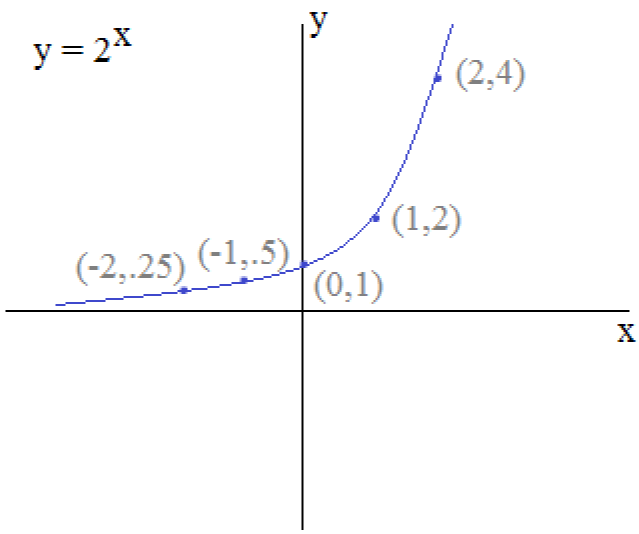
Exponents & Exponential Equations



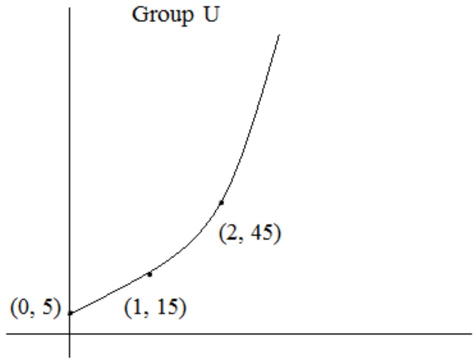
x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

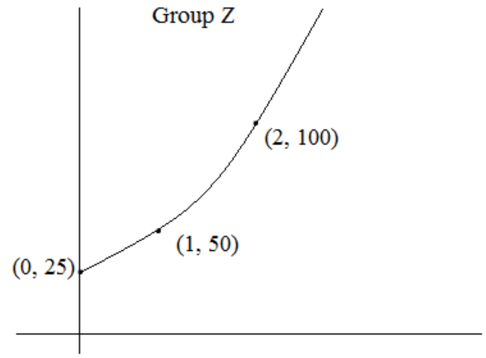
Topics include growth, decay, models, transformations, rule of 72, and more.

mathplane.com

Exponential Functions: Growth Factors

Example:





1) Compare the "growth factors": which one appears to grow faster?

Although Group Z (2nd graph) starts at a higher level, Group U (1st graph) is steeper..

Group U appears to grow faster!

2) Find the exponential functions of each graph.

Since these are *exponential* functions, we'll use the template

$y = ab^x$

where a = initial value
b = growth factor

Group U: (0, 5) $5 = ab^0$

$5 = a(1)$

$a = 5$

(1, 15) $15 = ab^1$

$15 = (5)b$

$b = 3$

$\boxed{y = (5)(3)^x}$

Group Z: (0, 25) $25 = ab^0$

$25 = a$

(1, 50) $50 = ab^1$

$50 = (25)b$

$b = 2$

$\boxed{y = (25)(2)^x}$

3) What do the coordinates (0, 5) and (0, 25) represent?

Today's population ("at time 0")

4) After 5 years, which has a 1215 population?

Group U: $x = 5$ $y = (5)(3)^5$

$= 1215$

Group Z: $x = 5$ $y = (25)(2)^5$

$= 800$

Example: A population has a growth factor of 3. After 1 month there are 36 residents. After 3 months, there are 324 residents.

1) Write an equation that models the town's growth.

2) How many were original residents?

Using the exponential function: $y = ab^x$ OR $y = ab^x$

$36 = a(3)^1$ OR $324 = a(3)^3$

$a = 12$ $a = 12$

The function is $P = 12(3)^t$

where P = population
t = time (months)

and, when t = 0, there are 12 original residents...

Here are some examples and applications:

Scientific Notation

Example: $\dfrac{3.1 \times 10^8}{12.4 \times 10^4}$

Divide the parts separately... $3.1/12.4 = .25$ $\quad 10^8 / 10^4 = 10^4$

then, combine and simplify... $.25 \times 10^4 = \boxed{2.5 \times 10^3}$ or 2500

notice, since we move the decimal to the right one space (and make the number bigger), we decrease the exponent above the 10 by one...

Geometric Sequence

Example: Write the geometric sequence 2, 8, 32, 128 as an exponential model.

Determine the initial value... 2
Identify the "common ratio"... $8/2 = 4 \quad 32/8 = 4 \quad 128/32 = 4$

The common ratio is 4...
Each number is 4 times the previous term...

Using the template $y = ab^x$

we construct an explicit sequence $\boxed{T_n = 2(4)^{n-1}}$ where n is a natural number representing each term in the infinite sequence...

Growth/Decay (Depreciation)

Example: Since 1990 tuition has increased 8% per year. In 1995 the tuition was \$2300. Predict the cost in 2025.

Instead of using 1990, 1995, and 2025, we'll use "years since 1990"... So, 1990 is 0, 1995 is 5, and 2025 is 35...

$y = ab^x$

a: The initial value is unknown...

b: The growth rate is 8%, which is a growth factor of 1.08... (each term is 1.08 times the previous)

x: Each year since 1990

To find the initial tuition value (a), we'll use substitution:

$2300 = a(1.08)^5$

$2300 = 1.47a$

$a = 1565.34$

so, $y = 1565.34(1.08)^x$

The cost in 2025 is $y = 1565.34(1.08)^{35} = \boxed{\$23{,}144}$ Yikes!

