Word Problems? No Problem!

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The University of Texas at Austin

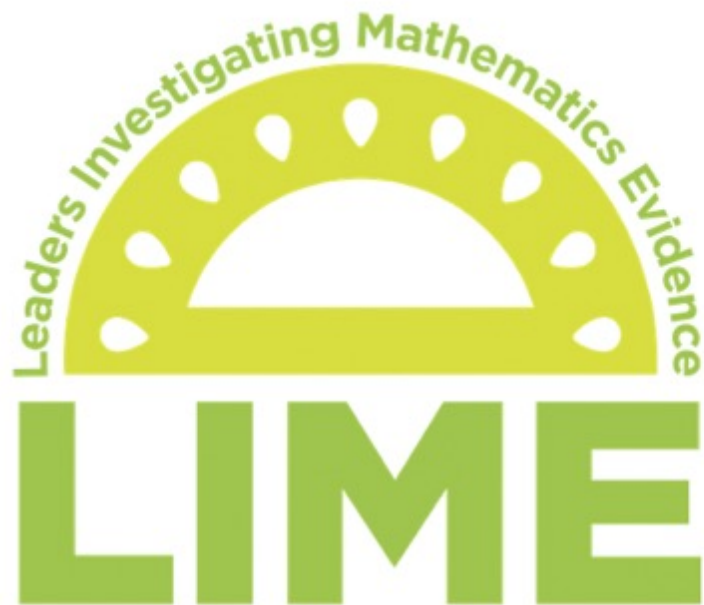www.sarahpowellphd.com

srpowell@utexas.edu

@sarahpowellphd

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Describe the mathematics you support.

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Schedule for Today

9:00-10:25	- How students solve word problems - Word-problem solving difficulties - Ineffective word-problem strategies - Effective practice: Attack strategies
10:25-10:35	BREAK
10:35-11:50	- Effective practice: A focus on schemas - Total problems - Difference problems - Change problems
11:50-1:00	LUNCH
1:00-2:25	- Equal groups problems - Comparison problems - Ratios/proportions problems
2:25-2:35	BREAK
2:35-4:00	- Identifying word-problem schemas on the STAAR - Multi-step word problems - Wrap-up

How Students Solve Word Problems

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

Determine the misconceptions and errors.

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

Undetermined

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

43

Undetermined

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

Donna

Undetermined

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

96 peper fold

Repeated information

M A + H

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

Repeated information

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

Identified incorrect information – wrong operation

Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$96$$
$$+25$$
$$\boxed{121} \checkmark$$

Identified correct information – wrong operation

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ \times\ 25 \\ \hline 121 \end{array}$$

Identified correct information – wrong operation

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$96$$
$$+ \ 25$$
$$\boxed{121}$$

Identified correct information – wrong operation

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ 25 \\ \hline 121 \end{array}$$

Identified correct information – wrong operation

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} {\scriptstyle +1} \\ 96 \\ + 25 \\ \hline 121 \end{array}$$

Identified correct information – wrong operation

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$96 + 25 = 121$$

Identified correct information – wrong operation

MA+H

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} \overset{1}{9}6 \\ +\ 25 \\ \hline \boxed{121} \end{array}$$

Identified correct information – wrong operation

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$-\frac{96}{25}$$

$$\boxed{71}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ -25 \\ \hline 71 \end{array}$$
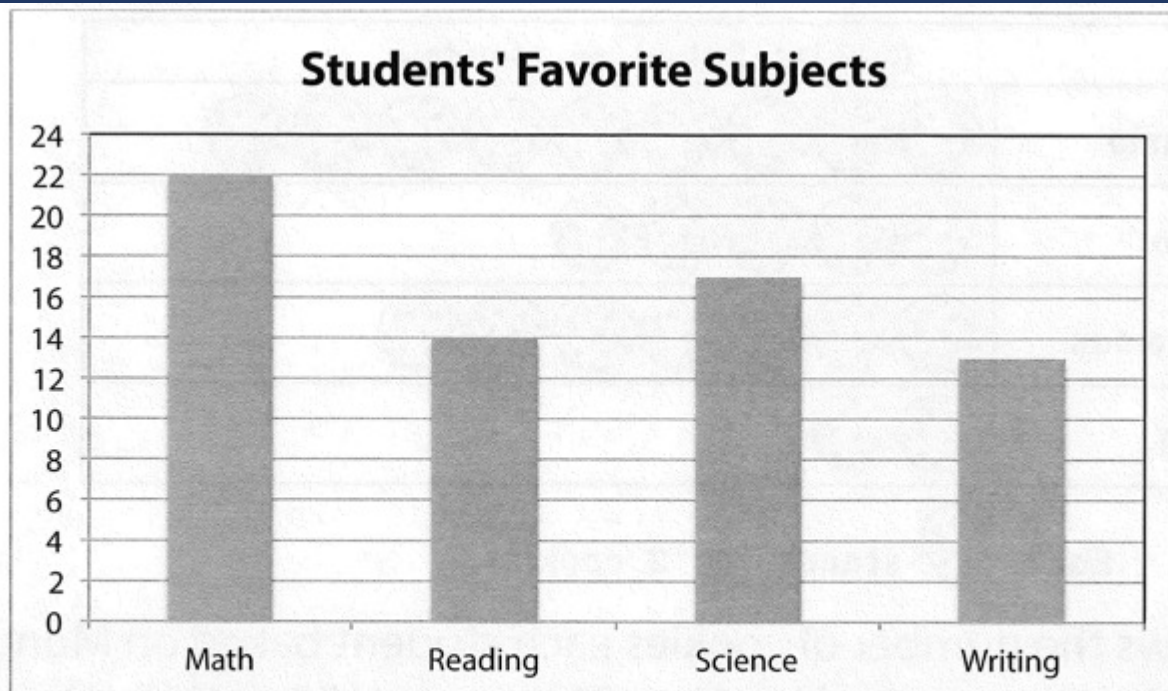
Correct

Undetermined

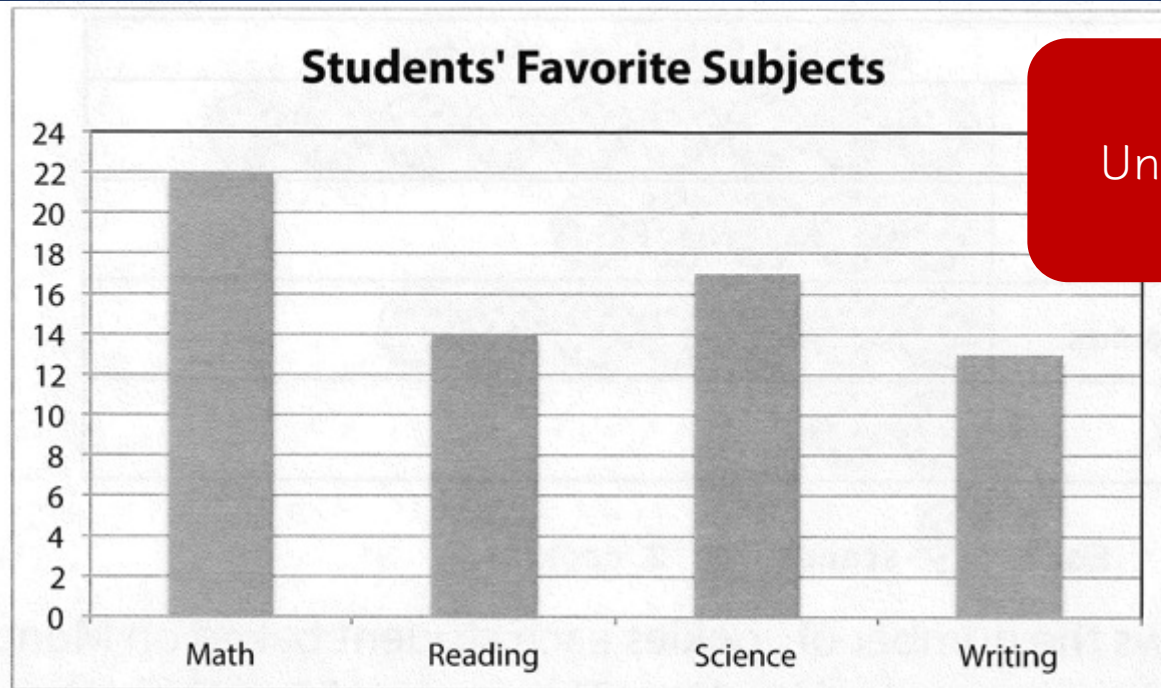Repeated information

Wrong information

Wrong (misapplied) operation

J.

Students' Favorite Subjects

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

J.

Students' Favorite Subjects
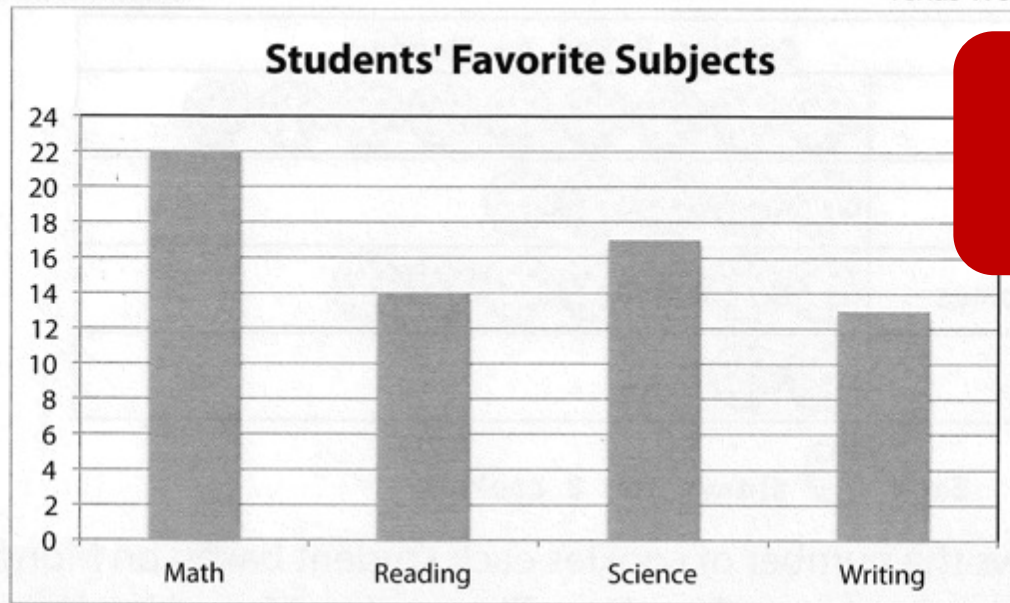
The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

