

Addition and Subtraction

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Say hello.

Describe one thing from our
Early Numeracy session which
you've put into action.



November 2022

Early Numeracy

- Counting principles
- Connecting number
- Comparison of numbers
- Addition and subtraction concepts

January 2023

Addition and Subtraction

- Addition computation
- Subtraction computation
- Addition and subtraction fluency
- Addition and subtraction word problems

March 2023

Place value and money

- Understanding tens and ones
- Representing thousands, hundreds, tens, and ones
- Money

April 2023

Geometry

- Identification of shapes
- Composing and decomposing shapes



Focus on addition computation

Focus on subtraction computation

Increase addition and subtraction fluency

Teach addition and subtraction word problems



Instructional Platform



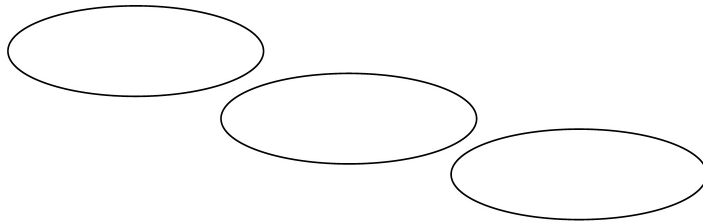


Addition and Subtraction

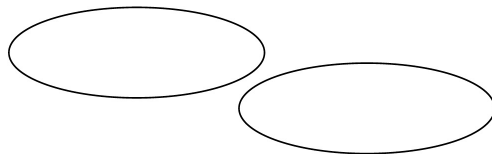
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Instructional Platform

Instructional Delivery



Instructional Strategies



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

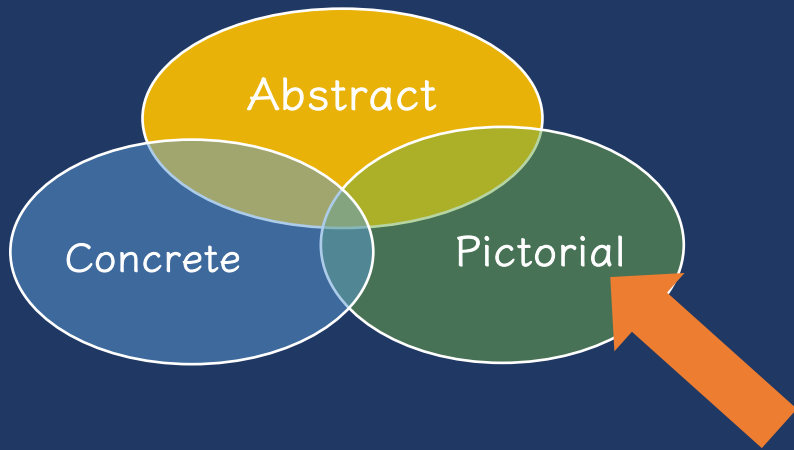
Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



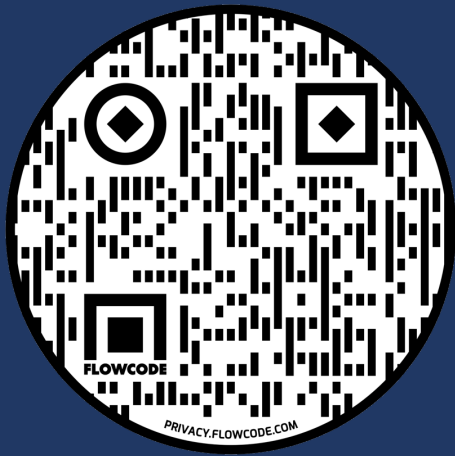


Virtual Manipulatives

Help students see and learn math using different tools!

Number & Operations	Place Value
Fractions & Decimals	Integers & Algebra
Geometry	Time & Money
Data & Probability	Extras

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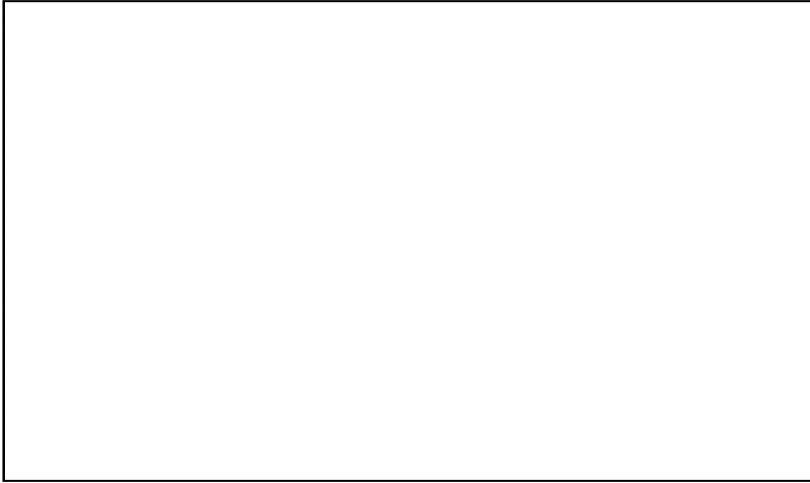
Fractions & Decimals	fraction strips	fraction strips	fraction strips	Cuisenaire rods
	fraction circles	geoboard	geoboard	geoboard
	two-color counters	decimal strips	place value disks	percentage strips
				pattern blocks



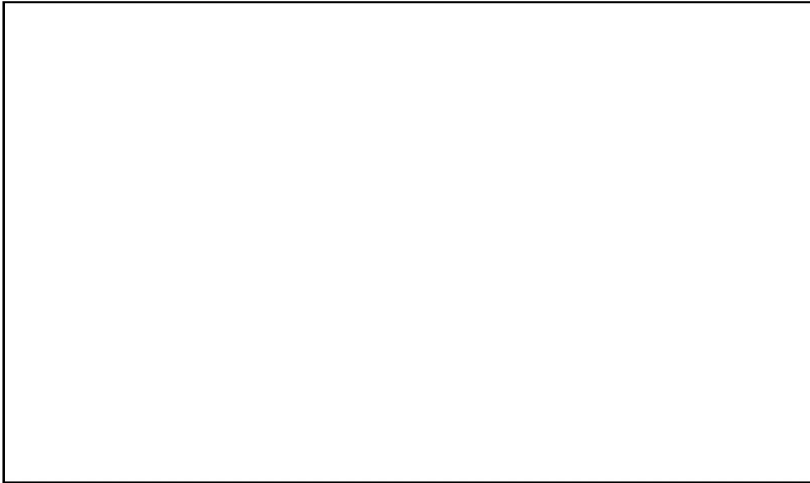
Addition and Subtraction Concepts



Addition Concepts



Subtraction Concepts



100 addition facts

Single-digit addends sum to a single- or double-digit number

$$\begin{array}{r} 5 \\ + 4 \\ \hline 9 \end{array}$$

(addend)
(addend)
(sum)



