How to Design and Deliver Effective Math Intervention

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Evidence-based mathematics resources for educators

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IDEAs that Work
Office of Special Education Programs

NSF

T.L.L. TEMPLE FOUNDATION
EST 1962
BUILDING A THRIVING DEEP EAST TEXAS

spencer

GREATER TEXAS FOUNDATION

NATIONAL ACADEMY of EDUCATION

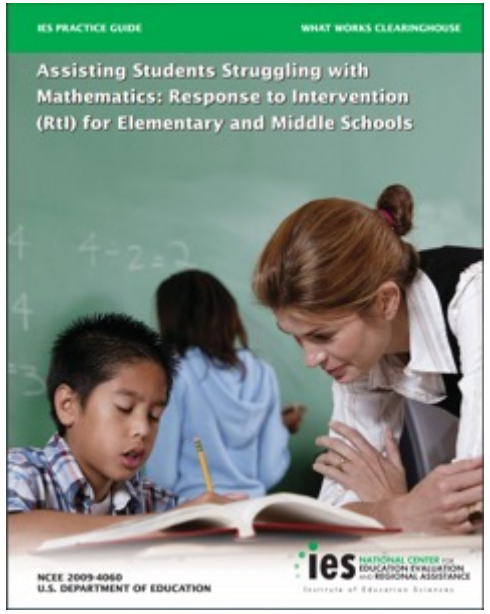
TEA
Texas Education Agency

~~no or negative evidence~~

Evidence-Based Mathematics Practices

WWC Practice Guides:

- Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools



COMING SOON (2020-2021):

- *Assisting Students Struggling with Mathematics: Intervention in the Elementary and Middle School Grades*

ies INSTITUTE OF EDUCATION SCIENCES

https://ies.ed.gov/ncee/wwc/PracticeGuides

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PRACTICE GUIDE

Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools

Released: April 2009
PDF (4.0 MB)

Recommendations | Details | Panel

Taking early action may be key to helping students struggling with mathematics. The eight recommendations in this guide are designed to help teachers, principals, and administrators use Response to Intervention for the early detection, prevention, and support of students struggling with mathematics.

1 Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk. — MODERATE EVIDENCE

2 Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. — MINIMAL EVIDENCE

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3 Instruction during the intervention should be explicit and systematic. — STRONG EVIDENCE

4 Interventions should include instruction on solving word problems that is based on common underlying structures. — STRONG EVIDENCE

5 Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas. — MODERATE EVIDENCE

6 Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts. — MODERATE EVIDENCE

7 Monitor the progress of students receiving supplemental instruction and other students who are at risk. — MINIMAL EVIDENCE

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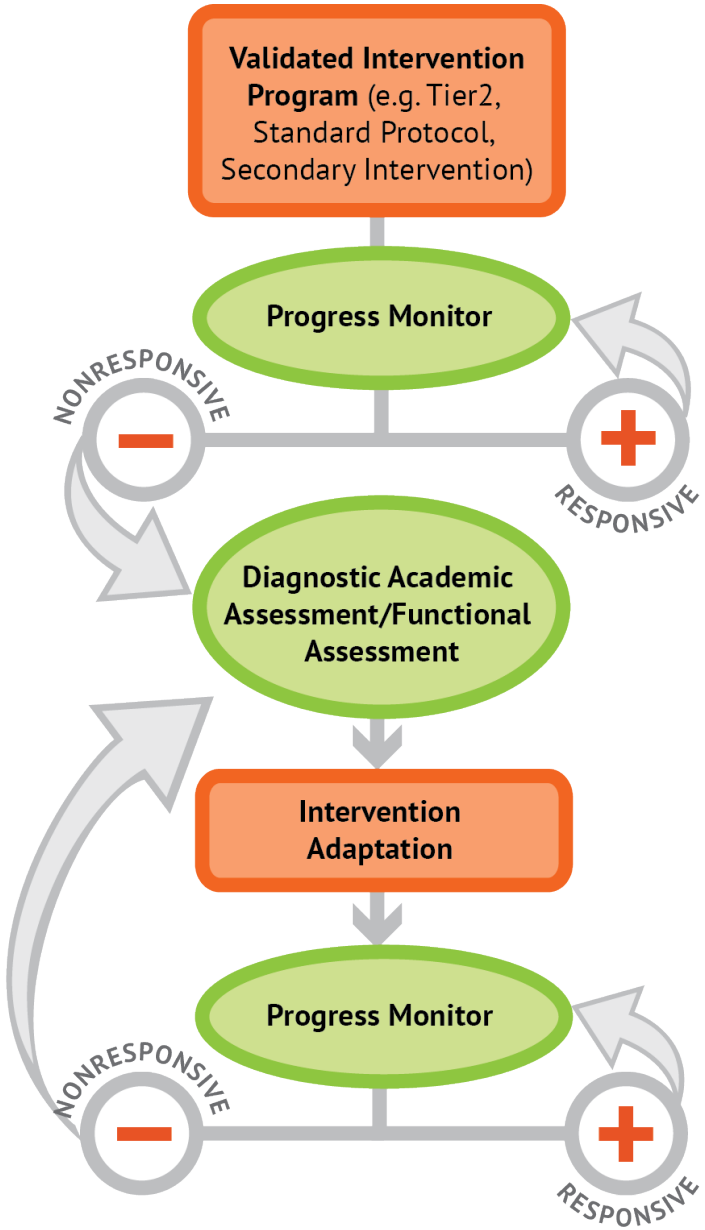
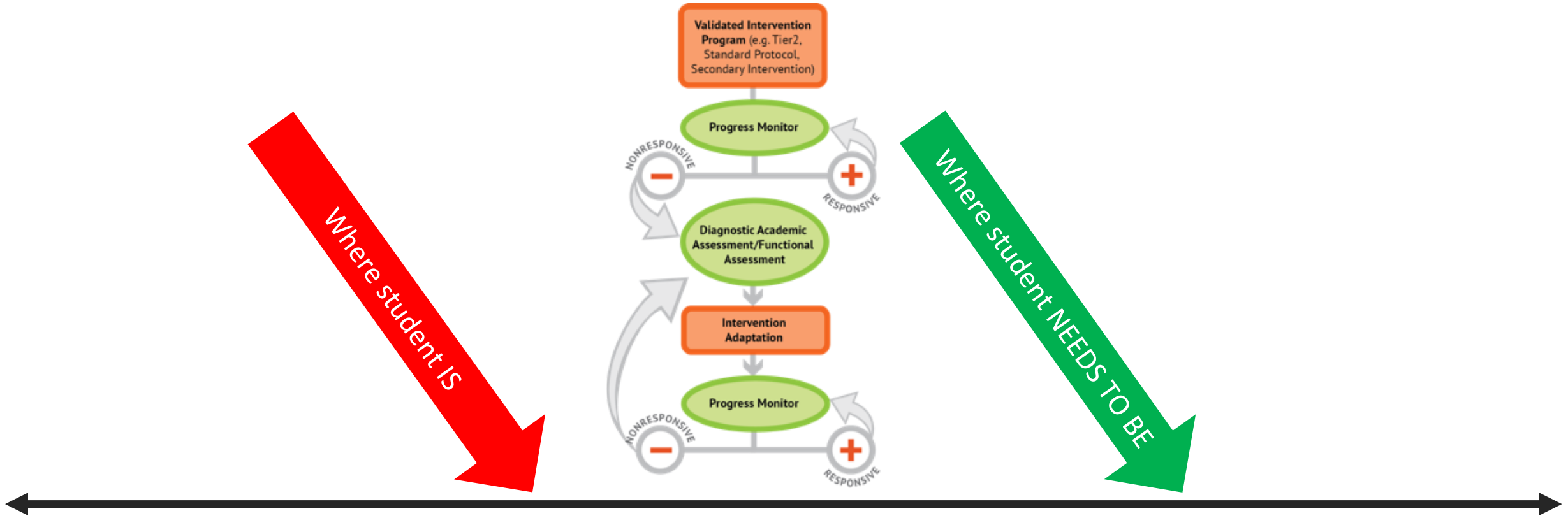
8 Include motivational strategies in tier 2 and tier 3 interventions. — MINIMAL EVIDENCE

▾ Show More

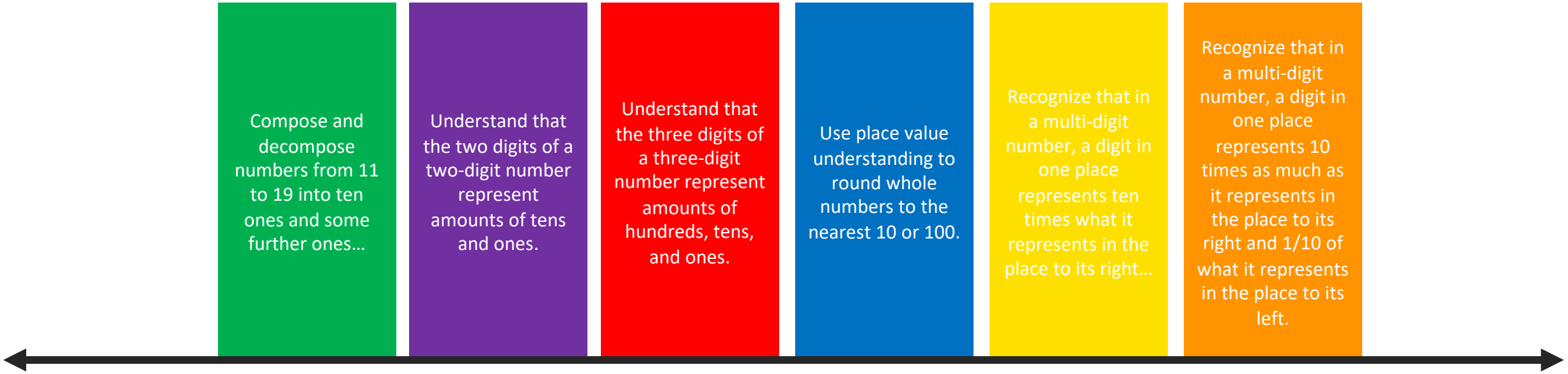
https://ies.ed.gov/ncee/wwc/PracticeGuides



https://www.intensiveintervention.org


Where student IS
Validated Intervention Program (e.g. Tier2, Standard Protocol, Secondary Intervention)
Progress Monitor
NONRESPONSIVE
RESPONSIVE
Diagnostic Academic Assessment/Functional Assessment
Intervention Adaptation
Progress Monitor
NONRESPONSIVE
RESPONSIVE
Where student NEEDS TO BE

Design


Compose and decompose numbers from 11 to 19 into ten ones and some further ones…
Understand that the two digits of a two-digit number represent amounts of tens and ones.
Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.
Use place value understanding to round whole numbers to the nearest 10 or 100.
Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right…
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.


Where student IS
Where student NEEDS TO BE
Solve addition and subtraction word problems, and add and subtract within 10…
Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20…
Use addition and subtraction within 100 to solve one- and two-step word problems…
Use multiplication and division within 100 to solve word problems…
Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations…
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators …
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions…
Solve real-world and mathematical problems involving the four operations with rational numbers.
Solve real-world and mathematical problems leading to two linear equations in two variables.


Where student IS
Where student NEEDS TO BE
Explain why addition and subtraction strategies work, using place value and the properties of operations.
Understand that the two digits of a two-digit number represent amounts of tens and ones.
Use addition and subtraction within 100 to solve one- and two-step word problems…
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.
Apply properties of operations as strategies to multiply and divide….
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division…
Use multiplication and division within 100 to solve word problems.
Fluently add and subtract multi-digit whole numbers using the standard algorithm.
Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.
Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
Fluently multiply multi-digit whole numbers using the standard algorithm.
Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations…

CCSS
WHERE TO FOCUS
GRADES K–8
MATHEMATICS

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.
** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

https://achievethecore.org/category/774/mathematics-focus-by-grade-level

Table A.2. Grades 3–5 Curriculum Focal Points and Connections Compared with the Expectations of the Content Standards in *Principles and Standards for School Mathematics*

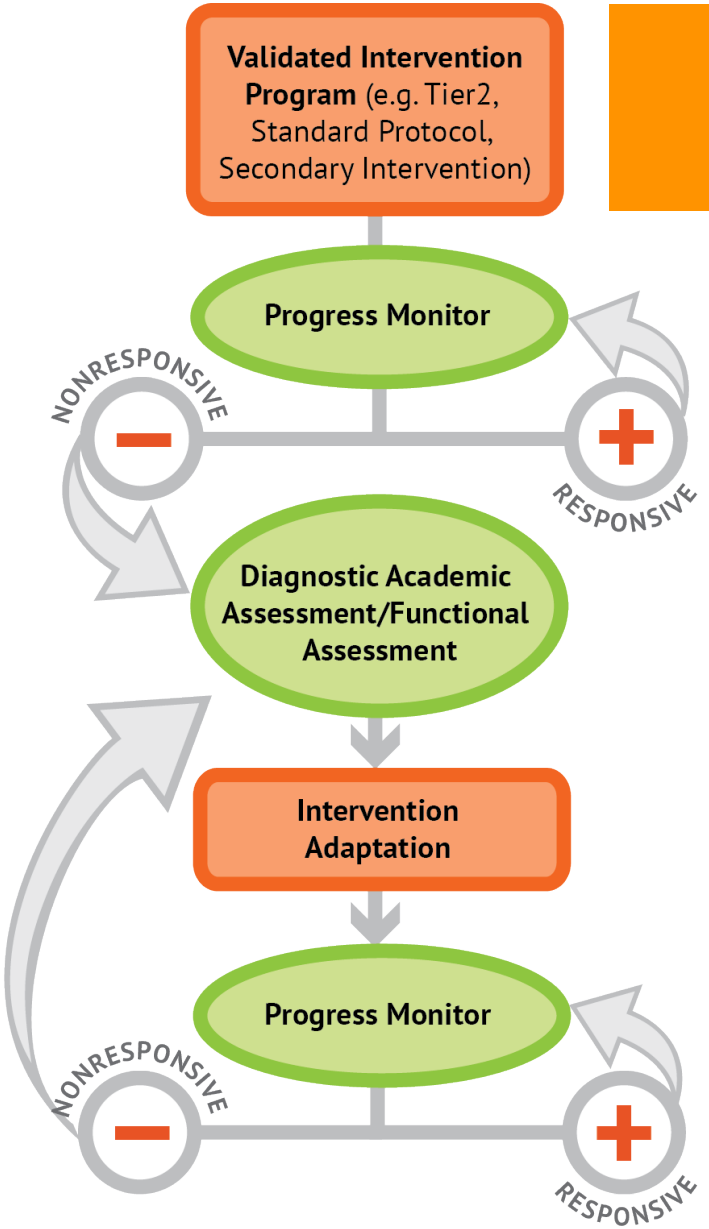
Grade 3 · Grade 4 · Grade 5 · Outside grades 3–5 · Not identified at any level, pre-K–8