60

MA+H

J.
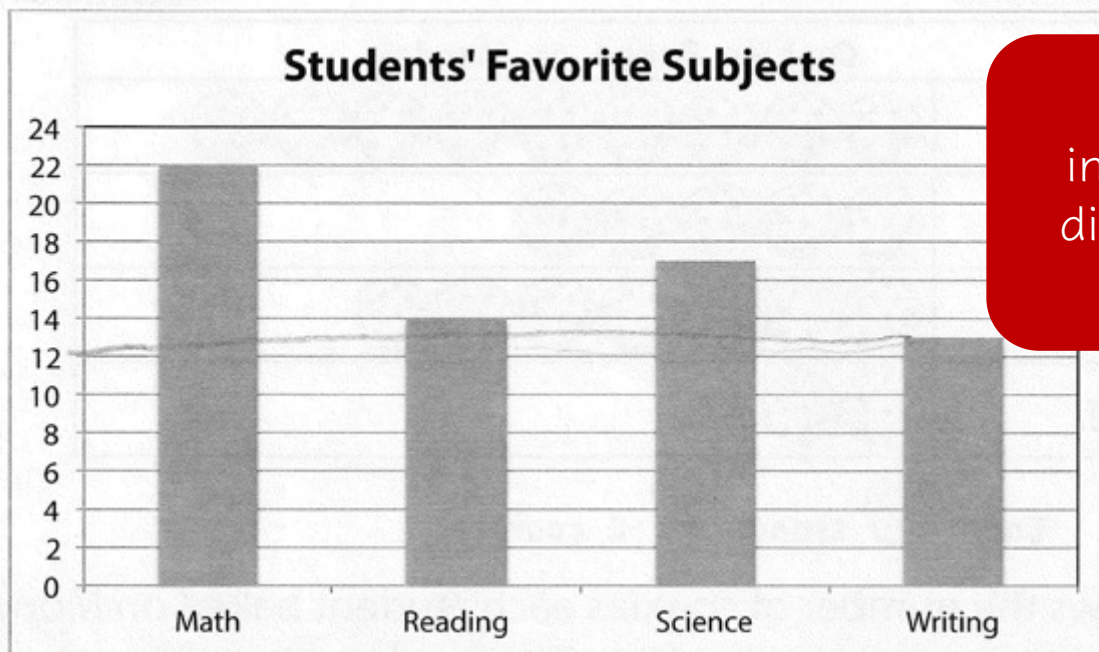
Students' Favorite Subjects

Undetermined

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

they choose ⑧% more than
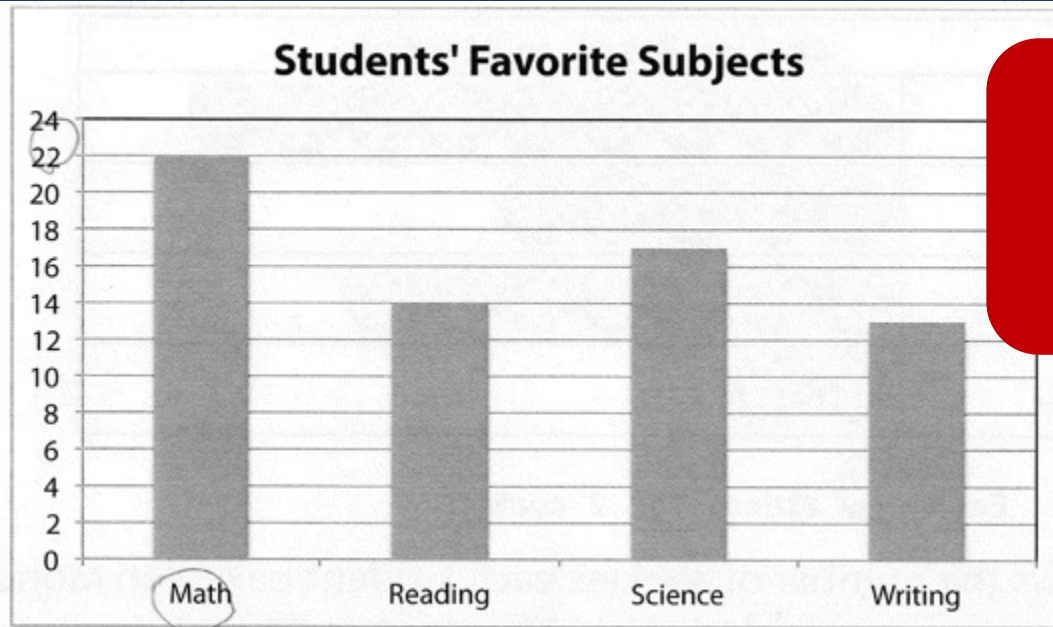
reading

J.

Students' Favorite Subjects

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?
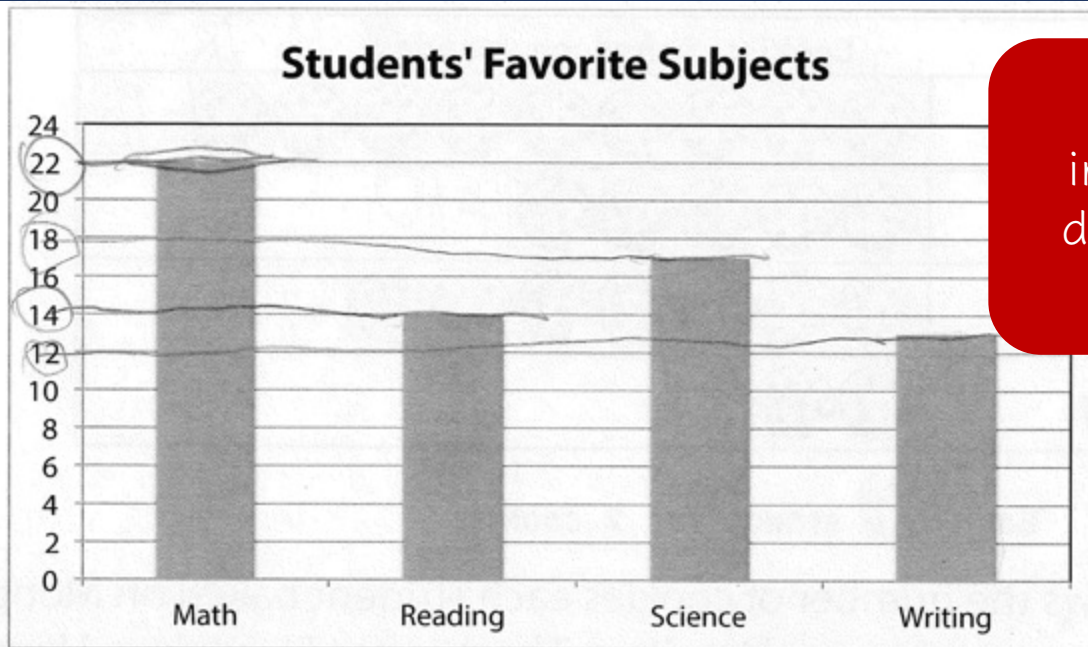
22 math   12 writing

Identified information – didn't provide answer

J.

Students' Favorite Subjects

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

22
Math

Identified information – didn't provide answer

J.

Students' Favorite Subjects

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

the students choses they

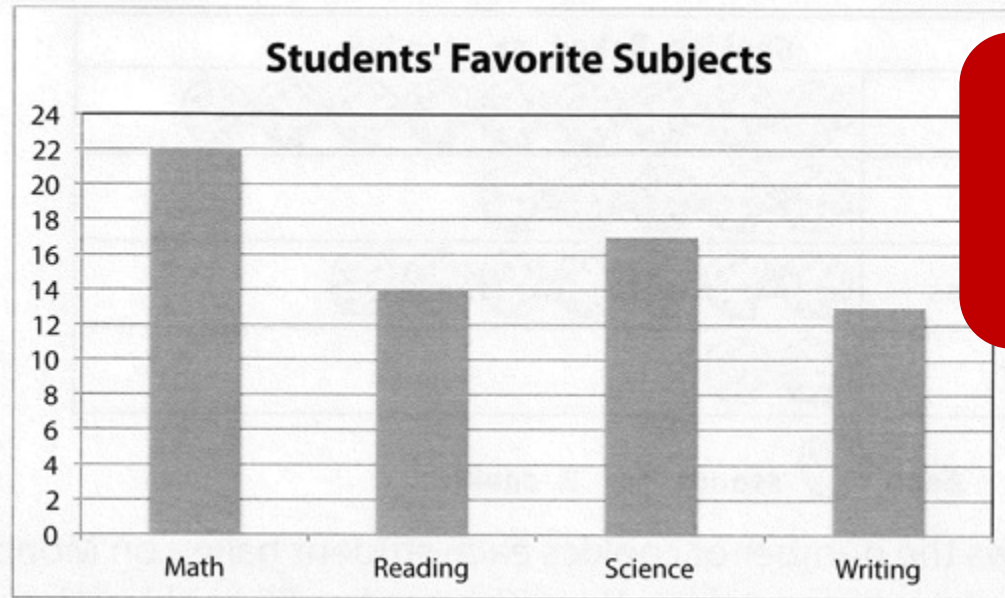love more math dicas es

⊘ tu 22 math

Identified information – didn't provide answer

Students' Favorite Subjects

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

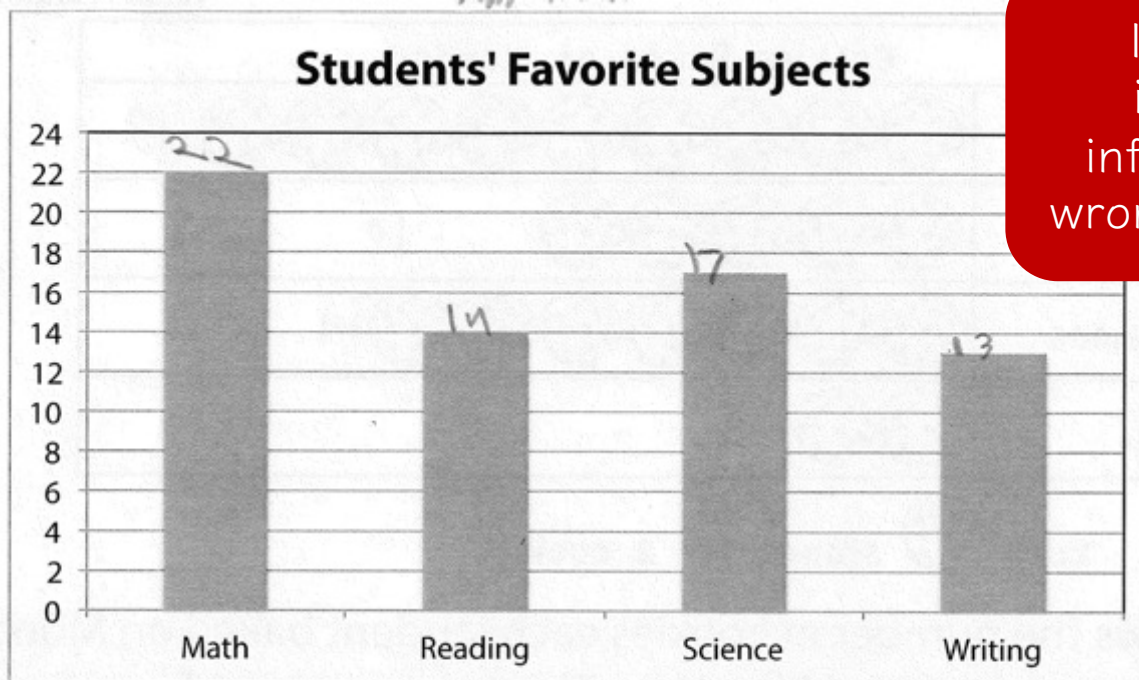The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

J.

