Analysis of New and Existing Methods of Reducing Intercarrier Interference Due to Carrier Frequency Offset in OFDM

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Abstract—**Orthogonal frequency division multiplexing (OFDM) is very sensitive to frequency errors caused by frequency differences between transmitter and receiver local oscillators. In this paper, this sensitivity is analyzed in terms of the complex weighting coefficients which give the contribution of each transmitter subcarrier to each demodulated subcarrier. Previously described windowing and self-intercarrier interference (ICI) cancellation methods are analyzed in terms of these weighting coefficients. New ICI cancellation schemes with very much improved performance are described. A condition for orthogonality of windowing schemes is derived in terms of the discrete Fourier transform (DFT) of the windowing function.**

Index Terms—**Discrete Fourier transform, intercarrier interference, interference suppression, orthogonal frequency division multiplexing, synchronization.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is being considered for data transmission in a number of environments [1], [2]. One limitation of OFDM in many applications is that it is very sensitive to frequency errors caused by frequency differences between the local oscillators in the transmitter and the receiver [3]–[5]. Carrier frequency offset causes a number of impairments including attenuation and rotation of each of the subcarriers and intercarrier interference (ICI) between subcarriers [4]. A number of methods have been developed to reduce this sensitivity to frequency offset, including windowing of the transmitted signal [6], [7] and use of self ICI cancellation schemes [8].

This paper analyzes in detail, for a perfect Nyquist channel, the ICI resulting from carrier frequency offset. Expressions are derived for each demodulated subcarrier at the receiver in terms of each transmitted subcarrier and N complex weighting factors. Windowing and ICI cancellation schemes can be related and described in terms of these complex weighting factors. New ICI cancellation schemes which give greater ICI cancellation are developed. A new condition for the orthogonality of windowing schemes is derived in terms of the discrete Fourier transform (DFT) of the windowing function.

II. Analysis of ICI

A. Structure of OFDM System

Fig. 1 shows the structure of the OFDM communication system being considered. In this OFDM system there are N subcarriers and the symbol period is T. In the ith symbol period, the N complex values $a_{0,i} \cdots a_{N-1,i}$ modulate the N subcarriers.

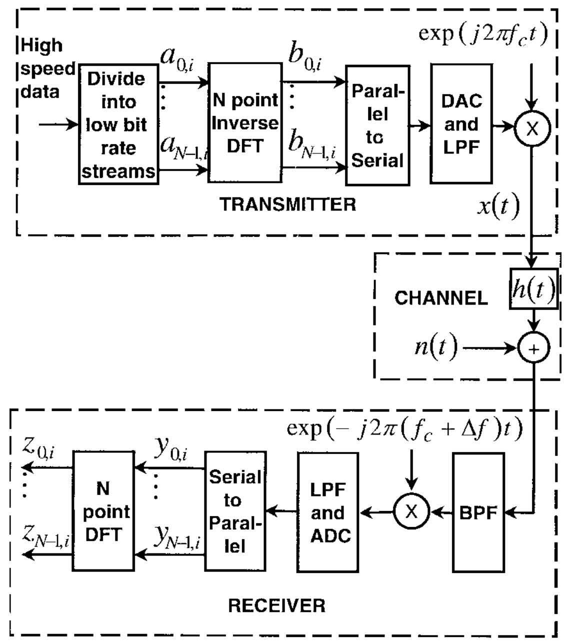

Fig. 1. Structure of an OFDM communication system.

Systems using a number of different types of modulation of subcarriers within OFDM, such as phase shift keying (PSK) and quadrature amplitude modulation (QAM) have been described in the literature. This analysis does not depend on the mapping of the data to be transmitted to the complex values $a_{0,i} \cdots a_{N-1,i}$, and is therefore applicable to all forms of modulation which can be used within OFDM.

This analysis considers only the impairments due to carrier frequency offset. Other authors [9] have analyzed more general models, but these do not clearly reveal the structure on which ICI cancellation depends. Frequency offset alone does not cause intersymbol interference (ISI). Often a cyclic prefix is used in OFDM to eliminate the ISI and ICI caused by errors in sampling time or distortion in the channel. This use of a cyclic prefix is not considered in this analysis.

