## Best Practices for Fluency and Vocabulary January 12, 2023

## MA+:

$M A+1:$

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## Say hello.

Describe the mathematics you support.

When you see this icon, it's time to use the Chat Box.

Raise your Zoom hand at anytime.

Drop questions in the Chat Box at anytime.

## $0 \quad 0$ $\square$ Schedule for Today

| November | Word-Problem Solving |
| :--- | :--- |
| January 12th | Fact Fluency <br> Computational Fluency <br> Mathematical Language |
| January | Elementary Check-in <br> Data-Based Decision Making |
| January | Secondary Check-in <br> Data-Based Decision Making |
| February | Communities of Practice |
| March | Communities of Practice |
| April | Communities of Practice |

## Increase the fact fluency of your students

## Increase the computational fluency of your students

Increase the focus on mathematics vocabulary



## Instructional Platform

INSTRUCTIONAL DELIVERY


INSTRUCTIONAL STRATEGIES
Fluency building
Problem solving
instruction


## Building Fluency

Fluency is doing mathematics easily and accurately.

Fluency in mathematics makes mathematics easier.

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both conceptual learning and procedural learning.


Fact Fluency

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## 100 addition facts

Single-digit addends sum to a single- or double-digit number

> | 5 | (addend) |
| ---: | :--- |
| +4 | (addend) |
| 9 | $($ sum $)$ |

## Total

## Addition

Count one set, count another set, put sets together, count sum


$$
2+3=5
$$

## Change

## Addition

Start with a set, add the other set, count sum


$$
2+3=5
$$

## Total

## Parts put together into a total

Karly saw 4 cardinals and 5 blue jays. How many birds did Karly see?

## Addition

## Addition

An amount that increases or decreases

Premila had \$4. Then they earned \$5 for cleaning their room. How much money does Premila have now?
$3+9=$

If you have brown eyes: What's a Total story to show addition?
If you don't have brown eyes: What's a Change/Join story to show addition?

100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

> (minuend)
> (subtrahend)
> (difference)

## Change

Start with a set, take away from that set, count difference

$$
5-3=2
$$

## Difference

Compare two sets, count difference


$$
5-3=2
$$

## Change

An amount that increases or decreases

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

## Difference

Greater and lesser amounts compared for a
difference

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)

## Subtraction

If you would chose beaches: What's a Change/Separate story to show subtraction?
If you would chose mountains: What's a Difference story to show subtraction?
$x A+H$

100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

$$
\begin{aligned}
2 & \text { (factor) } \\
\times 3 & \text { (factor) } \\
\hline 6 & \text { (product) }
\end{aligned}
$$

Show the groups, show the amount for each group, count product

$3 \times 2=6$

## Equal Groups

Show the groups, show the amount for each group, count product
$3 \times 2=6$

## Comparison

Show a set, then multiply the set


$$
3 \times 2=6
$$

Groups multiplied by number in each group for a product

Rhiannon has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Rhiannon have altogether?

## Comparison

Set multiplied by a number of times for a product

Vivienne picked 12 apples. Jessica picked 2 times as many apples as Vivienne. How many apples did Jessica pick?

## $2 \times 5=$

## If you wear glasses:

 What's an Equal Groups story to show multiplication? If you don't wear glasses: What's a Comparison story to show multiplication?
## Division

## 90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

$$
\begin{array}{cccc}
8 & \div & 4 & 2 \\
\text { (dividend) } & \text { (divisor) } & \text { (quotient) }
\end{array}
$$

## Equal Groups (Partitive Division)

Show the dividend, divide equally among divisor, count quotient


앙

## 0



아

$$
8 \div 2=4
$$

Show the dividend, make groups of the divisor, count groups

$$
8 \div 2=4
$$

## Equal Groups

Groups multiplied by number in each group for a product
Stefanie has 12 apples. She wants to share them equally among her 2 friends. How many apples will each friend receive?

Nicole has 12 apples. She put them into bags with 6 apples each. How many bags did Nicole use?

## $12 \div 4=$

If you watch Stranger Things: What's a Partitive story to show division?

If you watch Ted Lasso:
What's a Quotative story to show division?

## Addition <br> Subtraction

Multiplication
Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$
\begin{array}{r}
5 \\
+\quad 8 \\
\hline
\end{array} \begin{array}{r}
6 \\
\times \quad 7 \\
\\
\hline
\end{array} \quad 56
$$







| Addition | Subtraction |
| :---: | :---: |
| Multiplication | Division |

What fact fluency is important for your students?
What are five ways to help students build fact fluency?