Students' Favorite Subjects

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$22$$
$$+ 12$$
$$\overline{34}$$

$$\boxed{34}$$

Identified correct information – wrong operation

J.

Students' Favorite Subjects

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

J.

Students' Favorite Subjects

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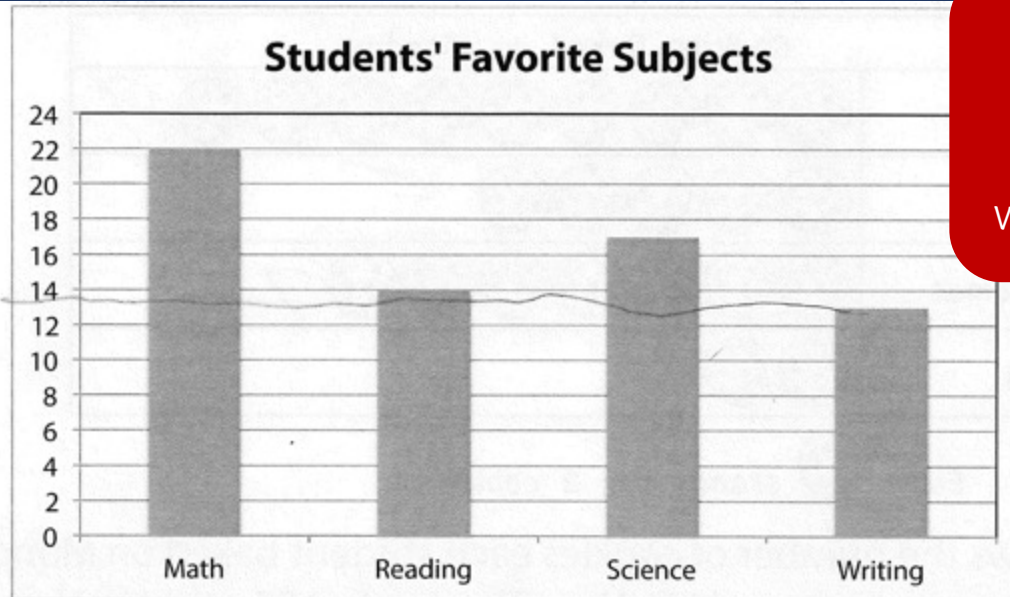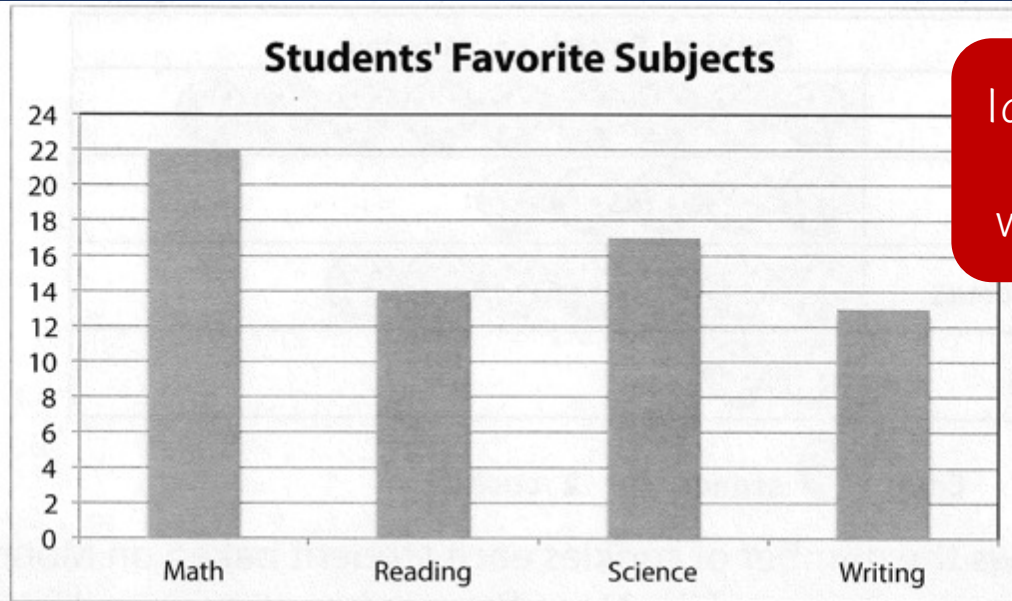
$$\begin{array}{r} {}^{1}\cancel{2}\cancel{2}^{12} \\ -\ 15 \\ \hline \textcircled{7} \end{array}$$

J.

Students' Favorite Subjects

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$22 - 12 = 10$$

J.

Students' Favorite Subjects

24
22
20
18
16
14
12
10
8
6
4
2
0

Math   Reading   Science   Writing

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

Identified incorrect information

J.

Students' Favorite Subjects

The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?
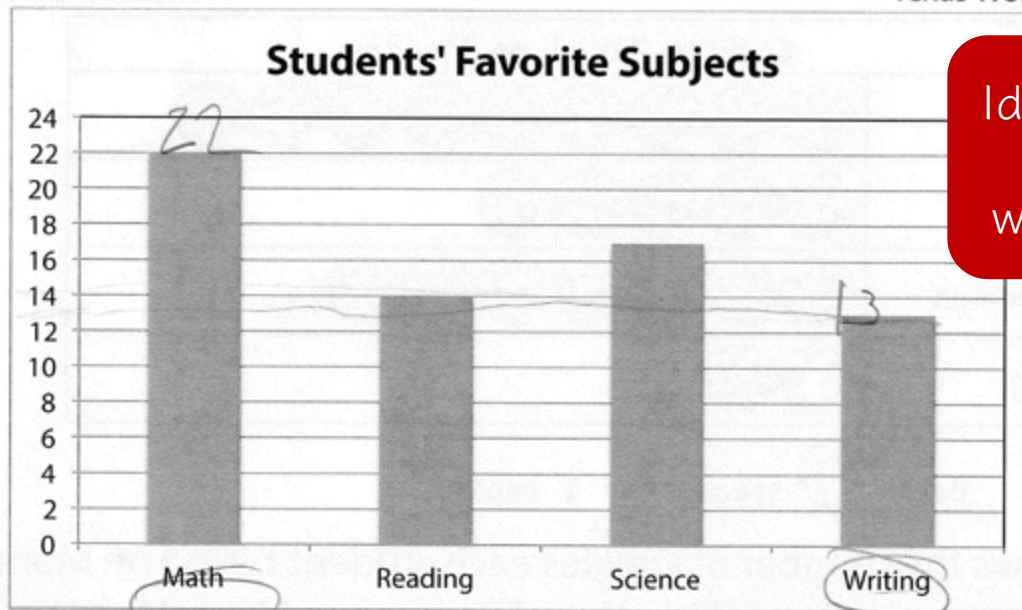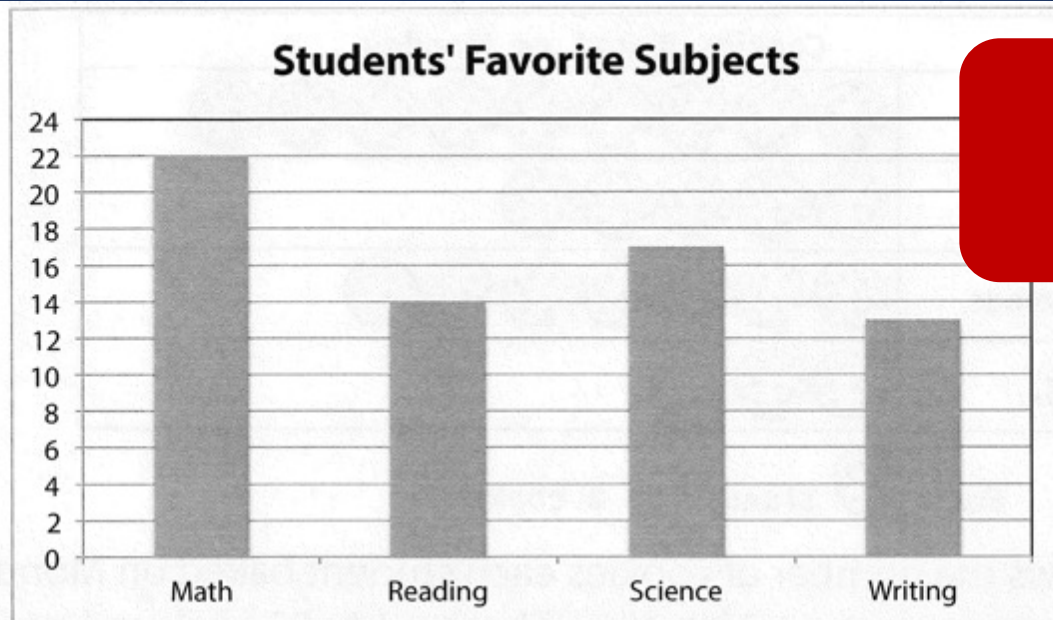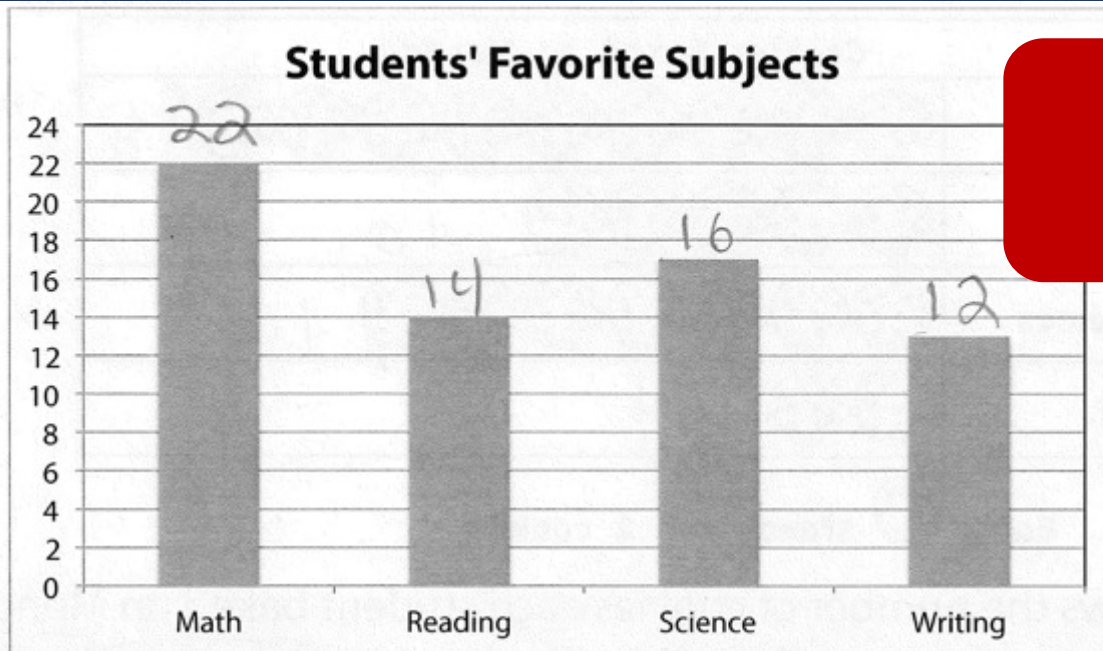
$$
\begin{array}{r}
1\ \cancel{2}\cancel{2}\ 12 \\
-\ 1\ 3 \\
\hline
0
\end{array}
$$

Correct

Undetermined

Didn't provide answer

Wrong (misapplied) operation

Wrong information

Subtraction error

MATH

Undetermined

Didn't provide answer

Repeat information from problem

Wrong information

Wrong (misapplied) operation

Addition error

Subtraction error

MATH

Word Problems? No Problem!

