Updates own average strategy with computed best response

$$\Pi_{k+1} \in \Pi_k + \alpha_{k+1} \left(\mathrm{BR}_{\epsilon_{k+1}} [\Pi_k] - \Pi_k + M_{k+1} \right)$$

Original fictitious play: $\alpha_k = \frac{1}{k}, \epsilon_k = 0, M_k = 0$

Reinforcement Learning from Self-Play

Fictitious play & reinforcement learning

- Fictitious players choose best (payoff-maximizing) strategies in hindsight.
- Reinforcement learners learn optimal (reward-maximizing) policies from their experience.

Reinforcement Learning from Self-Play

Extensive-Form Fictitious Play (XFP)

Refs: Koller et al. 1994; Von Stengel 1996; Heinrich et al. 2015

- Compute best responses with dynamic programming

$$\beta_{t+1} \in \mathrm{BR}(\pi_t),$$

- Update average strategies

$$\pi^i_{t+1}(u) = \pi^i_t(u) + w_{t+1}(u) \left(\beta^i_{t+1}(u) - \pi^i_t(u) \right)$$

weighted by conditional probability (of playing best response at respective information state)

$$w^i_{t+1}(u) = \frac{\alpha_{t+1} x_{\beta^i_{t+1}}(\sigma_u)}{(1 - \alpha_{t+1}) x_{\pi^i_t}(\sigma_u) + \alpha_{t+1} x_{\beta^i_{t+1}}(\sigma_u)}$$

Reinforcement Learning from Self-Play

Fictitious Self-Play

Refs: Heinrich et al. 2015

	XFP	FSP
Best response	Dynamic programming in whole tree	Reinforcement learning from sampled trajectories
Average strategy	Explicit update in whole tree	Imitation learning from sampled trajectories of past best responses

Fictitious Self-Play

Learning a best response

- Each agent plays its average strategy, π
- Each agent learns a best response strategy by maximizing and evaluating its Q values with off-policy reinforcement learning

$$Q^i(u, a) \approx \mathbb{E}_{\beta^i, \pi^{-i}} \left[\sum_{k=t}^{T} R_{k+1} \,\middle|\, U_t = u, A_t = a \right]$$

$$\text{e.g. } \beta^i = \text{greedy}(Q^i)$$

Fictitious Self-Play

Learning the average strategy

To learn

$$\Pi_k^i = \sum_{j=1}^{k} w_j B_j^i$$

1. Sample a proportion of w_j episodes from each β_j^i respectively
2. Store experienced state-action pairs, (u_t, a_t), in a memory
3. Train the average strategy with imitation learning from the behaviour data in memory

Fictitious Self-Play

Memorizing the Average Strategy

Infinite stream of state-action pairs (u_t, a_t) can be memorized in an online fashion and with finite memory capacity:

- Counting model, $N_k(u, a)$

$$\pi_k(a \mid u) = \frac{N_k(u, a)}{N_k(u)}$$

- Reservoir sampling
  - Tracks a finite random sample of a possibly large or infinite stream of items (here, best responses)

Reinforcement Learning from Self-Play

Neural Fictitious Self-Play

Refs: Heinrich&Silver 2016

1. Train an action-value network, $Q(s, a \,|\, \theta^Q)$, that approximates best response with neural fitted Q-learning (DQN)
   - $\beta = \epsilon$-greedy (Q)
2. Train an average-policy network, $\Pi(s, a \,|\, \theta^\Pi)$, that approximates own average behaviour with imitation learning
   - $\pi = \Pi$

Neural Fictitious Self-Play

Sampling Experience

Each agent

- Uses policy $\sigma = \begin{cases} \epsilon\text{-greedy}\,(Q)\,, & \text{with probability } \eta \\ \Pi, & \text{with probability } 1-\eta \end{cases}$
- Stores its transitions $(u_t, a_t, r_{t+1}, u_{t+1})$ in reinforcement learning memory $\mathcal{M}_{RL}$
- Stores its behaviour tuples (s_t, a_t) in supervised learning memory $\mathcal{M}_{SL}$, when following its best response strategy (ϵ-greedy$\,(Q)$)

Neural Fictitious Self-Play

Reinforcement learning of best response

Refs: Mnih et al. 2015

Train $Q(s, a \mid \theta^Q)$ with SGD on

$$\mathcal{L}\left(\theta^Q\right) = \mathbb{E}_{(s,a,r,s') \sim \mathcal{M}_{RL}} \left[\left(r + \max_{a'} Q(s', a' \mid \theta^{Q'}) - Q(s, a \mid \theta^Q) \right)^2 \right]$$