Curriculum Focal Points and Connections	Expectations of the Content Standards
Grade 3 Curriculum Focal Points	
Number and Operations and ***Algebra:*** **Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts** Students understand the meanings of multiplication and division of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication, and successive subtraction, partitioning, and sharing for division). They use properties of addition and multiplication (e.g., commutativity, associativity, and the distributive property) to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts. By comparing a variety of solution strategies, students relate multiplication and division as inverse operations.	***Number and Operations, Grades 3–5***
	Understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals
	Recognize equivalent representations for the same number and generate them by decomposing and composing numbers
	Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and [in Grade 6 Curriculum Focal Points] as divisions of whole numbers
	Use models, benchmarks, and equivalent forms to judge the size of fractions
	Recognize and generate equivalent forms of commonly used fractions, decimals, and [in Grade 7 Curriculum Focal Points] percents
	Explore numbers less than 0 by extending the number line and through familiar applications
	Describe classes of numbers according to characteristics such as the nature of their factors
	Understand various meanings of multiplication and division
	Understand the effects of multiplying and dividing whole numbers
	Identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems
	Understand and use properties of operations, such as the distributivity of multiplication over addition
	Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 × 50
Number and Operations: **Developing an understanding of fractions and fraction equivalence** Students develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models, including the number line, to identify equivalent fractions.	
Geometry: **Describing and analyzing properties of two-dimensional shapes** Students describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes. Students investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space and the use of those attributes and properties in solving problems, including applications involving congruence and symmetry.	

https://www.nctm.org/curriculumfocalpoints/

Deliver

Instructional Platform



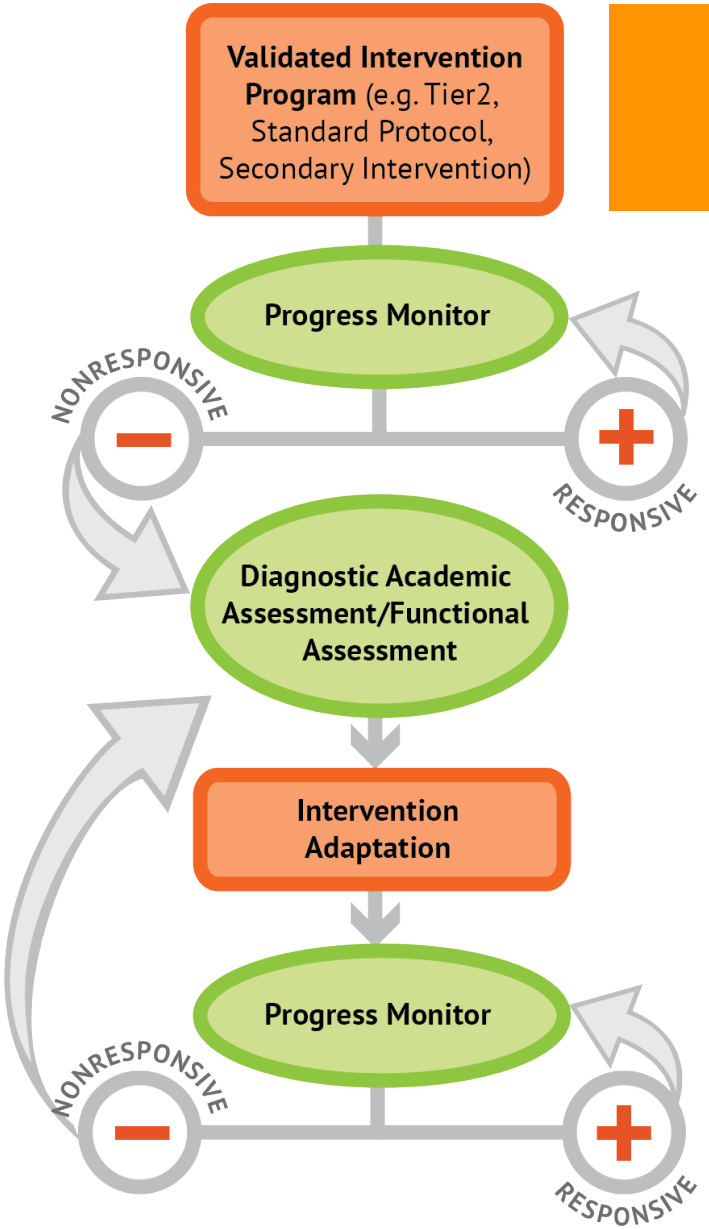
INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

Instructional Platform



INSTRUCTIONAL DELIVERY

Explicit instruction

INSTRUCTIONAL STRATEGIES

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

https://intensiveintervention.org/intensive-intervention-features-explicit-instruction

Goal and importance

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

"Let's continue working with our three-dimensional shapes and volume. Understanding volume and calculating volume helps with measuring capacity."

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Goal and importance

Model steps

"To solve 26 plus 79, I first decide about the operation. Do I add, subtract, multiply or divide?"

"The plus sign tells me to add. So, I'll add 26 plus 79. I'll use the partial sums strategy. First, I add 20 plus 70. What's 20 plus 70?"

"20 plus 70 is 90. I write 90 right here under the equal line. Where do I write 90?"

"Then I add 6 plus 9. What's 6 plus 9?"

"How did you add 6 plus 9?"

"6 plus 9 is 15. So, I write 15 here under the equal line."

"Finally, we add the partial sums: 90 and 15. 90 plus 15 is 105. So, 26 plus 79 equals 105. What's 26 plus 79?"

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Goal and importance

Model steps

With examples

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

$24 / 6$

$28 \div 7$

$35 \overline{)\ 5}$

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Goal and importance

Model steps

With examples

With non-examples

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

$32 \div 8$ $42 \div 7$ $25 - 5$

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
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- Providing immediate specific feedback
- Maintaining a brisk pace

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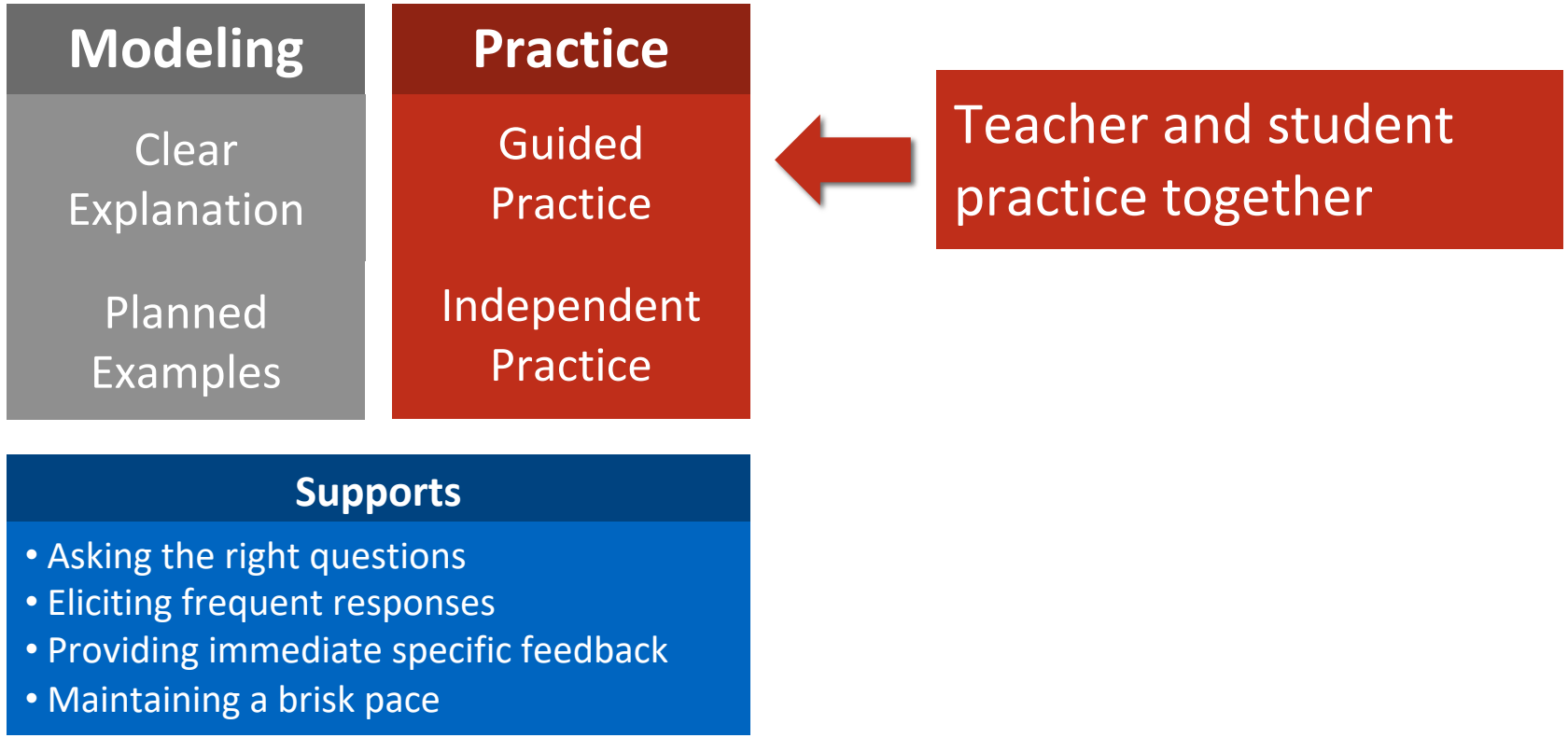
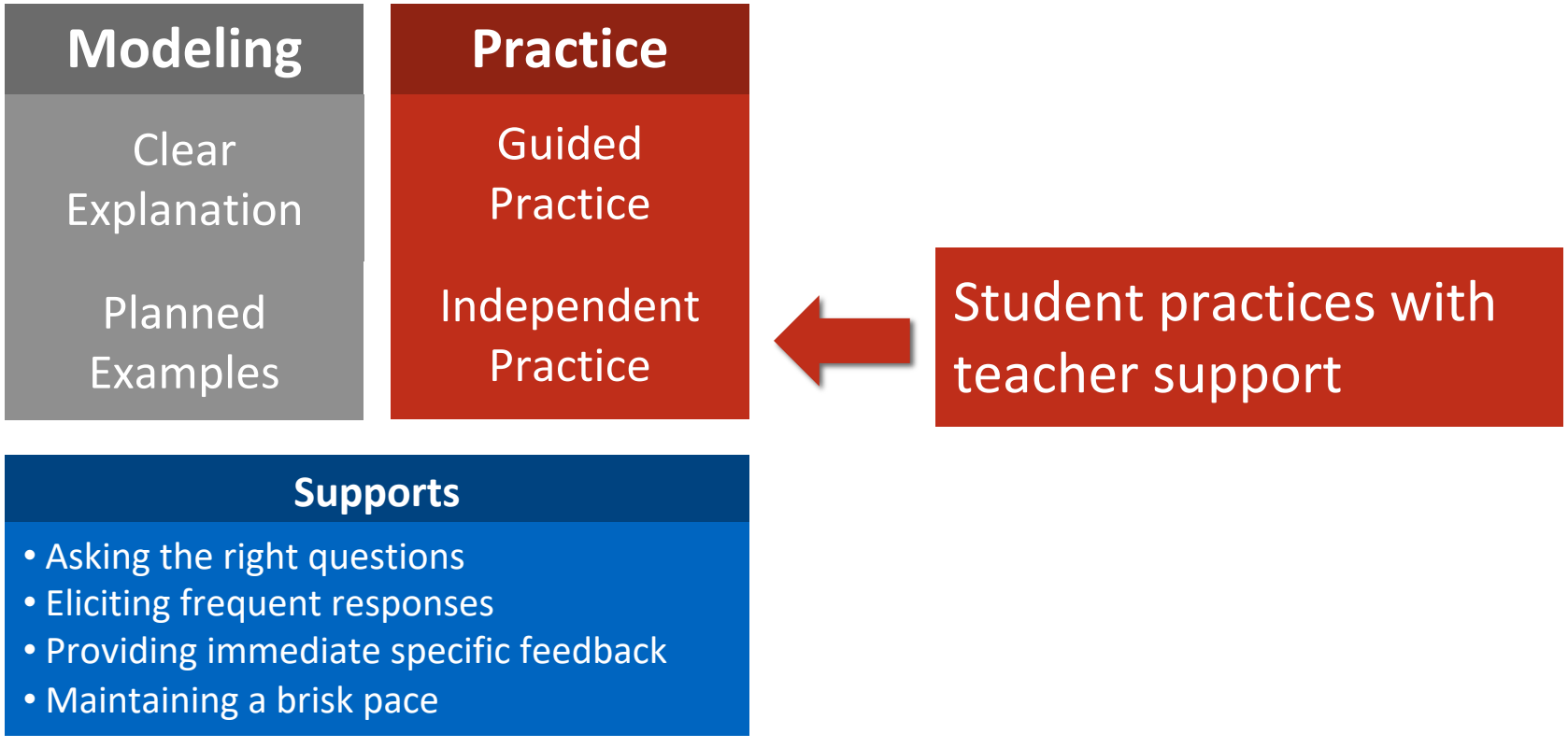

Modeling
Clear Explanation
Planned Examples
Practice
Guided Practice
Independent Practice
Teacher and student practice together
Supports
• Asking the right questions
• Eliciting frequent responses
• Providing immediate specific feedback
• Maintaining a brisk pace


Modeling
Clear Explanation
Planned Examples
Practice
Guided Practice
Independent Practice
Student practices with teacher support
Supports
• Asking the right questions
• Eliciting frequent responses
• Providing immediate specific feedback
• Maintaining a brisk pace

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

https://intensiveintervention.org/intensive-intervention-features-explicit-instruction

Low-level and high-level

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

"What is 7 times 9?"