Estimate: using "rule of 72"
at 8%, tuition will double
every 72/8 = 9 years...
1995, 2004, 2013, 2022, ..
It will double approx. 3+ times..

2300	1995
4600	2004
9200	2013
18400	2022
36800	2029

⟸ 23144

Example: The value of your car decreases exponentially. You paid \$27,000, and the value drops 26% each year. What is the value of the car after 7 years?

$y = ab^x$

a: the initial value is \$27,000

b: since the value drops 26% each successive year, it's 'growth factor' is .74... (1 - .26)
Each successive year is .74 the previous...

x: 7 years...

$y = \$27{,}000(.74)^7 = \boxed{\$3{,}280.85}$

Example: A soccer tournament has 256 teams. If 1/2 are eliminated every round, how many teams will remain after 4 rounds?

$y = 256(1/2)^r$

teams remaining — completed rounds

Since 1/2 are eliminated, each round (i.e. successive term) is 1/2 the previous round...

$y = 256(1/2)^4$

$= 256(1/16) = \boxed{16}$

Exponents and Exponential Functions Quiz

I. Exponent Rules: Simplify the following

a) $9^2 =$ b) $9^{-2} =$ c) $9^0 =$ d) $9^{\frac{1}{2}} =$ e) $9^{-\frac{1}{2}} =$

f) $-5^2 =$ g) $(-5)^2 =$ h) $(3a^2)^3 =$ i) $a^2 \cdot b^3 =$ j) $\left(\frac{2}{3}\right)^{-2} =$

II. Exponents and Roots: Fill in the blanks

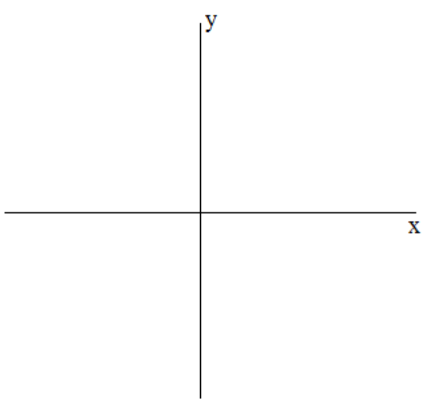
a) $5^{\square} = 25$ b) $\square^2 = 16$ c) $5^{\square} = .2$ d) $x^{\square} = 1$ e) $\frac{1}{7}^{\square} = 49$

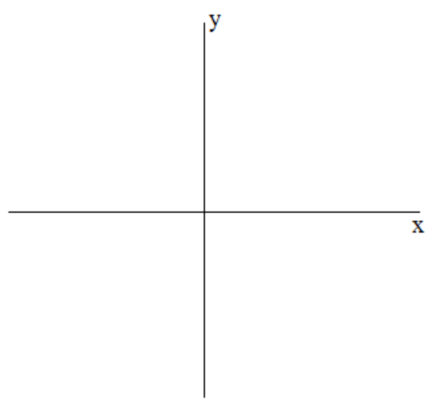
f) $(\square)^3 = -8$ g) $8^{\square} = 1/8$ h) $\left(\square b^2\right)^{\square} = 8b^6$ i) $2\left(c^2\right)^{\square} = \frac{2}{c^6}$

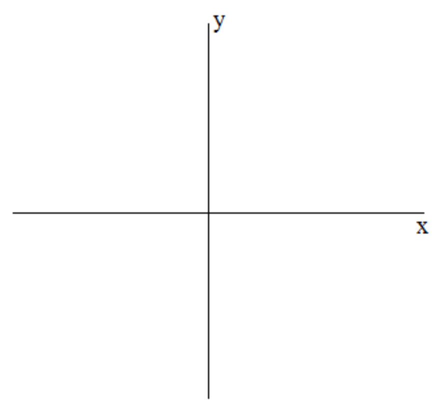
III. Exponential Functions: $y = ab^x$

1) Identify 5 points in each function; then, graph:

a) $y = 2^x$ b) $y = \frac{4^x}{2}$ c) $y = -3^x$







2) Match each graph with its function:

Example: $y = 2^x$ A

i. $y = 2^x + 4$

ii. $y = -2^x$

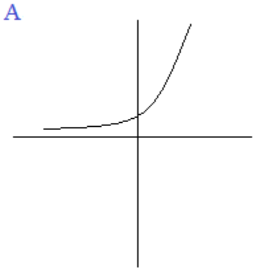
iii. $y = 2^{-x}$

iv. $y = -2^x + 4$

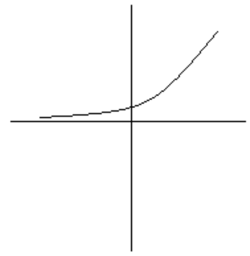
v. $y = 5 \cdot 2^x$

vi. $y = (\frac{1}{2})2^x$

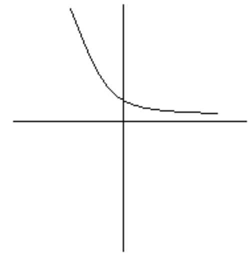
A



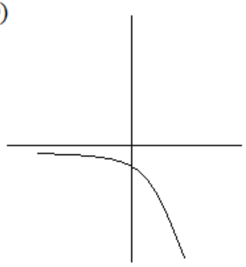
B



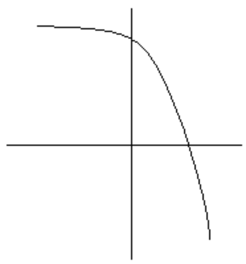
C.