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Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.

- $\frac{1}{3}$ are covered with chocolate chips.

- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

C 25

D 20

Solve the problem

What skills are necessary to solve this problem?

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.

- $\frac{1}{3}$ are covered with chocolate chips.

- $\frac{3}{10}$ are covered with coconut.

- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

C 25

D 20

How would you solve this problem?

What skills are necessary to solve this problem?

MA+H

Problem-Solving Difficulties

Teaching Problem Solving

Reading the problem

Understanding the vocabulary

Identifying important information

Ignoring irrelevant information

Reading charts and graphs

Selecting operation(s)

Doing the computation(s)
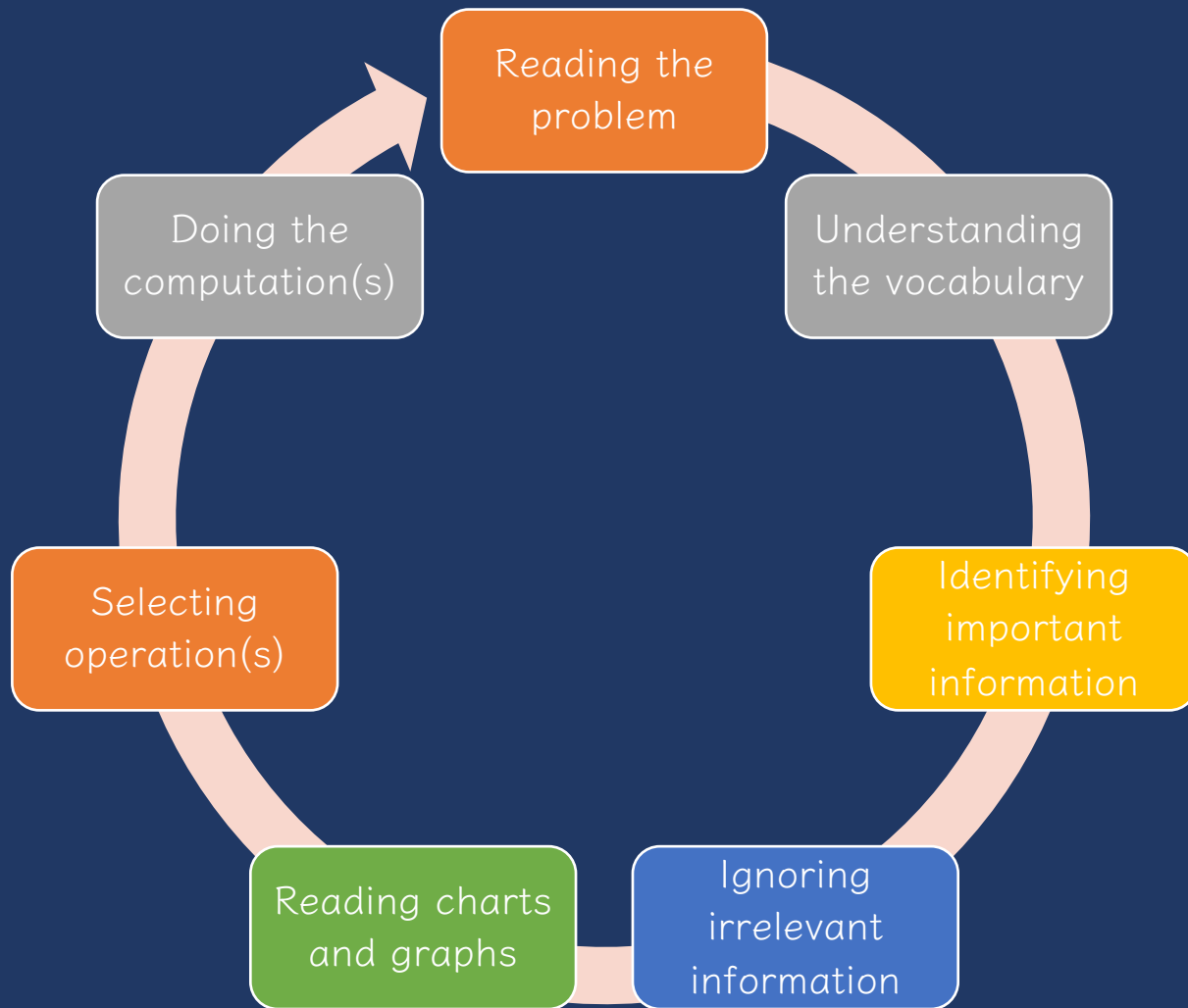
Ineffective Strategies

1.Keywords tied to operations

Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?

Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords[a]		Multiple keywords[a]		Keyword(s) led to correct solution[a]	
	n	%	*n*	%	*n*	%	*n*	%	*n*	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

[a]When a problem featured a keyword.

Mr. Rivera's taxable income is $20 each hour before taxes are taken out.
Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of $11.50.

- The video game cost $4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?

M✕A+H

Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema[a]		Any keyword		Keyword(s) led to correct solution[b]	
	n	%	*n*	%	*n*	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

[a]Sum across schemas does not equal 100 because each word problem featured more than one schema.

[b]When a problem featured a keyword.

Important notes about keywords

Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")

But, *do not* tie a keyword to a specific operation!

MA+H

2. Presenting problems by operation

Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. How many books does Noah have in all?

2. Bonnie found 8 rocks on her sidewalk and 7 rocks in her backyard. How many rocks did Bonnie find in all?

3. Edward had 5 toy cars. He got 8 more toy cars. How many toy cars did Edward have in all?

4. Mariela collected 11 feathers. Then she found 3 more. How many feathers did Mariela have in all?

5. LaMonte made 14 cookies. Then he made 5 more cookies. How many cookies did LaMonte have in all?

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LONG DIVISION WORD PROBLEMS

1. Zookeeper Al wants to give each monkey in the zoo an equal number of bananas. There are 37 monkeys in the zoo and 567 bananas. How many bananas does each monkey get? And How many are left over for him to eat himself?

2. Betty has oranges and needs to pack them up equally in 23 boxes. How many oranges go in each box and how much does she have left over?

3. Miss King has 1376 pages of scrap paper. She wants to make them into scrap paper packets for her 32 students. How many pages will each packet have? How many extra pages will she have left over?

4. Mr. Chong has 1,440 pages of scrap paper. He instead wants to make packets of 40 pages each but forgets to check if that will be enough for his 37 students. Will there be enough packets per student? If not how much more scrap paper does he need?

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Effective Strategies

Teaching Problem Solving

Have an attack strategy
Teach word-problem schemas

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.

- $\frac{1}{3}$ are covered with chocolate chips.

- $\frac{3}{10}$ are covered with coconut.

- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

A 100

B 33

C 25

D 20

What was your process for working through this problem?

Attack Strategy

SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.

UPS✓

UNDERSTAND
Read and explain.

PLAN
How will you solve the problem?

SOLVE
Set up and do the math!

✓CHECK
Does your answer make sense?

Have an attack strategy

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development.
Solve the equation.

Have an attack strategy

RICE

Read and record the problem.
Illustrate your thinking.
Compute.
Explain your thinking.

STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve the problem.

Review your answer.

Have an attack strategy

SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.

Have an attack strategy

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.

SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

MATH

Have an attack strategy

Share your favorite attack strategy.

Combining an Attack Strategy with a Focus on Schemas

Teach word-problem schemas

Total

Equal Groups

Difference

Comparison

Change

Ratios/Proportions

Additive Word Problems

Meanings of Addition

Meanings of Subtraction

Addition: Total (Part-Part-Whole, Combine)

Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

Total

Parts put together into a total

Karly saw 4 cardinals and 5 blue jays. How many birds did Karly see?

$$4 + 5 = ?$$

Addition: Join (Change Increase)

Start with a set, add the other set, count sum

$$2 + 3 = 5$$

MATH
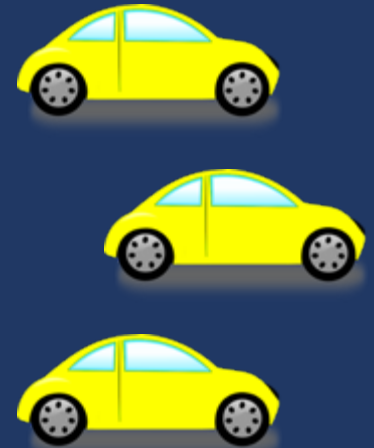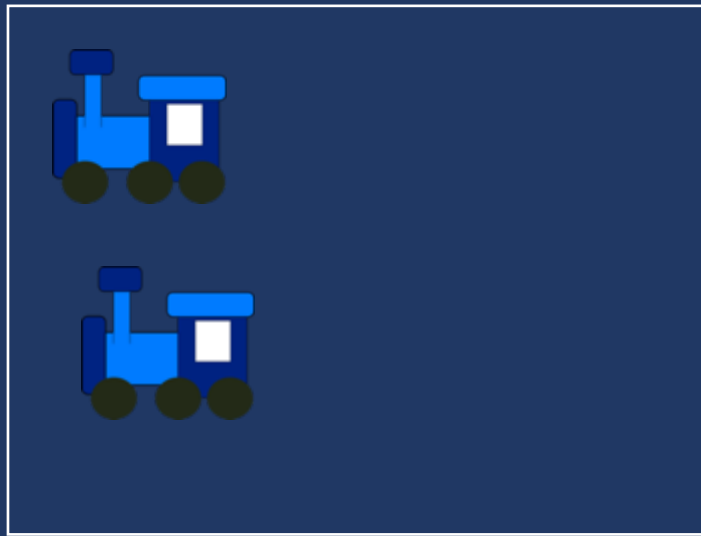
Change

An amount that **increases** or decreases

Premila had $4. Then they earned $3 for cleaning their room. How much money does Premila have now?

$$4 + 3 = ?$$

M A T H

Total Versus Change (Join)

$$3 + 9 = \underline{}$$

If you have brown eyes:

What's a Total story to show addition?