B. Derivation of Expressions for the Complex Weighting Coefficients

The signal at the output of the OFDM transmitter resulting from the ith transmitted symbol is given by

$$x(t) = \exp(j2\pi f_c t) \sum_{k=0}^{N-1} b_{k,i}\, p\left(t - \frac{kT}{N}\right) \tag{1}$$

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where f_c is the carrier frequency and $p(t)$ is the impulse response of the low-pass filter in the transmitter. At the receiver, the signal is mixed with a local oscillator signal which is Δf above the correct frequency f_c. Ignoring the effects of noise, the demodulated signal is then given by

$$y(t) = \exp j(2\pi \Delta f t + \theta_0) \sum_{k=0}^{N-1} b_{k,i} q\left(t - \frac{kT}{N}\right) \quad (2)$$

where $q(t)$ is the combined impulse response of the channel and of the transmitter and receiver filters. θ_0 is the phase offset between the phase of the receiver local oscillator and the carrier phase at the start of the received symbol.

Assuming that $q(t)$ satisfies the Nyquist criterion for samples taken at intervals T/N and that $y(t)$ is sampled at the optimum instants, then the samples input to the receiver DFT are given by

$$y_{k,i} = \exp(j\theta_0) b_{k,i} \exp\left(\frac{j2\pi k \Delta f T}{N}\right). \quad (3)$$

The result of the DFT of these samples is given by

$$z_{m,i} = \sum_{k=0}^{N-1} y_{k,i} \exp\left(-\frac{j2\pi km}{N}\right). \quad (4)$$

Substituting the value of $y_{k,i}$ from (3) and after some manipulation, it can be shown that

$$z_{m,i} = \frac{1}{N} \exp(j\theta_0) \sum_{l=0}^{N-1} a_{l,i} \sum_{k=0}^{N-1} \exp\left(\frac{j2\pi k(l - m + \Delta f T)}{N}\right). \quad (5)$$

Using the properties of geometric series, this can alternatively be expressed as

$$z_{m,i} = \frac{1}{N} \exp(j\theta_0) \sum_{l=0}^{N-1} a_{l,i} \frac{\sin(\pi(l - m + \Delta f T))}{\sin(\frac{\pi(l-m+\Delta f T)}{N})} \times \exp\left(j\left(\frac{N-1}{N}\right)(l - m + \Delta f T)\right). \quad (6)$$

If $\Delta f = 0$, then $z_{m,i} = \exp(j\theta_0) a_{m,i}$, and each decoded complex value is simply the phase rotated version of the transmitted value. The amount of rotation depends on the phase offset between the transmitter and receiver local oscillators.

If $\Delta f \neq 0$, then ICI will occur and each output data symbol will depend on all of the input values. The analysis of ICI can be simplified by defining N complex weighting coefficients, $c_0 \cdots c_{N-1}$, which give the contribution of each of the N input values $a_{0,i} \cdots a_{N-1,i}$ to the output value $z_{m,i}$

$$z_{m,i} = \exp(j\theta_0) \sum_{l=0}^{N-1} c_{l-m} a_{l,i} \quad (7)$$

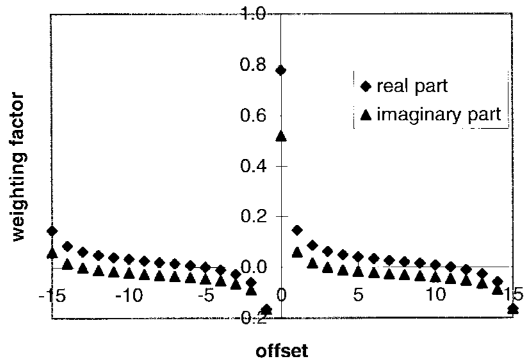

Fig. 2. Real and imaginary components of the complex weighting factors $c_0 \cdots c_{N-1}$ for the case of $\Delta f T = 0.2$ and $N = 16$.