Total

Addition

Count one set, count another set, put sets together, count sum



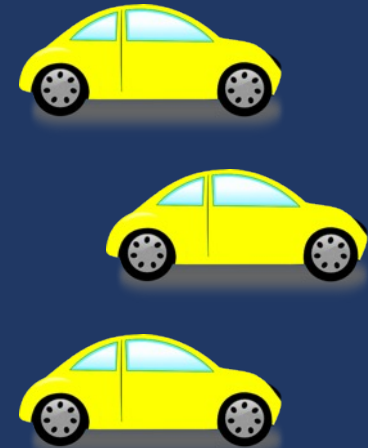
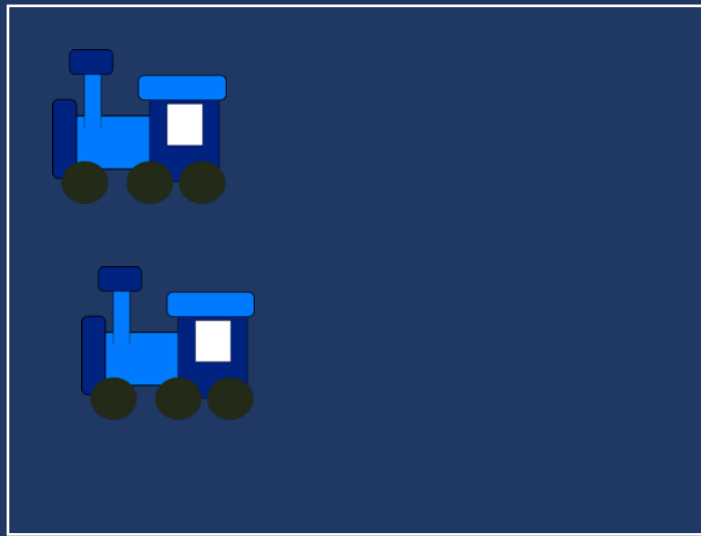
$$2 + 3 = 5$$



Change

Addition

Start with a set, add the other set, count sum



$$2 + 3 = 5$$

100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{r} 16 \\ - \quad 8 \\ \hline 8 \end{array}$$

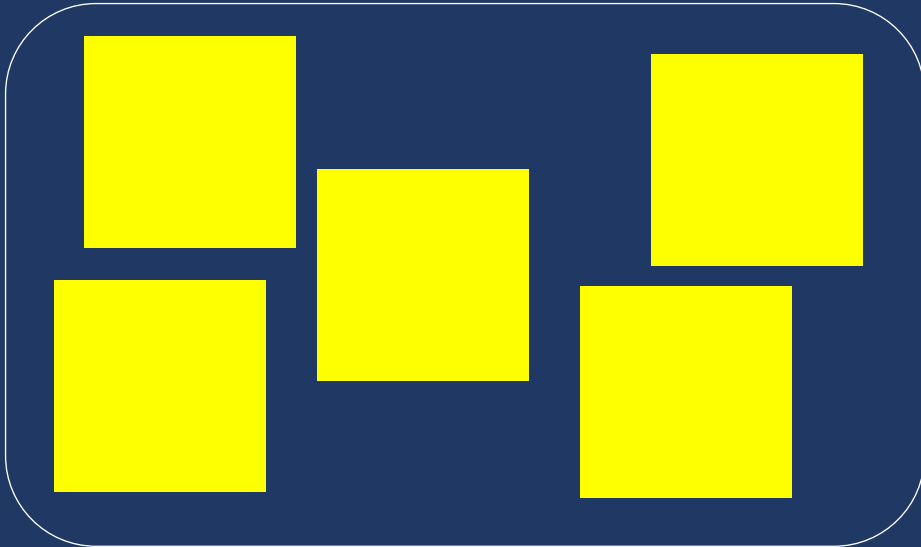
(minuend)
(subtrahend)
(difference)



Change

Subtraction

Start with a set, take away from that set, count difference



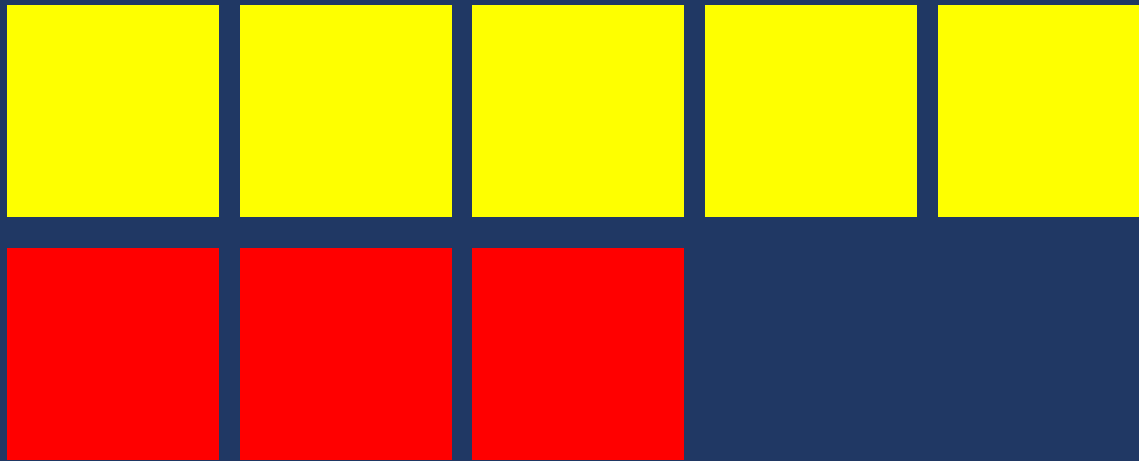
$$5 - 3 = 2$$



Difference

Subtraction

Compare two sets, count difference



$$5 - 3 = 2$$



Focus on addition computation

Focus on subtraction computation

Increase addition and subtraction fluency

Teach addition and subtraction word problems



Addition Computation



Addition Computation

$$227 + 185 =$$



Traditional

A.

$$\begin{array}{r} 1 \\ 74 \\ + 18 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 1 1 \\ 725 \\ + 365 \\ \hline 1,090 \end{array}$$



$$\begin{array}{r} 227 \\ + 185 \\ \hline \end{array}$$



Partial Sums

A.

$$\begin{array}{r} 74 \\ + 18 \\ \hline 80 \\ + 12 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ + 365 \\ \hline 1,000 \\ 80 \\ + 10 \\ \hline 1,090 \end{array}$$



$$\begin{array}{r} 227 \\ + 185 \\ \hline \end{array}$$



Opposite Change

A.

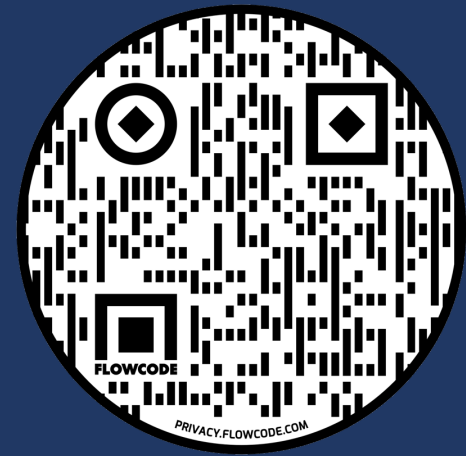
$$\begin{array}{r} 74 \\ + 18 \\ \hline \end{array} \xrightarrow{+4} \begin{array}{r} 70 \\ + 22 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ + 365 \\ \hline \end{array} \xrightarrow{-5} \begin{array}{r} 730 \\ + 360 \\ \hline 1,090 \end{array}$$



$$\begin{array}{r} 227 \\ + 185 \\ \hline \end{array}$$



- (1) Model an addition computation problem using the strategy of your choice.
- (2) Discuss the strategies and tools you would use for addition computation.