## Traditional



## Partial Sums

A.

$$
\begin{array}{r}
74 \\
+\quad 18 \\
\hline 80 \\
+12 \\
\hline 92
\end{array}
$$

в. 725

$$
\begin{array}{r}
365 \\
+1,000 \\
+\quad 80 \\
+\quad 10 \\
\hline 1,090
\end{array}
$$

Opposite Change
A. $74 \xrightarrow{-4} 70$

$$
+18 \frac{+4}{\rightarrow+22} 92
$$

8. $725 \stackrel{+5}{\longrightarrow} 730$
$+365^{-5} \xrightarrow{\frac{5}{1,960}}$


## Traditional



## Partial Differences

$$
\text { A. } \begin{array}{r}
62 \\
-\quad 17 \\
\hline+50 \\
-\quad 5 \\
\hline 45
\end{array}
$$

в. 305
-96
-300
$+300$
-9 0
$-1$
209

## $\square$

232 $\begin{array}{r}232 \\ -\quad 164 \\ \hline\end{array}$

## Same Change

$$
\text { A. } \begin{array}{r}
62 \stackrel{+3}{\longrightarrow} 65 \\
-\quad 17 \xrightarrow{+3-20} \\
\hline 45
\end{array}
$$

8. $\quad 305 \xrightarrow{+4} 309$
$-96 \xrightarrow{+4}-100$

232
$\begin{array}{r}232 \\ -\quad 164 \\ \hline\end{array}$

Add Up


Traditional

| 1 |  |
| ---: | ---: |
| $x$ | 11 |
| 24 | 132 |
| $\times 43$ |  |
| 172 | $\times 53$ |
| +960 | +6900 |
| 1,032 | 6,996 |

## Partial Products

A.
24
$\times, 43$
$\times 800$
60
60
$\begin{array}{r}160 \\ +\quad 12 \\ \hline 1,032\end{array}$
B.
132
53
$\times \quad 00$
5000
1500
300
90
66
$+\quad 6996$


Area (Array)



Lattice




Traditional


Partial Quotients
A. $12 \begin{array}{r}158 \\ -120 \\ \begin{array}{r}38 \\ -36 \\ 2\end{array} \\ \\ \\ \\ \\ \hline 10 \mathrm{RZ}\end{array}$

$$
\text { B. } \begin{array}{r}
4 \begin{array}{r}
8970 \\
-680 \\
\hline 290 \\
-170 \\
\hline 1720 \\
102 \\
\hline 18
\end{array} \\
\hline 20 \\
\hline 28 R 18
\end{array}
$$

Lattice


Division as Fractions



| Addition | Subtraction |
| :---: | :---: |
| Multiplication | Division |

What computational fluency is important for your students? What are your plans for creating practice time for students to practice computational fluency?

Mathematical
Language

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Mathematical Language




1. Some math terms are shared with English but have different meanings

right
degree
2. Some math terms are shared with English but have different meanings
3. Some math words are shared with English with similar meanings (but a more precise math meaning)

4. Some math terms are shared with English but have different meanings
5. Some math words are shared with English with similar meanings (but a more precise math meaning)
6. Some math terms are only used in math

7. Some math terms are shared with English but have different meanings
8. Some math words are shared with English with similar meanings (but a more precise math meaning)
9. Some math terms are only used in math
10. Some math terms have more than one meaning
$\square$

Rubenstein \& Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
variable vs.
variably cloudy
6. Some math terms are shared with English but have different meanings
7. Some math words are shared with English with similar meanings (but a more precise math meaning)
8. Some math terms are only used in math
9. Some math terms have more than one meaning
10. Some math terms are similar to other content-area terms with different meanings
11. Some math terms are homographs

## eight vs. ate

sum vs. some


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3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
factor vs. multiple
hundreds vs. hundredths
```
numerators vS. denominator
```

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
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4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings

## mesa vs.

tabla

1. Some math terms are shared with English but have different meanings
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3. Some math terms are only used in math
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8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
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14. Some math terms are similar to other content-area terms with different meanings
15. Some math terms are homographs
16. Some math terms are related but have distinct meanings
17. An English math term may translate into another language with different meanings
skip count vs. multiples
18. English spelling and usage may have irregularities
19. Some math concepts are verbalized in more than one way
20. Some math terms are shared with English but have different meanings
21. Some math words are shared with English with similar meanings (but a more precise math meaning)
22. Some math terms are only used in math
23. Some math terms have more than one meaning
24. Some math terms are similar to other content-area terms with different meanings
25. Some math terms are homographs
26. Some math terms are related but have distinct meanings
27. An English math term may translate into another language with different meanings
28. English spelling and usage may have irregularities

## rhombus vs. diamond

10. Some math concepts are verbalized in more than one way
11. Informal terms may be used for formal math terms
12. Some math terms are shared with English but have different meanings
13. Some math words are shared with English with similar meanings (but a more precise math meaning)
14. Some math terms are only used in math
15. Some math terms have more than one meaning
16. Some math terms are similar to other content-area terms with different meanings
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18. Some math terms are related but have distinct meanings
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21. Some math concepts are verbalized in more than one way
22. Informal terms may be used for formal math terms

## Use formal math language

## Use terms precisely



What number is in the tens place?

