Best Practices for Fluency and Vocabulary January 12, 2023





Sarah R. Powell, Ph.D.

Associate Professor The University of Texas at Austin







srpowell@utexas.edu







Say hello.

Describe the mathematics you support.





When you see this icon, it's time to use the Chat Box.

Raise your Zoom hand at anytime.

Drop questions in the Chat Box at anytime.





November	Word-Problem Solving
January 12th	Fact Fluency Computational Fluency Mathematical Language
January	Elementary Check-in Data-Based Decision Making
January	Secondary Check-in Data-Based Decision Making
February	Communities of Practice
March	Communities of Practice
April	Communities of Practice



Increase the fact fluency of your students

Increase the computational fluency of your students

Increase the focus on mathematics vocabulary



Instructional Platform













Fluency Building



Building Fluency

Fluency is doing mathematics easily and accurately. Fluency in mathematics makes mathematics easier. Fluency provides less stress on working memory. Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both conceptual learning and procedural learning.





Knowing

formulas

Fact Fluency



Fluency	
Addition	Subtraction
Multiplication	Division







100 addition facts

Single-digit addends sum to a single- or double-digit number







Addition

Count one set, count another set, put sets together, count sum



2 + 3 = 5





Addition

Start with a set, add the other set, count sum



2 + 3 = 5



Parts put together into a total

Karly saw 4 cardinals and 5 blue jays. How many birds did Karly see?



An amount that **increases** or decreases

Premila had \$4. Then they earned \$5 for cleaning their room. How much money does Premila have now?



3 + 9 = 1

If you have brown eyes: What's a Total story to show addition? If you don't have brown eyes: What's a Change/Join story to show addition?



100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number



(minuend) (subtrahend) (<u>difference</u>)





Subtraction

Start with a set, take away from that set, count difference



5 - 3 = 2





Compare two sets, count difference



Subtraction

5 - 3 = 2



An amount that increases or **decreases**

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?



Greater and lesser amounts compared for a difference

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)



Subtraction

If you would chose beaches:
What's a Change/Separate story to show subtraction?
If you would chose mountains:
What's a Difference story to show subtraction?



- 5

100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product





Multiplication

Show the groups, show the amount for each group, count product



$3 \times 2 = 6$



Show the groups, show the amount for each group, count product



$3 \times 2 = 6$





Show a set, then multiply the set

0 1 2 3 4 5 6 7 8 9 10 II

$3 \times 2 = 6$



Multiplication

Groups multiplied by number in each group for a product

Rhiannon has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Rhiannon have altogether?



Set multiplied by a number of times for a product

Vivienne picked 12 apples. Jessica picked 2 times as many apples as Vivienne. How many apples did Jessica pick?



 $2 \times 5 =$

Multiplication

If you wear glasses:
What's an Equal Groups story to show multiplication?
If you don't wear glasses:
What's a Comparison story to show multiplication?





90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

 $8 \div 4 = 2$ (dividend) (divisor) (quotient)



Equal Groups

(Partitive Division)

Division

Show the dividend, divide equally among divisor, count quotient





Equal Groups

(Quotative Division)

Division

Show the dividend, make groups of the divisor, count groups




Groups multiplied by number in each group for a product

Stefanie has 12 apples. She wants to share them equally among her 2 friends. How many apples will each friend receive?

Nicole has 12 apples. She put them into bags with 6 apples each. How many bags did Nicole use?



$12 \div 4 =$

If you watch Stranger Things: What's a Partitive story to show division?
If you watch Ted Lasso: What's a Quotative story to show division?



Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor





Fact Fluency Building Strategies



Computational Fluency Building Strategies



Cover, Copy, Co	ompare			Taped Pro	blems	
9	8 × 6		6 × 5	8 × 6	7 × 9	6 <u>× 8</u>
x 0 54 7	48 6		9 × 8	8 × 5	7 × 8	6 × 6
× 8 56 9	$3_{6+3=}$ 1+7=	File Folder	7 × 7	6 × 9	5 × 9	8 × 4
× 9 81 6	$x = \frac{6+4=}{7+3=}$ 2+7= 5+6=		$\frac{1}{10} \qquad 9$	6 × 9	9 × 5	8 × 7
× 7 42	4+7 = 7+8 = 6+7 =	1	11 6 11 6 × 7 5	8 × 8	4 × 8	5 × 7
8 <u>× 8</u> 64	7 + 9 = 7 + 6 = 8 + 7 =	1. 16 13	3			
	7 + 0 = 9 + 6 =	15 7				
	6 + 0 = 6 + 8 =	15 6 14				
pyriaht 2023 Sarah R. Powe	II. Ph.D.				X A	

























Addition	Subtraction
Multiplication	Division



What fact fluency is important for your students?What are five ways to help students build fact fluency?