Focus on addition computation

Focus on subtraction computation

Increase addition and subtraction fluency

Teach addition and subtraction word problems



Subtraction Computation



Subtraction Computation

$$232 - 164 =$$



Traditional

A.

$$\begin{array}{r} 5 \\ \cancel{6}2 \\ - 17 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 29 \\ \cancel{30}5 \\ - 96 \\ \hline 209 \end{array}$$



$$\begin{array}{r} 232 \\ - 164 \\ \hline \end{array}$$



Partial Differences

A.

$$\begin{array}{r} 62 \\ - 17 \\ \hline + 50 \\ - 5 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ - 96 \\ \hline + 300 \\ - 90 \\ - 1 \\ \hline 209 \end{array}$$



$$\begin{array}{r} 232 \\ - 164 \\ \hline \end{array}$$

Same Change

A.

$$\begin{array}{r} 62 \\ - 17 \\ \hline \end{array} \xrightarrow{+3} \begin{array}{r} 65 \\ - 20 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ - 96 \\ \hline \end{array} \xrightarrow{+4} \begin{array}{r} 309 \\ - 100 \\ \hline 209 \end{array}$$



$$\begin{array}{r} 232 \\ - 164 \\ \hline \end{array}$$

Add Up

A.

$$\begin{array}{r} 62 \\ - 17 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ 20 \\ 60 \\ \hline 62 \end{array} \quad \begin{array}{r} 3 \\ 40 \\ 2 \\ \hline 45 \end{array}$$

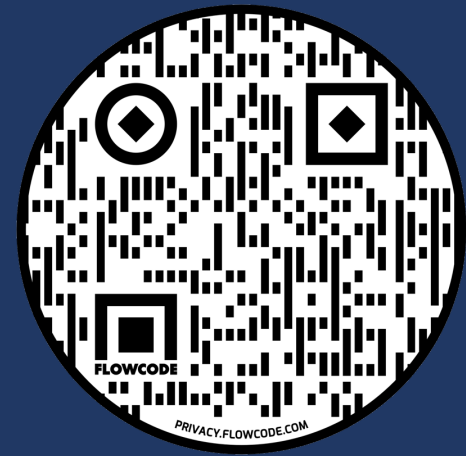
B.

$$\begin{array}{r} 305 \\ - 96 \\ \hline \end{array}$$

$$\begin{array}{r} 96 \\ 100 \\ 300 \\ \hline 305 \end{array} \quad \begin{array}{r} 4 \\ 200 \\ 5 \\ \hline 209 \end{array}$$



$$\begin{array}{r} 232 \\ - 164 \\ \hline \end{array}$$



- (1) Model a subtraction computation problem using the strategy of your choice.
- (2) Discuss the strategies and tools you would use for subtraction computation.

Focus on addition computation

Focus on subtraction computation

Increase addition and subtraction fluency

Teach addition and subtraction word problems



Addition and Subtraction Fluency



Addition and Subtraction Fluency



Fluency is
doing
mathematics
easily and
accurately.

Fluency in
mathematics
makes
mathematics
easier.

Fluency
provides less
stress on
working
memory.

Fluency
helps
students
build
confidence
with
mathematics.

With fluency, it is important to emphasize both
conceptual learning and procedural learning.



Addition	Subtraction
Multiplication	Division

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

Identifying shapes

Knowing multiples

Knowing formulas



Addition	Subtraction
Multiplication	Division

memorization
automaticity

easy use of strategies
accuracy



Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend

$$\begin{array}{r} 5 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - 4 \\ \hline \end{array}$$



Cover, Copy, Compare

$\begin{array}{r} 9 \\ \times 6 \\ \hline 54 \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline 48 \end{array}$
$\begin{array}{r} 7 \\ \times 8 \\ \hline 56 \end{array}$	$\begin{array}{r} 6 \\ \times 5 \\ \hline 30 \end{array}$
$\begin{array}{r} 9 \\ \times 9 \\ \hline 81 \end{array}$	$\begin{array}{r} 3 \\ \times 3 \\ \hline 9 \end{array}$
$\begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array}$	$\begin{array}{r} 3 \\ \times 3 \\ \hline 9 \end{array}$
$\begin{array}{r} 8 \\ \times 8 \\ \hline 64 \end{array}$	$\begin{array}{r} 3 \\ \times 3 \\ \hline 9 \end{array}$

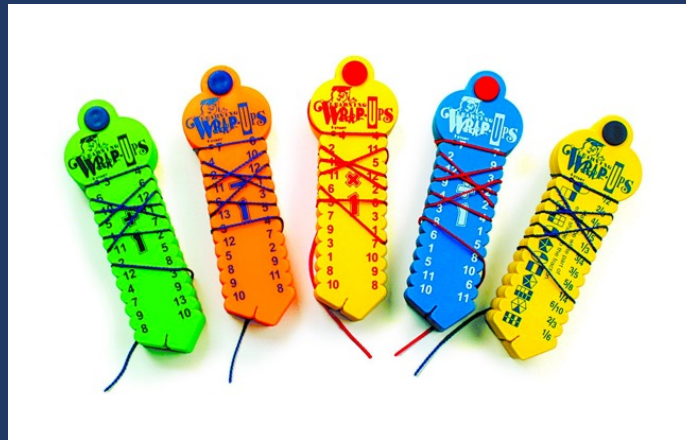
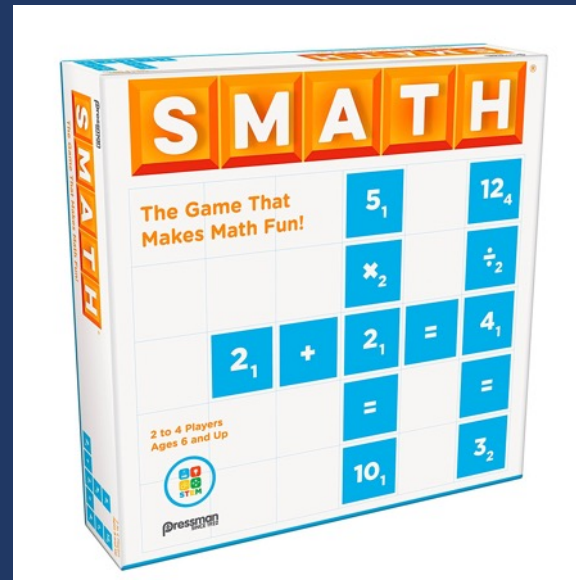
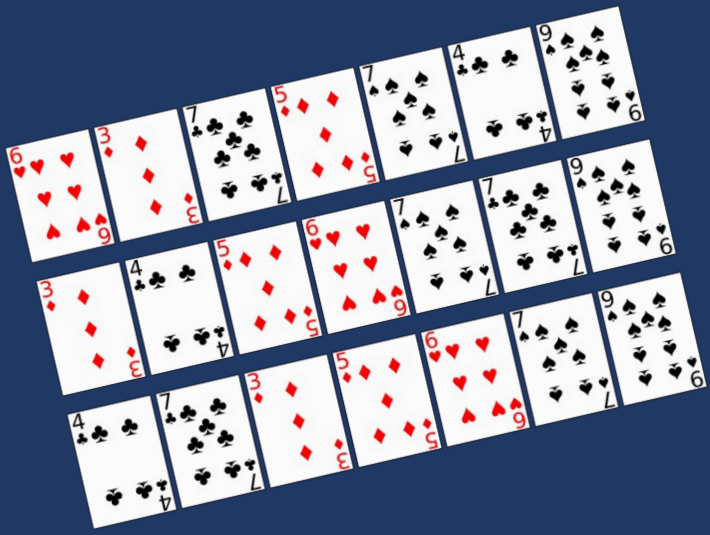
File Folder

