Overview of Experiments for Magnetic Torque

General Description of Apparatus

The Magnetic Torque instrument consists of a pair of Helmholtz like coils with a brass air bearing mounted in the middle. (The coils are not perfectly Helmholtz but do give a very uniform field at the middle.) When doing the experiments, students can calculate the magnetic field as a function of current using the information on the number, radius and separation of the coils. We also have calibrations to give the students.

$$B = 1.36 \times 10^{-3}\ \frac{\text{tesla}}{\text{amp}}\ I$$

$$\frac{dB}{dz} = 1.69 \times 10^{-2}\ \frac{\text{tesla / m}}{\text{amp}}\ I$$

A snooker ball floats with minimal friction on the air which comes out of a small hole at the base of the bearing. A small magnetized disc (which is treated as an ideal magnetic dipole) is embedded in the middle of the ball. The axis of its magnetic moment is oriented along the black handle which sticks out of the ball. The handle is used for spinning the ball. The system is balanced so that the handle does not cause a net gravitational torque on the ball.
Figure 1 is a schematic picture of the ball.

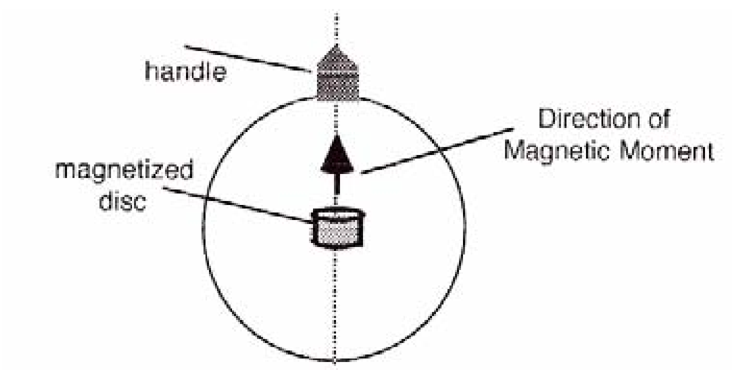


Figure 1 - Schematic Drawing of Snooker Ball With Imbedded Magnet

The actual value of the magnetic moment, μ, can be determined by three different experiments which are done with the ball in the air bearing. Each experiment uses completely independent measurements and has its own systematic errors. This is important because experimental physicists are never content with a single measurement of an important physical parameter. (Using a Hall Effect Probe to measure the magnetic field as a function of distance along the z axis gives a fourth independent way to determine the magnetic moment of the ball.) Before beginning, a student must measure the mass and diameter of the ball so that the moment of inertia can be calculated. For calculations, we assume the ball has a uniform density.

The following pages describe how to demonstrate the experiments that can be performed with the Magnetic Torque Apparatus. Although certainly effective as a demonstration, this instrument is most impressive when used in a laboratory setting.

Experiment I - Magnetic Torque Balancing Gravitational Torque

This experiment balances gravitational torque, r x mg, and magnetic torque, μ x B.

- Find the mass of a clear plastic disc, then slide it onto the longer aluminum rod.
- Insert the end of the aluminum rod which contains a small iron chip into the handle. (The iron keeps the rod from slipping.) Position the plastic disc quite close to the handle.
- Place the ball on the air bearing and adjust the current for balance.
- Move the disc out in increments and determine the current that must be used each time to reestablish balance.

The mathematical relationships for this experiment are shown in the right hand column below. (B is found from current, I, using the calibration supplied.) The graph of Torque vs. B does not go through 0. The intercept is due to the torque of the rod. The intercept does not matter, however, because the slope is the source of μ. As shown in Figure 2, the magnetic moment and the aluminum rod are collinear and both mg and B are in a vertical direction. The sine terms of the cross products are, therefore, equal and the vector equation becomes algebraic.

This experiment also provides a "functional definition" of magnetic moment: A one unit magnetic moment experiences one newton-meter of torque when perpendicular to a one tesla magnetic field.