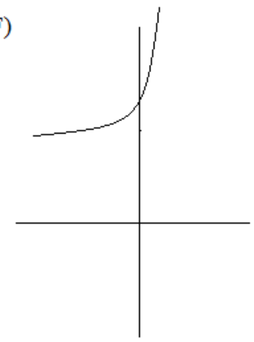
D)



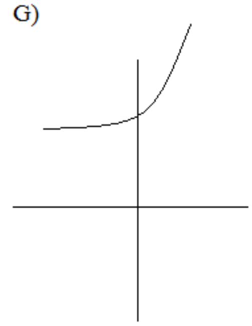
E)



F)



G)



3) Determine whether an equation models growth or decay:

a) $y = (1.1)^x$ b) $y = 2(.7)^x$ c) $y = (\frac{7}{3})^x - 5$ d) $y = (.9)^{-x}$

4) Find the *exponential* and *linear* functions that pass through:

a) (0, 3) and (1, 12) b) (1, −8) and (2, − 32) c) (0, −5) and (2, − 20)

Exponents and Exponential Functions Quiz

IV. Scientific Notation

1) Convert to Scientific Notation

a) 234,567 b) 527.021 c) .00045 d) 20^3

2) Convert to Standard Notation

a) 1.8808×10^3 b) 4.0007×10^{-7} c) $4.6\overline{6} \times 10^{5}$ d) 5.60005×10^4

3) Simplify

a) $\frac{20{,}000}{.004} =$ b) $(3.44 \times 10^6) \div 2 =$ c) $(4.5 \times 10^{-4}) \cdot (3.1 \times 10^{-3}) =$

V. Applications/Word Problems

1) A bacteria has a *half-life* of 3 years. If the current mass is 500 mg.,

   a) write an equation to model the weight (y) over years (t)

   b) what is the mass of the bacteria after 3 years? 9 years?

   c) what is the mass of the bacteria after 20 years?

2) A bank offers an annual interest rate of 6%. If you deposit $1000,

   a) write a model to express the amount of money (A) over years (t)

   b) how much will you have after 5 years? (compounding annually)

   c) how much would you have after 5 years if the interest were compounded quarterly? monthly?

3) The "rule of 72" is a method for estimating 'doubling time' of an investment. $\text{Doubling time} = \frac{72}{\text{interest rate}}$

Compare the rule of 72 with a true measurement, using an interest rate of 10%

4) A math company was established in 2007. Its value has grown exponentially, where after one year, it was worth \$10,000; and, its worth after the second year was \$25,000... How much was the company worth on *Day 1*? How much will it be worth in 2015?

5) A cost of a government agency is modeled by the equation $y = 25(1.12)^t$

a) What is the annual percentage growth?

b) How much does it cost now?

c) How much will the cost be 5 years from now?

VI: Miscellaneous:

1) What is $(-1)^{23}$?

2) What is the y-intercept of $y = 5^{(x+3)}$?

3) What is the x-intercept of $y = 3^x - 9$?

Exponents and Exponential Functions Quiz

4) $4^{14} + 4^{14} + 4^{14} + 4^{14} =$

(simplify to one term)

5) $(x + 1)^2 = x^2 + 1$ Always, sometimes, or never?

6) The sum of the first 25 perfect squares: $1^2 + 2^2 + 3^2 + \ldots + 25^2 = 5525$

What is $2^2 + 4^2 + 6^2 + \ldots + 50^2$?

7) How many (positive) perfect squares are less than 10,000?

8) $-1^{1998} + (-1)^{1999} + 1^{2000} - 1^{2001}$

9) Express as 2^n

a) $\frac{1}{2}(2^{13})$

b) $2^3 \cdot 4 \cdot \frac{1}{16}$

c) $(2^3)^6 \div 4^3$

d) $2^{25} - 2^{24} - 2^{23}$

10) If $a = -4$, find $x^2 - x^{-1} + 3x$

11) Simplify:

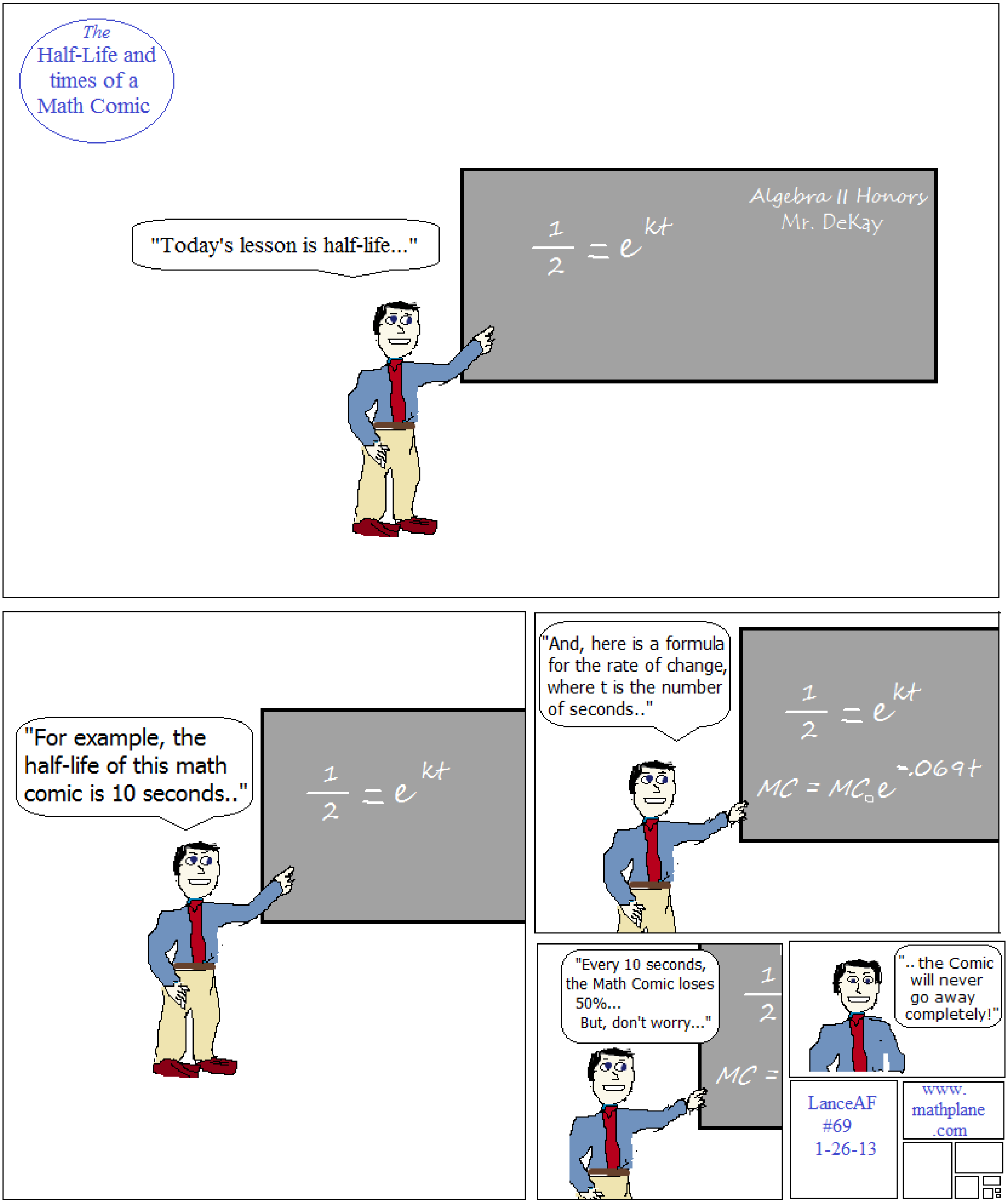
a) -4^{-2}

b) $(-4)^{-2}$

c) $(-2)^{-3}$

d) $\dfrac{1}{(-4)^{-2}}$

e) $\dfrac{1}{(-\frac{1}{2})^{-3}}$


The Half-Life and times of a Math Comic
"Today's lesson is half-life..."
Algebra II Honors
Mr. DeKay
$\frac{1}{2} = e^{kt}$
"For example, the half-life of this math comic is 10 seconds.."
"And, here is a formula for the rate of change, where t is the number of seconds.."
$MC = MC_0 e^{-.069t}$
"Every 10 seconds, the Math Comic loses 50%... But, don't worry..."
".. the Comic will never go away completely!"
LanceAF
#69
1-26-13
www.
mathplane
.com

Exponents and Exponential Functions Quiz

Solutions

I. Exponent Rules: Simplify the following

a) $9^2 =$

$9 \times 9 = \boxed{81}$

b) $9^{-2} =$

$\frac{1}{9^2} = \boxed{\frac{1}{81}}$

c) $9^0 =$

$\boxed{1}$

d) $9^{\frac{1}{2}} =$

$\sqrt{9} = \boxed{3}$

$9^{(1/2)} \cdot 9^{(1/2)} = 9^1$ ✓

$3 \cdot 3 = 9$

e) $9^{-\frac{1}{2}} =$

$\frac{1}{9^{(1/2)}} = \boxed{\frac{1}{3}}$

f) $-5^2 =$

$(-1) \cdot (5)^2 = \boxed{-25}$

(order of operations: exponents, followed by multiplication)

g) $(-5)^2 =$

$(-5) \cdot (-5) = \boxed{25}$

h) $(3a^2)^3 =$

$(3a^2)(3a^2)(3a^2) =$

$\boxed{27a^6}$

i) $a^2 \cdot b^3 =$

$\boxed{a^2 b^3}$

j) $\left(\frac{2}{3}\right)^{-2} =$

$\frac{1}{\left(\frac{2}{3}\right)^2}$

$\left(\frac{3}{2}\right)^2 = \boxed{\frac{9}{4}}$

II. Exponents and Roots: Fill in the blanks

a) $5^{\square} = 25$

$\boxed{2}$

b) $\square^2 = 16$

$\boxed{4}$

c) $5^{\square} = .2$

$5^x = \frac{1}{5}$

$\boxed{x = -1}$

d) $x^{\square} = 1$

$\boxed{0}$

e) $\frac{1}{7}^{\square} = 49$

$\boxed{-2}$

f) $(\square)^3 = -8$

$\boxed{-2}$

$(-2)(-2)(-2) = -8$ ✓

g) $8^{\square} = 1/8$

$\boxed{-1}$

h) $\left(\square b^2\right)^{\square} = 8b^6$

since $\left(b^2\right)^x = b^6$

the $\boxed{\text{right box is 3..}}$

Then, since the right box is 3,

$y^3 = 8$

so, the $\boxed{\text{left box is 2}}$

i) $2\left(c^2\right)^{\square} = \frac{2}{c^6}$

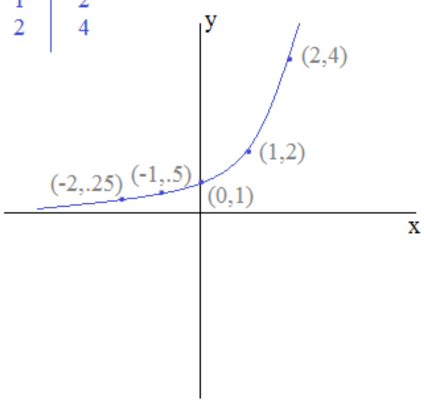
$\boxed{-3}$

III. Exponential Functions: $y = ab^x$

1) Identify 5 points in each function; then, graph:

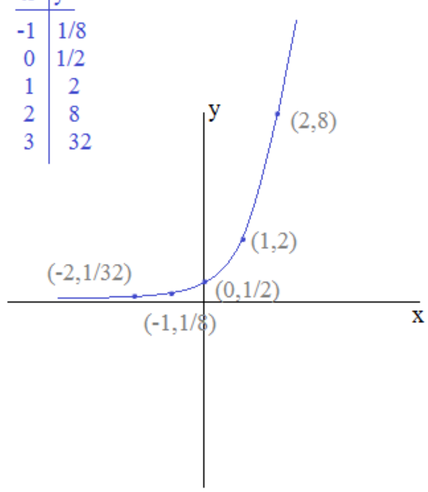
a) $y = 2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4



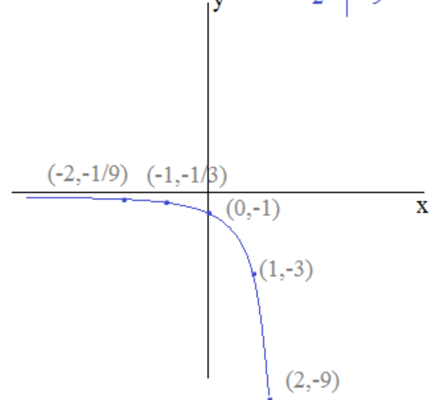
b) $y = \frac{4^x}{2}$ $\frac{1}{2}4^x$

x	y
-1	1/8
0	1/2
1	2
2	8
3	32



c) $y = -3^x$

x	y
-2	-1/9
-1	-1/3
0	-1
1	-3
2	-9



Solutions

2) Match each graph with its function:

Example: $y = 2^x$ A

i. $y = 2^x + 4$ G

ii. $y = -2^x$ D

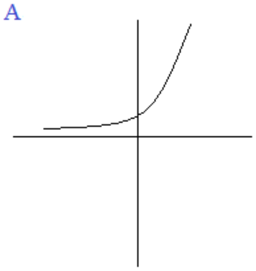
iii. $y = 2^{-x}$ C

iv. $y = -2^x + 4$ E

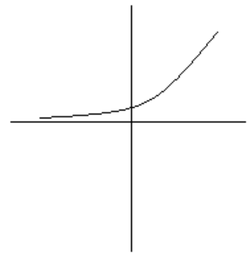
v. $y = 5 \cdot 2^x$ F

vi. $y = (\frac{1}{2})2^x$ B

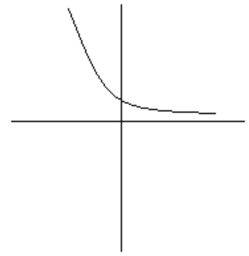
A



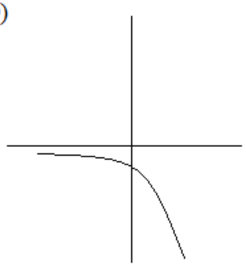
B



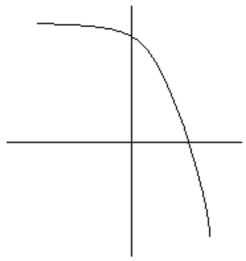
C.



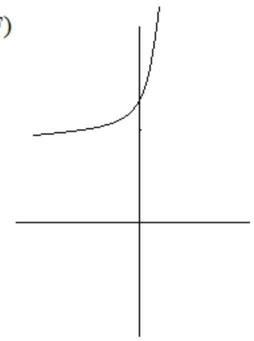
D)



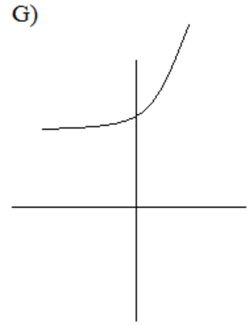
E)



F)



G)



3) Determine whether an equation models growth or decay:

a) $y = (1.1)^x$

Growth

b) $y = 2(.7)^x$

Decay

c) $y = (\frac{7}{3})^x - 5$

Growth

d) $y = (.9)^{-x}$

$y = (9/10)^{-x}$

$= (10/9)^x$

Growth

4) Find the *exponential* and *linear* functions that pass through:

a) $(0, 3)$ and $(1, 12)$

linear equation: slope $= \frac{(12 - 3)}{(1 - 0)} = 4$

$\boxed{y = 4x + 3}$

exponential equation: $y = ab^x$

substitute first point: $(3) = ab^{(0)}$

$3 = a(1) \quad a = 3$

substitute second point: $(12) = ab^{(1)}$

$12 = ab$

$12 = (3)b \quad b = 4$

$\boxed{y = 3(4)^x}$

b) $(1, -8)$ and $(2, -32)$

linear: slope $= \frac{(-8 - (-32))}{(1 - 2)} = -24$

$\boxed{y + 8 = -24(x - 1)}$

exponential: $y = ab^x$

first point: $-8 = ab^{(1)} \quad -8 = ab \quad a = \frac{-8}{b}$

second point: $-32 = ab^{(2)}$

$-32 = \left(\frac{-8}{b}\right)b^2 \quad -32 = -8b$

$b = 4$

$ab = -8 \quad$ so, $a = -2$

$\boxed{y = -2(4)^x}$

c) $(0, -5)$ and $(2, -20)$

linear: slope $= \frac{(-5 - (-20))}{(0 - 2)} = \frac{-15}{2}$

$\boxed{y = \frac{-15}{2}x - 5}$

exponential:

first point: $-5 = ab^0 \quad a = -5$

second point: $(-20) = ab^2$

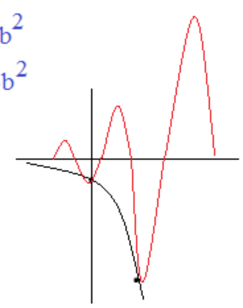
$(-20) = -5b^2$

$b = \pm 2$

$\boxed{y = -5(2)^x}$

$y = -5(-2)^x$

periodic function



NOTE: To check solutions, just plug in the points!!