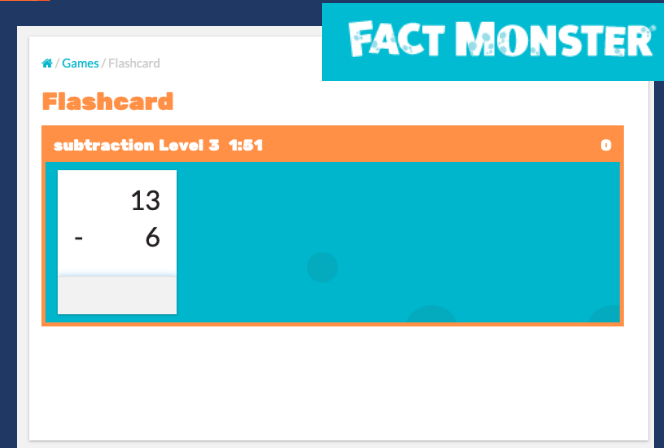
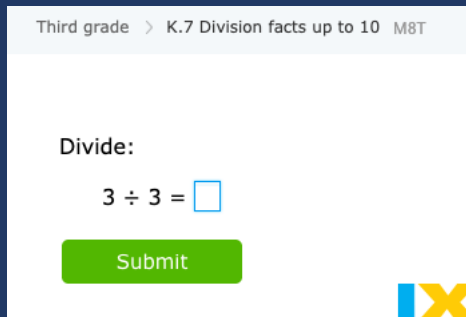
$6+3=$	9
$1+7=$	8
$6+4=$	10
$7+3=$	10
$2+7=$	9
$5+6=$	11
$4+7=$	11
$7+8=$	15
$6+7=$	13
$7+9=$	16
$7+6=$	13
$8+7=$	15
$7+0=$	7
$9+6=$	15
$6+0=$	6
$6+8=$	14

Taped Problems

$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$
$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$
$\begin{array}{r} 9 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$







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DAILY and BRIEF



Addition	Subtraction
Multiplication	Division



Describe three activities to help students with fact fluency.

Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ - 724 \\ \hline \end{array}$$



Addition	Subtraction
Multiplication	Division



Describe one activity to increase computational fluency.

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



What are your strengths with modeling addition and subtraction?

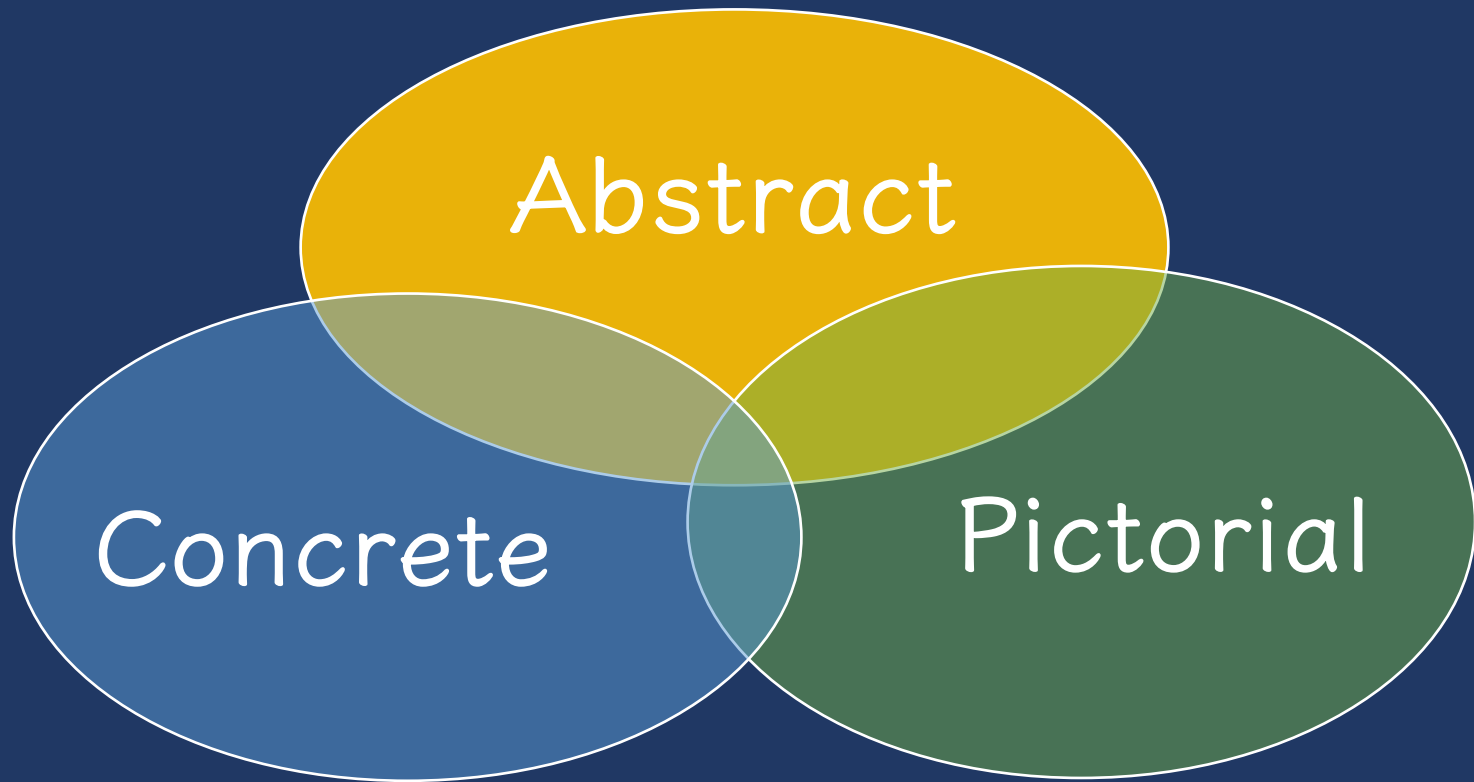
What are your opportunities for growth?

Use formal math language

Use terms precisely



What are five essential math vocabulary for addition and subtraction?



What are the representations you'll use to teach addition and subtraction?

