How to Design and Deliver Effective Math Intervention
evidence-based practice

evidence-based intervention

evidence-based strategy

promising practice

no or negative evidence
Evidence-Based Mathematics Practices

WWC Practice Guides:

- Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools

COMING SOON (2020-2021):

- Assisting Students Struggling with Mathematics: Intervention in the Elementary and Middle School Grades

https://ies.ed.gov/ncee/wwc/PracticeGuides
Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools

Taking early action may be key to helping students struggling with mathematics. The eight recommendations in this guide are designed to help teachers, principals, and administrators use Response to Intervention for the early detection, prevention, and support of students struggling with mathematics.

1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.

2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8.

3. Instruction during the intervention should be explicit and systematic.

4. Interventions should include instruction on solving word problems that is based on common underlying structures.

5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.

8. Include motivational strategies in tier 2 and tier 3 interventions.
Validated Intervention Program (e.g. Tier2, Standard Protocol, Secondary Intervention)

Progress Monitor

Diagnostic Academic Assessment/Functional Assessment

Intervention Adaptation

Progress Monitor

NONRESPONSIVE

RESPONSIVE

NONRESPONSIVE

RESPONSIVE
Where student IS

Where student NEEDS TO BE
Design
Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Use place value understanding to round whole numbers to the nearest 10 or 100.

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Compose and decompose numbers from 11 to 19 into ten ones and some further ones...
Solve addition and subtraction word problems, and add and subtract within 10...

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20...

Use addition and subtraction within 100 to solve one- and two-step word problems...

Use multiplication and division within 100 to solve word problems...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations...

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators...

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions...

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve real-world and mathematical problems leading to two linear equations in two variables.

Where student needs to be: When student is:

Solve addition and subtraction word problems, and add and subtract within 10...
Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Apply properties of operations as strategies to multiply and divide...

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...

Use multiplication and division within 100 to solve word problems.

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.

Fluently multiply multi-digit whole numbers using the standard algorithm.

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations...
An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

<table>
<thead>
<tr>
<th>K</th>
<th>1</th>
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<th>4</th>
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<tbody>
<tr>
<td></td>
<td>Represent and solve problems involving addition and subtraction</td>
<td>Represent and solve problems involving addition and subtraction</td>
<td>Represent &amp; solve problems involving multiplication and division</td>
<td>Represent &amp; solve problems involving multiplication and division</td>
<td>Understand the place value system</td>
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<td>Add and subtract within 20</td>
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<td>Add and subtract within 20</td>
<td>Add and subtract within 20</td>
<td>Perform operations with multi-digit whole numbers and decimals to hundredths</td>
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<td>Understand place value</td>
<td>Use place value understanding and properties of operations to add and subtract</td>
<td>Use place value understanding and properties of operations to add and subtract</td>
<td>Use place value understanding and properties of operations to perform multidigit arithmetic</td>
<td>Generalize place value understanding for multidigit whole numbers</td>
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<td><strong>Understand place value</strong></td>
<td>Measure and estimate lengths in standard units</td>
<td>Solve problems involving the four operations, and identify &amp; explain patterns in arithmetic</td>
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<td>Extend understanding of fraction equivalence and ordering</td>
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<td>Use place value understanding and properties of operations to add and subtract</td>
<td>Relate addition and subtraction to length</td>
<td>Solve problems involving measurement and estimation of intervals of time, liquid volumes, &amp; masses of objects</td>
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<td>Build fractions from unit fractions by applying and extending previous understandings of operations</td>
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<td>Measure lengths indirectly by iterated length units</td>
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<td>Measure lengths indirectly by iterated length units</td>
<td>Understand decimal notation for fractions, and compare decimal fractions</td>
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* Indicates a cluster that is well thought of as a part of a student’s progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia’s draft documents.