If you don't have brown eyes:

What's a Change/Join story to show addition?

Subtraction: Separate (Change Decrease)

Start with a set, take away from that set, count difference
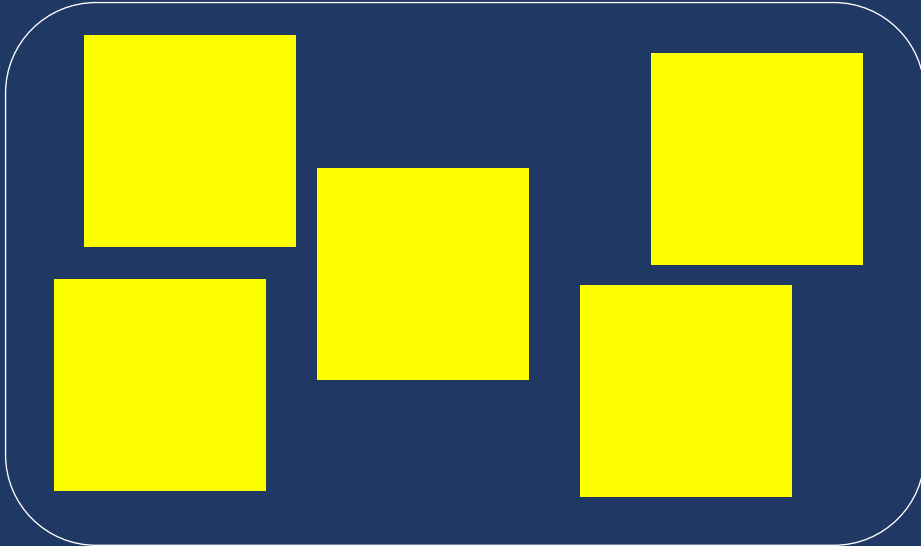
$$5 - 3 = 2$$

Change

An amount that increases or **decreases**

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

$$9 - 2 = ?$$

Subtraction: Difference (Compare)

Compare two sets, count difference
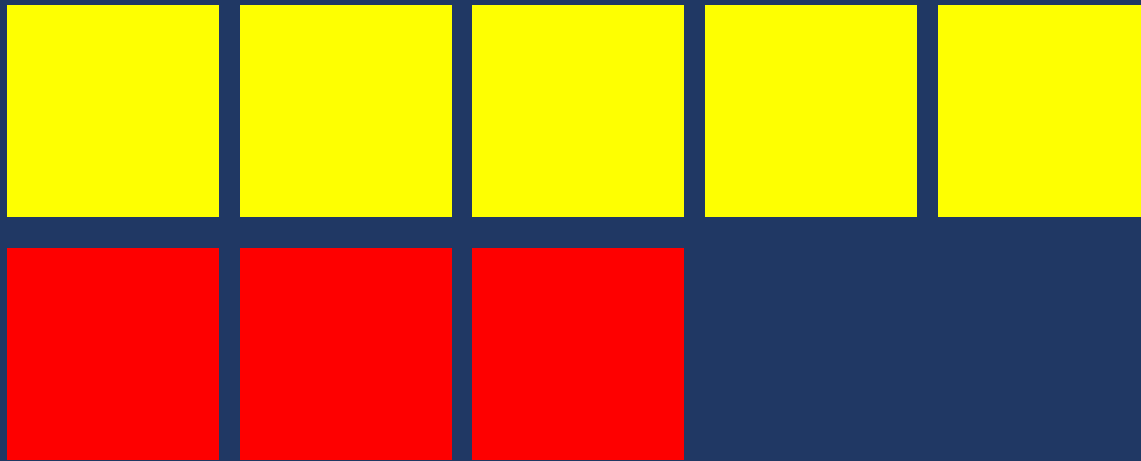
$$5 - 3 = 2$$

Difference

Greater and less amounts compared for a difference

Rachel has 9 pencils. Jodie has 4 pencils. How many more pencils does Rachel have? (How many fewer does Jodie have? What's the difference between Rachel's and Jodie's pencils?)

$$9 - 4 = ?$$

MATH

Change (Separate) versus Difference

$$9 - 5 = \underline{\quad}$$

If you weren't born in Texas:

What's a Change/Separate story to show subtraction?

If you were born in Texas:

What's a Difference story to show subtraction?

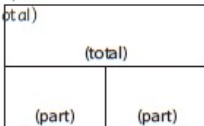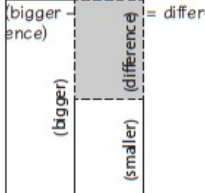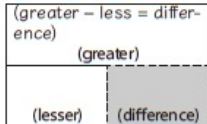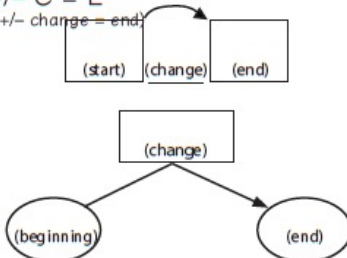| Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total) (total) (part) (part)	**Sum unknown:** Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	**Part unknown:** Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		**More than two parts:** Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B − s = D (bigger − ence) = differ- (bigger) (smaller) (difference) G − L = D (greater − less = difference) (greater) (lesser) (difference)	**Difference unknown:** Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	**Bigger/greater unknown:** Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	**Smaller/lesser unknown:** Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/− C = E (start +/− change = end) (start) (change) (end) (change) (beginning) (end)	**End (increase) unknown:** Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now? **End (decrease) unknown:** Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	**Change (increase) unknown:** Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting? **Change (decrease) unknown:** Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	**Start (increase) unknown:** Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with? **Start (decrease) unknown:** Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	**Multiple changes:** Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?

Powell & Fuchs (2018).
Material collected from: Griffin & Jitendra, 2009; Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, 2002; Kintsch & Greeno, 1985; Van de Walle, Karp, & Bay-Williams, 2013.

Total

Parts put together into a **total**

Daniela saw **3** canoes and **8** kayaks. How many boats did Daniela see?

Daniela saw **11** boats. If **3** of the boats were canoes, how many were kayaks?

Daniela saw **11** boats. **8** of the boats were kayaks, how many were canoes?

Total

Part

Part

"Are parts put together for a total?"

$$P1 + P2 = T$$

(total)

(part)   (part)

MA+H
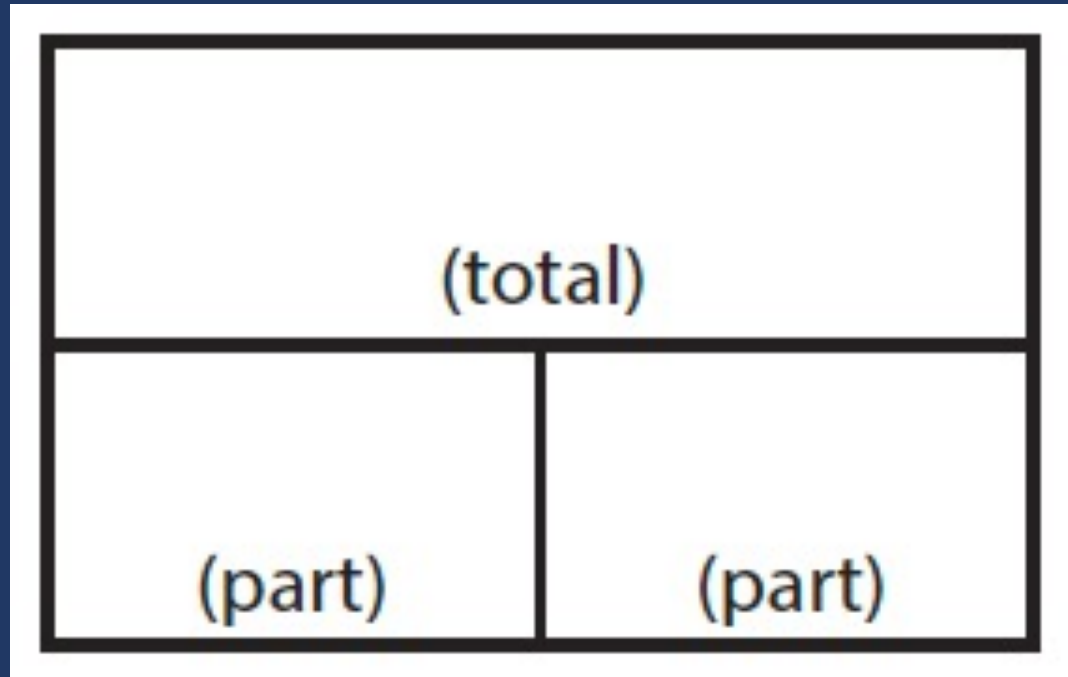
Total

Additive Word Problems

A.
Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.
In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.
Sam mows lawns and made $560 last week. She made $95 on Monday, $135 on Tuesday, and $70 on Wednesday. How much did Sam make on Thursday and Friday?

NOTES ABOUT TOTAL PROBLEMS:

Total

Write a Total problem with an unknown total.

Write a Total problem with an unknown part.

(Label the type on the back of the card.)

Difference

Greater and lesser amounts compared for a difference

Adrianna has 10 pencils. Tracy has 4 pencils. How many more pencils does Adrianna have?

Difference

Adrianna has 6 more pencils than Tracy. If Tracy has 4 pencils, how many does Adrianna have?

Greater amount

Tracy has 6 fewer pencils than Adrianna. Adrianna has 10 pencils. How many pencils does Tracy have?

Lesser amount

Total

"Are parts put together for a total?"

Difference

"Are amounts compared for a difference?"