"Which shape has 6 sides?"

"What do you do when you see a word problem?"

"Why do you have to regroup?"

"How would you solve this problem?"

"Why do you have to use zero pairs?"

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

“Turn and discuss the formula for perimeter with your partner.”

“Write the multiplication problem on your whiteboard.”

“In your math journal, draw a picture to help you remember to term *parallelogram*.”

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

"Good work using your word-problem attack strategy."

"Let's look at that again. Tell me how you added in the hundreds column."

Modeling	Practice	
Clear Explanation	Guided Practice	Low-level and high-level
Planned Examples	Independent Practice	Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.
		Affirmative and corrective
		Planned and organized

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

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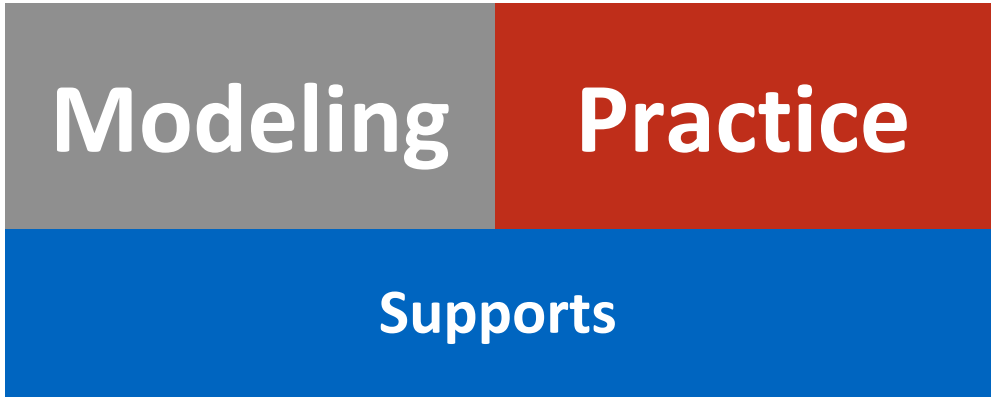
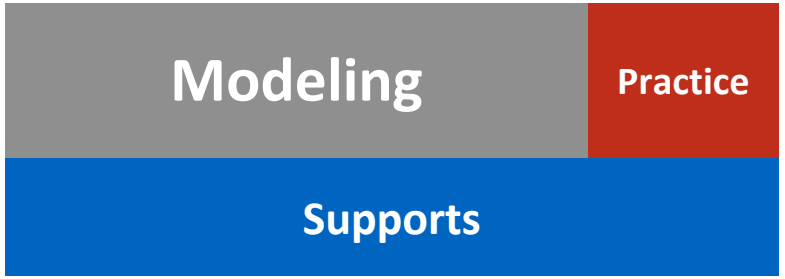
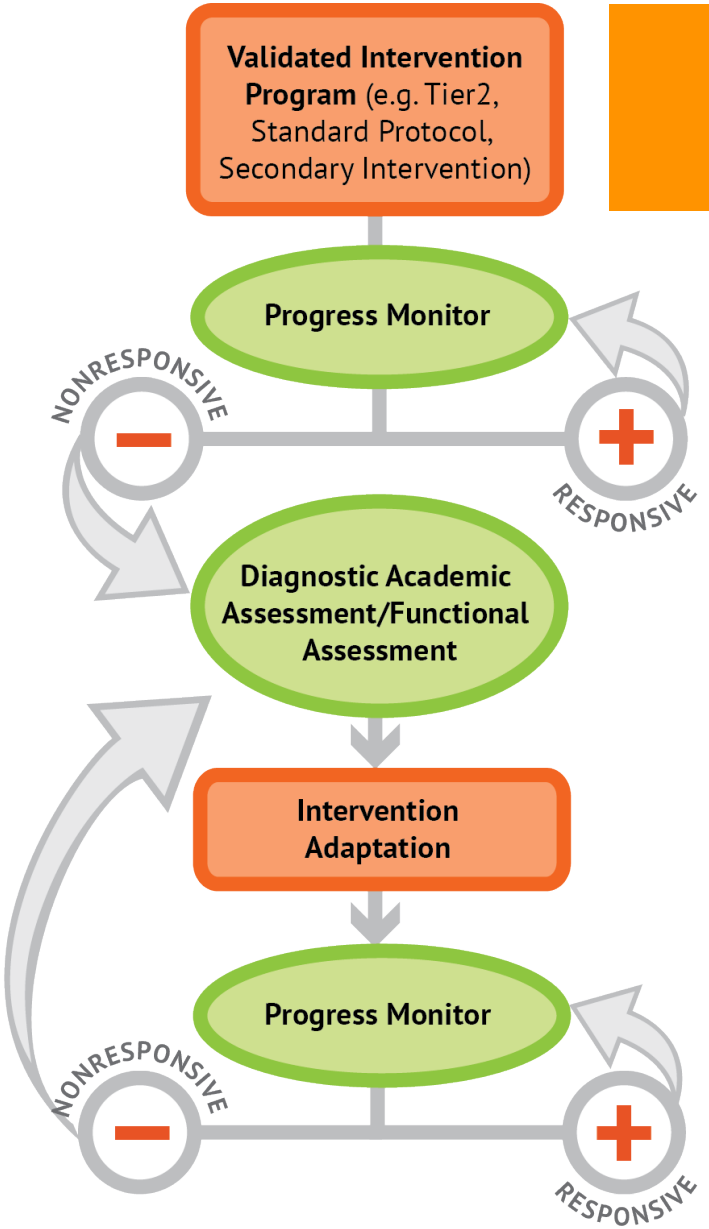

Modeling
Practice
Supports


Modeling
Practice
Supports


Modeling
Practice
Supports

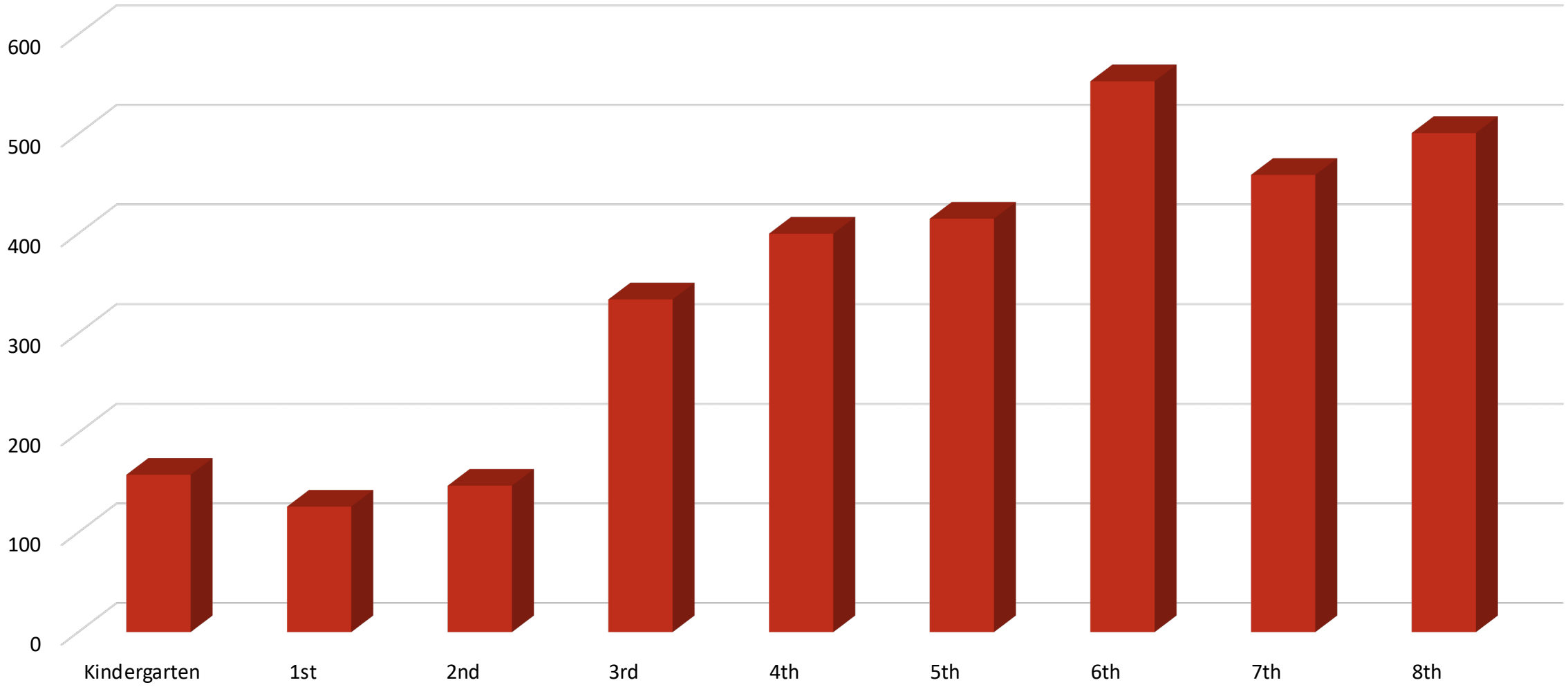
Instructional Platform



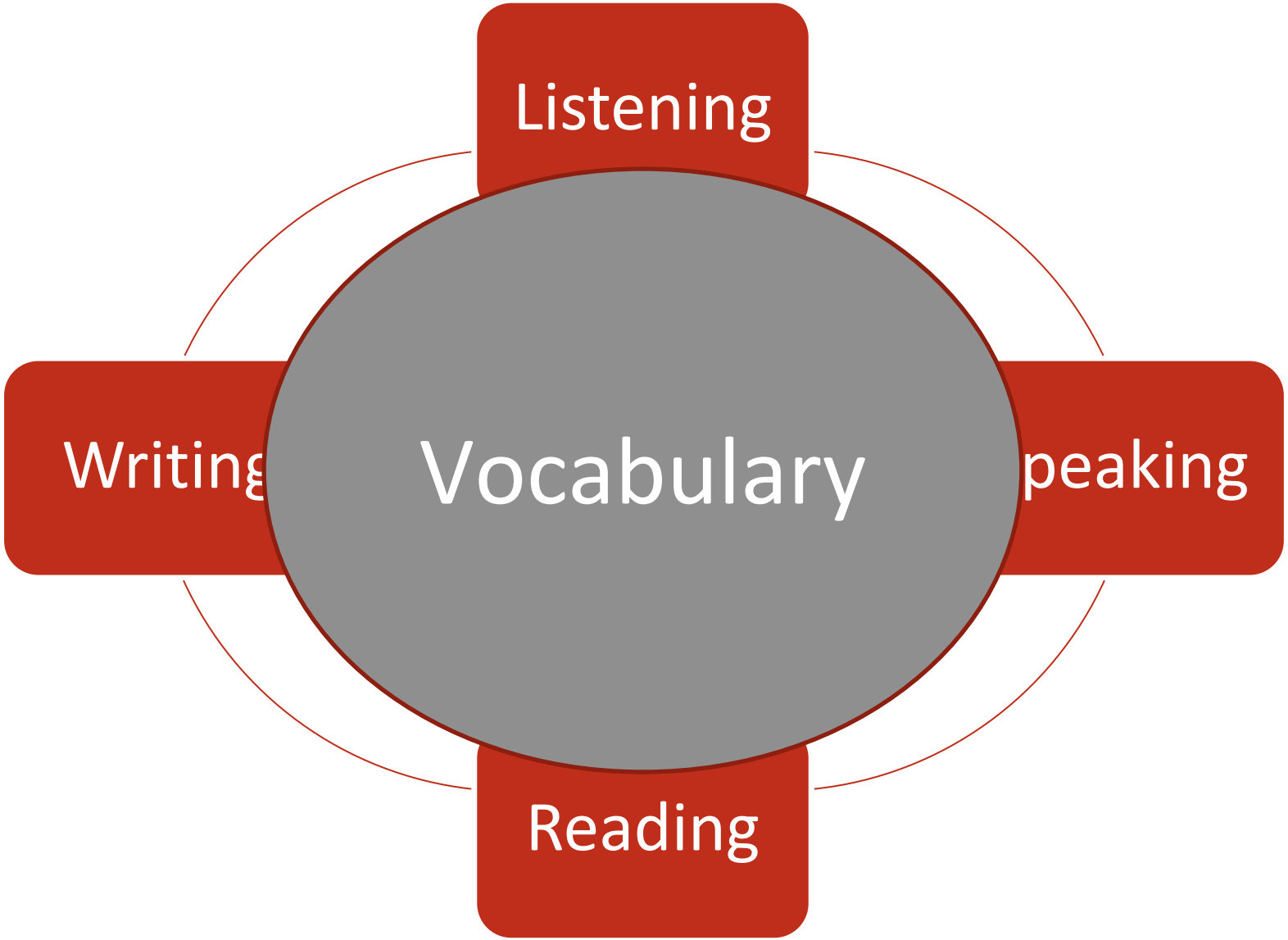
INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language

INSTRUCTIONAL STRATEGIES



Powell, Bos, & Lin (2019)


Listening
Writing
Vocabulary
peaking
Reading

1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)

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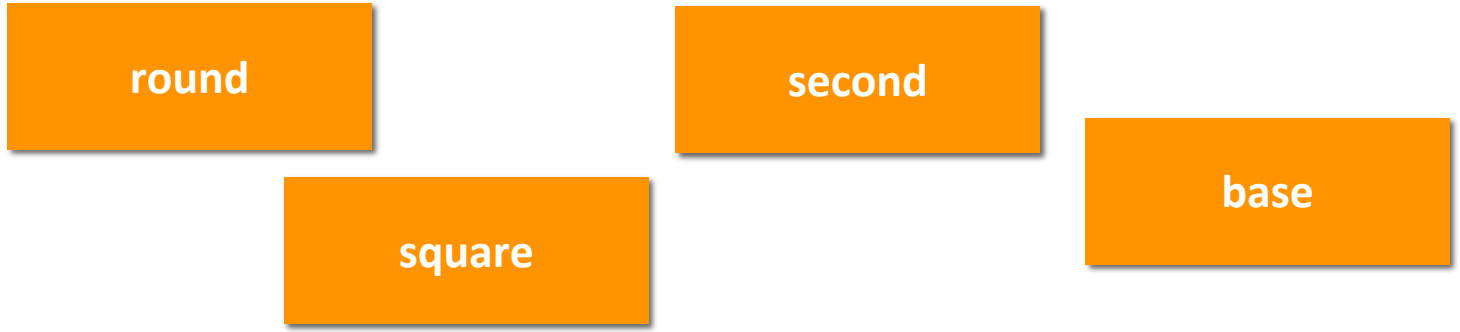
4. Some math terms have more than one meaning