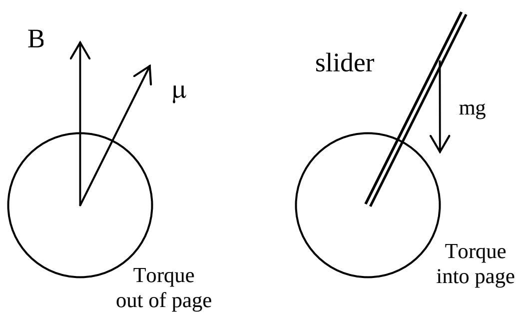


Figure 2 – Vector Diagrams of Magnetic and Gravitational Torques.

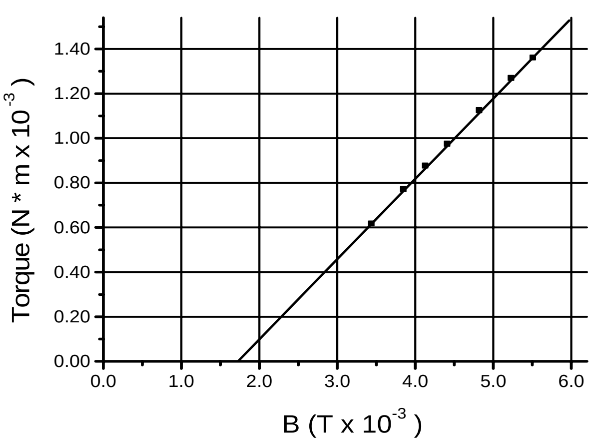


Figure 3 – Gravitational Torque on Slider vs. Magnetic Field

Gravitational and magnetic torques balance.

$$\tau_{gravitational} = -\tau_{magnetic}$$
$$\vec{r} \times m\vec{g} = -\vec{\mu} \times \vec{B}$$
$$rmg = \mu B$$

This term rmg, however, gives only the torque from the plastic slider. It neglects the torque of the rod.

$$\tau_{\text{slider}} + \tau_{\text{rod}} = \mu B$$
$$\tau_{\text{slider}} = \mu B - \tau_{\text{rod}}$$

The negative intercept does not affect the slope and does not interfere with finding μ.

μ.= 0.36 ± .01 N-m/tesla

μ.= 0.36 ± .01 A-m^2

Experiment II - Finding μ from SHM of a Spherical Pendulum

- Place the ball on the air bearing and set the current in the coil at a maximum.
- Rotate the handle 15-20° from the vertical and release it. The ball then performs simple harmonic or sinusoidal motion.
- Find the period with a stop watch.
- To demonstrate that the period depends on the magnetic field, lower the current dramatically. The increased period is very obvious. For measurements be sure to keep deflections from the vertical < 20°.

For a mass on a spring, the period, T, depends on $\sqrt{m/k}$. Here, T depends on $\sqrt{I/\mu B}$. The moment of inertia can be calculated, and B, which acts as a restoring torque, is again determined from the current in the coils. The restoring torque also depends on the value of μ. The graph goes nicely through zero and μ is easily gotten from the slope if T^2 is plotted against 1/B. An overview of the relevant equations and the graph of student data are shown below

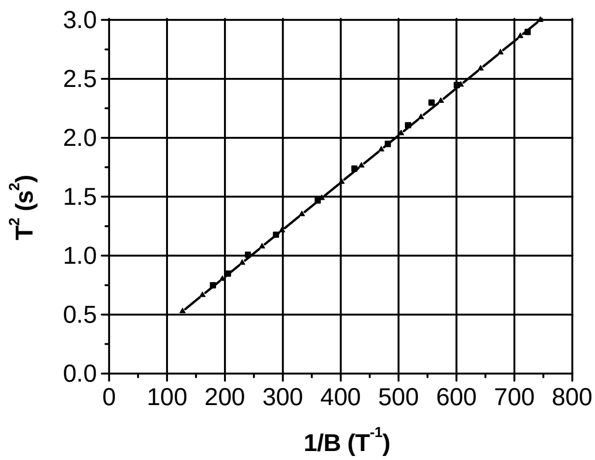


Figure 4 – Period Square vs. Inverse of Magnetic Field

$$I_{cm} = \frac{2}{5}mr^2 \quad \text{and} \quad L = I\omega$$

$$\sum \tau = -\mu \times B$$

For small angles where $sin\theta \approx \theta$

$$\tau = -\mu B\theta$$

For this physical pendulum the period is:

$$T = 2\pi\sqrt{\frac{I_{cm}}{\mu B}}$$

$$T^2 = 4\pi^2 \frac{I_{cm}}{\mu B}$$

$\mu = 0.40 \pm .01$ A-m^2

Experiment III: Finding μ from Processional Motion

- With the current off, spin the ball so that the axis is just off the vertical. Spin the ball by holding the handle in the thumb and fingers and twisting or snapping your fingers. Steady the nutation by barely touching the side of the handle with the edge of your nail or some other low friction object.
- Use the strobe to find the rotational frequency. The white spot on the handle will appear to stand still when the strobe light is at the correct frequency. (I never try to do this exactly for a demo.)
- Once the frequency is known, turn the current to 3 amps.
- Time one full precession with a stop watch.
- For qualitative demonstrations, vary the current, vary the spin rate, switch the field to "Down."

For a given μ, the precession period depends on rotation frequency and magnetic field, both of which are variable. The equation is: $\Omega_{precession} = \mu B/L$. We chose to do the experiment using a constant rotation frequency. With the current off and the strobe at a fixed frequency, we set the ball spinning quite fast. When the ball slowed to match the strobe frequency, the current was set to a particular value and the precession was timed. This was repeated for a variety of currents. Spin frequency decreases slowly enough to be treated as constant for a single precession. (Trials could also be done allowing both parameters to change and precession frequency plotted against B/L.) An overview of the relevant equations and a graph of the data are shown**.**

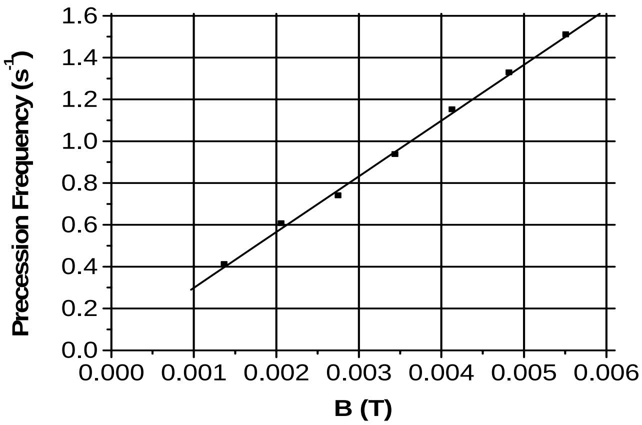


Figure 5 - Precession Frequency vs. Magnetic Field
For Fixed Angular Momentum

$$\vec{L} = I\vec{\omega}$$

$$\vec{\mu} \times \vec{B} = \frac{d\vec{L}}{dt}$$

$$\mu B \sin\theta' = \Omega_p L$$

$$\Omega_p = \frac{\mu}{L} B$$

Either B or L could be varied to determine μ. In the experiment shown, L was constant and B was varied. The experimenter waited until the sphere reached a fixed ω, then timed one precession

$\mu = 0.38 \pm .01$ A-m^2

Using a Hall Effect Probe to find μ

The ball can be set on its side so that the direction of the magnetic moment is horizontal. The magnetic field can be plotted using a Hall Effect Probe capable of reading small fields. The magnetic moment can be found by plotting the magnetic field B against $1/r^3$. Equations and graphs are shown below.