where

$$c_{l-m} = \frac{1}{N} \sum_{k=0}^{N-1} \exp\left(\frac{j2\pi k(l - m + \Delta f T)}{N}\right) = \frac{1}{N} \frac{\sin \pi(l - m + \Delta f T)}{\sin \pi(\frac{l-m+\Delta f T}{N})} \times \exp j\pi \frac{(N-1)(l - m + \Delta f T)}{N}. \quad (8)$$

The decoded complex value $z_{m,i}$ therefore consists of a wanted component which is due to $a_{m,i}$ but which is subject to a change in amplitude and phase given by c_0, where

$$c_0 = \frac{1}{N} \sum_{k=0}^{N-1} \exp\left(\frac{j2\pi k \Delta f T}{N}\right). \quad (9)$$

c_0 depends on the normalized frequency offset $\Delta f T$ but is independent of m. In other words, all subcarriers experience the same degree of attenuation and rotation of the wanted component.

In addition, the decoded complex value is subject to ICI. This is the sum of components dependent on each of values $a_{l,i}$, $l \neq m$. The contribution of each $a_{l,i}$ depends on the normalized frequency offset $\Delta f T$ and on $(l - m) \bmod N$. It does not depend directly on m.

Fig. 2 shows the complex weighing factors $c_{-N+1} \cdots c_0 \cdots c_{N-1}$ for the case of $\Delta f T = 0.2$ and $N = 16$. Note that as the coefficients depend on the distance mod N between the subcarriers, there are only N distinct coefficients, $c_0 \cdots c_{N-1}$. The graphs are smooth, there are no sudden changes in the weighting coefficients as the distance moves from -15 to 15, except between -1 and 0, and between 0 and 1.

Fig. 3 shows how, for $\Delta f T = 0.1$, the value of c_0 depends on the number of subcarriers in the OFDM system. For $N > 8$ there is little change in the power of the wanted subcarrier. Because carrier frequency offset does not change the total power in the received signal, this also means that the total ICI power changes little with N.

III. Self ICI Cancellation Schemes

Zhao and Häggman [8] have described a method of reducing sensitivity to frequency errors which they call self ICI cancellation. The method maps the data to be transmitted onto

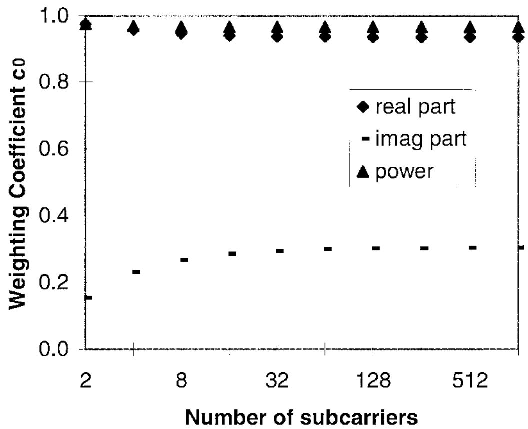

Fig. 3. c_0 as function of number of subcarriers for $\Delta fT = 0.1$.

adjacent pairs of subcarriers rather than onto single subcarriers, so that $a_{0,i} = -a_{1,i},\ a_{2,i} = -a_{3,i}, \cdots a_{N-2,i} = -a_{N-1,i}$. This results in cancellation of most of the ICI in the values $z_{0,i} \cdots z_{N-1,i}$.

For example, the decoded value for the zeroth carrier is given by

$$z_{0,i} = \exp(j\theta_0)\{(c_0 - c_1)a_{0,i} + (c_2 - c_3)a_{2,i} + \cdots + (c_{N-2} - c_{N-1})a_{N-2,i}\}. \quad (10)$$

The ICI now depends on the difference between the adjacent weighting coefficients rather than on the coefficients themselves. As the difference between adjacent coefficients is small, this results in substantial reduction in ICI. If adjacent coefficients were equal, then the ICI would be completely cancelled. Thus this process can be considered as cancelling out the component of ICI which is *constant* between adjacent pairs of coefficients. ICI cancellation depends only on the coefficients being slowly varying functions of offset. It does depend not on the absolute values of the coefficients and so improves the performance for any frequency offset.