Exponents and Exponential Functions Quiz

Solutions

IV. Scientific Notation

1) Convert to Scientific Notation

a) 234,567

move decimal 5 spaces to the left

2.34567×10^5

b) 527.021

2 spaces to the left

5.27021×10^2

c) .00045

4 spaces to the *right*

4.5×10^{-4}

d) 20^3

$(2 \times 10)^3$

8×10^3

2) Convert to Standard Notation

a) 1.8808×10^3

1880.8×10^0

1880.8

b) 4.0007×10^{-7}

move decimal 7 spaces to the left; increase exponent by 7

.00000040007

c) $4.6\overline{6} \times 10^5$

$\underbrace{4.666666\ldots}_{\text{5 spaces}} \times 10^5$

$46666.6\overline{6}$

d) 5.60005×10^4

56000.5

3) Simplify

a) $\dfrac{20{,}000}{.004} =$

$\dfrac{2.0 \times 10^4}{4.0 \times 10^{-3}} = .5 \times 10^7$

$\boxed{5{,}000{,}000}$

b) $(3.44 \times 10^6) \div 2 =$

$\boxed{1.72 \times 10^6 \text{ or } 1{,}720{,}000}$

c) $(4.5 \times 10^{-4}) \cdot (3.1 \times 10^{-3}) =$

$4.5 \times 3.1 = 13.95$

$10^{-4} \times 10^{-3} = 10^{-7}$

$\boxed{1.395 \times 10^{-6} \text{ or } .000001395}$

V. Applications/Word Problems

1) A bacteria has a *half-life* of 3 years. If the current mass is 500 mg.,

a) write an equation to model the weight (y) over years (t)

b) what is the mass of the bacteria after 3 years? 9 years?

c) what is the mass of the bacteria after 20 years?

a) exponential equation: $y = ab^x$

$\boxed{y = 500\left(\frac{1}{2}\right)^{t/3}}$

t = time (years)
y = weight
500 = initial amount
1/2 = decay/growth rate (each period)

b) after 3 years, 250 mg

after 9 years, 62.5 mg

years	# of half-lifes	mass
0	0	500
3	1	250
6	2	125
9	3	62.5
12	4	31.25
15	5	15.625
18	6	7.8125
21	7	3.90625

c) if t = 20 years,

$y = 500\left(\frac{1}{2}\right)^{20/3}$

$y = 500 \cdot (.5)^{6.\overline{6}}$

$\boxed{= 4.92 \text{ mg}}$

2) A bank offers an annual interest rate of 6%. If you deposit $1000,

a) write a model to express the amount of money (A) over years (t)

b) how much will you have after 5 years? (compounding annually)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = future amount
P = principal amount
r = (nominal) annual interest rate
n = number of times of compounding
t = years

c) how much would you have after 5 years if the interest were compounded quarterly? monthly?

a) $A = P(1 + r)^t$

$\boxed{A = 1000(1.06)^t}$

b) After 5 years:

$A = 1000(1.06)^5$

$\boxed{= \$1338.23}$

c) quarterly compounding: n = 4

over 5 years, compounded 20 times at a rate of 1.5% each time

$A = 1000(1.015)^{4(5)}$

$\boxed{= \$1346.86}$

monthly compounding: n = 12

over 5 years, the money is compounded 60 times at .5% each time..

$A = 1000(1.005)^{12(5)}$

$\boxed{= \$1348.85}$

Solutions

3) The "rule of 72" is a method for estimating 'doubling time' of an investment. $\text{Doubling time} = \dfrac{72}{\text{interest rate}}$

Compare the rule of 72 with a true measurement, using an interest rate of 10%

Using the 'rule of 72', it would take roughly

$\dfrac{72}{10} = 7.2$

7.2 years for an investment compounded annually at 10% to double..

$A = P(1 + r)^t$

Assume $P = \$10{,}000$
$r = 10\%$
$t = 7.2$ years

$A = \$10{,}000(1.10)^{7.2}$

In 7.2 years, $10,000 would *almost* double... $\longrightarrow$ $= \$19{,}862.20$

How long would it take?

$20{,}000 = 10{,}000(1.10)^t$

using logarithms

$2 = (1.1)^t$

$\log 2 = \log(1.1)^t$

$t = \dfrac{\log 2}{\log(1.1)} = 7.2725$ years

4) A math company was established in 2007. Its value has grown exponentially, where after one year, it was worth $10,000; and, its worth after the second year was $25,000... How much was the company worth on *Day 1*? How much will it be worth in 2015?

x (years)	y (worth)
0 (2007)	?
1 (2008)	10,000
2 (2009)	25,000
8 (2015)	?

(1, 10,000)
(2, 25,000)

exponential growth

$y = ab^x$

$10{,}000 = ab^1$ $\quad$ $25{,}000 = ab^2$

$a = \dfrac{10{,}000}{b}$ $\quad$ $25{,}000 = \dfrac{10{,}000}{b}\, b^2$

$b = 2.5$ $\quad$ $a = 4000$

$y = \$4000(2.5)^t$

Its initial value (on day 1)

$y = \$4000(2.5)^0$

$\boxed{= \$4000}$

In 2015, the company will be in year 8

$y = \$4000(2.5)^8$

$\boxed{= \$6{,}103{,}515}$

5) A cost of a government agency is modeled by the equation $y = 25(1.12)^t$

a) What is the annual percentage growth? $\boxed{12\%}$

b) How much does it cost now? $\boxed{25}$

c) How much will the cost be 5 years from now? $y = 25(1.12)^5$

$y \cong 25(1.7623) = \boxed{44.06}$

VI: Miscellaneous:

1) What is $(-1)^{23}$?