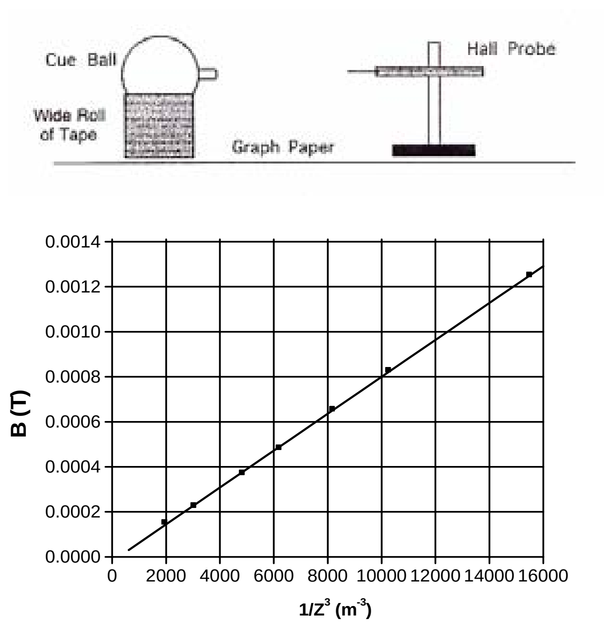


Figure 6 – Magnetic Field vs. $1/z^3$

From the Biot-Savart law, the axial magnetic field of a current loop of radius r and current I is:

$$B_z = \frac{\mu_o I r^2}{2(r^2 + z^2)^{3/2}}$$

At "far field" with z>>r: $$B_z = \frac{\mu_o I r^2}{2z^3}$$

A permanent magnetic dipole can be modeled as a current loop where: $$I\pi r^2 = \mu$$

The far field of the permanent magnetic dipole is then: $$B_z = \frac{\mu_o \mu}{2z^3}$$

$\mu = 0.41 \pm .01$A-m^2

Experiments with the Magnetic Moment on a Spring

Another interesting set of experiments can be done using the plastic tower with the spring inside. A magnet, just like the one inside the ball, is set in a gimbal (the plastic holder free to rotate) and hung from the spring. The spring can be calibrated using one gram metal spheres which adhere to the magnet.

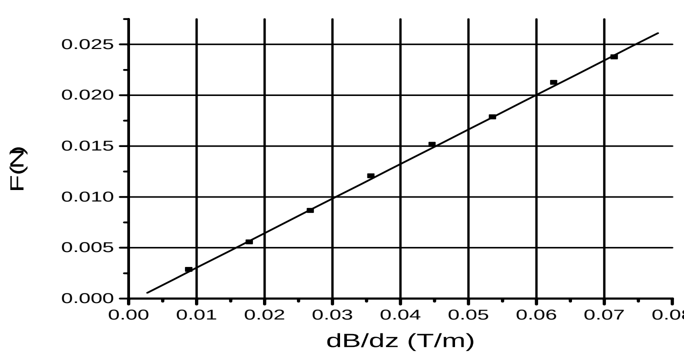
- With the current off, adjust the location of the magnet-gimbal until it is at the center of the coils. The location is adjusted by raising and lowering the brass rod which comes out of the top of the tube. An arrow on the gimbal shows the direction of the magnetic moment.
- Turn on the current just enough to align the magnet with the field. Ask people watching to predict what will happen when the current is increased. To most people's surprise, nothing happens. In a uniform field there is only a torque, not a force.
- Reverse the field to the opposite direction, using the up/down toggle switch. The magnet spins and stops with the arrow now facing down. It remains, however, in the same vertical location.

  To get a force on a magnet, there must be a field gradient, a field that changes with distance.
- Create a field gradient by moving the toggle switch on the front panel marked Gradient to ON. With Gradient On, the current in the top coil is reversed with respect to the lower coil. There is then a 0 field at the center of the coils BUT there is a field gradient.
- With the gradient on, the magnet is pulled down. Decreasing the current reduces the stretch because dB/dz, and thus the force, depends on the current. If the up/down toggle is used, the gradient reverses and the magnet jumps the other way. Flip the toggle using only a small current or the magnet itself turns over is again pulled down. When taking a series of measurements, the flipping can be prevented by tightening the eye screw so that it touches the magnet. Then, the gimbal cannot rotate.

Making Measurements to find the Magnetic Moment.