Computational Fluency









Traditional



■ ■ 725 <u>+365</u> **1,090**

227

185

Partial Sums

Α. 74 + 18 80 +12 92

в. 725 <u>+ 365</u> 1,000 10 1,090

227

185

+

Opposite Change

^B 725
$$\xrightarrow{+5}$$
 730
+ 365 $\xrightarrow{-5}$ + 360
I,090







Traditional

ø^l2 Α. 7

в.

29 \$\$95 - 96 20

232

164

Partial Differences

^{A.} 62	^{в.} 305
<u> </u>	<u> </u>
+50	+300
- 5	-90
45	-1
15	209

232

164

Same Change

62 +3 65 17 +3 -20 Α.

305 +4, 309 - 96 +4, 100 в. 209



Add Up

96 Β, 17 305 Α. 62 $\begin{array}{r}
 100 \\
 300 \\
 200 \\
 305 \\
 + 5
 \end{array}$ $\begin{array}{r} 20 & 3 \\ 60 & 40 \\ 62 & + 2 \\ 45 \end{array}$ 96 17 209

232

164







Traditional

1 24 132 Α. в. × 43 • • 72 • 960 1,032 × 53 396 +6600 6996

13

47

Partial Products



13

47

Area (Array)

A
$$24$$

 $\times 43$
 100
 100
 123
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 100
 1000
 1000
 1000
 1000
 1000
 1000
 1000
 1000
 1000

Lattice



13

47







Traditional

$$\begin{array}{r} 13 R2 \\
 12)158 \\
 -12 \\
 38 \\
 -36 \\
 2
\end{array}$$

$$\begin{array}{r} 8 \\
 34)8970 \\
 -68 \\
 28 \\
 -68 \\
 28 \\
 -68 \\
 -272 \\
 18 \\
\end{array}$$

804

•

12

Partial Quotients

804

12

Lattice

<u>**13</u> R2** 12)158</u> ^{в.} 34)970 12 0,1 34 0 5 B

28 R 18

Ο

804

12

Division as Fractions

804

12

Fact Fluency Building Strategies



Computational Fluency Building Strategies



Addition	Subtraction
Multiplication	Division



What computational fluency is important for your students? What are your plans for creating practice time for students to practice computational fluency?



Mathematical Language



Mathematical Language

Instead of that	Say this











1. Some math terms are shared with English but have different meanings



Rubenstein & Thompson (2002)


- 1. Some math terms are shared with English but have different meanings
- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)



Rubenstein & Thompson (2002)



- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)
- 3. Some math terms are only used in math



Rubenstein & Thompson (2002)



- 1. Some math terms are shared with English but have different meanings
- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)
- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning



Rubenstein & Thompson (2002)



- 1. Some math terms are shared with English but have different meanings
- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)
- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings







- 1. Some math terms are shared with English but have different meanings
- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)
- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs







- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)
- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

- 6. Some math terms are homographs
- 7. Some math terms are related but have distinct meanings

factor vs. multiple hundreds vs. hundredths

numerators vs. denominator

Rubenstein & Thompson (2002)



- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)
- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

- 6. Some math terms are homographs
- 7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

mesa vs. tabla

Rubenstein & Thompson (2002)



- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)
- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

- 6. Some math terms are homographs
- 7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)



- 1. Some math terms are shared with English but have different meanings
- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)
- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

- 6. Some math terms are homographs
- 7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

10. Some math concepts are verbalized in more than one way

skip count vs. multiples

XA++

one-fourth

vs. one

quarter

- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)
- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

- 6. Some math terms are homographs
- 7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

10. Some math concepts are verbalized in more than one way

11. Informal terms may be used for formal math terms

rhombus vs. diamond

> vertex vs. corner



- 1. Some math terms are shared with English but have different meanings
- 2. Some math words are shared with English with similar meanings (but a more precise math meaning)
- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning
- 5. Some math terms are similar to other content-area terms with different meanings
- 6. Some math terms are homographs
- 7. Some math terms are related but have distinct meanings
- 8. An English math term may translate into another language with different meanings
- 9. English spelling and usage may have irregularities
- 10. Some math concepts are verbalized in more than one way
- 11. Informal terms may be used for formal math terms



Use formal math language

Use terms precisely







What number is in the tens place?

What digit is in the tens place? What is the value of the digit in the tens place?

Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.