** Indicates an area on similarity ideas from geometry to show that triangles can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.
Table A.2. Grades 3–5 Curriculum Focal Points and Connections Compared with the Expectations of the Content Standards in *Principles and Standards for School Mathematics*

<table>
<thead>
<tr>
<th>Curriculum Focal Points and Connections</th>
<th>Expectations of the Content Standards</th>
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<tr>
<td>Grade 3 Curriculum Focal Points</td>
<td>Number and Operations, Grades 3–5</td>
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</table>
| **Number and Operations** and **Algebra:** Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts | - Understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals  
- Recognize equivalent representations for the same number and generate them by decomposing and composing numbers  
- Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and (in Grade 6 Curriculum Focal Points) as divisions of whole numbers  
- Use models, benchmarks, and equivalent forms to judge the size of fractions  
- Recognize and generate equivalent forms of commonly used fractions, decimals, and (in Grade 7 Curriculum Focal Points) percents  
- Explore numbers less than 1 by extending the number line and through familiar applications  
- Describe classes of numbers according to characteristics such as the nature of their factors  
- Understand various meanings of multiplication and division  
- Understand the effects of multiplying and dividing whole numbers  
- Identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems  
- Understand and use properties of operations, such as the distributivity of multiplication over addition  
- Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as $30 \times 50$ |
| Students understand the meanings of multiplication and division of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication, and successive subtraction, partitioning, and sharing for division). They use properties of addition and multiplication (e.g., commutativity, associativity, and the distributive property) to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts. By comparing a variety of solution strategies, students relate multiplication and division as inverse operations. |  |
| Number and Operations: Developing an understanding of fractions and fraction equivalence |  |
| Students develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models, including the number line, to identify equivalent fractions. |  |
| Geometry: Describing and analyzing properties of two-dimensional shapes |  |
| Students describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes. Students investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space and the use of those attributes and properties in solving problems, including applications involving congruence and symmetry. |  |
Deliver
Instructional Platform

**INSTRUCTIONAL DELIVERY**
- Explicit instruction
- Precise language
- Multiple representations

**INSTRUCTIONAL STRATEGIES**
- Fluency building
- Problem solving instruction

**Validated Intervention Program** (e.g., Tier 2, Standard Protocol, Secondary Intervention)

- Progress Monitor
- Diagnostic Academic Assessment/Functional Assessment
- Intervention Adaptation
- Progress Monitor

Nonresponsive → Responsive

Responsive → Nonresponsive
Instructional Platform

**INSTRUCTIONAL DELIVERY**

Explicit instruction

**INSTRUCTIONAL STRATEGIES**
<table>
<thead>
<tr>
<th>Modeling</th>
<th>Practice</th>
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<tr>
<td>Clear Explanation</td>
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<tr>
<td>Planned Examples</td>
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**Supports**
- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace
Goal and importance

“Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends.”

“Let’s continue working with our three-dimensional shapes and volume. Understanding volume and calculating volume helps with measuring capacity.”

Supports
- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace
"To solve 26 plus 79, I first decide about the operation. Do I add, subtract, multiply or divide?"
"The plus sign tells me to add. So, I’ll add 26 plus 79. I’ll use the partial sums strategy. First, I add 20 plus 70. What’s 20 plus 70?"
"20 plus 70 is 90. I write 90 right here under the equal line. Where do I write 90?"
"Then I add 6 plus 9. What’s 6 plus 9?"
"How did you add 6 plus 9?"
"6 plus 9 is 15. So, I write 15 here under the equal line."
"Finally, we add the partial sums: 90 and 15. 90 plus 15 is 105. So, 26 plus 79 equals 105. What’s 26 plus 79?"
“Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends.”

24 \div 6

28 \div 7

35 \div 5
“Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends.”

\[ 32 \div 8 \quad 42 \div 7 \quad 25 - 5 \]
## Modeling
- Clear Explanation
- Planned Examples

## Practice
- Guided Practice
- Independent Practice

### Supports
- Asking the right questions
- Eliciting frequent responses
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https://intensiveintervention.org/intensive-intervention-features-explicit-instruction
Teacher and student practice together

**Modeling**
- Clear Explanation
- Planned Examples

**Practice**
- Guided Practice
- Independent Practice

**Supports**
- Asking the right questions
- Eliciting frequent responses
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Student practices with teacher support

**Modeling**
- Clear Explanation
- Planned Examples

**Practice**
- Guided Practice
- Independent Practice

**Supports**
- Asking the right questions
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### Modeling
- Clear Explanation
- Planned Examples

### Practice
- Guided Practice
- Independent Practice

### Supports
- Asking the right questions
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<td>• Maintaining a brisk pace</td>
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Low-level and high-level

“What is 7 times 9?”