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3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

divide vs. Continental Divide

variable vs. variably cloudy

Rubenstein & Thompson (2002)

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6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)

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6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

Rubenstein & Thompson (2002)

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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

mesa vs. tabla

Rubenstein & Thompson (2002)

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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)

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10. Some math concepts are verbalized in more than one way

one-fourth vs. one quarter

skip count vs. multiples

Rubenstein & Thompson (2002)

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10. Some math concepts are verbalized in more than one way

11. Informal terms may be used for formal math terms

rhombus vs. diamond

vertex vs. corner

Rubenstein & Thompson (2002)

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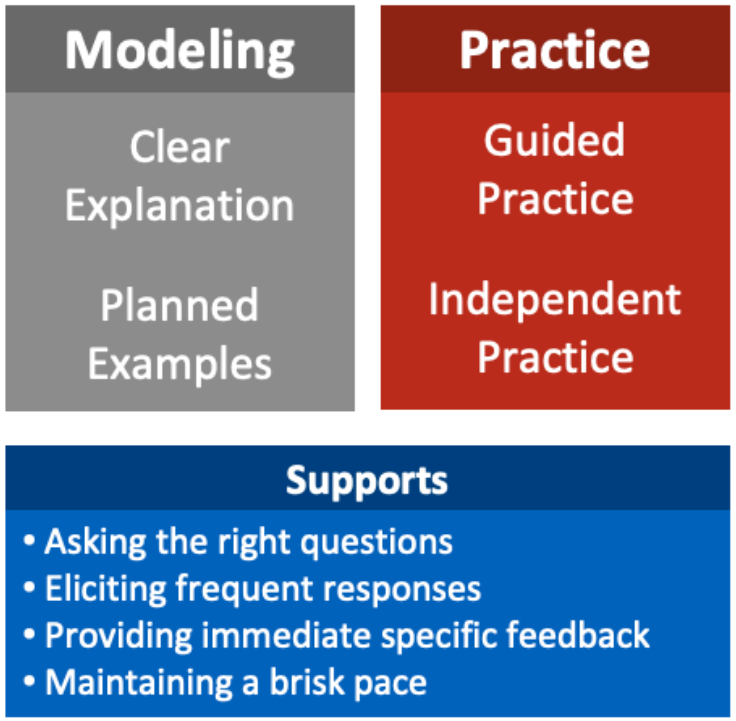
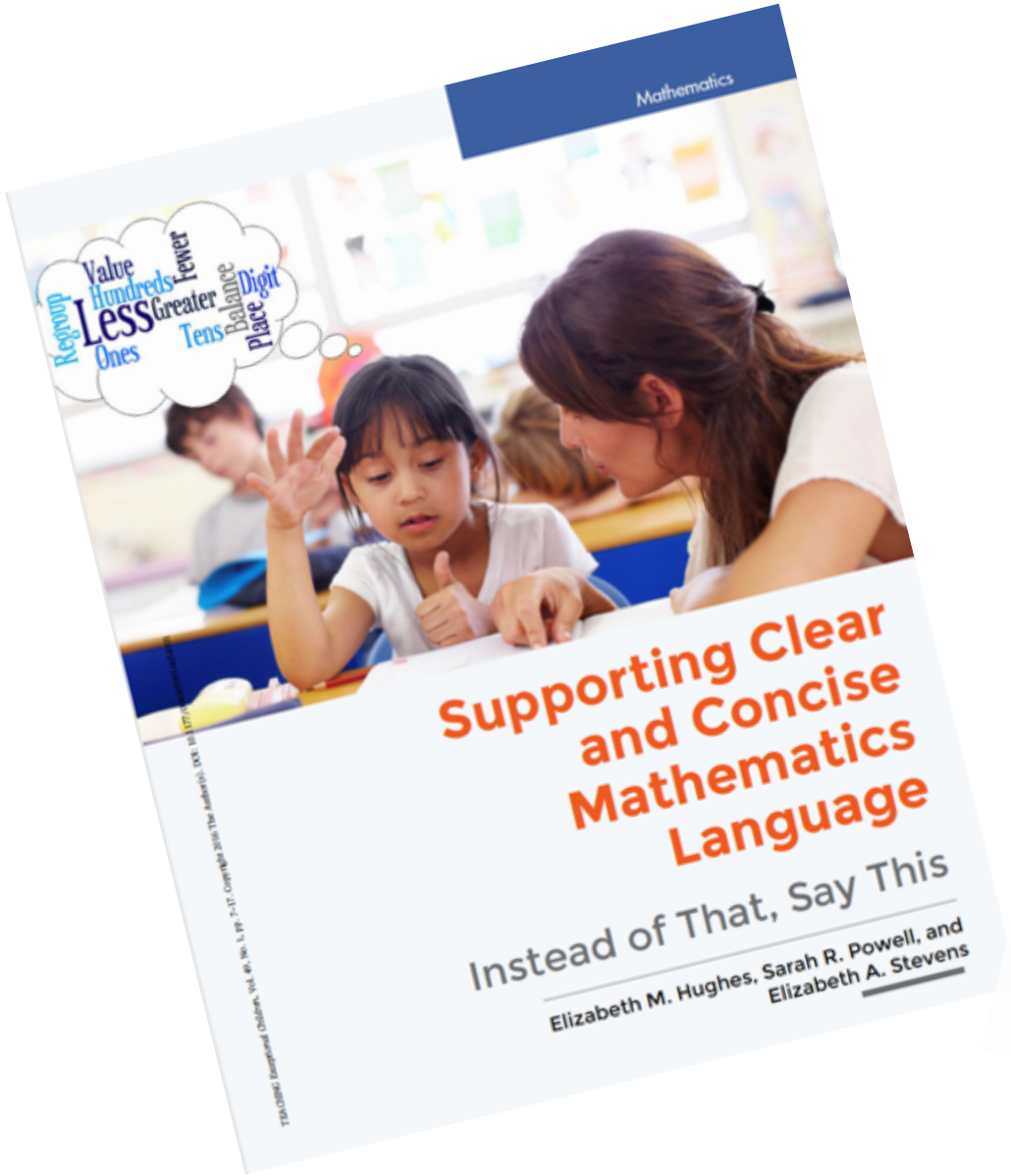
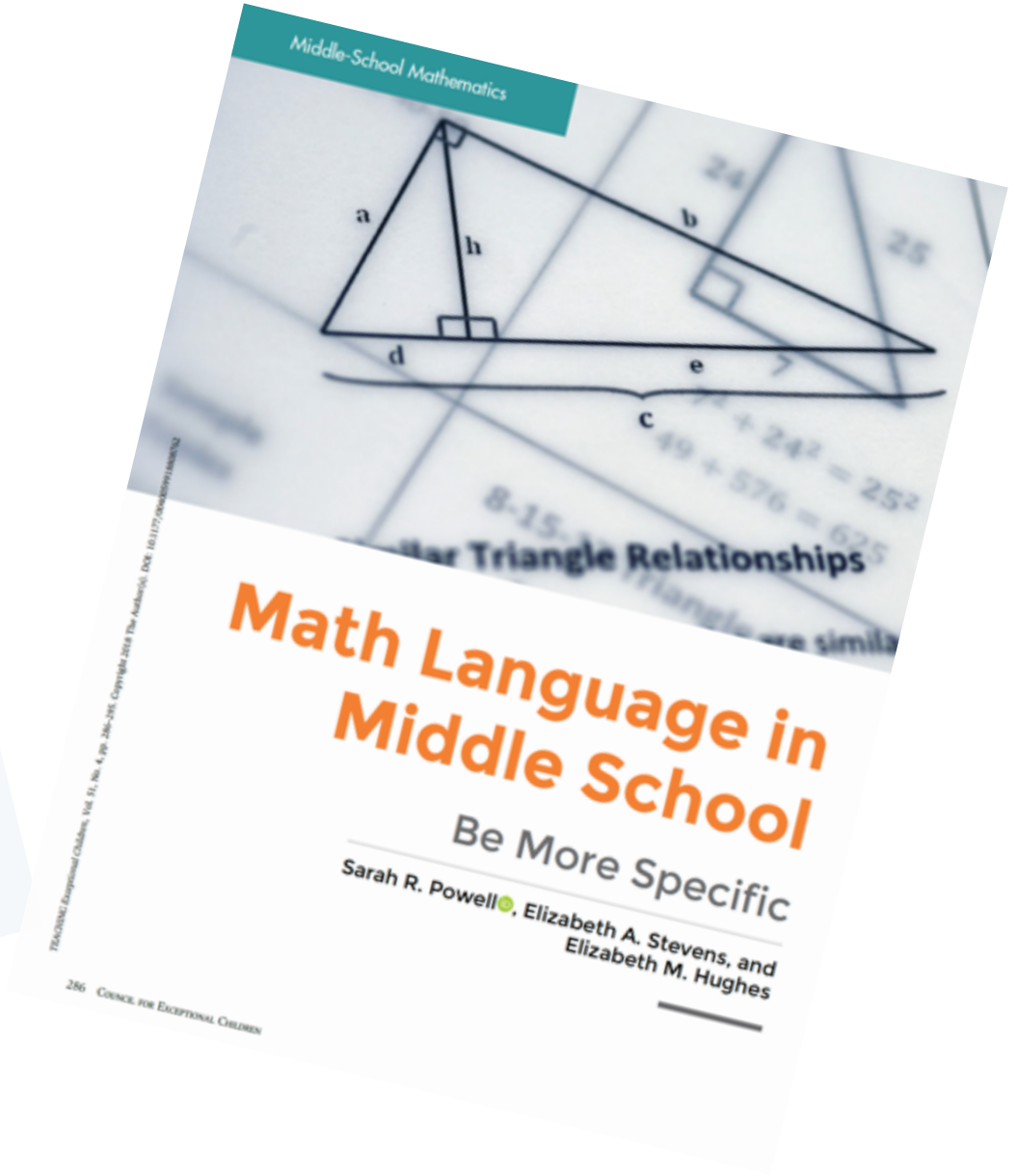
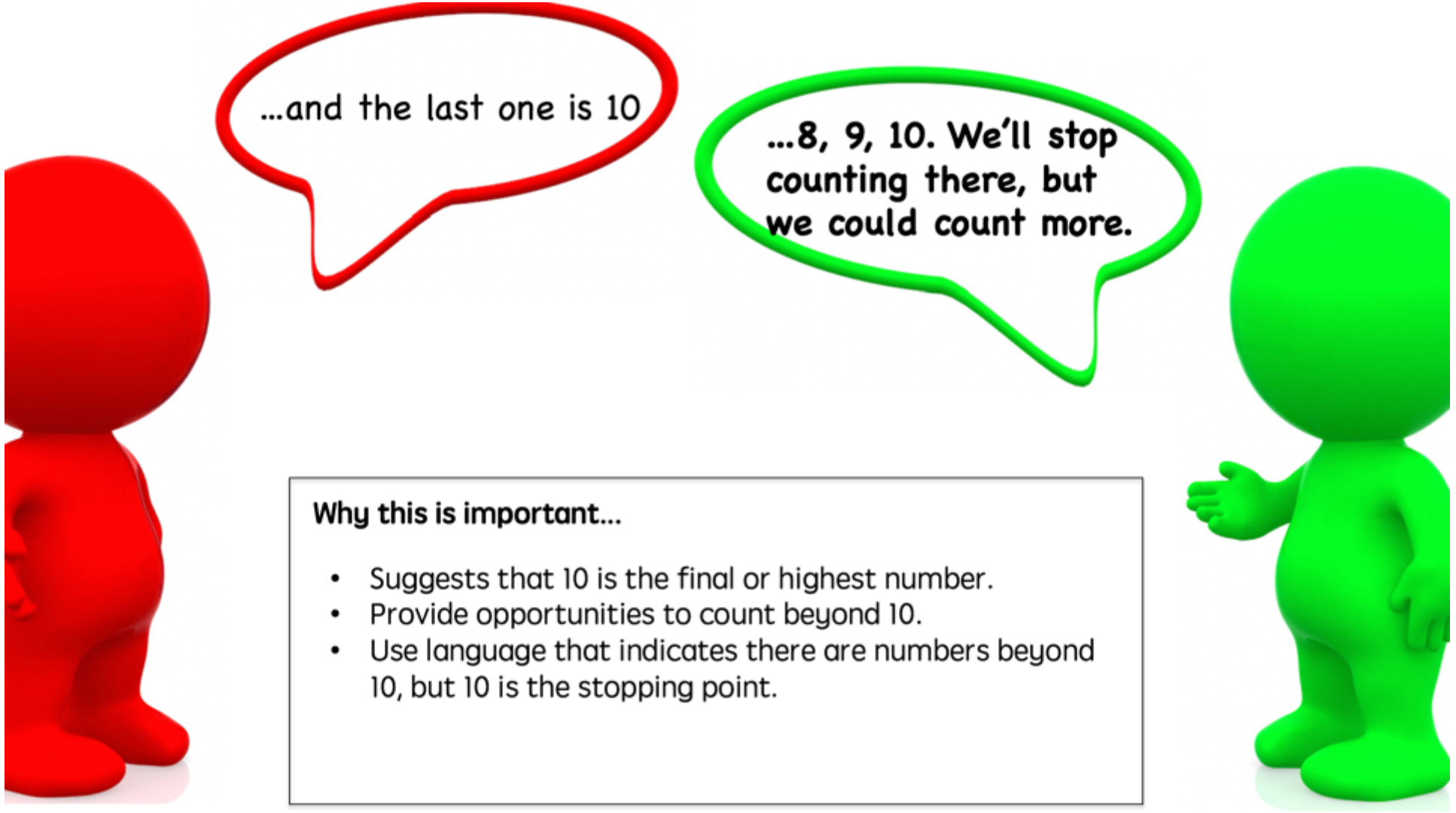


