Practice paper C1
Exercise 1, Question 1

Question:

(a) Write down the value of $16^{\frac{1}{2}}$. (1)

(b) Hence find the value of $16^{\frac{3}{2}}$. (2)

Solution:

(a) $16^{\frac{1}{2}} = \sqrt{16} = 4$

(b) $16^{\frac{3}{2}} = \left(16^{\frac{1}{2}} \right)^3 = 4^3 = 64$

Practice paper C1
Exercise 1, Question 2

Question:

Find $\int (6x^2 + \sqrt{x})\,dx$. (4)

Solution:

$$\int \left(6x^2 + x^{\frac{1}{2}} \right) dx$$

$$= 6\,\frac{x^3}{3} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2x^3 + \frac{2}{3}x^{\frac{3}{2}} + c$$

Practice paper C1
Exercise 1, Question 3

Question:

A sequence $a_1, a_2, a_3, \ldots\ a_n$ is defined by
$a_1 = 2, a_{n+1} = 2a_n - 1$.

(a) Write down the value of a_2 and the value of a_3. (2)

(b) Calculate $\sum_{r=1}^{5} a_r$. (2)

Solution:

(a) $a_2 = 2a_1 - 1 = 4 - 1 = 3$
$a_3 = 2a_2 - 1 = 6 - 1 = 5$

(b) $a_4 = 2a_3 - 1 = 10 - 1 = 9$
$a_5 = 2a_4 - 1 = 18 - 1 = 17$

$$\sum_{r=1}^{5} a_r = a_1 + a_2 + a_3 + a_4 + a_5 = 2 + 3 + 5 + 9 + 17 = 36$$

Practice paper C1
Exercise 1, Question 4

Question:

(a) Express $(5 + \sqrt{2})^2$ in the form $a + b\sqrt{2}$, where a and b are integers. (3)

(b) Hence, or otherwise, simplify $(5 + \sqrt{2})^2 - (5 - \sqrt{2})^2$. (2)

Solution:

(a) $(5 + \sqrt{2})^2 = (5 + \sqrt{2}) (5 + \sqrt{2}) = 25 + 10\sqrt{2} + 2 = 27 + 10\sqrt{2}$

(b) $(5 - \sqrt{2})^2 = (5 - \sqrt{2}) (5 - \sqrt{2}) = 25 - 10\sqrt{2} + 2 = 27 - 10\sqrt{2}$

$(5 + \sqrt{2})^2 - (5 - \sqrt{2})^2$
$= (27 + 10\sqrt{2}) - (27 - 10\sqrt{2})$
$= 27 + 10\sqrt{2} - 27 + 10\sqrt{2}$
$= 20\sqrt{2}$

Practice paper C1
Exercise 1, Question 5

Question:

Solve the simultaneous equations:
$x - 3y = 6$
$3xy + x = 24$ (7)

Solution:

$x - 3y = 6$
$x = 6 + 3y$
Substitute into $3xy + x = 24$:
$3y(6 + 3y) + (6 + 3y) = 24$
$18y + 9y^2 + 6 + 3y = 24$
$9y^2 + 21y - 18 = 0$
Divide by 3:
$3y^2 + 7y - 6 = 0$
$(3y - 2)(y + 3) = 0$
$y = \frac{2}{3}, y = -3$
Substitute into $x = 6 + 3y$:
$y = \frac{2}{3} \Rightarrow x = 6 + 2 = 8$
$y = -3 \Rightarrow x = 6 - 9 = -3$
$x = -3, y = -3$ or $x = 8, y = \frac{2}{3}$

Practice paper C1
Exercise 1, Question 6

Question:

The points *A* and *B* have coordinates $(-3, 8)$ and $(5, 4)$ respectively.
The straight line l_1 passes through *A* and *B*.

(a) Find an equation for l_1, giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers. (4)

(b) Another straight line l_2 is perpendicular to l_1 and passes through the origin. Find an equation for l_2. (2)

(c) The lines l_1 and l_2 intersect at the point *P*. Use algebra to find the coordinates of *P*. (3)

Solution:

(a) Gradient of $l_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 8}{5 - (-3)} = -\frac{4}{8} = -\frac{1}{2}$

Equation for l_1:

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{2}\left(x - 5\right)$

$y - 4 = -\frac{1}{2}x + \frac{5}{2}$

$\frac{1}{2}x + y - \frac{13}{2} = 0$

$x + 2y - 13 = 0$

(b) For perpendicular lines, $m_1 m_2 = -1$

$m_1 = -\frac{1}{2}$, so $m_2 = 2$

Equation for l_2 is $y = 2x$

(c) Substitute $y = 2x$ into $x + 2y - 13 = 0$:

$x + 4x - 13 = 0$

$5x = 13$

$x = 2\frac{3}{5}$

$y = 2x = 5\frac{1}{5}$

Coordinates of *P* are $\left(2\frac{3}{5}, 5\frac{1}{5}\right)$

Practice paper C1
Exercise 1, Question 7

Question:

On separate diagrams, sketch the curves with equations:

(a) $y = \frac{2}{x}$, $-2 \leq x \leq 2, x \neq 0$ (2)

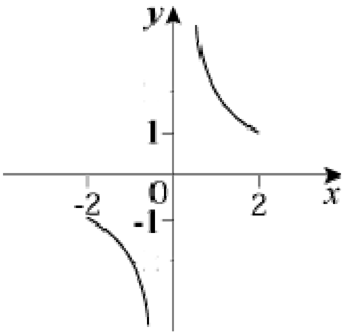
(b) $y = \frac{2}{x} - 4$, $-2 \leq x \leq 2, x \neq 0$ (3)

(c) $y = \frac{2}{x+1}$, $-2 \leq x \leq 2, x \neq -1$ (3)

In each part, show clearly the coordinates of any point at which the curve meets the *x*-axis or the *y*-axis.

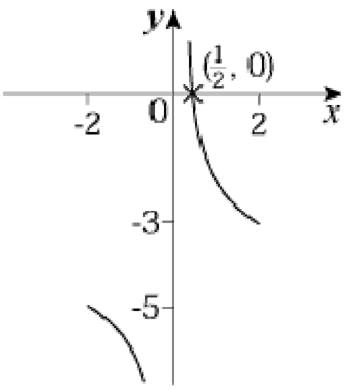
Solution:

(a)



$y = \frac{2}{x}$

(b) Translation of -4 units parallel to the *y*-axis.



$y = \frac{2}{x} - 4$

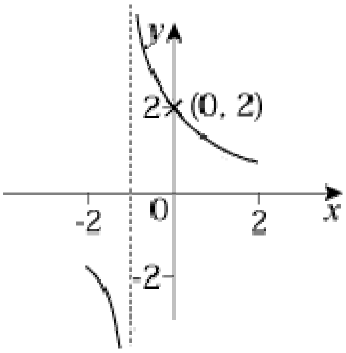
Curve crosses the *x*-axis where $y = 0$:

$$\frac{2}{x} - 4 = 0$$

$$\frac{2}{x} = 4$$

$$x = \frac{1}{2}$$

(c) Translation of -1 unit parallel to the *x*-axis.



$$y = \frac{2}{x + 1}$$

The line $x = -1$ is an asymptote.
Curve crosses the *y*-axis where $x = 0$:

$$y = \frac{2}{0 + 1} = 2$$

Practice paper C1
Exercise 1, Question 8

Question:

In the year 2007, a car dealer sold 400 new cars. A model for future sales assumes that sales will increase by x cars per year for the next 10 years, so that $(400 + x)$ cars are sold in 2008, $(400 + 2x)$ cars are sold in 2009, and so on. Using this model with $x = 30$, calculate:

(a) The number of cars sold in the year 2016. (2)

(b) The total number of cars sold over the 10 years from 2007 to 2016. (3)
The dealer wants to sell at least 6000 cars over the 10-year period.
Using the same model:

(c) Find the least value of x required to achieve this target. (4)

Solution:

(a) $a = 400$, $d = x = 30$
$T_{10} = a + 9d = 400 + 270 = 670$
670 cars sold in 2010

(b) $S_n = \frac{1}{2} n \left[2a + \left(n - 1 \right) d \right]$
So $S_{10} = 5 [(2 \times 400) + (9 \times 30)] = 5 \times 1070 = 5350$
5350 cars sold from 2001 to 2010

(c) S_{10} required to be at least 6000:
$\frac{1}{2} n \left[2a + \left(n - 1 \right) d \right] \geq 6000$
$5 (800 + 9x) \geq 6000$
$4000 + 45x \geq 6000$
$45x \geq 2000$
$x \geq 44 \frac{4}{9}$
To achieve the target, $x = 45$.