Focus on addition computation

Focus on subtraction computation

Increase addition and subtraction fluency

Teach addition and subtraction word problems



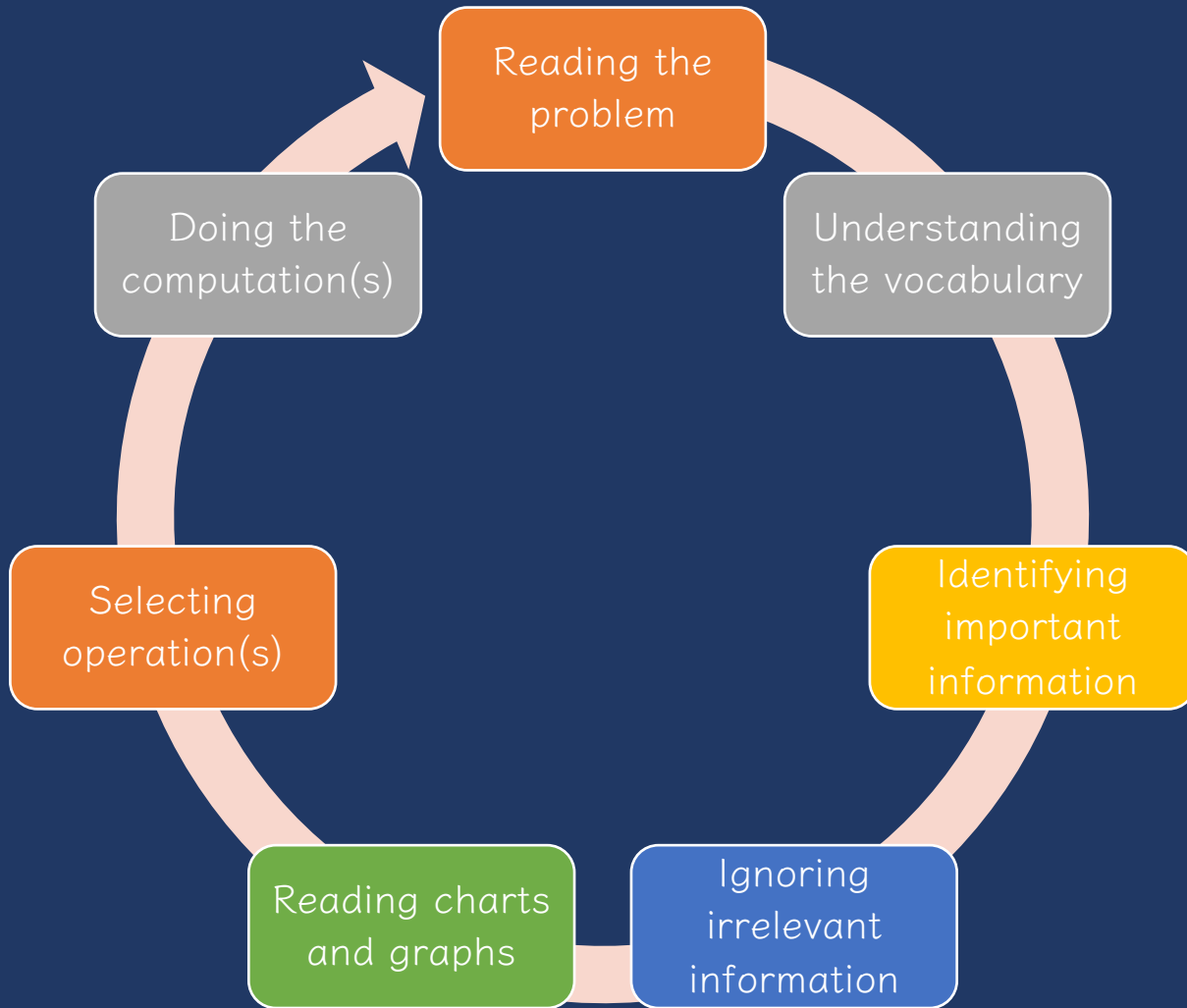
Addition and Subtraction Word-Problem Solving



Word-Problem Solving

Teaching Problem Solving





Ineffective Strategies





1. Keywords tied to operations



Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?

Key Words Used in Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> add all together or altogether and both combined how many in all how much in all increased by plus sum together total 	<ul style="list-style-type: none"> change decreased by difference fewer or fewer than how many are left (or have left) how many did not have how many how much taller, heavier, less or less lost minus need to reduce remain subtract take away

+

OPERATION cue words

ADDITION	SUBTRACTION
<ul style="list-style-type: none"> and total join more than in all sum increased 	<ul style="list-style-type: none"> less than decreased remaining left fewer take away difference minus
MULTIPLICATION	DIVISION
<ul style="list-style-type: none"> product times as many as of by equal groups 	<ul style="list-style-type: none"> quotient each broken into distributed evenly parts

Math POSTER

ADDITION	MULTIPLICATION
<ul style="list-style-type: none"> -sum -total -more than -plus 	<ul style="list-style-type: none"> -both -combined -increased by -perimeter -product -per -double -every -each -by
SUBTRACTION	DIVISION
<ul style="list-style-type: none"> -difference -remain -left -less than -minus -how many more 	<ul style="list-style-type: none"> -fewer than -decrease -give away -reduce -discount -how many more -quotient -divide by -into -split -out of -shared -per -every -each -evenly -equal groups -half

Problem Solving Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add together 	<ul style="list-style-type: none"> are not decrease difference fewer, larger, shorter left less than minus remain take away

key words

combined

addition: sum, both

in all: together, perimeter

total: plus, add

more than

triple: factor, product

multiply: each, per, in all, multiple, area, double, times

average

division: equal groups, half, split, shared, evenly, quotient, divide, each

distribute

Math Operation - Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add altogether and both in all sum total increase 	<ul style="list-style-type: none"> difference fewer than gave/take away decreased by how many more show much longer/smaller/shorter minus remaining
Multiplication	Division
<ul style="list-style-type: none"> area product Each by - of - per Times double, twice, triple total increase 	<ul style="list-style-type: none"> quotient divide into equal parts/share equally per amount of each

Math Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> plus sum add total all together increase more combine 	<ul style="list-style-type: none"> subtract minus difference left left over decrease take away fewer 	<ul style="list-style-type: none"> times product factor double groups each area rows 	<ul style="list-style-type: none"> quotient split share divide separate each average equal groups



Math Words Poster Set

★★★★★ (4.1)

75%

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Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords ^a		Multiple keywords ^a		Keyword(s) led to correct solution ^a	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

^aWhen a problem featured a keyword.





Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema ^a		Any keyword		Keyword(s) led to correct solution ^b	
	n	%	n	%	n	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

^aSum across schemas does not equal 100 because each word problem featured more than one schema.

^bWhen a problem featured a keyword.



Mr. Rivera's taxable income is \$20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?



Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords
("What does *more than* tell you about?")



But, do not tie a keyword to a specific operation!





2. Presenting problems by operation



Name: _____

Date: _____

Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. How many books did Noah have in all?
2. Bonnie found 8 rocks on her front yard and 7 rocks in her backyard. How many rocks did she find in all?
3. Edward had 5 toy cars. He got 3 more toy cars. How many toy cars did Edward have in all?
4. Mariela collected 11 feathers. She found 3 more feathers. How many feathers did she have in all?
5. LaMonte made 14 cookies. He made 7 more cookies. How many cookies did LaMonte have in all?

Division Word Problems

1. Zookeeper Al had 567 bananas. He gave an equal number of bananas to 9 monkeys in the zoo and 567 bananas. How many bananas did each monkey get? And how many are left over?
2. Betty has 427 oranges. She wants to pack them up equally in 23 boxes. How many oranges will she have in each box and how much does she have left over?
3. Mr. King has 1376 pages of paper. He wants to give 32 pages to each student. How many students can he give paper to? How many extra pages will he have left over?
4. Mr. King has 1376 pages of paper. He wants to give 32 pages to each student. He instead gives 30 pages to each student. Will there be enough paper for all the students? How much more scrap paper does he need?



Effective Strategies



Teach an attack strategy

Teach about schemas



Attack Strategy

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.

UPS ✓
UNDERSTAND
Read and explain.

PLAN
How will you solve the problem?

SOLVE
Set up and do the math!

✓**CHECK**
Does your answer make sense?



RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.



STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve the problem.

Review your answer.

RICE

Read and record the problem.

Illustrate your thinking.

Compute.

Explain your thinking.



SUPER

Slowly read the story problem twice.

Underline the question and circle the numbers you need.

Picture it. Draw the scenario to show what is happening.

Explain the problem with a number sentence.

Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.

Highlight or underline key information.

Identify the question by drawing a circle around it.

Now solve the problem. Show your work.

Examine your work for precision, accuracy, and clarity.

Share your answer by writing a sentence.



SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

Box action words.

Evaluate steps.

Solve and check.



UPS✓

UNDERSTAND

Read and explain.

PLAN

How will you solve the problem?