MATH

Difference

$$G \quad - \quad L \quad = \quad D$$

(bigger) (difference) (smaller)

(greater) (lesser) (difference)

Fuchs et al. (2008); Griffin & Jitendra (2009)

MA+H
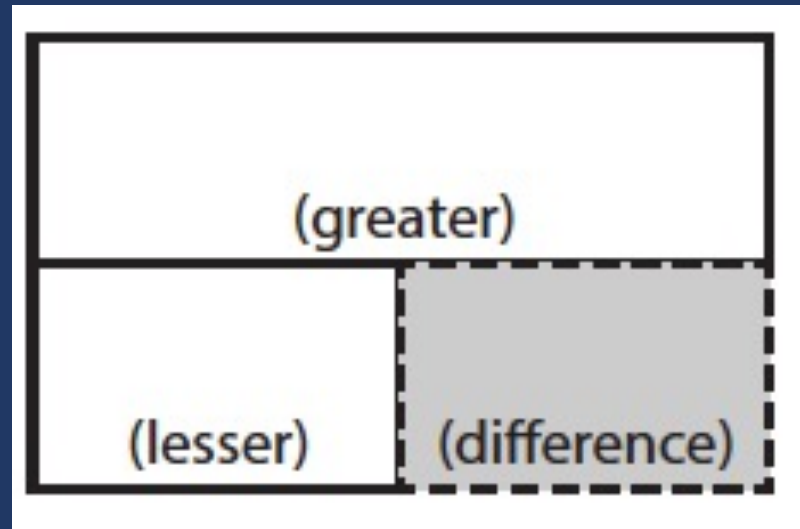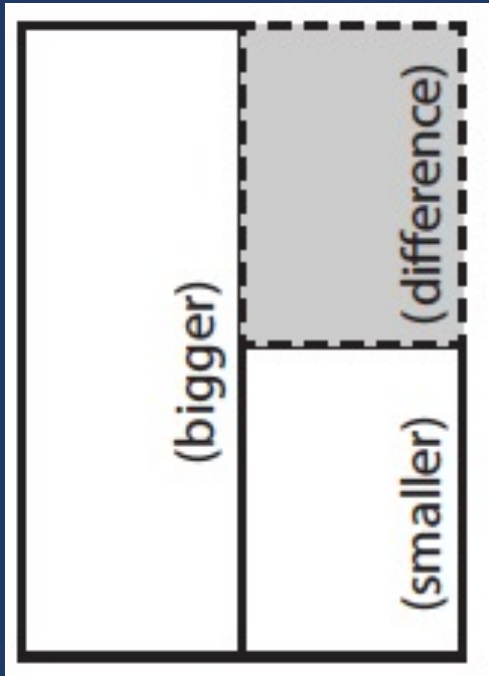
Difference

Additive Word Problems

D.
Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?

E.
Damian's dog eats 5 1/2 cups of dog food each week. Monte's dog eats 4 1/2 cups more each week than Damian's dog. How much does Monte's dog eat in a week?

F.
The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?

NOTES ABOUT DIFFERENCE PROBLEMS:

Difference

Write a Difference problem with an unknown difference.

Write a Difference problem with an unknown greater OR lesser amount.

(Label the type on the back of the card.)

M∆+H

Change

Join

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

End amount

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Change amount

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

Start amount

MATH

An amount that increases or **decreases**

Samantha baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Samantha eat?

Change amount

Samantha baked some cookies. She ate 3 of the cookies and has 17 cookies left. How many cookies did Samantha bake?

Start amount

MA+H

Total

"Are parts put together for a total?"

Difference

"Are amounts compared for a difference?"

Change

"Does an amount increase or decrease?"

MATH

Change

$$ST \quad +/- \quad C \quad = \quad E$$

(start) (change) (end)

(change)

(beginning) (end)

MATH
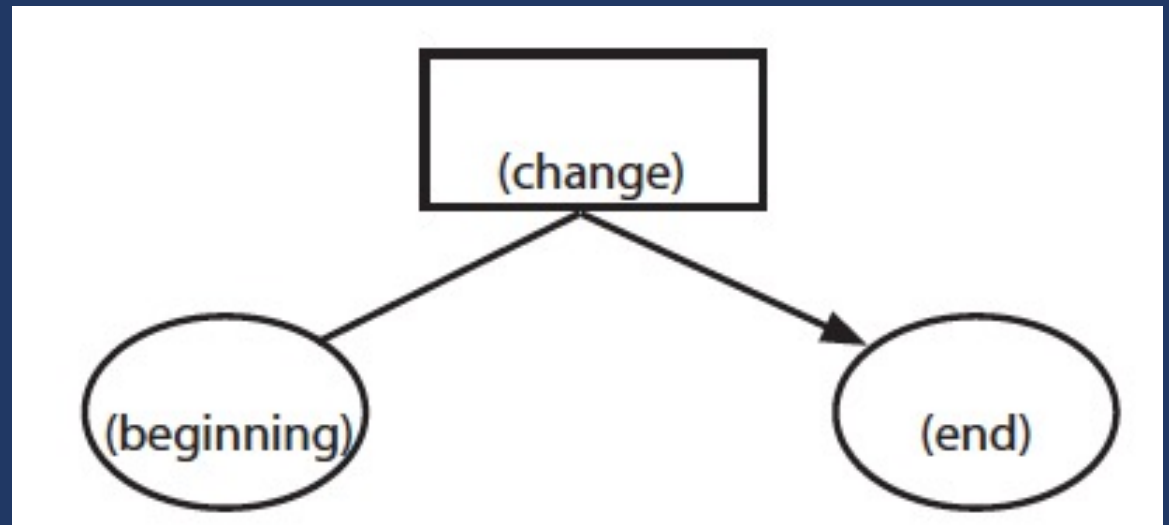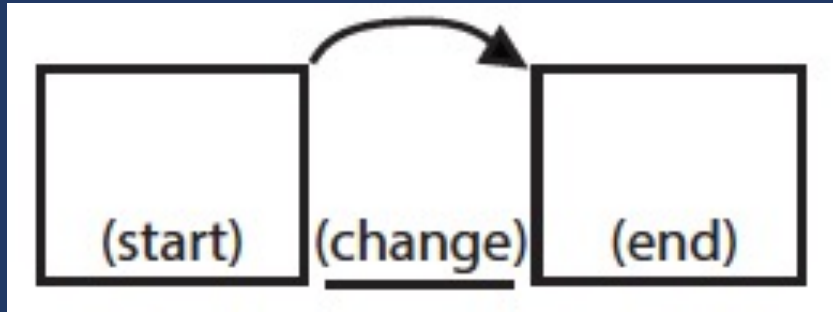
Change

Additive Word Problems

G.
A plant was 3 3/4 inches tall at the beginning of June. By the end of July, the plant was 9 1/8 inches tall. How many inches did the plant grow in 2 months?

H.
Martina has some money in her bank account. Then, she spent $135.69 and has a balance of -$24.80. How much money did Martina have to begin with?

I.
Hui saved $70 in January. In February, she spent $64 of the money she saved. She saved $92 more in March. How much has Hui saved by the end of March?

NOTES ABOUT CHANGE PROBLEMS:

Change

Write a Change problem with an unknown end amount. (Increase or decrease!)

Write a Change problem with an unknown change amount. (Increase or decrease!)

Write a Change problem with an unknown start amount. (Increase or decrease!)

(Label the type on the back of the card.)

MA+H

Schema Quiz Time!

Pablo goes to a stamp show where he can share, buy, and sell stamps.

26. Part A

The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.

Recycled Plastic

March   April   May   June   July

Each ▯ means 20 pounds.
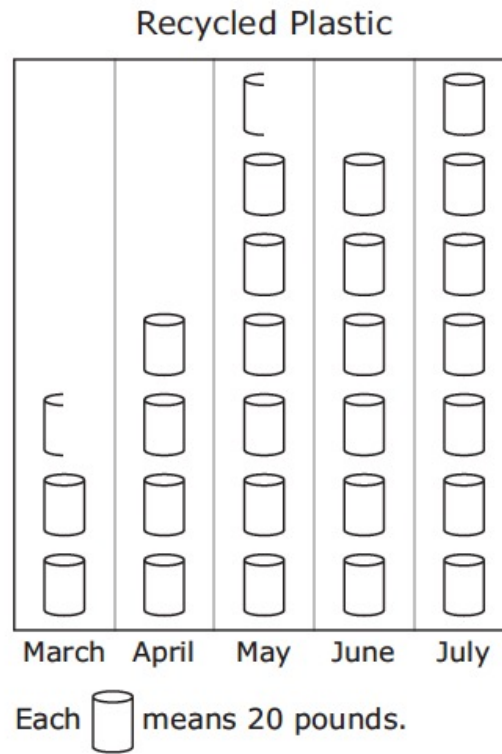
Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Total

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.

Package Delivery

10. **Part A**

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

Ⓐ 300

Ⓑ 340

Ⓒ 350

Ⓓ 360

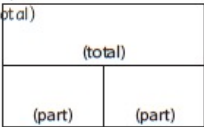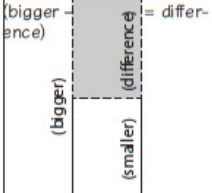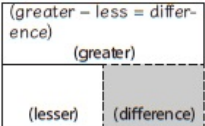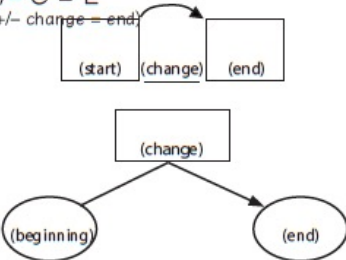| Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total) (total) / (part) (part)	**Sum unknown:** Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	**Part unknown:** Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		**More than two parts:** Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B − s = D (bigger − smaller = difference) G − L = D (greater − less = difference)	**Difference unknown:** Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	**Bigger/greater unknown:** Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	**Smaller/lesser unknown:** Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/− C = E (start +/− change = end) (start) (change) (end) (change) (beginning) (end)	**End (increase) unknown:** Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now? **End (decrease) unknown:** Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	**Change (increase) unknown:** Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting? **Change (decrease) unknown:** Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	**Start (increase) unknown:** Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with? **Start (decrease) unknown:** Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	**Multiple changes:** Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?

Powell & Fuchs (2018).
Material collected from: Griffin & Jitendra, 2009; Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, 2002; Kintsch & Greeno, 1985; Van de Walle, Karp, & Bay-Williams, 2013.

MATH

Multiplicative Word Problems

Meanings of Multiplication

Meanings of Division

Multiplication: Equal Groups

Show the groups, show the amount for each group, count product

$$3 \times 2 = 6$$
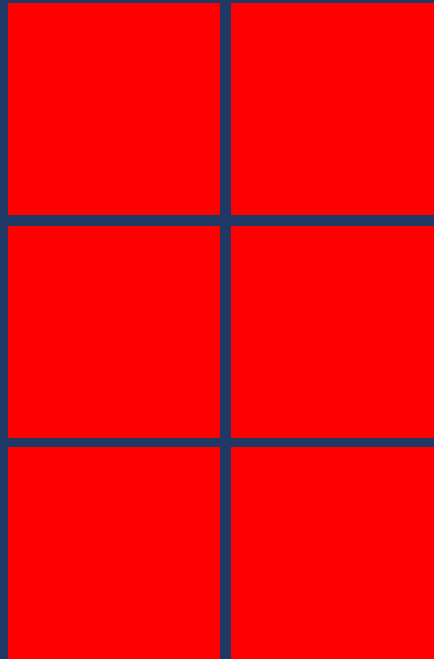
Groups multiplied by **number in each group** for a **product**

- Rhiannon has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Rhiannon have altogether?

$$2 \times 6 = ?$$

Multiplication: Comparison

Show a set, then multiply the set

$$3 \times 2 = 6$$

Set multiplied by a number of times for a product

- Vivienne picked 6 apples. Jessica picked 2 times as many apples as Vivienne. How many apples did Jessica pick?

$$6 \times 2 = \,?$$

Equal Groups versus Comparison

$$2 \times 5 = \underline{}$$

If you have glasses on:

What's an Equal Groups story to show multiplication?

