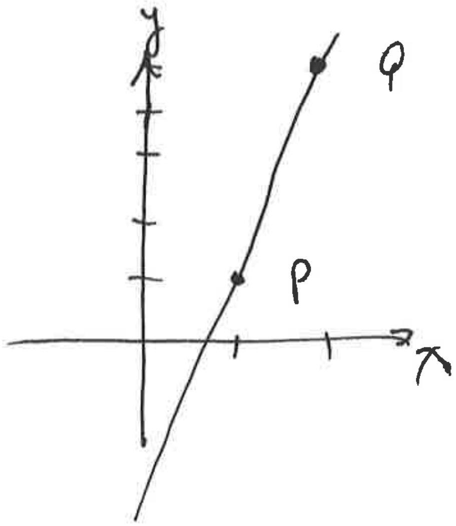
Math # 2471 – Calc 2

Lines & Plane

Consider the line connecting $P(1,1) \to Q(2,5)$



so $\dfrac{\Delta y}{\Delta x} = \dfrac{5-1}{2-1} = 4$

$y - 1 = 4(x-1)$

Another way of looking at this is start at $P(1,1)$ and move in the direction of the vector $\overline{PQ}$

$\overline{PQ} = \langle 2-1, 5-1 \rangle = \langle 1, 4 \rangle$

so parametrically the line is

$x = 1 + t$

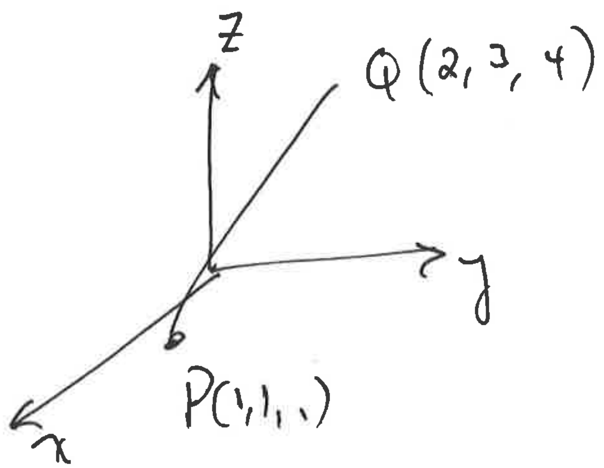
$y = 1 + 4t$

as along as we keep the ratio 1–4

eliminate $t = x - 1$

$y = 1 + 4(x-1)$ same

so does this easily extend to 3 D



$\overline{PQ} = \langle 1, 2, 3 \rangle$ direction of line

Parametric form of line

↓ from vector $\overline{PQ}$

$$x = 1 + t$$
$$y = 1 + 2t$$
$$z = 1 + 3t$$

Isolate t

$$t = x - 1 = \frac{y-1}{2} = \frac{z-1}{3}$$ ← called symmetric form

Ex Find eq^n of line through $P(1, 3, -1)$ and parallel to the line $x = 5 - t$, $y = 6 + 2t$, $z = -t$

The vector is

$\langle -1, 2, -1 \rangle$

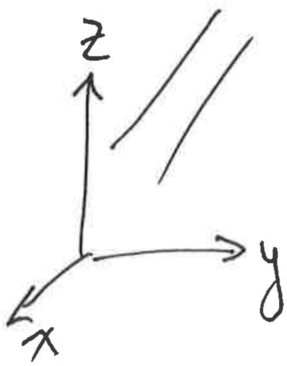
$$x = 1 - s$$
$$y = 3 + 2s$$
$$z = -1 - s$$