SOLVE

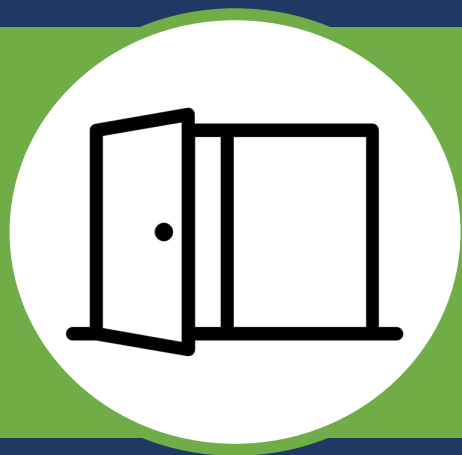
Set up and do the math!

✓CHECK

Does your answer make sense?

Created by: Sarah Powell (srpowell@austin.utexas.edu)





Share your favorite attack strategy.

Describe how you will use the attack strategy in your teaching,



Teach an attack strategy

Teach about schemas



Total

Difference

Change

Equal Groups

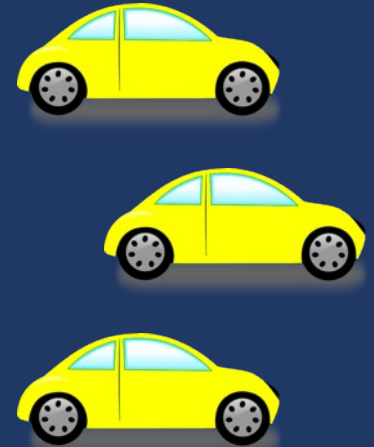
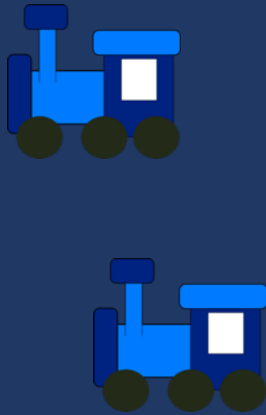
Comparison

Ratios/Proportions



Total

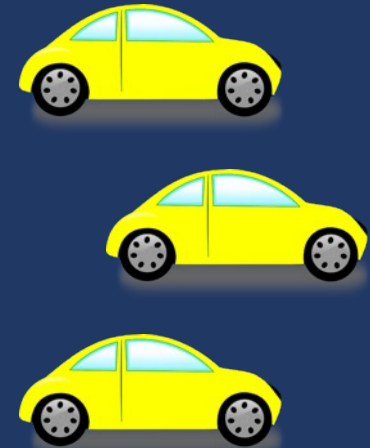
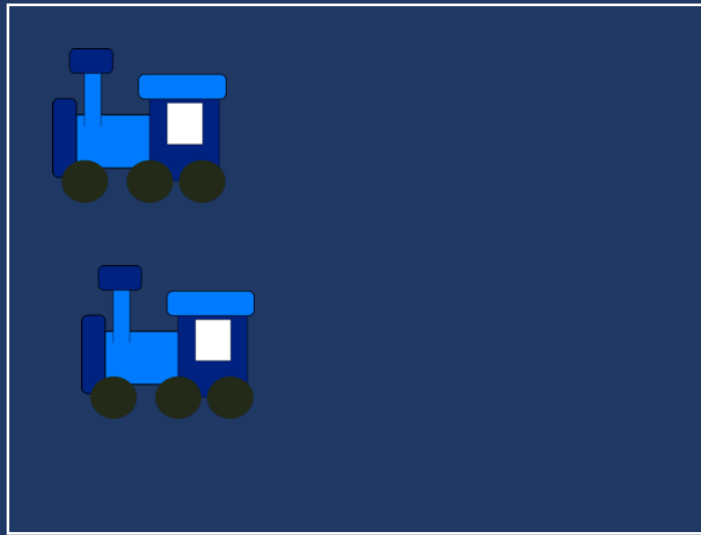
Count one set, count another set, put sets together, count sum



$$2 + 3 = 5$$

Change

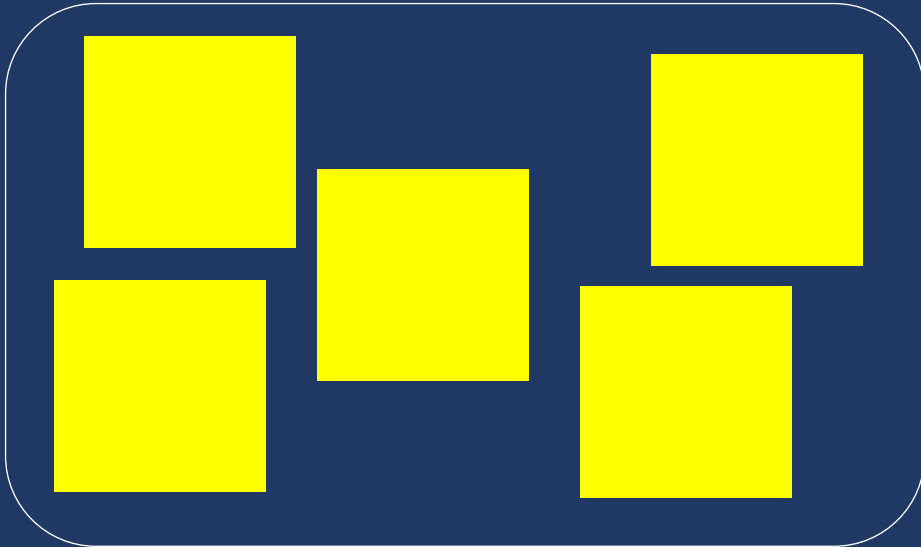
Start with a set, add the other set, count sum



$$2 + 3 = 5$$

Change

Start with a set, take away from that set, count difference

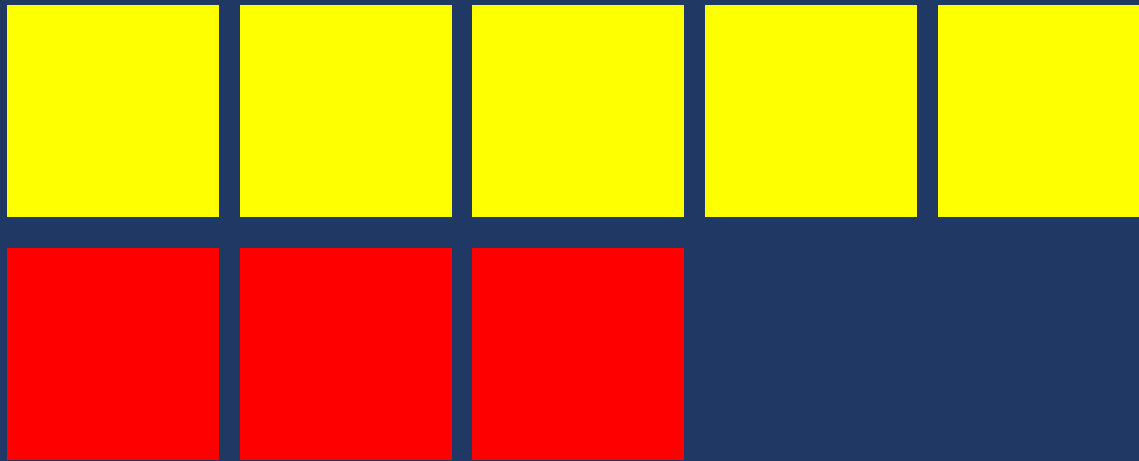


$$5 - 3 = 2$$







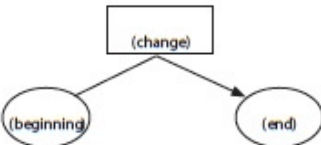
Difference

Compare two sets, count difference



$$5 - 3 = 2$$



Schema and Definition	Equations and Graphic Organizers	Examples	Variations
<p>Total (Combine; Part-part-whole) Parts combined for a sum</p>	<p>$P1 + P2 = T$ (part + part = total)</p> 	<p>Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?</p> <p>Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?</p>	<p>More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?</p>
<p>Difference (Compare) Sets compared for a difference</p>	<p>$B - s = D$ (bigger - smaller = difference)</p>  <p>$G - L = D$ (greater - less = difference)</p> 	<p>Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?</p> <p>Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?</p> <p>Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?</p>	<p>(None)</p>
<p>Change (Join; Separate) An amount that increases or decreases</p>	<p>$ST \pm C = E$ (start +/- change = end)</p>  	<p>End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?</p> <p>Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?</p>	<p>Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?</p>
		<p>End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?</p> <p>Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?</p>	<p>Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?</p>



Total

Additive Word Problems

A.
Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.
In March and April, it rained a total of 11 inches. If it rained 3 inches in March, how many inches did it rain in April?