If you don't have glasses on:

What's a Comparison story to show multiplication?

Division

90 division basic facts

- Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

$$8 \div 4 = 2$$

(dividend) (divisor) (quotient)

MA+H

Division: Equal Groups (Partitive Division)

Show the dividend, divide equally among divisor, count quotient

$$8 \div 2 = 4$$

Division: Equal Groups (Quotative Division)

Show the dividend, make groups of the divisor, count groups
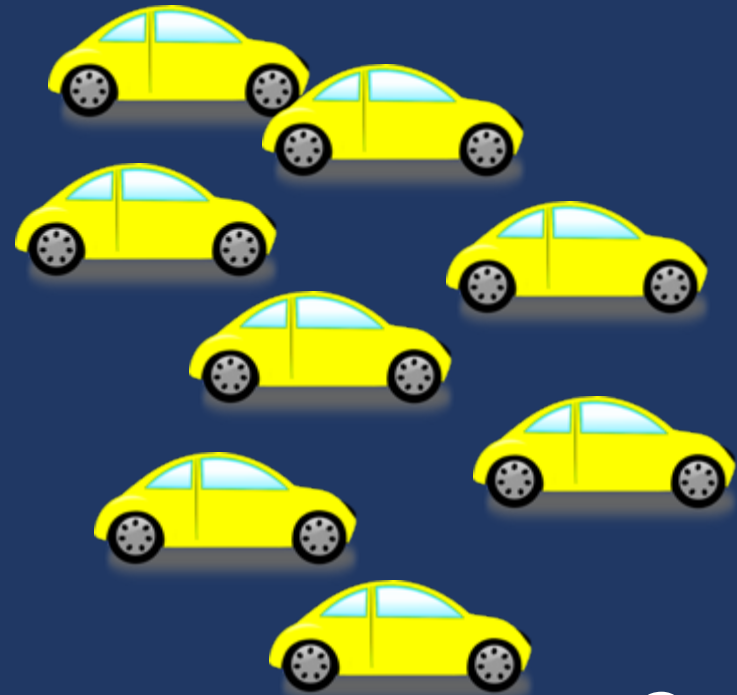
$$8 \div 2 = 4$$

Equal Groups

Groups multiplied by number in each group for a product

- Stefanie has 12 apples. She wants to share them equally among her 2 friends. How many apples will each friend receive?

$$2 \times ? = 12$$

- Nicole has 12 apples. She put them into bags containing 6 apples each. How many bags did Nicole use?

$$? \times 6 = 12$$

Partitive versus Quotative

$$12 \div 4 = \underline{}$$

If you watched *Friends*:

What's a Partitive story to show division?

If you watched *Seinfeld*:

What's a Quotative story to show division?

Teach word-problem schemas

Total		Equal Groups
Difference		**Comparison**
Change		**Ratios/Proportions**

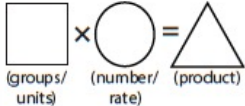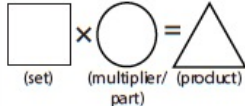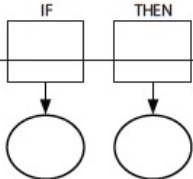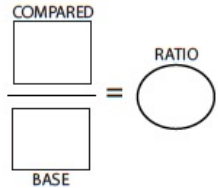| Schema and Definition	Graphic Organizers	Examples				Variations
Equal Groups (Vary) A number of equal sets or units	□ × ○ = △ (groups/ units) (number/ rate) (product)	**Product unknown:** Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	**Groups unknown:** Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	**Number unknown:** Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?		**With rate:** Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set	□ × ○ = △ (set) (multiplier/ part) (product) IF THEN □ □	**Product unknown:** Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	**Set unknown:** Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	**Times unknown:** Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?		**With fraction:** Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions	↓ ↓ ○ ○ COMPARED □ ————— = ○ RATIO □ BASE	**Subject unknown:** Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes? **Base unknown:** Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	**Object unknown:** Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words? **Compared unknown:** Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	**Ratio unknown:** Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?		**With percentage:** Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly? **With unit rate:** Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

Material collected from: Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.

Equal Groups

Groups multiplied by number in each group
for a product

Toni has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Toni have altogether?

Toni has **24** crayons. They want to place them equally into **2** boxes. How many crayons will Toni place in each box?

Toni has **24** crayons. They put them into boxes with **12** crayons each. How many boxes did Toni use?

Groups

Number in each group

Product

MATH

"Are there groups with an equal number in each group?"

MATH

Equal Groups

$$GR \times N = P$$

Equal Groups

Multiplicative Word Problems

A.
Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?

B.
Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light blubs did Jane buy?

C.
Zachary has 3 feet of string. He makes braclets, and each bracelet needs 5 1/4 inches of string. How many bracelets could Zachary make?

NOTES ABOUT EQUAL GROUPS PROBLEMS:

Equal Groups

Write an Equal Groups problem with an unknown product.

Write an Equal Groups problem with unknown groups.

Write an Equal Groups problem with an unknown number within each group.

(Label the type on the back of the card.)

Comparison

Set multiplied by a number of times for a product

Brooke ran 6 minutes. Shaleeni ran 4 times longer than Brooke. How many minutes did Shaleeni run?

Set

Number of times

Product

Equal Groups

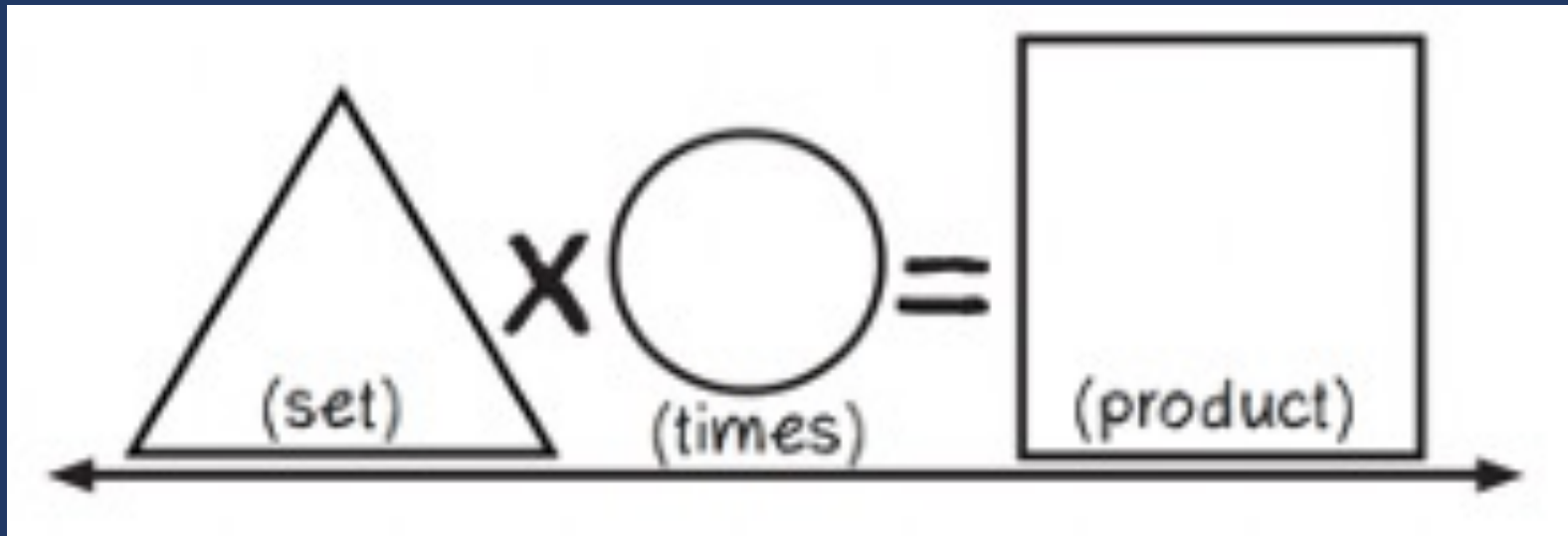"Are there groups with an equal number in each group?"

Comparison

"Is a set compared a number of times?"

MA+H

$$S \times T = P$$

(set) × (times) = (product)

MATH

Comparison

Multiplicative Word Problems

D.
Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?

E.
Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?

NOTES ABOUT COMPARISON PROBLEMS:

F.
Sally typed 56 words in 2 minutes. At this rate, how many words could Sally type in 7 minutes?

G.
An airplane's altitude changed –378 feet over 7 minutes. What was the mean change of altitude in feet per minute?

Comparison

Write a Comparison problem with an unknown product.

(Label the type on the back of the card.)

MATH

Ratios/Proportions

Description of relationships among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

Xin et al. (2005)

Equal Groups

"Are there groups with an equal number in each group?"

Comparison

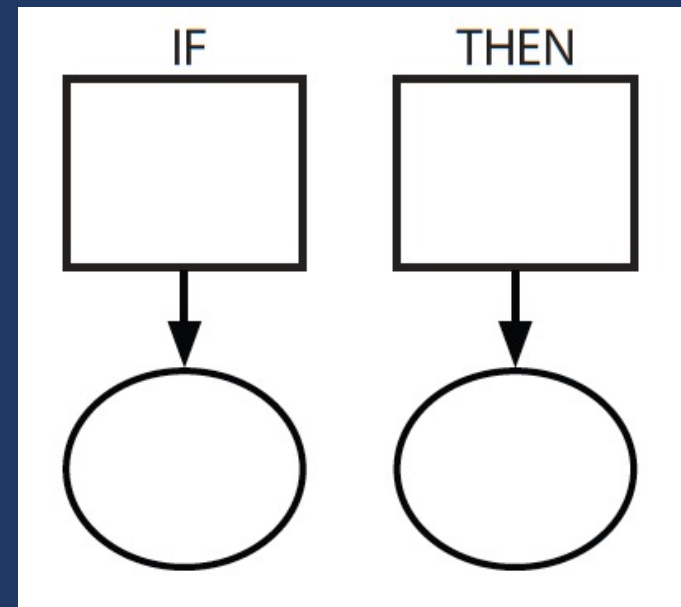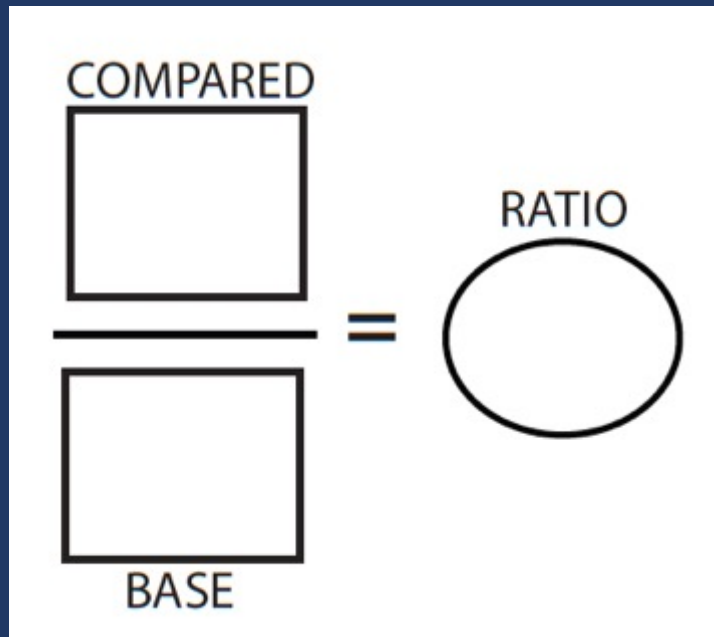"Is a set compared a number of times?"