s- parameter

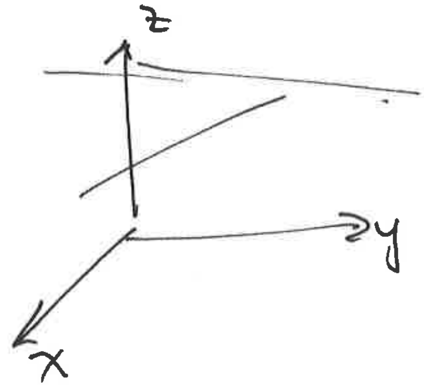
Intersection of lines

Possibilities

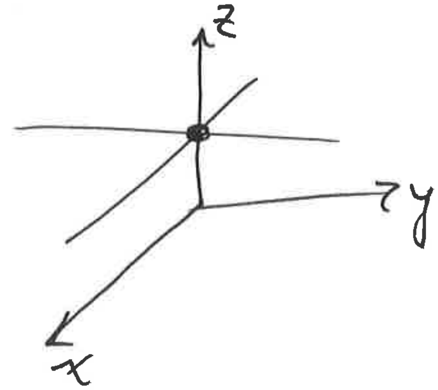
(1) parallel



(2) skew



(3) intersect



Determine whether the lines intersect

$x = 1+t$ $\qquad$ $x = -1+2s$

$y = 2-t$ $\qquad$ $y = -11+3s$

$z = 6-3t$ $\qquad$ $z = -3-s$

1st equate x's & y's

$1+t = -1+2s$ $\qquad$ $t = 2s-2$

$2-t = -11+3s$

$2-2s+2 = -11+3s$

$-5s = -11-4 = -15$

$s = 3$

$t = 6-2 = 4$

so $t = 4 \quad s = 3$

$x = 5 \quad x = 5$

$y = -2 \quad y = -2$

Now z's $\quad z_1 = 6-3(4)$

$= 6-12 = -6$

$z_2 = -3-s$

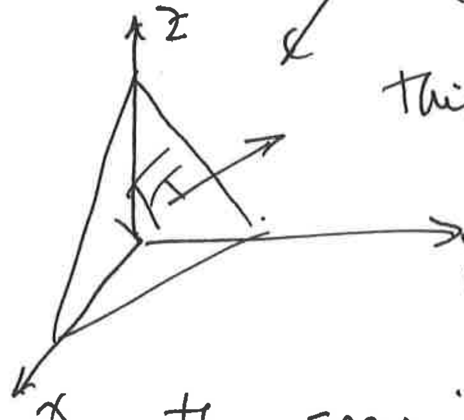
$= -3-3$

$= -6$

Same so they intersect

Planes

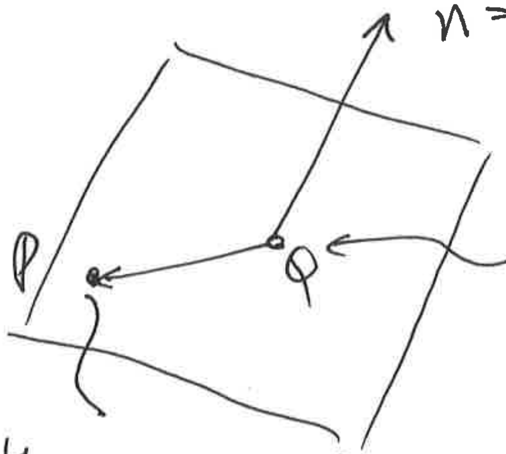
Flat surface



this is characterized by a single vector $\perp$ to every vector in the plane

the special vector is called the normal vector

$\bar{n} = \langle a, b, c \rangle$



P Q ← pick a pt (x_0, y_0, z_0)

another arbitrary pt (x, y, z) – create vector

$$\overline{PQ} = \langle x - x_0,\ y - y_0,\ z - z_0 \rangle$$

Now $\bar{n} \cdot \overline{PQ} = 0$ since $\bar{n} \perp \overline{PQ}$

$$\Rightarrow \langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

general form for a plane

Ex Find the eq^n of the plane through $P(1,1,1)$ with normal $\bar{n} = \langle 1,2,3 \rangle$

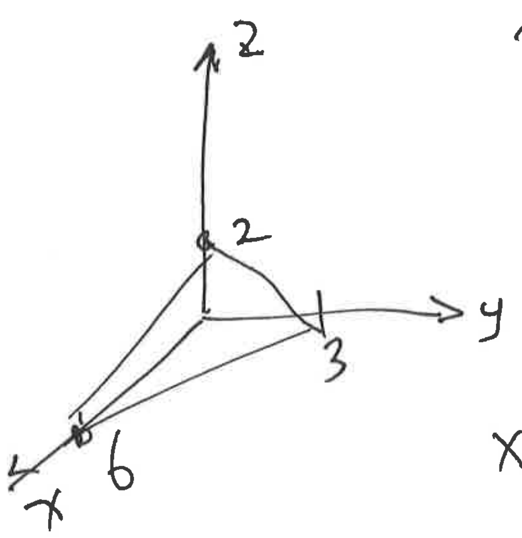
Go to general form

$$1(x-1) + 2(y-1) + 3(z-1) = 0$$

$$x - 1 + 2y - 2 + 3z - 3 = 0$$

$$\boxed{x + 2y + 3z = 6}$$ alternate form

Drawing Planes – find out where plane crosses x, y, z axes



Set $x = y = 0$ find z:

so $0 + 0 + 3z = 6$

$z = 2$

$x = z = 0$ $2y = 6$ $y = 3$

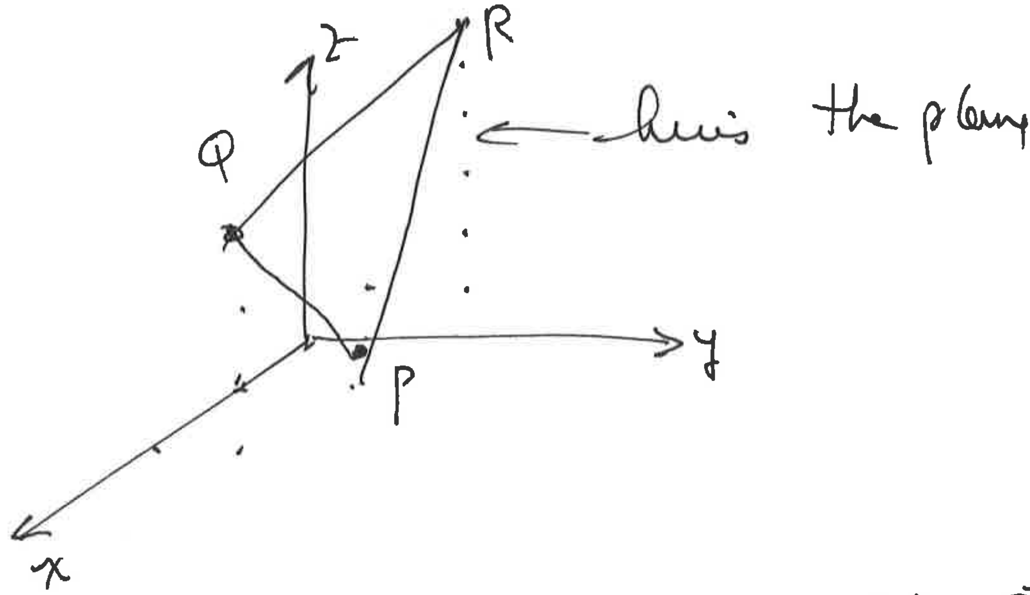
$y = z = 0 \Rightarrow x = 6$

Then connect the dot to get the plane

Sometimes the normal is not given.

Ex Find the eq^n of the plane through

$P(1,1,1)$ $Q(2,1,3)$ $R(-1,2,5)$



to find the normal find vectors $\overline{PQ}$ $\overline{PR}$

$\overline{PQ} = \langle 1, 0, 2 \rangle$ $\qquad \overline{PR} = \langle -2, 1, 4 \rangle$

if we cross these 2 we'll get the normal

$$\overline{PQ} \times \overline{PR} = \begin{vmatrix} i & j & k \\ 1 & 0 & 2 \\ -2 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix} i - \begin{vmatrix} 1 & 2 \\ -2 & 4 \end{vmatrix} \bar{j} + \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} \bar{k}$$

$$= -2\bar{i} - 8\bar{j} + k = \langle -2, -8, 1 \rangle$$

Plane $\quad -2(x-1) - 8(y-1) + (z-1) = 0$ $\quad$ we used P

so $-2x+2-8y+8+z-1=0$

$-2x-8y+z=-2-8+1=-9$

or $2x+8y-z=9$

check $P(1,1,1)$ L.S. $2+8-1=9$ ✓

$Q(2,1,3)$ L.S. $4+8-3=9$ ✓

$R(-1,2,5)$ L.S. $-2+16-5=16-7=9$ ✓

Ex Find eq^n of plane that contains the point $P(1,1,1)$ $Q(3,2,5)$ and the vector $\bar{u}$ $\langle 1,1,1\rangle$

so create second vector $\overline{PQ}=\langle 2,1,4\rangle$

$$\overline{PQ}\times\bar{u}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 1\\ 2 & 1 & 4\end{vmatrix}=\begin{vmatrix}1 & 1\\ 1 & 4\end{vmatrix}\vec{i}-\begin{vmatrix}1 & 1\\ 2 & 4\end{vmatrix}\vec{j}+\begin{vmatrix}1 & 1\\ 2 & 1\end{vmatrix}\vec{k}$$

$$=3\vec{i}-2\vec{j}+(-1)\vec{k}$$

Plane $3(x-1)-2(y-1)-(z-1)=0$

$3x-2y-z=3-2-1=0$

$$\boxed{3x-2y-z=0}$$