Rubenstein & Thompson (2002)

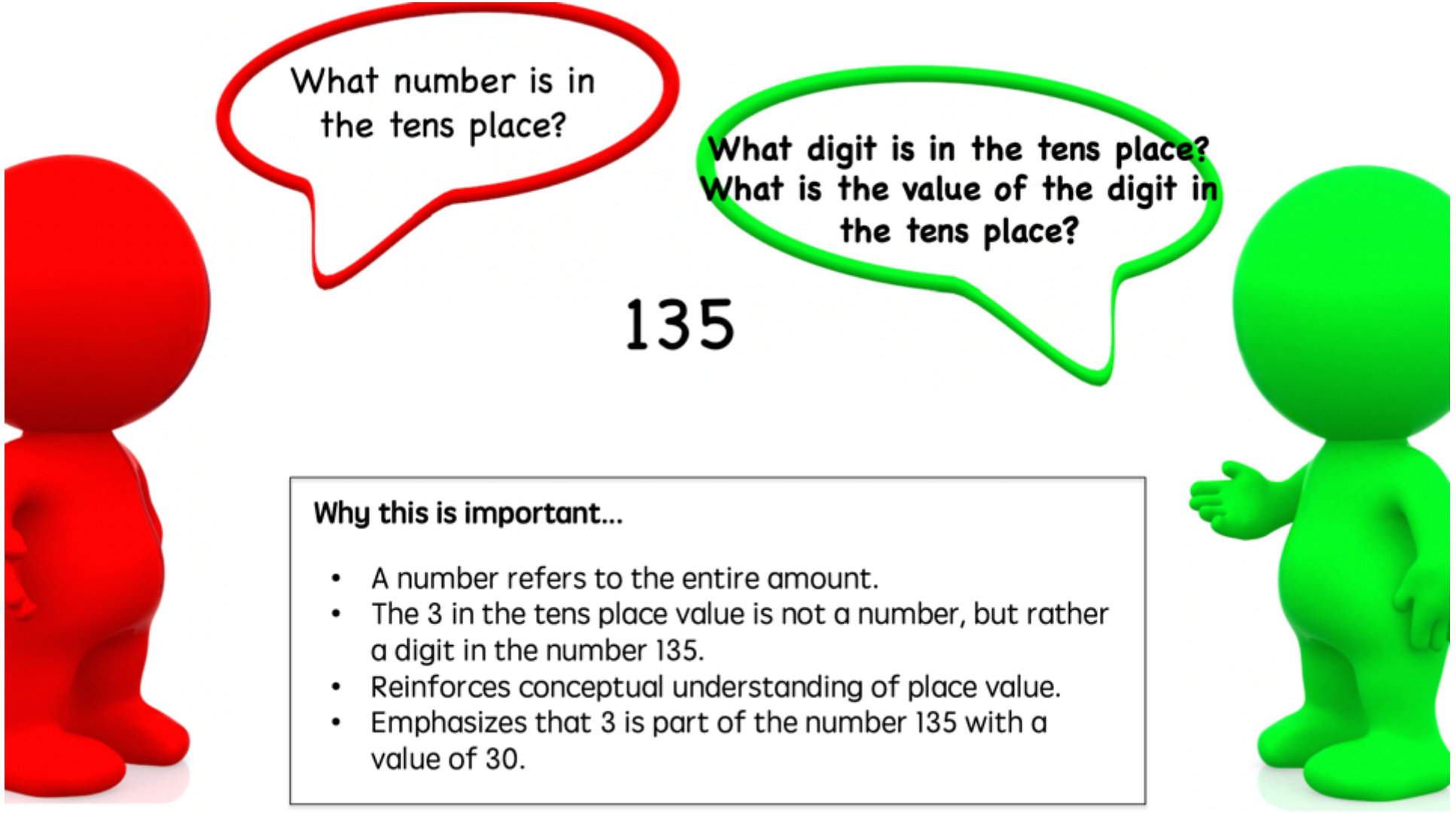

Mathematics
Regroup
Value
Hundreds
Fewer
Less
Greater
Ones
Tens
Balance
Digit
Place
Supporting Clear and Concise Mathematics Language
Instead of That, Say This
Elizabeth M. Hughes, Sarah R. Powell, and Elizabeth A. Stevens


Middle-School Mathematics
a
h
b
d
e
c
Triangle Relationships
Math Language in Middle School
Be More Specific
Sarah R. Powell, Elizabeth A. Stevens, and Elizabeth M. Hughes
286 Council for Exceptional Children



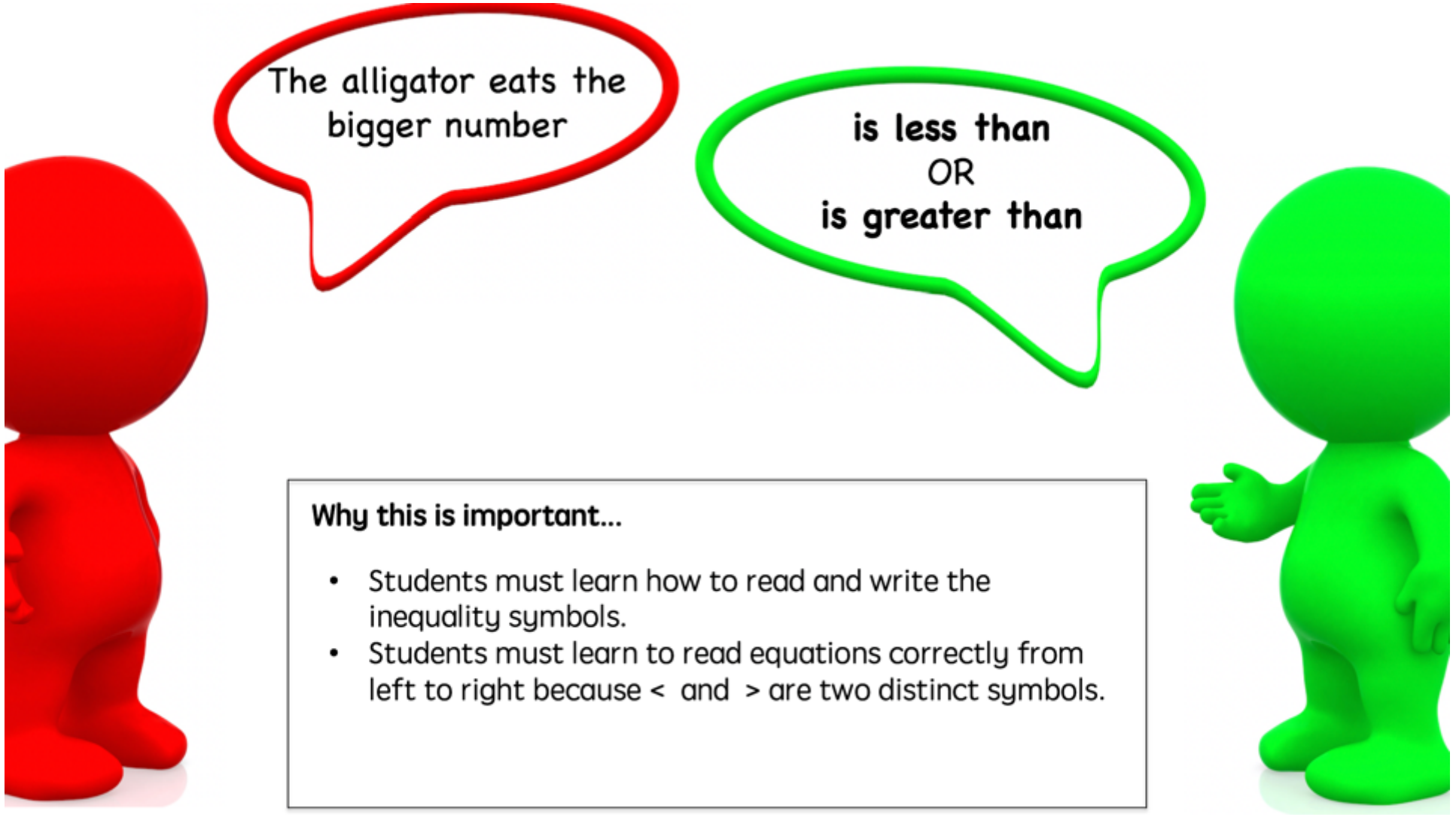
Why this is important...

- Suggests that 10 is the final or highest number.
- Provide opportunities to count beyond 10.
- Use language that indicates there are numbers beyond 10, but 10 is the stopping point.



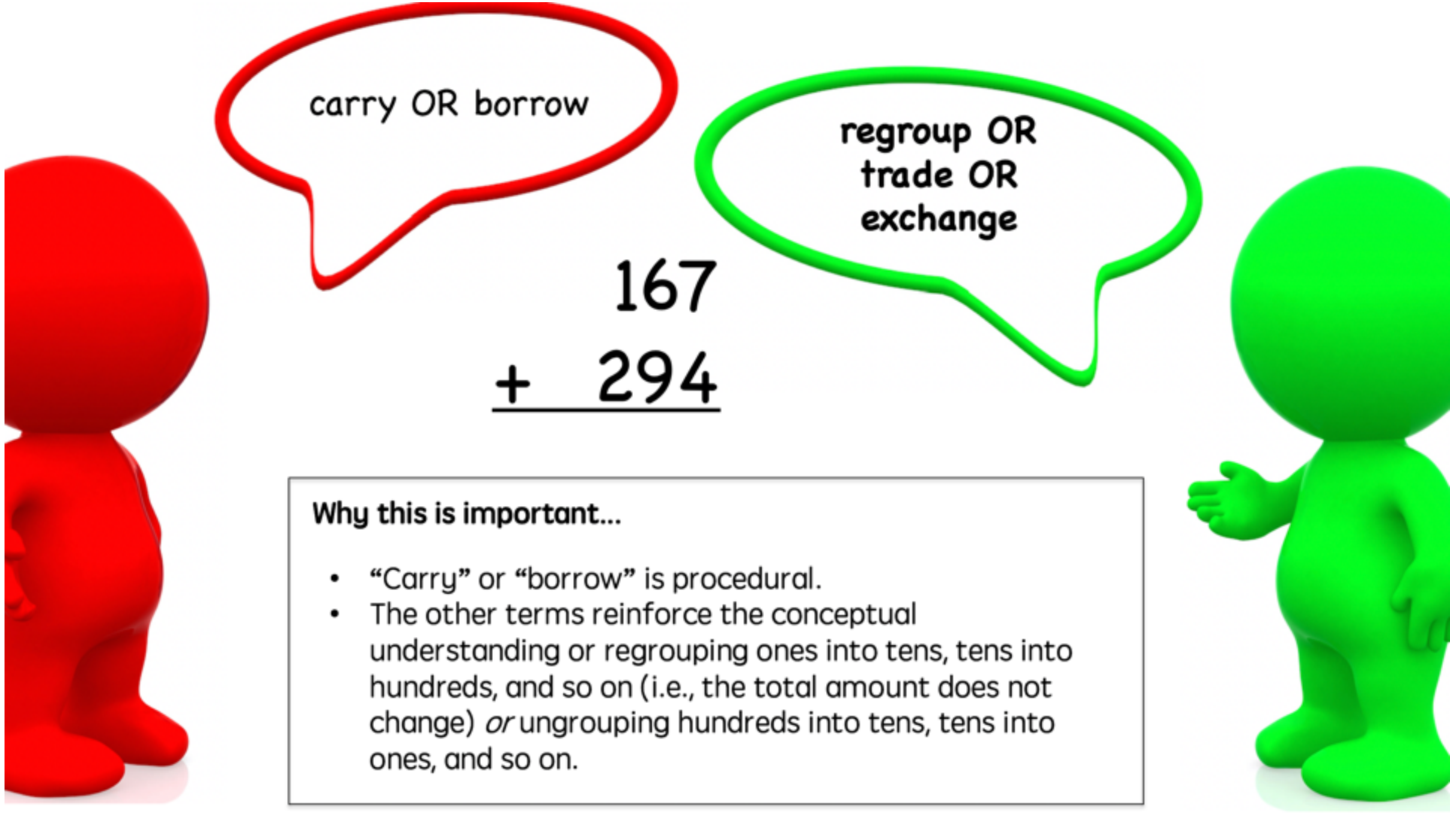
Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.



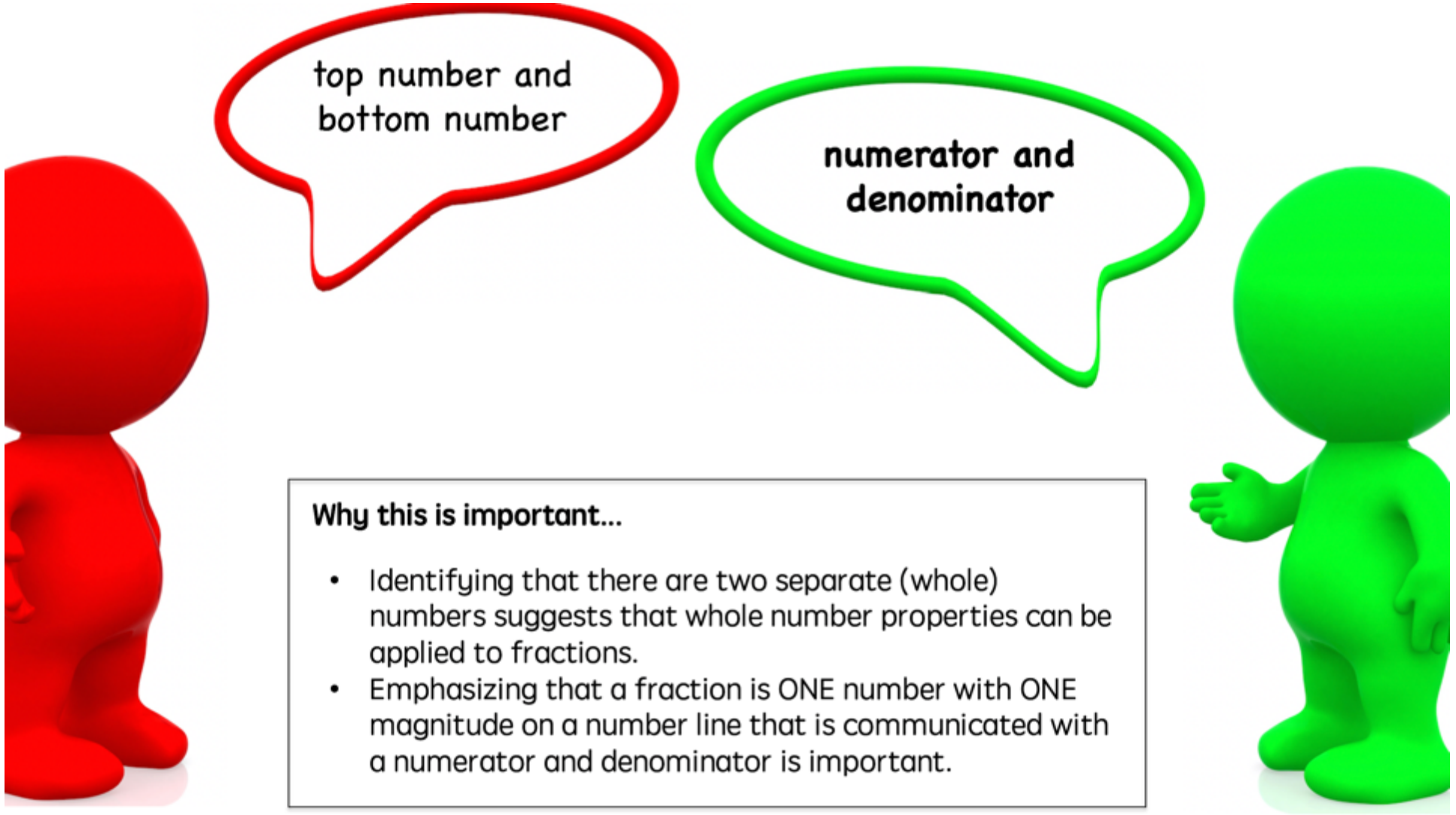
Why this is important...