What digit is in the tens place? What is the value of the digit in the tens place?

## 135

Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135 .
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30 .

The alligator eats the bigger number

## is less than OR

 is greater thanWhy this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.




Four point seven Four point oh seven

$$
\begin{array}{r}
4.7 \\
4.07
\end{array}
$$

Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.


flips, slides, turns


## reflections, translations, rotations

Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.



Identify examples of "Instead of $\qquad$ , say
$\qquad$ .

## Use formal math language

## Use terms precisely




Improper fraction Proportion

$$
\frac{8}{5}
$$

Mixed number

$$
1 \frac{3}{5}
$$

Proper fraction $\frac{2}{9}$

$$
\frac{2}{5}=\frac{8}{20}
$$

Ratio
$4: 3$
Unit fraction
$\frac{1}{6}$


Equation $9 x-4=7 x$
Expression 9x - 4
Formula $a^{2}+b^{2}=c^{2}$
Function $\quad f(x)$
Inequality $9 x-4>6 x$

## Quadrilaterals

Kite


Parallelogram


Rectangle $\square$

Rhombus


Square


Trapezoid


## Acute angle

Obtuse angle $\xrightarrow[\text { angle }]{\longrightarrow}$


Straight angle


## Acute triangle <br> 

Obtuse triangle


Right triangle


## Equilateral triangle

 $\Delta$Isosceles triangle


Scalene triangle


C

Adjacent angles


Alternate angles

Complementary angles


Corresponding angles


Supplementary angles

$$
=180^{\circ}
$$



Vertical angles


Congruent figures
$\square_{\text {Similar figures }}^{\square}$






I



## Use formal math language

## Use terms precisely



## 1. Use explicit instruction

##  <br> Over $\mathbf{1 5 0 , 0 0 0}$ in Print <br> explicit

Effective and Efficient Teaching

ANITAL. ARCHER
CHARLES A. HUGHES

## 2. Use graphic organizers



Dunston \& Tyminski (2013)
2. Use graphic organizers


## Dunston \& Tyminski (2013)


6. Equal: having the same amount or value.


## 4. Have students create glossaries

## Integer Definitions

Zero Pairs
A positive and negative cancel one another;

Positive
A number that is greater than zero.

## Absolute

Value
The distance of a number from zero on a number line; shown as ||

Negative
A number that is
less than zero. Identified by a minus sign.

Numerator: how many parts of the whole

Ex. 10
Odd number: a number not divided evenly by 2

- Ex. 1, 3, 5, 7, 9....

Percent: a specific number in comparison to 100 - 74\%

Polygon: any enclosed shape that is made up of 3 or more straight lines


## 5. Create a word wall



## 6. Preview vocabulary



Bay-Williams \& Livers (2009)

## 7. Cluster vocabulary

|  | Length | Weight |
| :---: | :---: | :---: |
| Meaning | How long something is | How heavy something is |
| Visual | 1 Yard | 2000 pounds $=1$ ton |
|  |  |  |
|  |  |  |

Livers \& Bay-Williams (2014)

## 7. Cluster vocabulary

| Rating | Word | Definition | Synonym(s) | Example | Sample Problem |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | $e x)^{\text {(esin }}$ | a mathematical phrase combining operations, numbers and/or variables. | phrase <br> algebraic expression | $\begin{array}{cc} 6 & \text { い'equal- } \\ 6 n & \text { no } \\ 6+n & \text { sign: } \end{array}$ | Lucia earns \$8 per haur for babysilting and gets a $\$ 5$ tip. Write an expression to represent the amount she would earn if she worked for $x$ hours. |
| $2$ | joishor | a quantity that can change ortake many values. <br> (refers to the letter orsymbol representing the quantity) | unknown |  | The variable $x$ vepresents the number of hours charlie work in a week. Write an expression to vepresent his earnings if he carns $\$ 9$ per park |
| 1 | $p^{1000^{x}}$ | the result when two or more numbers are multiplied | total <br> answer | $\begin{array}{r} 3 \times 2=\underset{\uparrow}{\uparrow} \\ \text { product } \end{array}$ | The product of 6 and a number is 24. What is the number? |
| 3 |  | the result of a division crefers to the number of times the divisor divides the dividend) | answer | $\begin{aligned} & 18 \div 2=9 \\ & 2 \sqrt{18}<\text { quitient } \end{aligned}$ | Estimate the quotient when 365 is divided by 12. |

## 8. Use mnemonics



Riccomini et al. (2015)


VOCABULARY CROSSWORD
ANSWERKEY


## 10. Use technology




## Increase the fact fluency of your students

## Increase the computational fluency of your students

Increase the focus on mathematics vocabulary

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