To maximize the overall SNR, the values $z_{0,i}, \cdots z_{N-1,i}$ should be subtracted in pairs, because this results in the addition of the wanted signal components. This also further reduces the ICI

$$z_{0,i} - z_{1,i} = \exp(j\theta_0)\{(-c_{-1} + 2c_0 - c_1)a_{0,i} + (-c_1 + 2c_2 - c_3)a_{2,i} + \cdots + (-c_{N-3} + 2c_{N-2} - c_{N-1})a_{N-2,i}\}. \quad (11)$$

The remaining ICI depends on factors of the form $(-c_1 + 2c_2 - c_3)$. If the three weighting coefficients in each factor were linear functions of offset, for example, if $c_2 = k + c_1$ and $c_3 = 2k + c_1$, where k is any constant as well as the gradient of the linear function, these factors would all be zero. This cancellation could be considered as canceling out the component of ICI which is due to the *linear* variation in weighting coefficient over groups of three adjacent coefficients.

When there is no carrier frequency offset

$$z_{0,i} - z_{1,i} = 2a_{0,i}\exp(j\theta_0). \quad (12)$$

Thus, in the absence of other impairments, all of the received power is decoded into wanted signal, and the ICI cancellation scheme gives no reduction in overall SNR compared with normal OFDM. A disadvantage of the method is that it is less bandwidth efficient than normal OFDM as only half as many complex values can be transmitted per symbol.

The ICI cancellation scheme also results in a form of windowing and the overall transmitted signal has a sinusoidal envelope of period $2T$. This is because pairs of sinusoids of frequency difference $1/T$ are being subtracted.

IV. Windowing

A. Windowing to Reduce Sensitivity to Linear Distortions

A number of authors [6], [7], [10], [11] have described the use of windowing in OFDM. These applications can be divided into two groups. In the first group, windowing is used to reduce the sensitivity to linear distortions [10], [11]. In the second, windowing is used to reduce the sensitivity to frequency errors [6], [7]. In the first group, the signal at the output of the IDFT in the transmitter is cyclically extended. The windowing function shapes the cyclic extension, but the original part of the signal remains unchanged. Where the only impairments in the system are due to frequency differences between the local oscillators, this form of windowing has no effect on the system performance. This form of windowing will not be considered further.

B. Windowing to Reduce Sensitivity to Frequency Offset

The second form of windowing involves cyclically extending by v samples the time domain signal associated with each symbol. The whole of the resulting signal is then shaped with the window function. Fig. 4(a) shows a block diagram of a typical system. Note that the transform in the receiver is N point whereas that in the transmitter is $N/2$ point. The $N/2$ inputs to the transmitter transform have been labeled $a_{0,i}, a_{2,i} \cdots a_{N-2,i}$ so that windowing can be more readily analyzed and related to ICI cancellation. If $v = N/2$, then N points of the received signal are used as input to the DFT, if $v < N/2$, then the signal corresponding to each symbol is zero padded at the receiver to give length N. The outputs of the DFT with even-numbered subscripts are used as estimates of the transmitted data and the odd-numbered ones are discarded. Because not all of the received signal power is being used in generating data estimates, the method has a reduced overall SNR compared with OFDM without windowing. The value of the SNR loss depends on the form of windowing.

A number of different windows, including the Hanning window [6], windows satisfying the Nyquist criterion [7], and the Kaiser window [7] have been described in the literature. All of these windows give some reduction in the sensitivity to frequency offset. But only Nyquist windows (of which the Hanning window is one particular example) have no ICI for the case of no frequency offset [7].