Practice paper C1
Exercise 1, Question 9

Question:

(a) Given that
$x^2 + 4x + c = (x + a)^2 + b$
where a, b and c are constants:

(i) Find the value of a. (1)

(ii) Find b in terms of c. (2)
Given also that the equation $x^2 + 4x + c = 0$ has unequal real roots:

(iii) Find the range of possible values of c. (2)

(b) Find the set of values of x for which:

(i) $3x < 20 - x$, (2)

(ii) $x^2 + 4x - 21 > 0$, (4)

(iii) both $3x < 20 - x$ and $x^2 + 4x - 21 > 0$. (2)

Solution:

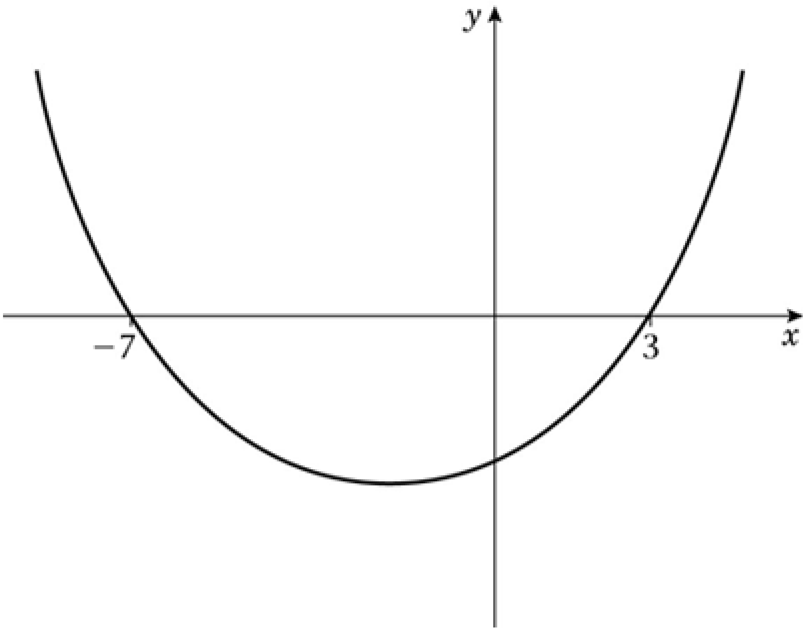
(a) (i) $x^2 + 4x + c = (x + 2)^2 - 4 + c = (x + 2)^2 + (c - 4)$
So $a = 2$

(ii) $b = c - 4$

(iii) For unequal real roots:
$(x + 2)^2 - 4 + c = 0$
$(x + 2)^2 = 4 - c$
$4 - c > 0$
$c < 4$

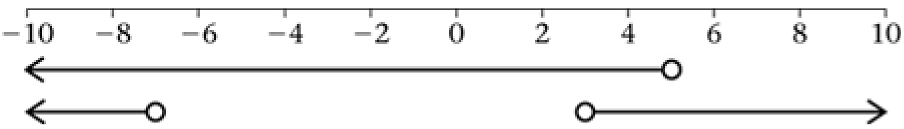
(b) (i) $3x < 20 - x$
$3x + x < 20$
$4x < 20$
$x < 5$

(ii) Solve $x^2 + 4x - 21 = 0$:
$(x + 7) (x - 3) = 0$
$x = - 7, x = 3$
Sketch of $y = x^2 + 4x - 21$:



$x^2 + 4x - 21 > 0$ when $x < -7$ or $x > 3$

(c)



Both inequalties are true when
$x < -7$ or $3 < x < 5$

(a) Show that $\frac{(3x-4)^2}{x^2}$ may be written as $P + \frac{Q}{x} + \frac{R}{x^{2'}}$ where P, Q and R are constants to be found. (3)

(b) The curve C has equation $y = \frac{(3x-4)^2}{x^2}$, $x \neq 0$. Find the gradient of the tangent to C at the point on C where $x = -2$. (5)

(c) Find the equation of the normal to C at the point on C where $x = -2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. (5)

Solution:

(a) $(3x-4)^2 = (3x-4)(3x-4) = 9x^2 - 24x + 16$

$\frac{(3x-4)^2}{x^2} = \frac{9x^2 - 24x + 16}{x^2} = 9 - \frac{24}{x} + \frac{16}{x^2}$

$P = 9, Q = -24, R = 16$

(b) $y = 9 - 24x^{-1} + 16x^{-2}$

$\frac{dy}{dx} = 24x^{-2} - 32x^{-3}$

Where $x = -2$, $\frac{dy}{dx} = \frac{24}{(-2)^2} - \frac{32}{(-2)^3} = \frac{24}{4} + \frac{32}{8} = 10$

Gradient of the tangent is 10.

(c) Where $x = -2$, $y = 9 - \frac{24}{(-2)} + \frac{16}{(-2)^2} = 9 + 12 + 4 = 25$

Gradient of the normal $= \frac{-1}{\text{Gradient of tangent}} = -\frac{1}{10}$

The equation of the normal at $(-2, 25)$ is

$y - 25 = -\frac{1}{10}\left[x - \left(-2\right)\right]$

Multiply by 10:

$10y - 250 = -x - 2$

$x + 10y - 248 = 0$