$(-1)(-1) = 1$
$(-1)(-1)(-1) = -1$
$(-1)(-1)(-1)(-1) = 1$

since exponent is odd, the output $\boxed{\text{is } 1}$

2) What is the y-intercept of $y = 5^{(x+3)}$?

y-intercept is the point where the equation crosses the y-axis --- i.e. where $x = 0$

$y = 5^{(0+3)}$
$y = 125$ $\quad$ $\boxed{(0, 125)}$

3) What is the x-intercept of $y = 3^x - 9$?

x-intercept is the point where the equation crosses the x-axis -- i.e. where $y = 0$

$0 = 3^x - 9$
$9 = 3^x$ $\quad$ $\boxed{(2, 0)}$
$x = 2$

Solutions

4) $4^{14} + 4^{14} + 4^{14} + 4^{14} =$

(simplify to one term)

$4 \cdot 4^{14} = \boxed{4^{15}}$

5) $(x+1)^2 = x^2 + 1$ Always, sometimes, or never?

$x^2 + 2x + 1 = x^2 + 1$

$2x = 0$

$x = 0$

Sometimes!

6) The sum of the first 25 perfect squares: $1^2 + 2^2 + 3^2 + ... + 25^2 = 5525$

What is $2^2 + 4^2 + 6^2 + ... + 50^2$?

$2^2 + 4^2 + 6^2 + ... + 50^2$ ------> $(1 \cdot 2)^2 + (2 \cdot 2)^2 + (3 \cdot 2)^2 + ... + (25 \cdot 2)^2 =$

$2^2 \cdot 1^2 + 2^2 \cdot 2^2 + 2^2 \cdot 3^2 + ... + 2^2 \cdot 25^2 =$

$2^2(1^2 + 2^2 + 3^2 + ... + 25^2) = 4 \cdot 5525 = \boxed{22{,}100}$

7) How many (positive) perfect squares are less than 10,000?

since 100 x 100 = 10,000, there are 99 perfect squares less than 10,000

8) $-1^{1998} + (-1)^{1999} + 1^{2000} - 1^{2001}$

note: $-1^{1998} = -1$ while $(-1)^{1998} = 1$

$-1 + (-1) + 1 - 1 = \boxed{-2}$

9) Express as 2^n

a) $\frac{1}{2}(2^{13})$

$2^{-1} \cdot 2^{13}$

$\boxed{2^{12}}$

b) $2^3 \cdot 4 \cdot \frac{1}{16}$

$2^3 \cdot 2^2 \cdot 2^{-4}$

$\boxed{2^1}$

c) $(2^3)^6 \div 4^3$

$\frac{2^{18}}{(2^2)^3} = \boxed{2^{12}}$

d) $2^{25} - 2^{24} - 2^{23}$

$2^{23}(2^2 - 2^1 - 2^0)$

$2^{23}(4 - 2 - 1) = \boxed{2^{23}}$

10) If $a = -4$, find $x^2 - x^{-1} + 3x$

$(-4)^2 - (-4)^{-1} + 3(-4)$

$16 - \frac{-1}{4} - 12 = \boxed{4\frac{1}{4}}$

11) Simplify:

a) -4^{-2}

$\boxed{-\frac{1}{16}}$

b) $(-4)^{-2}$

$\boxed{\frac{1}{16}}$

c) $(-2)^{-3}$

$\boxed{-\frac{1}{8}}$

d) $\frac{1}{(-4)^{-2}}$

$\frac{1}{\frac{1}{16}} = \boxed{16}$

e) $\frac{1}{(-\frac{1}{2})^{-3}}$

$\boxed{\frac{1}{-8}}$

Exponent Rules: Notes and Examples

Exponent *definition*:

$$X^A = X_1 \cdot X_2 \cdot X_3 \cdot \ \ldots \ \cdot X_{A-2} \cdot X_{A-1} \cdot X_A$$

Examples: $4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$

$$(-2)^7 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = -128$$

$$(-2)^6 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 64$$

Rule #1 ('Addition Rule')

$$X^A \cdot X^B = X^{A+B}$$

Examples:

$$X^3 \cdot X^5 = X^8$$

$$5^3 \cdot 5^2 = 125 \cdot 25 = 3125 = 5^5$$

Note:

$$Y^2 \cdot Y^4 = Y^6$$

$$\underbrace{(Y \times Y)}_{2} \cdot \underbrace{(Y \times Y \times Y \times Y)}_{4} = \underbrace{Y \times Y \times Y \times Y \times Y \times Y}_{\text{6 total Y's}}$$

Rule #2: ('Multiplication Rule')

$$(X^A)^B = X^{AB}$$

Examples: $(X^4)^3 = X^{12}$

$$(4^2)^4 = 4^8 = 16^4 = 65536$$

Note:

$$(Y^5)^3 = \underbrace{(Y \times Y \times Y \times Y \times Y)}_{5} \cdot \underbrace{(Y \times Y \times Y \times Y \times Y)}_{5} \cdot \underbrace{(Y \times Y \times Y \times Y \times Y)}_{5} = Y^{15}$$

3 groups of 5 Y's
total: 3 x 5 = 15 Y's

Rule #3: ('zero exponent')

$$X^0 = 1$$

Examples: $Y^0 = 1$

$$8^0 = 1$$

$$(3cd)^0 = 1$$

What is 0^0 ? $0^A = 0$ because $0 \cdot 0 \cdot 0 \cdot 0 \ldots = 0$
(if $A \neq 0$)