C. Windows Which Preserve Orthogonality

The conditions under which a window preserves orthogonality can be derived by considering the block diagram of Fig. 4(b). In the absence of noise and distortion, this is

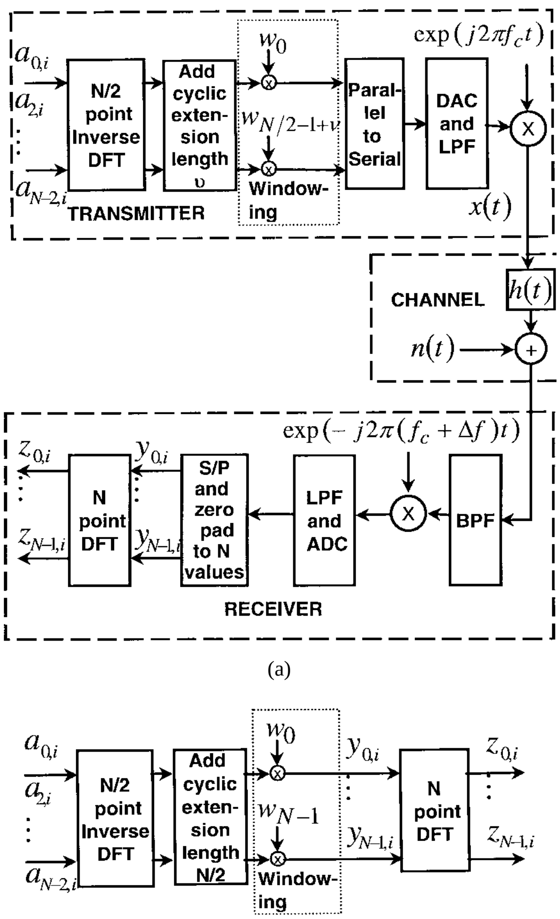

Fig. 4. (a) Windowing to reduce sensitivity to frequency offset. (b) Simplified block diagram of windowing.

equivalent to Fig. 4(a). The combination of cyclic extension, windowing, and zero padding is equivalent to a cyclic extension of length $N/2$ and windowing of length N, where some of the window coefficients may be zero. The $N/2$ point IDFT followed by the cyclic extension can be further simplified to an equivalent N point IDFT in which all the inputs with odd-numbered subscripts are zero.

For no distortion and orthogonality of the wanted outputs, that is those outputs with even subscripts, then $z_{k,i} = a_{k,i}$ for k even. For k odd, $z_{k,i}$ can take any value, and may depend on any of the inputs $a_{k,i}$ as these $z_{k,i}$ values are discarded. The windowing is a multiplication of the output of the IDFT output values with the windowing values. A well-known property of the DFT is that multiplication in the discrete time domain is equivalent to circular convolution in the discrete frequency domain and vice versa. Let $W_0 \cdots W_{N-1}$ be the N point DFT of $w_0 \cdots w_{N-1}$. Then $z_{0,i} \cdots z_{N-1,i}$ is the circular convolution of $a_{0,i} \cdots a_{N-1,i}$ with $W_0 \cdots W_{N-1}$. Using the fact that $a_{k,i} = 0$, for k odd it can readily be shown that the orthogonality condition is met if

$$\begin{aligned} &W_0 = 1, \\ &W_2, W_4 \cdots, W_{N-2} = 0, \\ &W_1, W_3 \cdots, W_{N-1} = \text{any value}. \end{aligned} \tag{13}$$

For the Hanning window, which is a cosine roll-off window with roll-off factor of o,

$$\begin{aligned} &W_0 = 1, \\ &W_2, W_4 \cdots, W_{N-2} = 0, \\ &W_1 = W_{N-1} = -0.5, \\ &W_3, W_5 \cdots, W_{N-3} = 0. \end{aligned} \tag{14}$$

This window results in linear cancellation of the ICI in the even-numbered outputs, $z_{0,i}, z_{2,i} \cdots z_{N-2,i}$, but each of the odd numbered outputs depend on two of the inputs $a_{0,i}, a_{2,i} \cdots a_{N-2,i}$. Therefore, these odd-numbered outputs are discarded and are not used to contribute to the useful signal. This windowing has the same performance with respect to ICI as the self ICI cancellation scheme with linear cancellation but has worse performance with respect to noise added in the channel.