C.
Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

NOTES ABOUT TOTAL PROBLEMS:



Total

Part-part-whole
Combine

Parts put together into a **total**

Dina saw **3** canoes and **8** kayaks. How many boats did Dina see?

Total

Dina saw **11** boats. If **3** of the boats were canoes, how many were kayaks?

Part

Dina saw **11** boats. **8** of the boats were kayaks, how many were canoes?

Part



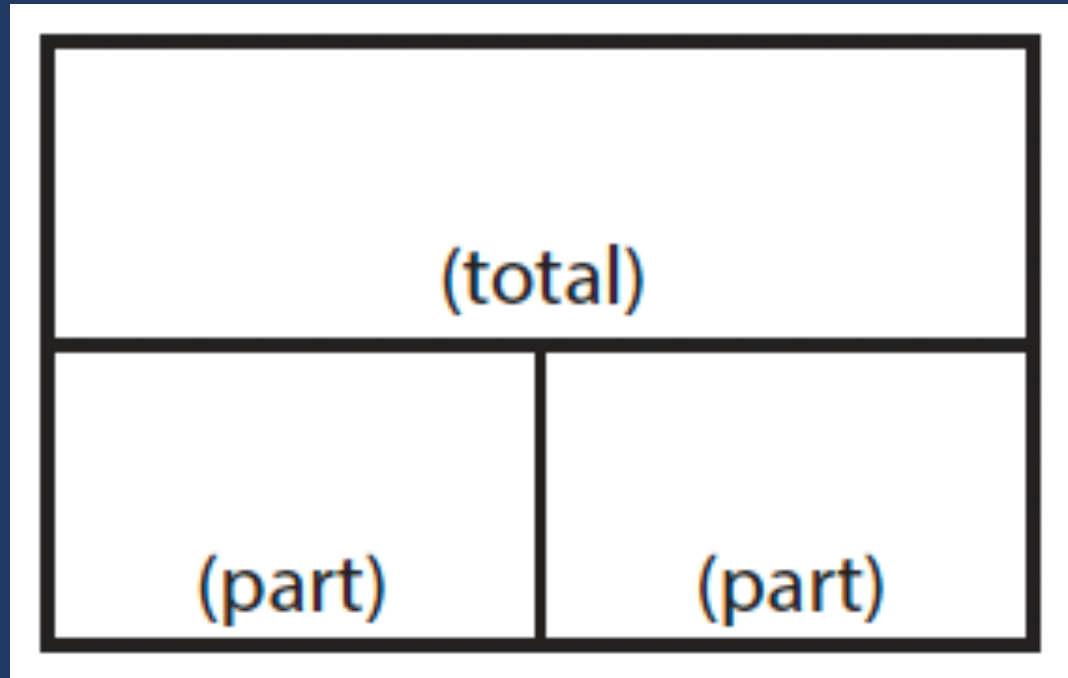
Total

“Are parts put together for a total?”



Total

$$P1 + P2 = T$$



Total

Additive Word Problems

A.
Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.
In March and April, it rained a total of 11 inches. If it rained 3 inches in March, how many inches did it rain in April?

C.
Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

NOTES ABOUT TOTAL PROBLEMS:



Total



Write a Total problem.



Difference

Compare

Greater and **lesser** amounts compared for a **difference**

Bethany has **10** pencils. Grant has **4** pencils. How many more pencils does Bethany have?

Difference

Bethany has **6** more pencils than Grant. If Grant has **4** pencils, how many does Bethany have?

Greater amount

Grant has **6** fewer pencils than Bethany. Bethany has **10** pencils. How many pencils does Grant have?

Lesser amount



Total

“Are parts put together for a total?”

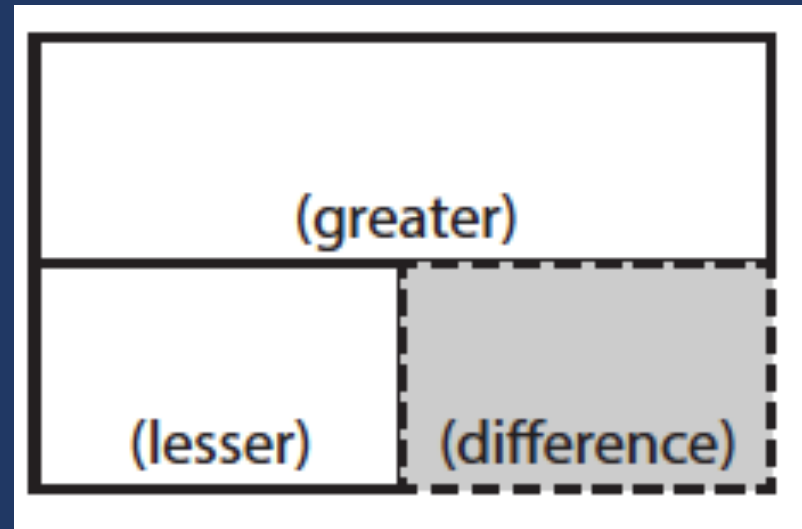
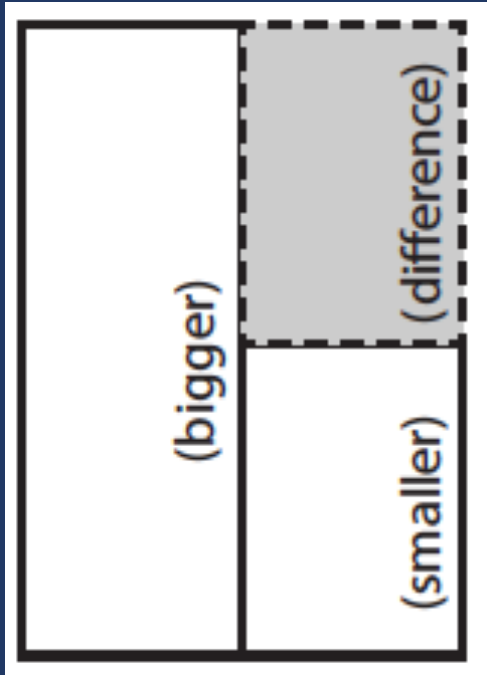
Difference

“Are amounts compared for a difference?”



Difference

$$G - L = D$$



Difference

Additive Word Problems

D.
Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?

E.
Damian's dog eats 9 cups of dog food each week. Monte's dog eats 4 cups less each week than Damian's dog. How much does Monte's dog eat in a week?

F.
The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?

NOTES ABOUT DIFFERENCE PROBLEMS:



Difference



Write a Difference problem.



Change

Join

An amount that **increases** or **decreases**

Maura had **6** notebooks. Then, she bought **3** notebooks. How many notebooks does Maura have now?