135

• Emphasizes that 3 is part of the number 135 with a value of 30.



The alligator eats the bigger number

is less than OR is greater than

Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.







top number and bottom number

numerator and denominator

Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.





rename OR find equivalent OR simplify

Why this is important...

• Reducing suggests that the quantity or magnitude of the new number will be less than the original number.



Four point seven Four point oh seven

Four and seven tenths Four and seven hundredths

Why this is important...

• Accurately shares the magnitude of the decimal.

4.7 4.07

• Emphasizes place value.













reflections, translations, rotations

Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.





minute hand and hour hand

Why this is important...

- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.



Mathematical Language

Instead of that	Say this	



Identify examples of "Instead of _____, say



Use formal math language

Use terms precisely



		1 4 / 1 . 1	D
Use	lerms	With	Precision



Strategies for Teaching Mathematics Language



Factor
1 × 8 = 8
2 × 4 = 8

$$f_{a_{cto_{r}}}$$
 $f_{a_{cto_{r}}}$
Multiple
8 × 1 = 8
8 × 2 = 16
multiples of 8
Multiples of 8



Improper fraction 8 5	Proportion $\frac{2}{5} = \frac{8}{20}$
Mixed number 1-3-5	Ratio 4:3
Proper fraction 2 9	Unit fraction $\frac{1}{6}$
	D







Equation
$$9x - 4 = 7x$$

Expression $9x - 4$
Formula $a^2 + b^2 = c^2$
Function $f(x)$
Inequality $9x - 4 > 6x$










































Use Terms With Precision

Strategies for Teaching Mathematics Language



Discuss terms you want your students to use with precision.



Use formal math language

Use terms precisely



Use Terms With Precision Strategies for Teaching Mathematics Language





1. Use explicit instruction





2. Use graphic organizers





Dunston & Tyminski (2013)



2. Use graphic organizers

Word	Lightbulb Word		
Definition	Picture		

Dunston & Tyminski (2013)



3. Have students create vocabulary cards



6. Equal: having the same amount or value.





4. Have students create glossaries



Numerator: how many parts of the whole



Odd number: a number not divided evenly by 2

- Ex. 1, 3, 5, 7, 9....

Percent: a specific number in comparison to 100

- 74%

Polygon: any enclosed shape that is made up of 3 or more straight lines





5. Create a word wall





6. Preview vocabulary

Dear Feisty Fifth Graders,

Today we have multiple opportunities to do exciting projects! For example, we are going to be doing a science experiment to see how the tilt of a ramp relates to how far a matchbox car will roll. There are several factors we will be looking at in this experiment. I look forward to hearing multiple ideas on how to set up this experiment.

One other thing that factors into our day is that we have an assembly before lunch. We will get to hear music from the high school play. I think we will hear multiple songs.

Sincerely, Ms. Livers

Here is a problem to start your day... in my letter I have used two words that are important math words for today's lesson. Can you find them and tell what they mean in this letter and what they mean when talking about numbers? (Answer this in your math notebook)

Bay-Williams & Livers (2009)



7. Cluster vocabulary



Livers & Bay-Williams (2014)



7. Cluster vocabulary

Rating	Word	Definition	Synonym(s)	Example	Sample Problem
2	expression	a mathematical phrase combining operations, numbers and/or variables.	Phrase algebraic expression	6 6n no equal- 6+n sign	Lucia earns \$8 per hour for babysilting and gets a\$5 tip. Write an expression to represent the amount she would earn if she worked
2	Josiable	a quantity that can Change ortake many Values. (refers to the letter or symbol representing the quantity)	Unknown	× D У T	The Variable x vepresents the number of hours charlie works in a week. Write an expression to represent his earnings if he earns \$9 per hours
1	Product	the result when two or more numbers are multiplied	total answer	3 × 2 = 6 T product	The <u>product</u> of 6 and a number is 24. What is the number?
3	quotient	the result of a clivision (refers to the number of times the divisor divides the dividend)	answer	$18 \div 2 = 9$ $9 \div 9$ 2)18 quotient	Estimate the quotient when 365 is divided by 12.

Marin (2018)



8. Use mnemonics



Riccomini et al. (2015)



9. Do word games









10. Use technology





Math Lingo



Use Terms With Precision

Strategies for Teaching Mathematics Language

Discuss your strategy for focusing on mathematical language in your teaching.



Increase the fact fluency of your students

Increase the computational fluency of your students

Increase the focus on mathematics vocabulary



Sarah R. Powell, Ph.D.

Associate Professor The University of Texas at Austin







srpowell@utexas.edu