Neural Fictitious Self-Play

Imitation learning of average strategy

Train $\Pi(s, a \mid \theta^{\Pi})$ with SGD on

$$\mathcal{L}(\theta^{\Pi}) = \mathbb{E}_{(s,a)\sim\mathcal{M}_{SL}}\left[-\log \Pi(s, a \mid \theta^{\Pi})\right]$$

Neural Fictitious Self-Play

Experiments in Limit Texas Hold'em

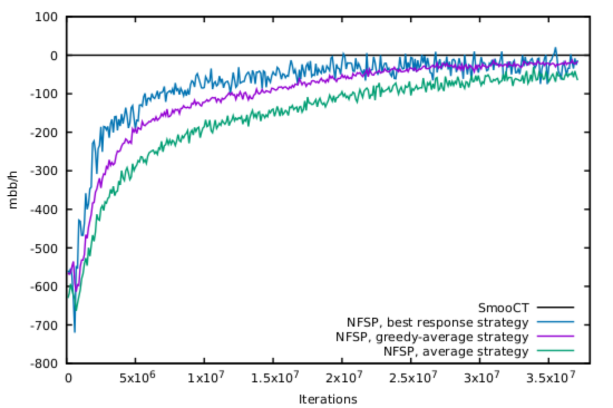


Figure: Win rates of NFSP against SmooCT in Limit Texas Hold'em. The estimated standard error of each evaluation is less than 10 mbb/h.

Neural Fictitious Self-Play

Experiments in Limit Texas Hold'em

Match-up	Win rate (mbb/h)
escabeche	-52.1 $\pm$ 8.5
SmooCT	-17.4 $\pm$ 9.0
Hyperborean	-13.6 $\pm$ 9.2

Table: Win rates of NFSP's greedy-average strategy against the top 3 agents of the ACPC 2014.

Neural Fictitious Self-Play

Visualization of a poker-playing neural network

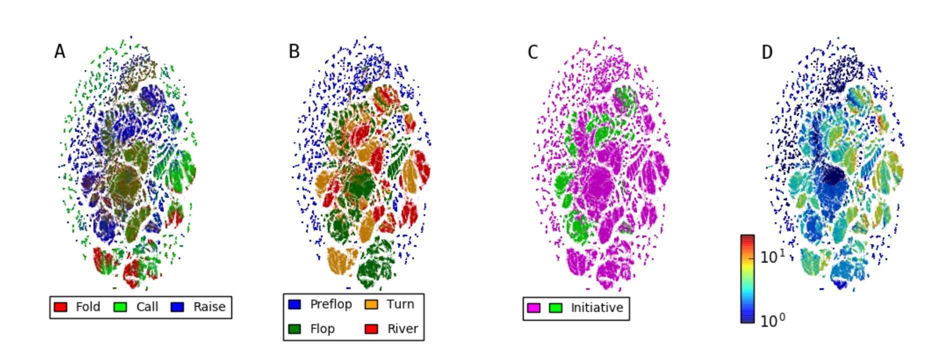
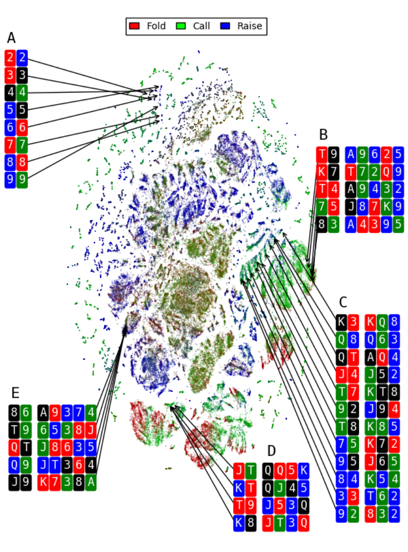


Figure: t-SNE embedding of first player's last hidden layer activations, coloured by A) action probabilities; B) round of the game; C) initiative feature; D) pot size in big bets (logarithmic scale).

Neural Fictitious Self-Play

Visualization of a poker-playing neural network

- A: Preflop
- B: Pairs on river, after check-calling down
- C: Pairs on flop, facing continuation bet after big-blind defense
- D: Straight draws on the turn
- E: Busted straight draws on the river, after bluffing on the turn



Reinforcement Learning from Self-Play

Why?

Self-play RL can be applied to a variety of poker games, including multi-player, without having to design abstractions. E.g. full ring stud poker or pot limit Omaha

Thanks!