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.



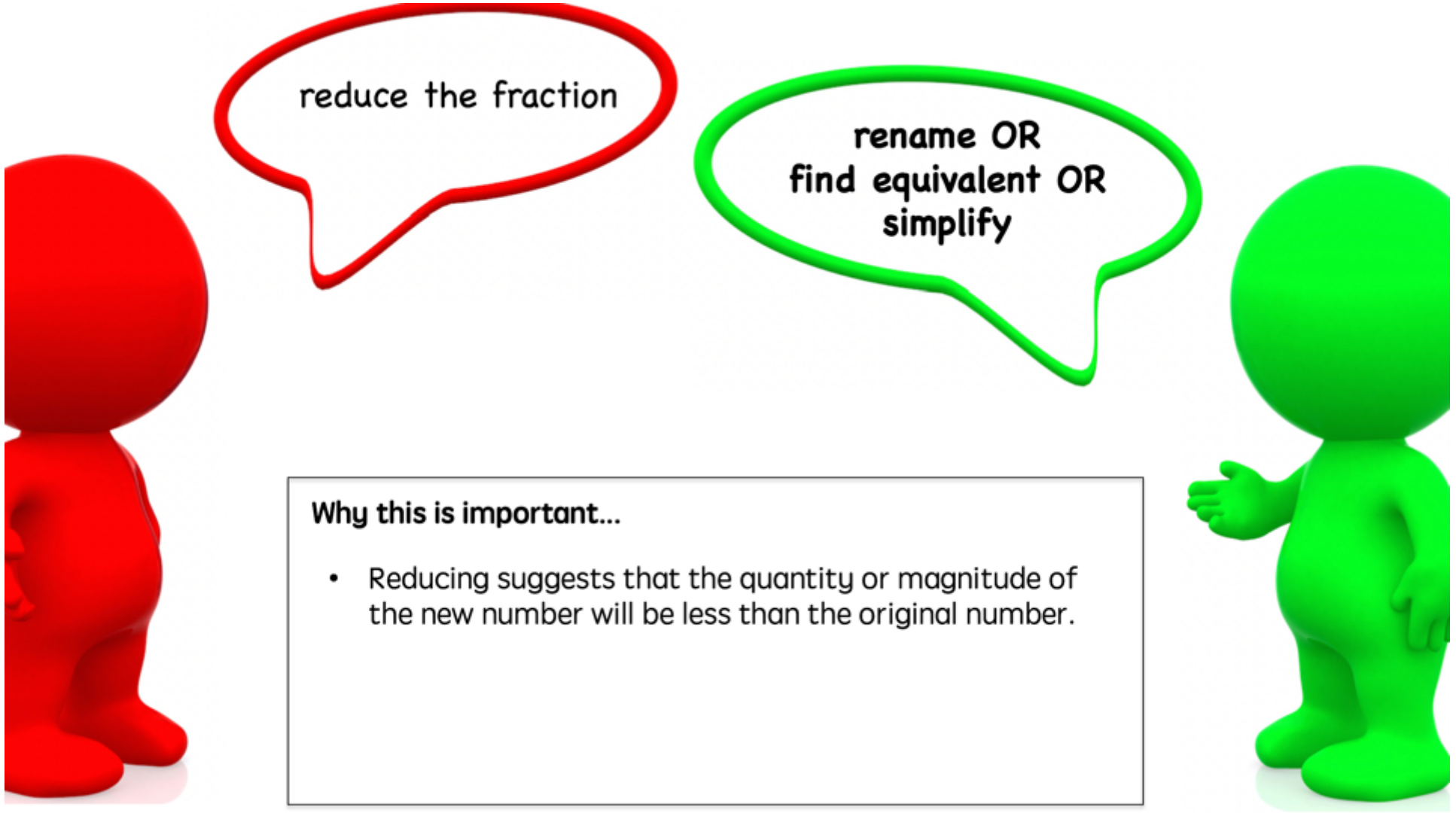
Why this is important...

- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



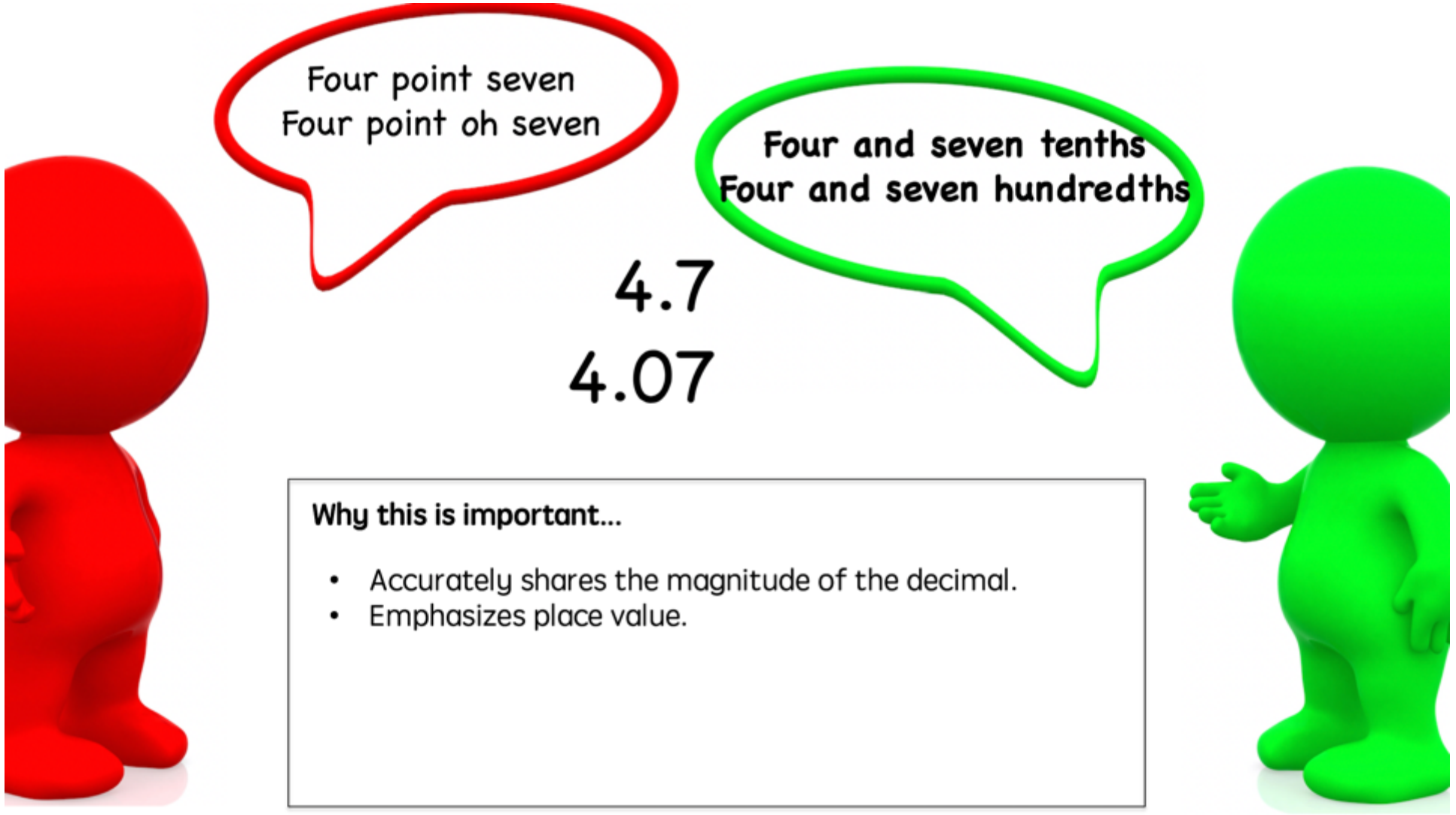
Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.



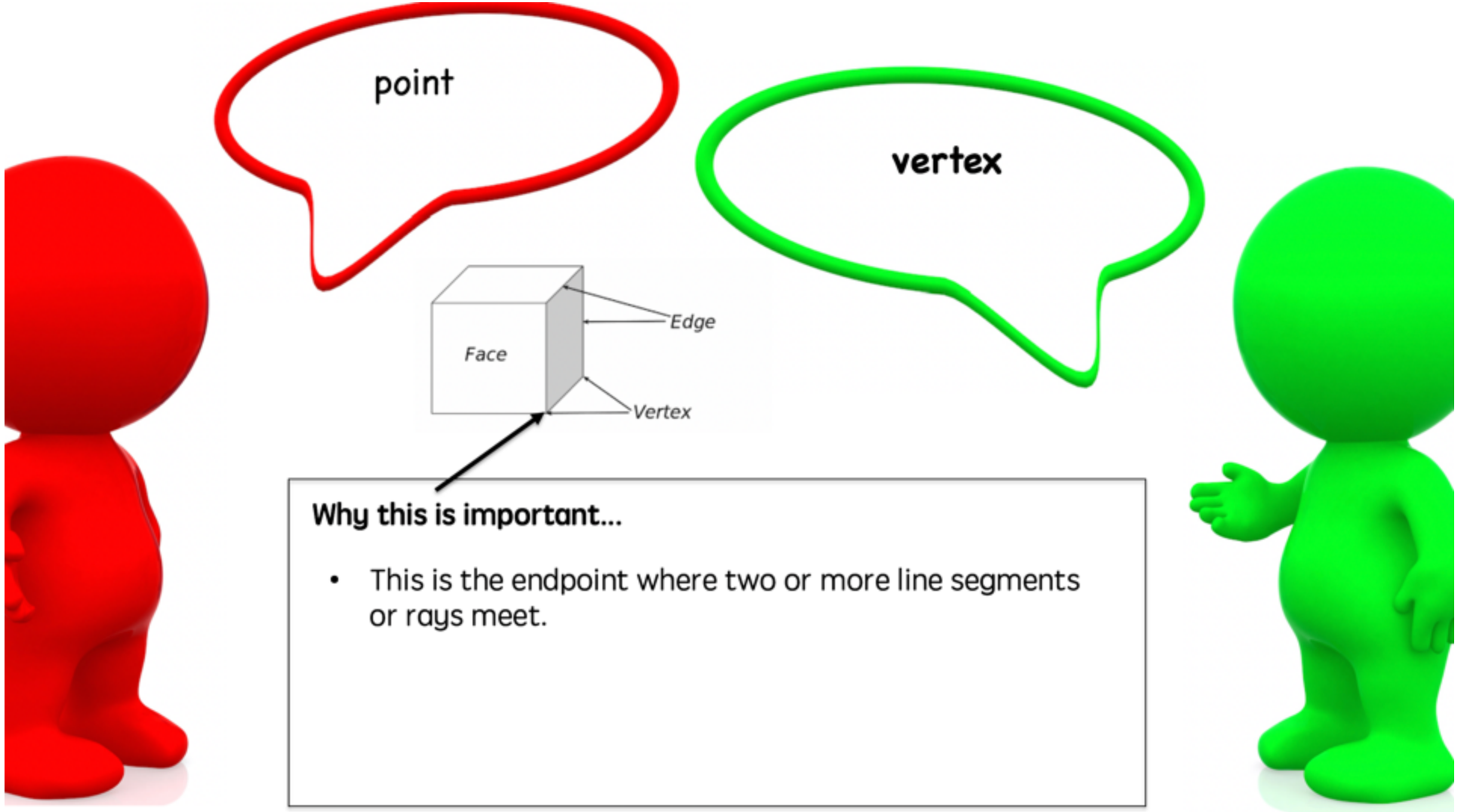
Why this is important...

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.