Ratios/Proportions

"Are there relationships among quantities - if this, then this?"

Ratios/Proportions

Description of relationships among quantities

Ratios/Proportions

Multiplicative Word Problems

H.
The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?

I.
Sara buys a sweater at a department store. The sweater costs $30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.

F.
Sally typed 56 words in 2 minutes. At this rate, how many words could Sally type in 7 minutes?

G.
An airplane's altitude changed -378 feet over 7 minutes. What was the mean change of altitude in feet per minute?

rita baked cookies and brownies. The of cookies to brownies was 3:5. If she 25 brownies, how many cookies did ke?

NOTES:

Write a Ratios problem.

Write a Proportions problem.

(Label the type on the back of the card.)

Schema Quiz Time!

MA+H

Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

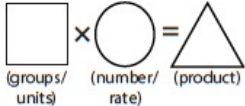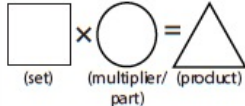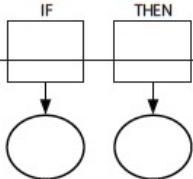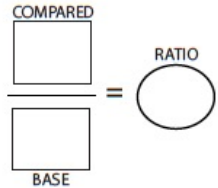What is the total number of granola bars Mr. Kowolski ordered?

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?

Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

Grade 4 Smarter Balanced

MATH

Schema and Definition	Graphic Organizers	Examples				Variations
Equal Groups (Vary) A number of equal sets or units	□ × ○ = △ (groups/ units) (number/ rate) (product)	**Product unknown:** Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	**Groups unknown:** Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	**Number unknown:** Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?		**With rate:** Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set	□ × ○ = △ (set) (multiplier/ part) (product) IF THEN	**Product unknown:** Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	**Set unknown:** Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	**Times unknown:** Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?		**With fraction:** Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions	COMPARED ── = ○ RATIO BASE	**Subject unknown:** Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	**Object unknown:** Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?			**With percentage:** Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
		Base unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	**Compared unknown:** Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	**Ratio unknown:** Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?		**With unit rate:** Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

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M✕A+H

Teach word-problem schemas

Total

Equal Groups

Difference

Comparison

Change

Ratios/Proportions

| Total | Difference | Change | Equal Groups | Comparison | Ratio/ Proportion |

Multi-Step Problems

Use your attack strategy

Use word-problem schemas

MATH

If needed, number the graph.

U
- **U**nderstand by reading
- **U**nderline the label

P
- (**P**arentheses) needed numbers
- **P**ut the numbers in order

S
- **S**chema(s)

Total	**Equal Groups**
Difference	**Equal Shares**
Change	

- **S**olve

✓²
- **C**heck the number answer
- **C**heck the label answer

MA+H

Multi-Step Problems

A.

Leslie had 3 pizzas. Each pizza was cut into 8 pieces. Leslie ate 2 pieces. How many pieces were left?

B.

Mr. Kahn gave away 8 blue balloons and 6 red balloons. He gave away 3 times the number of white balloons as red balloons. What was the total number of balloons Mr. Kahn gave away?

C.

An egg farm packages 264 total cartons of eggs each month. The farm has 3 different sizes of cartons.
The small carton hold 8 eggs, and 1/6 of the total cartons are small.
The medium carton holds 12 eggs, and 2/3 of the total cartons are medium.
The large carton holds 18 eggs, and the rest of the total cartons are large.

Determine how many each size of carton is needed each month. Then determine how many eggs are needed to fill the 264 cartons.

Multi-Step Problems

D.
Allison is saving to buy a $500 bicycle by working during summer vacation. The job pays her $8 for every 1 hour worked. Allison works exactly 20 hours each week. If she works for 4 weeks and buys the bicycle, how much money will she have left over?

E.
Jessica rented 1 video game and 3 movies for a total of $11.50. The video game cost $4.75 to rent. The movie costs the same amount each to rent. What amount, in dollars, did Jessica pay to rent each movie?

F.
Sal exercised by stretching and jogging 5 days last week. He stretched for a total of 25 minutes during the week. He jogged for an equal number of minutes each of the 5 days. He exercised for a total of 240 minutes.

Elena also exercised by stretching and jogging 5 days last week. She stretched for a total of 15 minutes during each day. She jogged for an equal number of minutes each of the 5 days. She exercised for a total of 300 minutes.

Determine the number of minutes Sal jogged each day last week and the number of minutes Elena jogged each day last week.
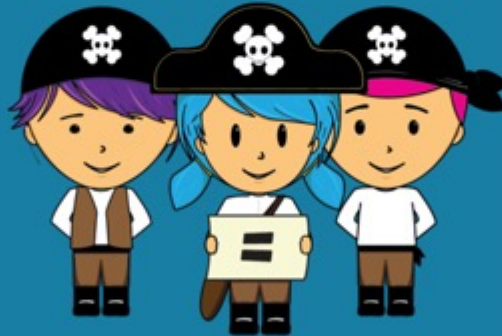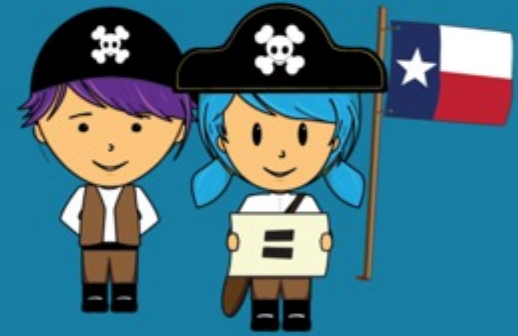
National Center on
INTENSIVE INTERVENTION
at American Institutes for Research ■

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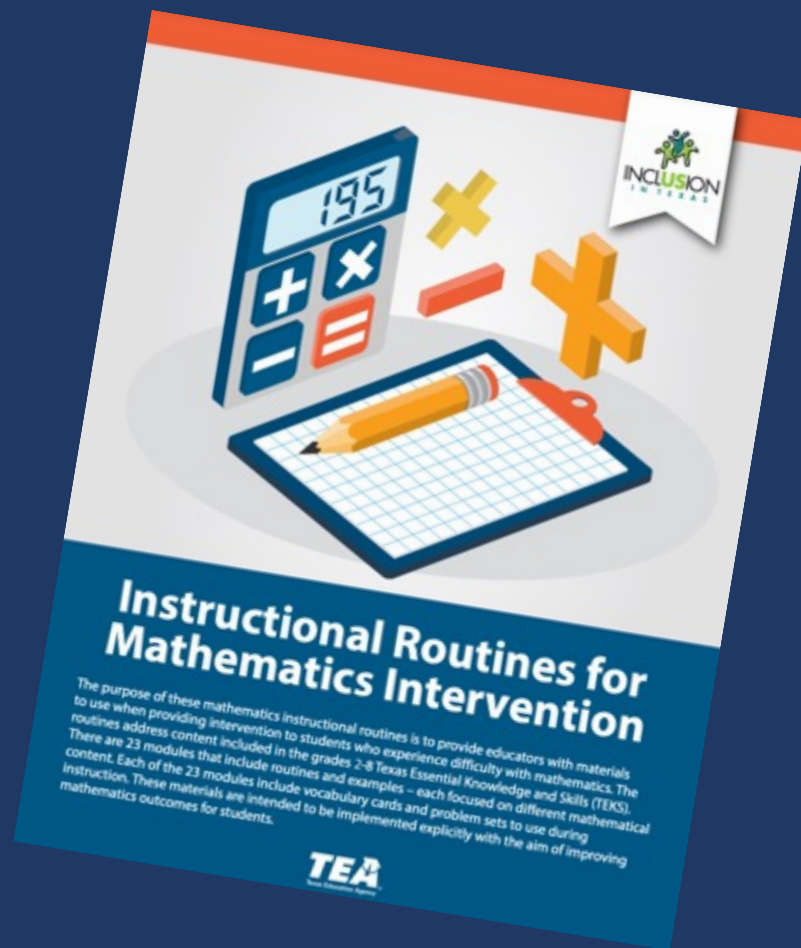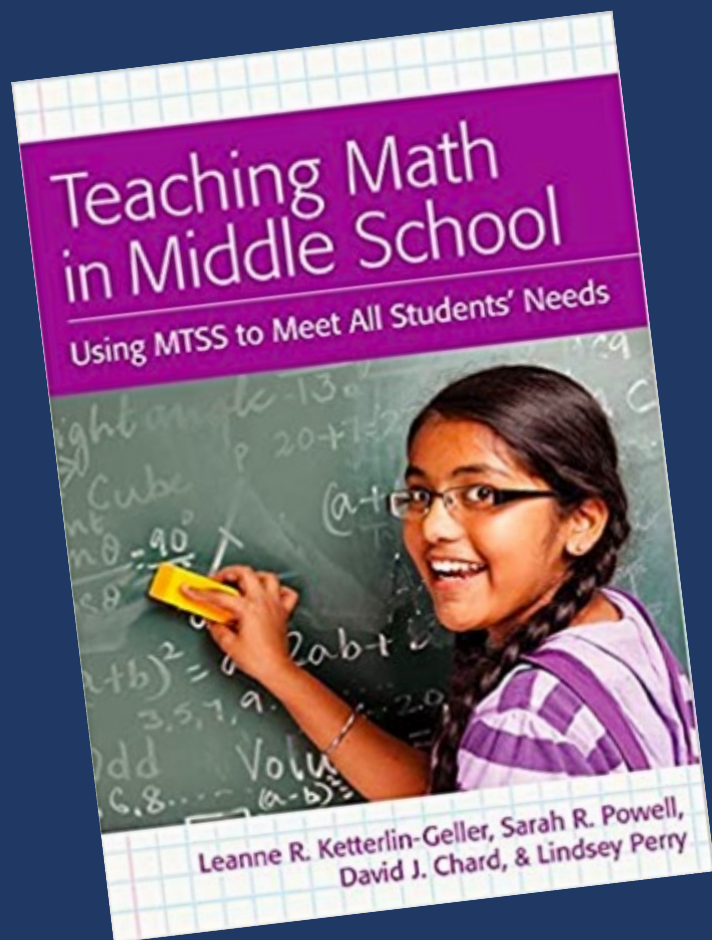# Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a high-leverage practice in special education , and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEEDAR Center , developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.

MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES

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