End amount

Maura had **6** notebooks. Then, she bought a few more notebooks. Now, Maura has **9** notebooks. How many notebooks did she buy?

Change amount

Maura had some notebooks. Then, she bought **3** notebooks. Now, Maura has **9** notebooks. How many notebooks did she have to start with?

Start amount



Change

Separate

An amount that increases or **decreases**

Adia baked **20** cookies. Then, she ate **3** of the cookies. How many cookies does Adia have now?

Adia baked **20** cookies. Then, she ate some of the cookies. Now, she has **17** cookies. How many cookies did Adia eat?

Adia baked some cookies. She ate **3** of the cookies and has **17** cookies left. How many cookies did Adia bake?

End amount

Change amount

Start amount



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

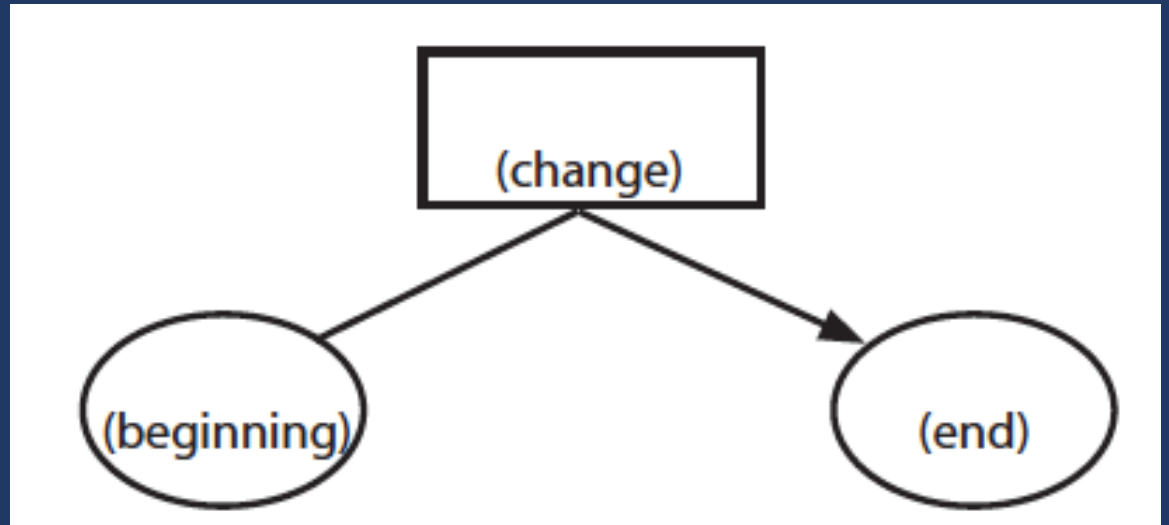
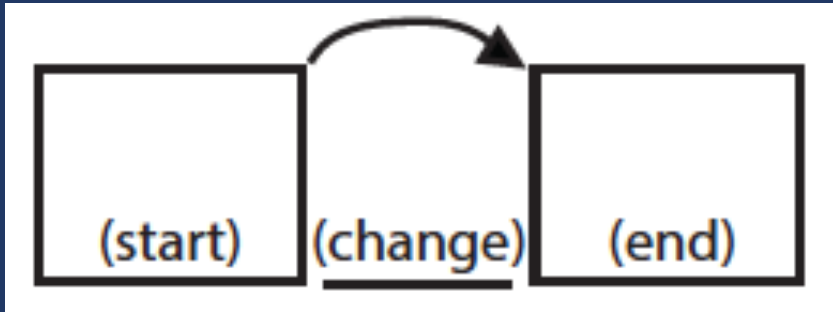
Change

“Does an amount increase or decrease?”



Change

$$ST \quad + / - \quad C \quad = \quad E$$



Change

Additive Word Problems

G.
A plant was 3 inches tall at the beginning of June. By the end of July, the plant was 9 inches tall. How many inches did the plant grow in 2 months?

H.
Martina has some money in her bank account. Then, she spent \$135 and has a balance of \$24. How much money did Martina have to begin with?

I.
Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?

NOTES ABOUT CHANGE PROBLEMS:



Change



Write a Change problem.





Schema Check!



Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

26. Part A

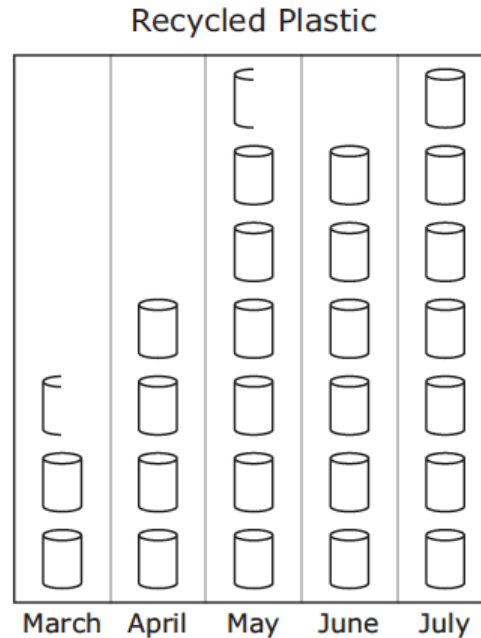
The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.


What is the total number of stamps that Pablo has after the first day of the stamp show?



Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Each  means 20 pounds.

Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Total





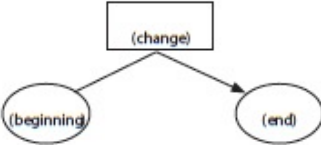
Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.



10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

- Ⓐ 300
- Ⓑ 340
- Ⓒ 350
- Ⓓ 360

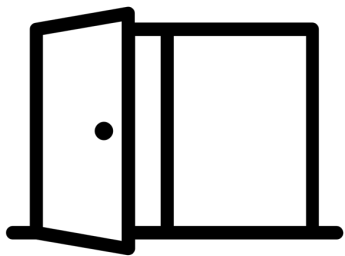
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		<p>End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?</p> <p>Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?</p>	<p>Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?</p>



Total

Difference

Change



Which of these schemas would be important to teach?

How do you plan to teach the schemas to your students?

What additional information or materials do you need?



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Total

Difference

Change

Josh ran 18 miles last week. He ran twice as many miles this week. How many miles did he run this week?



Total

Difference

Change

Josh ran 18 miles last week. He ran twice as many miles this week. How many miles did he run this week?



Total

Difference

Change

Josh ran 18 miles last week. He ran twice as many miles this week. How many miles did he run this week?



Teach an attack strategy

Teach about schemas



Explicit Instruction

Problem

Step-by-Step Explanation

1. Choose a math problem.
2. Write a step-by-step explanation. Focus on the language of math in your explanation. Consider the representations you will use.



Explicit Instruction

Problem

Practice Opportunities

High-Level Questions

Low-Level Questions

Affirmative Feedback

Corrective Feedback

1. Describe the practice opportunities you will use.
2. Write 3 high-level questions.
3. Write 3 low-level questions.
4. Write 2 ways to provide affirmative feedback.
5. Write 2 ways to provide corrective feedback.



November 2022

Early Numeracy

- Counting principles
- Connecting number
- Comparison of numbers
- Addition and subtraction concepts

January 2023

Addition and Subtraction

- Addition computation
- Subtraction computation
- Addition and subtraction fluency
- Addition and subtraction word problems

March 2023

Place value and money

- Understanding tens and ones
- Representing thousands, hundreds, tens, and ones
- Money

April 2023

Geometry

- Identification of shapes
- Composing and decomposing shapes



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