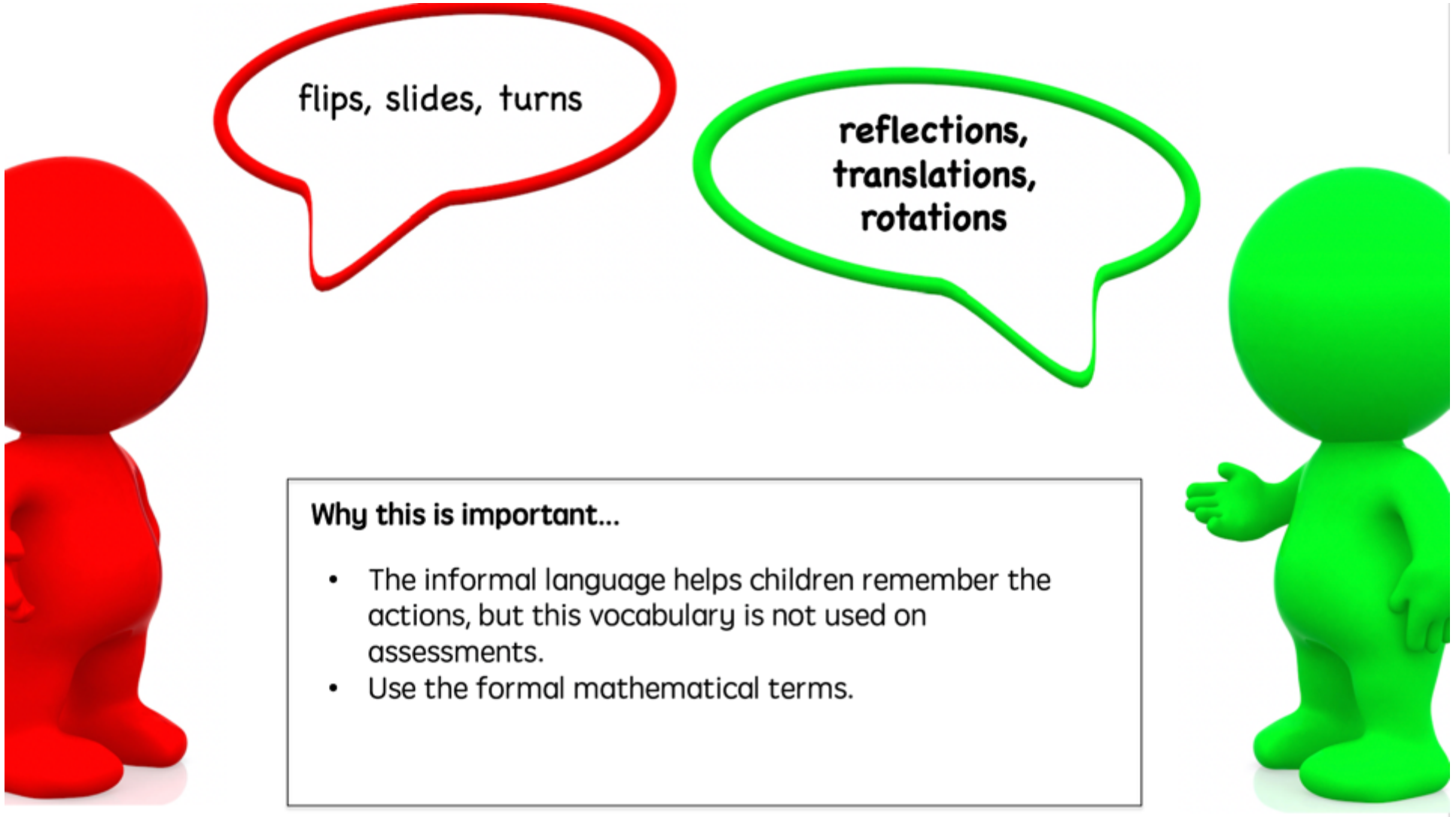
Why this is important...

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



Why this is important...

- This is the endpoint where two or more line segments or rays meet.



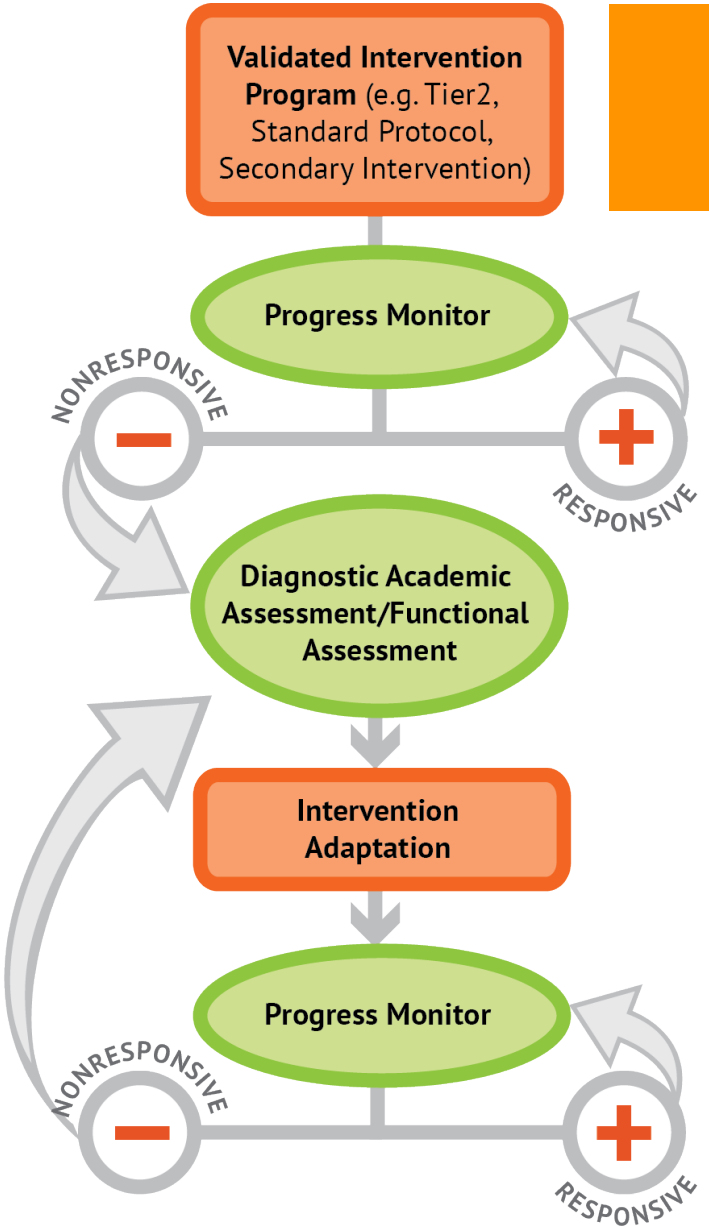
Why this is important...

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.

Use formal math language

Use terms precisely

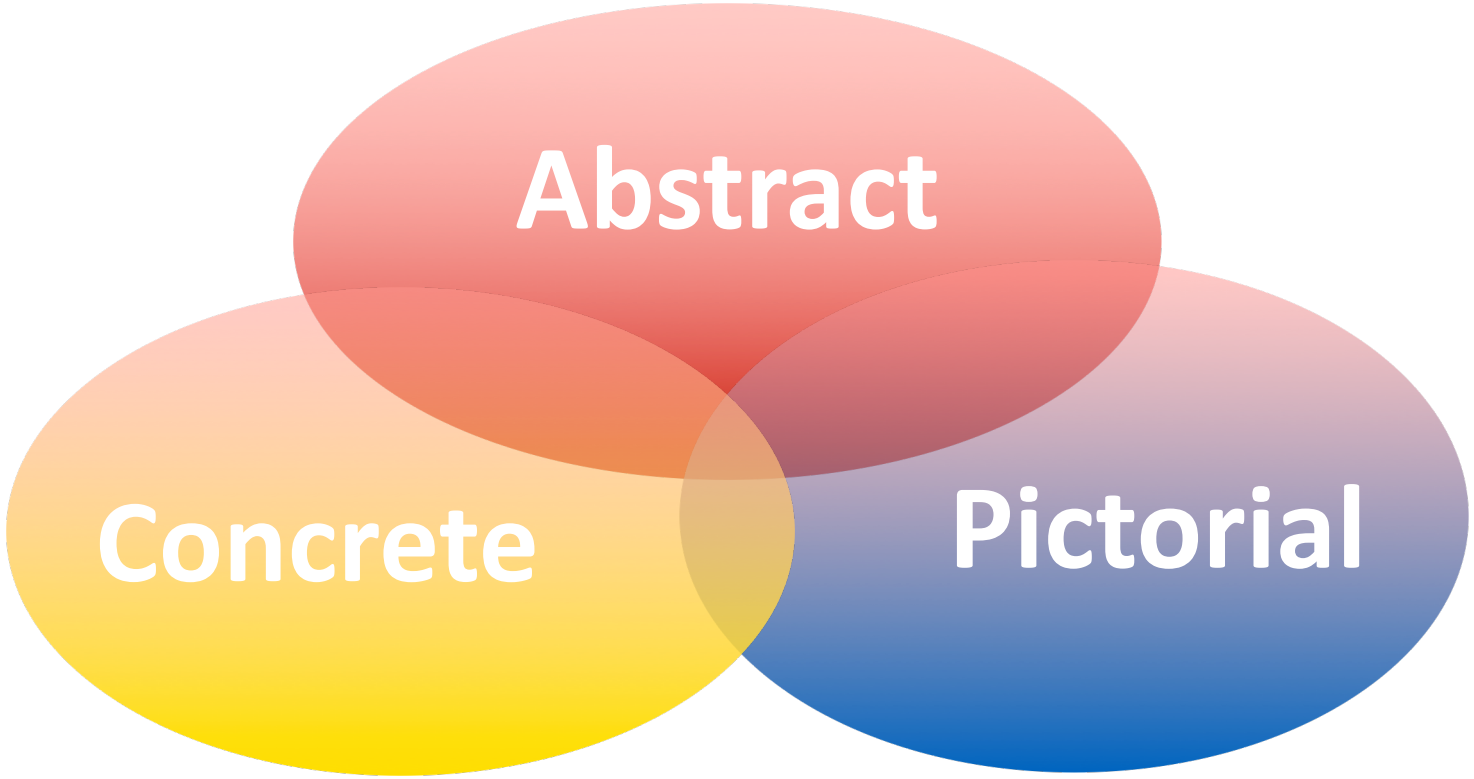
Instructional Platform

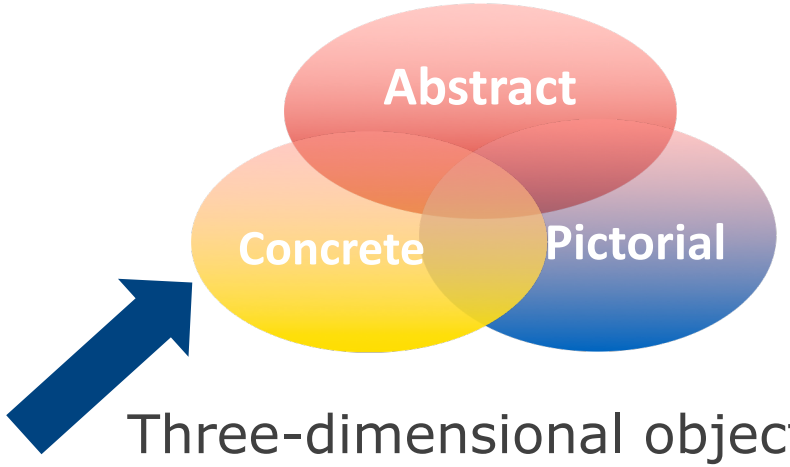


INSTRUCTIONAL DELIVERY

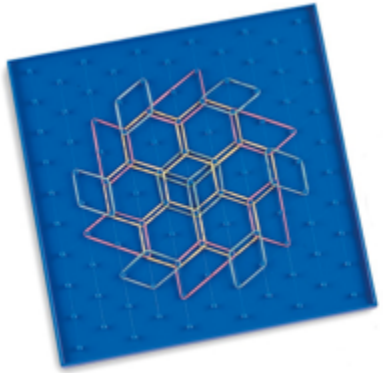
- Explicit instruction
- Precise language
- Multiple representations