V. New Higher Order ICI Cancellation Schemes

The concept of self ICI cancellation can be extended. In the method of Zhao and Häggman [8], data is mapped onto pairs of subcarriers. This results in cancellation of the component of ICI due to the *linear* variation in weighting coefficients over groups of three adjacent coefficients. By mapping data onto larger groups of subcarriers, higher order ICI cancellation can be achieved. For the general case of mapping onto groups of k subcarriers, the relative weightings of the subcarriers in the group are given by the coefficients of the polynomial expansion of $(1-x)^{k-1}$. Using a similar analysis to that for the linear case, it can be shown that when groups of k subcarriers are weighted in this way in both the transmitter and receiver, the component of ICI which is due to the variation in weighting coefficients described by a polynomial of order $(2k-3)$ over groups of $(2k-1)$ adjacent coefficients is cancelled.

For example, by using groups of three subcarriers, cancellation of the component of ICI which is due to the *cubic* variation in weighting coefficients over groups of five adjacent coefficients can be achieved. In this case, $-a_{0,i} = 0.5a_{1,i} = -a_{2,i}, -a_{3,i} = 0.5a_{4,i} = -a_{5,i}, \cdots$. At the receiver, the data is estimated from weighted sums of the form $-0.5z_{0,i} + z_{1,i} - 0.5z_{2,i}, -0.5z_{3,i} + z_{4,i} - 0.5z_{5,i}, \cdots$.

If the low-pass filter in the transmitter and the receiver are designed so that the receiver filter is matched to the transmitted filter, this form of weighting also results in overall matched filtering where the filtering is matched to the data which is being mapped onto the $a_{0,i} \cdots a_{N-1,i}$.

VI. Comparison of Performance of Different Methods of Reducing ICI Due to Frequency Offset

Fig. 5 shows how the ratio of mean wanted power to mean uncancelled ICI power varies as a function of normalized frequency offset for four different systems: standard OFDM and OFDM with cancellation of the constant, linear, and cubic components of ICI. The graphs are for $N = 32$, but graphs for any $N > 8$ would have almost identical form.

When ICI cancellation schemes are used, weighted groups of subcarriers are modulated rather than individual subcarriers.

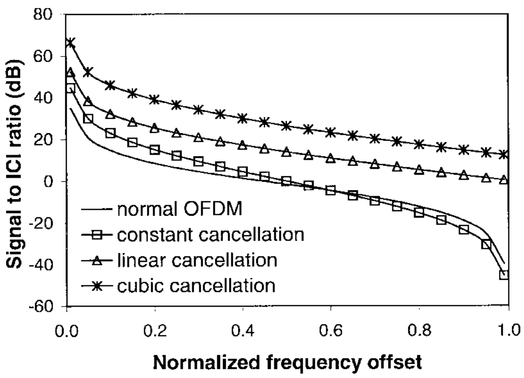

Fig. 5. Wanted signal power/uncancelled ICI power as a function of $/DeltafT$.

The graphs describe the performance for any modulation scheme which meets the condition that the data being mapped onto the weighted groups are independent identically distributed random variables and when N/k is integral so that all subcarriers are used. They do not describe exactly the case where different weighted groups of subcarriers are being modulated with different average powers, for example where spectral shaping is used, or where there is correlation between the variables modulating different groups of subcarriers. For N/k not integral, the ICI is less for subcarriers close to the unused subcarriers. The graphs shows the worst case ICI. All of the methods give a higher signal-to-uncancelled-ICI ratio than standard OFDM.

VII. Conclusion

An analysis of the effect of frequency errors in OFDM has been presented. The ICI due to carrier frequency offset has been described in terms of complex weighting coefficients.

The self ICI cancellation schemes and windowing schemes described by other authors have been analyzed in terms of the complex weighting factors. It is shown that the ICI cancellation scheme and cosine roll-off windowing, with a roll-off factor of one, cancel the component of ICI due to the linear variation of weighting coefficients over groups of three coefficients. New self ICI cancellation schemes have been derived which cancel higher order components. These have been shown to give very great reductions in the ICI due to frequency offset. These ICI cancellation methods map each complex value to be transmitted onto weighted groups of subcarriers. A disadvantage of these ICI methods is that fewer complex data values are transmitted per symbol period. A condition for the orthogonality of windowing schemes in terms of the DFT of the windowing function has been derived.

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