“How would you solve this problem?”

“Which shape has 6 sides?”

“What do you do when you see a word problem?”

“Why do you have to regroup?”

“How do you have to use zero pairs?”
Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

“Turn and discuss the formula for perimeter with your partner.”

“Write the multiplication problem on your whiteboard.”

“In your math journal, draw a picture to help you remember to term parallelogram.”
Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

“Good work using your word-problem attack strategy.”

“Let’s look at that again. Tell me how you added in the hundreds column.”
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**Supports**
- Asking the right questions
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**Low-level and high-level**
Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

**Affirmative and corrective**

**Planned and organized**
### Modeling
- Clear Explanation
- Planned Examples

### Practice
- Guided Practice
- Independent Practice

### Supports
- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace
Instructional Platform

INSTRUCTIONAL DELIVERY
- Explicit instruction
- Precise language

INSTRUCTIONAL STRATEGIES
1. Some math terms are shared with English but have different meanings

- base
- right
- degree
1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

Rubenstein & Thompson (2002)
1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

- trapezoid
- numerator
- parallelogram

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4. Some math terms have more than one meaning

- round
- square
- second
- base

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5. Some math terms are similar to other content-area terms with different meanings

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6. Some math terms are homographs

- eight vs. ate
- sum vs. some
- rows vs. rose
- base vs. bass

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7. Some math terms are related but have distinct meanings

- factor vs. multiple
- hundreds vs. hundredths
- numerators vs. denominator

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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings
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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

four vs. forty
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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

10. Some math concepts are verbalized in more than one way

(one-fourth vs. one quarter  
skip count vs. multiples)
1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

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7. Some math terms are related but have distinct meanings

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11. Informal terms may be used for formal math terms

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Rubenstein & Thompson (2002)
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### Models

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### Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Rubenstein & Thompson (2002)
Supporting Clear and Concise Mathematics Language

Instead of That, Say This

Elizabeth M. Hughes, Sarah R. Powell, and Elizabeth A. Stevens

Math Language in Middle School

Be More Specific

Sarah R. Powell, Elizabeth A. Stevens, and Elizabeth M. Hughes
...and the last one is 10

...8, 9, 10. We’ll stop counting there, but we could count more.

Why this is important...

- Suggests that 10 is the final or highest number.
- Provide opportunities to count beyond 10.
- Use language that indicates there are numbers beyond 10, but 10 is the stopping point.
What number is in the tens place?

What digit is in the tens place? What is the value of the digit in the tens place?

135

Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.
The alligator eats the bigger number

is less than
OR
is greater than

Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.
167
+ 294

Why this is important...

- “Carry” or “borrow” is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) or ungrouping hundreds into tens, tens into ones, and so on.
Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.
reduce the fraction

rename OR
find equivalent OR
simplify

Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.
Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.
Why this is important...

- This is the endpoint where two or more line segments or rays meet.
flips, slides, turns

reflections, translations, rotations

Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.
Use formal math language

Use terms precisely
Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction
Precise language
Multiple representations

INSTRUCTIONAL STRATEGIES
Three-dimensional objects
Two-dimensional images

Abstract

Concrete

Pictorial
Two-dimensional images
2 + 8 = 10

34 = 3 tens and 4 ones

x − 6 = 8

4,179 + 569
Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building

Diagram:
- Validated Intervention Program (e.g. Tier 2, Standard Protocol, Secondary Intervention)
- Progress Monitor
- Diagnostic Academic Assessment/Functional Assessment
- Intervention Adaptation
- Progress Monitor
Addition  Subtraction
Multiplication  Division
BRIEF (1-2 min) DAILY (everyday)

- Addition
- Subtraction
- Multiplication
- Division
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(place sum or product from bag here)

4 5
8 12
3 6
3 6
SMATH

The Game That Makes Math Fun!