- Calibrate the spring. (k is, conveniently, about 1N/m)
- With the current off, locate the gimbal at the center of the coils and note its position.
- Measure the length of the brass rod coming out of the top of the tube when the current is off.
- After setting the current, release the screw in the cap and gently pull up on the rod until the magnet is again at its original position. Re-measure the length of the rod to find the extension of the spring.
- Calculate the force needed to keep the magnet at the center when the gradient is on.
- Take measurements at a series of currents.
- Use the calibration provided to find dB/dz, and then find μ from a plot of Force vs. field gradient.

This experiment provides another operational definition of a magnetic moment: a one unit magnetic moment experiences a one newton force in a field gradient of one Tesla per meter. (This is like saying a one kg object is that object which requires a one newton net force to accelerate it at 1 m/s^2.)



Force on a magnetic moment depends on the field gradient.

$$F_{spring} = F_z$$

$$F_z = \mu \frac{dB_z}{dz}$$

$$\mu = 0.38 \pm .01 \ \frac{\text{N}}{\text{T/m}}$$

or

$$\mu = 0.38 \pm .01 \ \text{A-m}^2$$

Figure 7 – Force vs. dB/dz

A Classical Analog of the NMR Spin Flip

The last experiment you can do uses the horizontal magnetic field.

- Remove the ball and slip the horizontal permanent magnet assembly over the air bearing. Replace the ball and turn on the air bearing with the coil current off. You will see the ball orient itself along the horizontal field. Rotate the horizontal field and the ball will follow.
- Hold the ball upright and spin it. You will see the ball precess around the horizontal magnetic field until the handle hits the side of the bearing.
- Next, turn on the current as high as it goes. You can see the direction of the net field from the angle of the ball.
- Now, if you spin the ball, it will precess around the vector sum of the fields.
- At a moment when the handle of the ball lies in a plane parallel to the magnet ends, start turning the permanent magnet to keep the handle in this plane. If you do it right, the handle will actually precess around the horizontal field, just as it did when you spun it with the vertical field off.

Explanation: You are rotating the horizontal field at what is called the Larmor procession frequency. This is the exact frequency with which the ball would precess in the vertical field only. In this case, from the frame of reference of the rotating permanent magnet assembly, the vertical field "does not exist." Because the reference frame is moving at the same rate as the precession around the vertical, that precession around the vertical field has become "invisible." As a result, we are able to change the direction of the magnetic moment of the spinning ball. Your motion is "in resonance." If you move the permanent magnet assembly too fast, too slow, or in the opposite direction, the rotation around the horizontal does not happen.

This is the basis of the pulsed nuclear magnetic resonance which examines the spinning protons in an atom. In PNMR. there is the equivalent of our strong vertical field. A horizontal pulse which is in resonance is then used to tip the spins until they are perpendicular to the vertical field. In real PNMR, once the spins are "tipped," the entire horizontal magnetic field is turned off. The protons then precess freely around the vertical magnetic field. However, the protons interfere with one another. Eventually, they again realign themselves with the vertical field. The time for this to happen is called the "relaxation time" and can tell physicists and chemists a great deal about the kind of environment the proton is in. Using our real PNMR, we have watched how the relaxation time changes as an epoxy cures. In an MRI, the relaxation time of the protons in a particular spot in your body indicates the density of the tissue. A computer "images" that time into the x-ray like picture the doctor sees.

One question people who really know PNMR ask is "How come, in actual practice, you can use a radio frequency pulse to "tip" a spin in PNMR?" You do have an answer. A radio frequency (rf) pulse is actually simple harmonic. Now, a simple harmonic motion can be thought of as the vertical component of circular motion. Of course, you still have to deal with the horizontal component. But what if you use two circular motions going in opposite directions? The horizontal motions can cancel and the two vertical motions just make a stronger SHM.

Remember that the spinning ball does not respond if you rotate the horizontal magnetic field in the wrong direction. The rf pulse used in Pulsed NMR is a simple harmonic pulse at the same frequency as the proton's precession in the primary magnetic field. If the proton "experiences" the rf pulse as two counter rotating magnetic fields, the field rotating in the opposite direction of the precession is "ignored," while the one going in the right direction will make the proton tip its spin!!