$X^0 = 1$ (zero exponent rule)

$$\boxed{0^0 = 1}$$

Note: $Z^5 \cdot Z^{-5} = Z^0 = 1$

addition rule --- then, zero exponent rule

$$\frac{\overbrace{Z \times Z \times Z \times Z \times Z}^{Z^5}}{\underbrace{Z \times Z \times Z \times Z \times Z}_{Z^{-5}}} = 1$$

Rule #4: ('negatives' or 'reciprocal rule')

$$X^{(-A)} = \frac{1}{X^A}$$

Examples:

$$X^{-3} = \frac{1}{X^3}$$

$$5^{-2} = \frac{1}{25}$$ It is not equal to -25!!!

$$\left(\frac{1}{3}\right)^{-4} = 81$$

Note:

$$\boxed{Y^{(-A)} =} \; Y^{(-A)} \cdot \frac{Y^A}{Y^A} = \frac{Y^{(-A)} \cdot Y^A}{Y^A} = \frac{Y^{(-A+A)}}{Y^A} = \frac{Y^0}{Y^A} = \boxed{\frac{1}{Y^A}}$$

multiply by one — exponent addition rule — zero exponent

Rule #5: ('base rule')

$$X^A \cdot Y^A = (XY)^A$$

Examples:

$$5^3 \cdot 7^3 = 125 \text{ x } 343 = 42875 = 35^3$$

$$= (5 \text{ x } 5 \text{ x } 5) \text{ x } (7 \text{ x } 7 \text{ x } 7) = (5 \text{ x } 7) \text{ x } (5 \text{ x } 7) \text{ x } (5 \text{ x } 7)$$

$$4^{\frac{1}{2}} \cdot 16^{\frac{1}{2}} = 64^{(1/2)} = 8$$

$$\sqrt{4} \text{ x } \sqrt{16} = \sqrt{4 \text{ x } 16} = \sqrt{64}$$

Rule #6: ('rational exponents')

$$X^{(1/2)} = \sqrt{X} \qquad X^{\left(\frac{A}{B}\right)} = \sqrt[B]{X^A}$$

Examples:

$$25^{(1/2)} = \sqrt{25} = 5$$

$$8^{(1/3)} = \sqrt[3]{8} = 2$$ ('cubed root of 8')

$$121^{(.5)} = 11$$

Note:

$$Y^{(1/2)} \cdot Y^{(1/2)} = Y^1 \qquad \sqrt{Y} \cdot \sqrt{Y} = Y$$

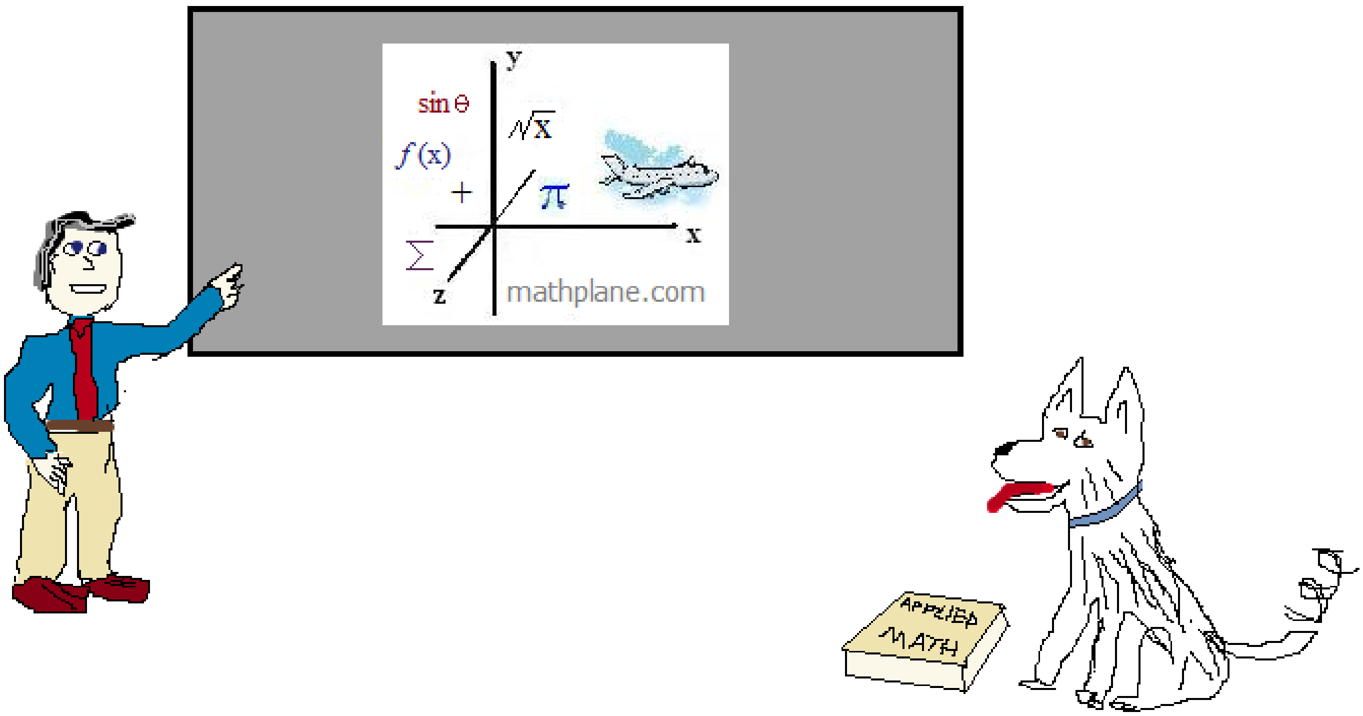
(addition exponent rule)

$$8^{(1/3)} \cdot 8^{(1/3)} \cdot 8^{(1/3)} = 8^1 = 8$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, and requests, let us know.

Cheers



Also, at Facebook, Google+, Pinterest, TES, and TeachersPayTeachers