2, +, 2, =, 4,

2, x, 2, =, 4,

2, - , 6, =, 3,

2 x 4 = 8

1 + 5 = 6

2, +, 2, =, 4,

2, x, 2, =, 4,

2, - , 6, =, 3,

2 x 4 = 8

1 + 5 = 6
BRIEF (1-2 min) DAILY (everyday)
INSTRUCTIONAL DELIVERY
- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES
- Fluency building
- Problem solving instruction
Don’t tie key words to operations

Do have an attack strategy

Do teach word-problem schemas
### Key Words Used in Math Word Problems

#### Addition Words
- add
- all together or altogether
- and
- both
- combined
- how many in all
- how much
- increased by
- plus
- sum
- together
- total

#### Subtraction Words
- change
- decreased by
- difference
- fewer or fewer than
- how many left (or have left)
- how many did not have
- how many (or much) more
- how much longer (shorter, taller, heavier, etc.)
- less or less than
- lost
- minus
- need to
- reduce
- remain
- subtract
- take away

#### Multiplication Words
- by (dimension)
- double
- each group
- equal groups
- equal sharing
- half (or other fractions)
- how many in each
- parts
- per
- percent
- quotient of
- ratio of
- separated
- are something equally

#### Division Words
- as much
- cut up
- each group has
- equal sharing
- half (or other fractions)
- how many in each
- parts
- per
- percent
- quotient of
- ratio of
- separated
- are something equally

---

**Addition**
- **Sum**
- **Total**
  - All together
  - How much did it cost altogether?

**Subtraction**
- **Sum**
- **Total**
- **Difference**
  - How much more...
  - How much taller...

**Division**
- **Quotient**
  - Average
  - Divided by
  - Each
  - Per
  - Each one

**Multiplication**
- **Product**
- **Times**
- **Multiplied**
  - How many per apple?
  - How much will fit into a certain size?
Word Problems: Subtraction

Read each problem. Write a number sentence and solve.

1. Mrs. Smith has 33 poodles and 18 boxers. How many more poodles does Mrs. Smith have?

   

2. The kennel holds 91 dogs. Mr. Glass has 67 dogs in the kennel now. How many spaces does he have left?

   

3. Mr. Kelly has 44 beagles. 26 of them are puppies. How many adult beagles does Mr. Kelly have?

   

4. Mrs. Green has 60 terriers. 25 of them are boys. How many terriers are girls?

   

5. There were 58 kittens at the pet shop on Friday. 29 of them were sold on Saturday. How many kittens were left?

   

6. Pat counted 22 lizards in the tank at the pet shop. 8 were sold later that day. How many lizards were left in the tank?
RIDGES

Read the problem.
Identify the relevant information.
Draw a picture.
Goal statement.
Equation development.
Solve the equation.

RIDE

Read the problem.
Identify the relevant information.
Determine the operation and unit for the answer.
Enter the correct numbers and calculate, then check the answer.
**Attack Strategy**

**UPS✓**

**Understand**
- Read the problem.
- Ask yourself:
  - What information do I know?
  - What is the question asking me to find?

**Plan**
- Choose a strategy, a tool or an approach.

**Solve**
- Show the math used to solve the problem.

**Check**
- Check your math.
- Ask yourself:
  - Did I answer the question asked?
  - Is my answer reasonable?
Don’t tie key words to operations

Do have an attack strategy

Do teach word-problem schemas
10 Key Mathematics Practices for All Elementary Schools
with strong evidence of effectiveness from high-quality research

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10 KEY MATH PRACTICES
for All Middle and High Schools
with strong evidence of effectiveness from high-quality research

1. Students are asked to solve meaningful problems by using representations and tools such as graphs, tables, and diagrams.

2. Students are asked to make their mathematical thinking transparent by talking about their strategies and solutions.

3. Students are asked to build connections between multiple representations of problems.

4. Students are asked to extend and explore the relationships between problems.

5. Students are asked to identify the properties of operations (e.g., commutative, associative, distributive) and use them to simplify calculations.

6. Students are asked to engage in collaborative problem solving.

7. Students are asked to apply mathematical concepts and strategies to solve real-world problems.

8. Students are asked to communicate their mathematical thinking to others.

9. Teachers are asked to provide students with feedback and opportunities to revise their work.

10. Teachers are asked to coordinate their instruction to ensure that students understand the connections between concepts and procedures.

The Meadows Center
The University of Texas at Austin
COLEGE OF EDUCATION
www.meadowcenter.org
Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators’ skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a high-leverage practice in special education, and DBI is a research-based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEDAR Center, developed course content focused on enhancing educators’ skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.