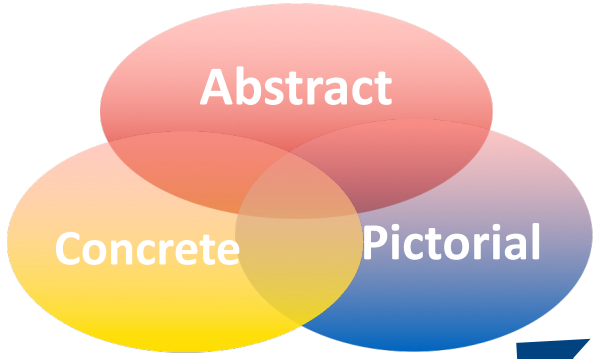
INSTRUCTIONAL STRATEGIES


Abstract
Concrete
Pictorial

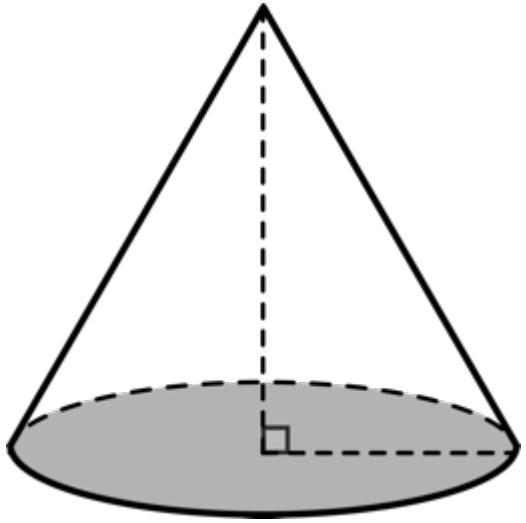


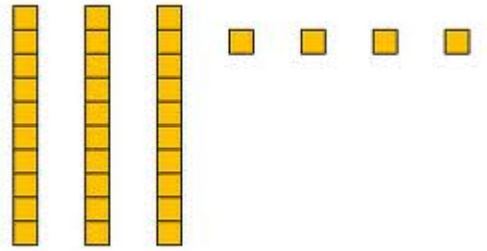
Three-dimensional objects

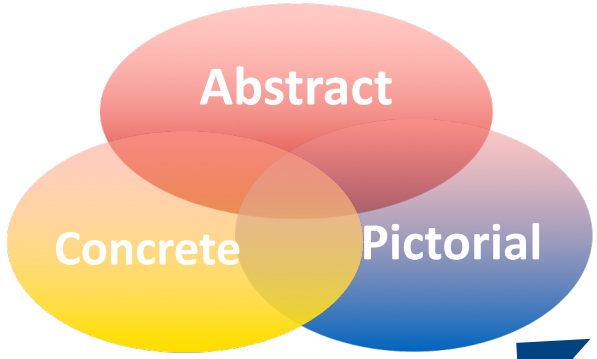




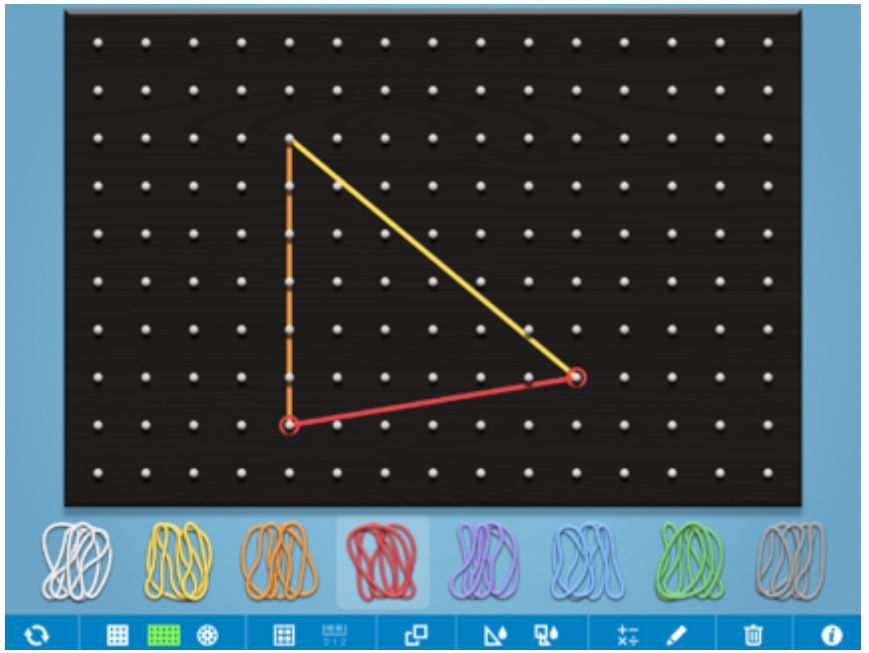
Two-dimensional images

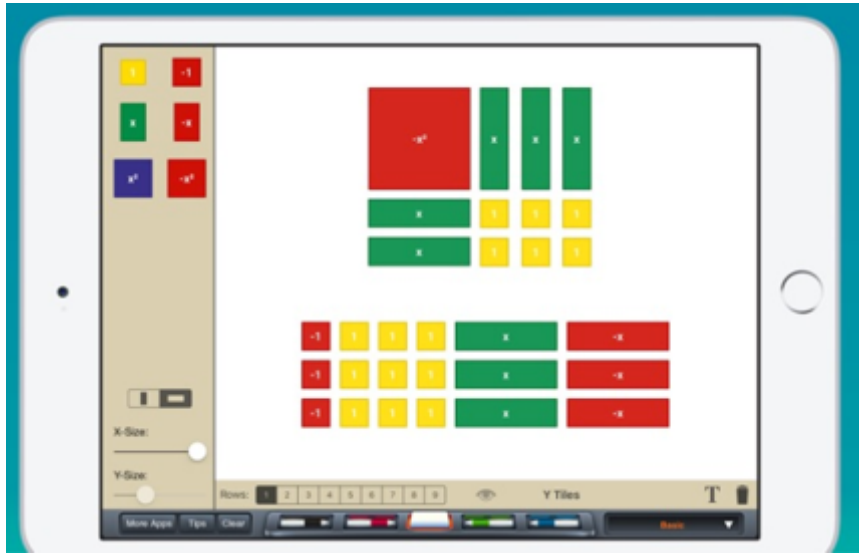


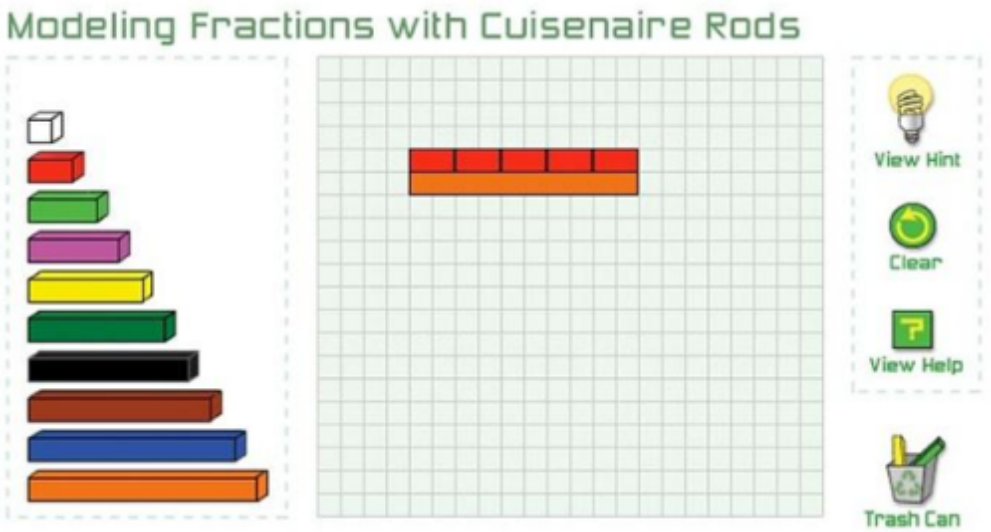


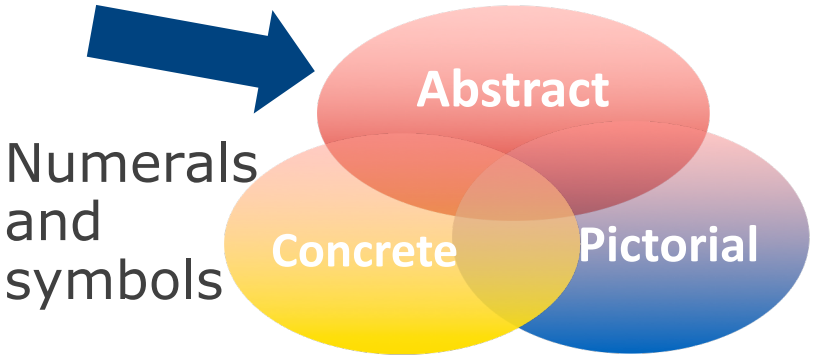


Two-dimensional images







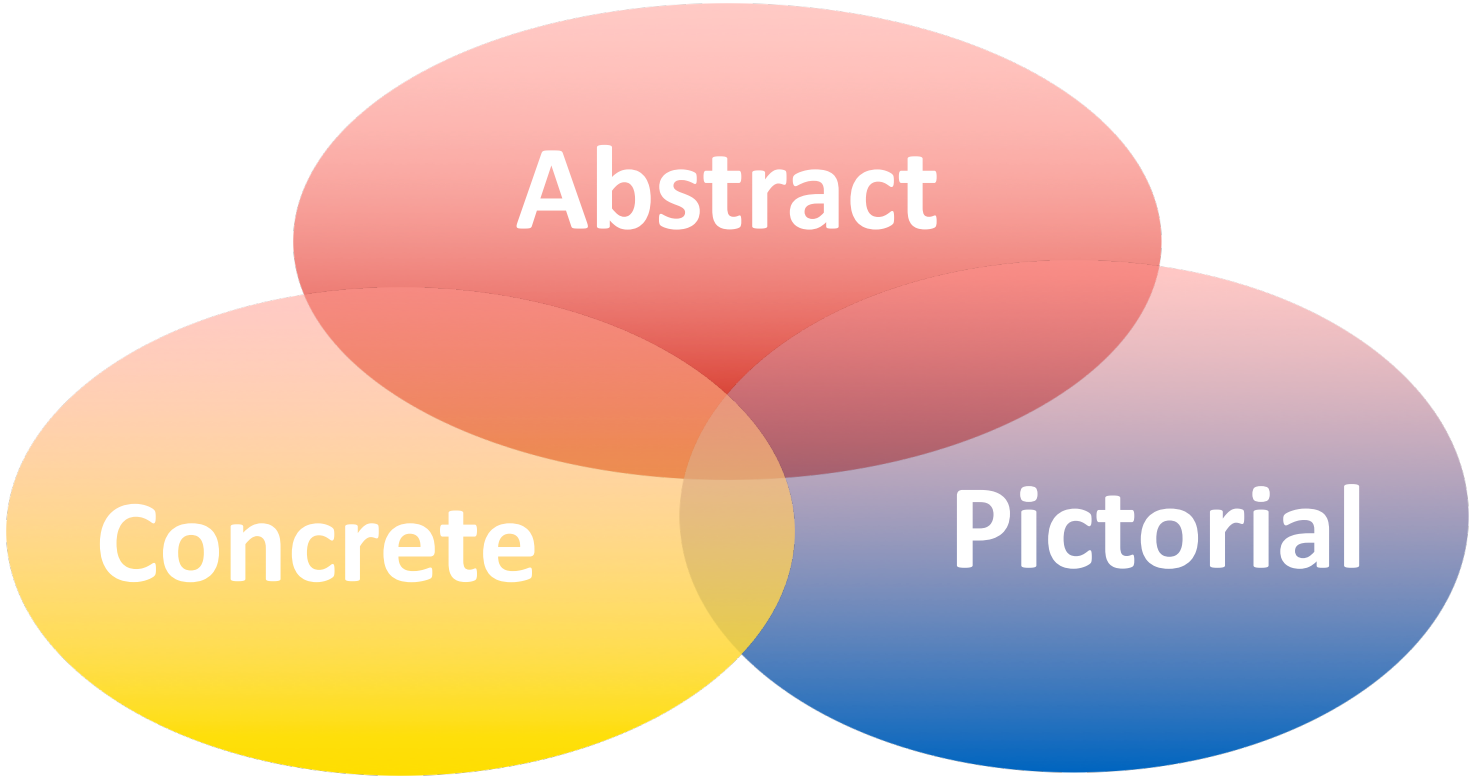


$2 + 8 = 10$

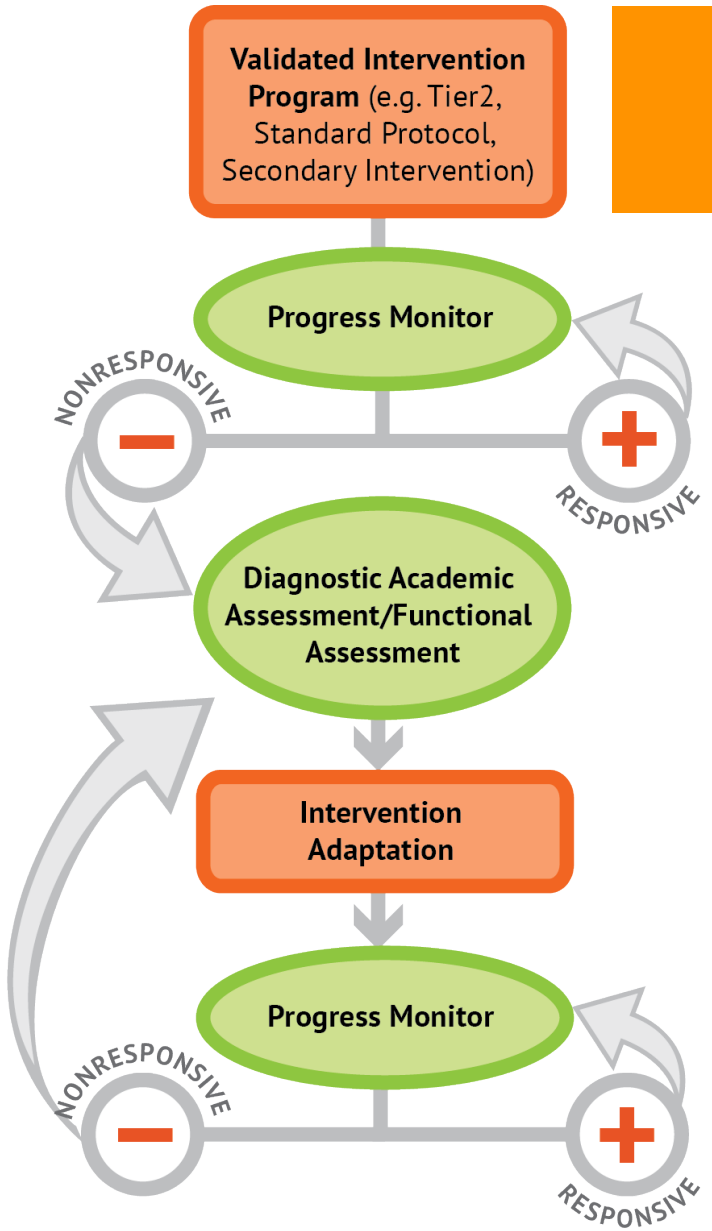
34 = 3 tens and 4 ones

$x - 6 = 8$

$$\begin{array}{r} 4{,}179 \\ +\ \ 569 \\ \hline \end{array}$$


Abstract
Concrete
Pictorial

Instructional Platform

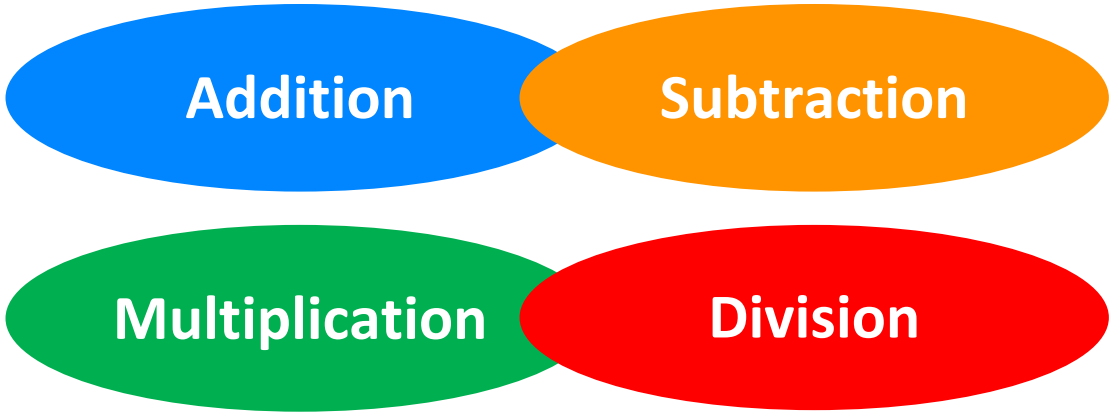


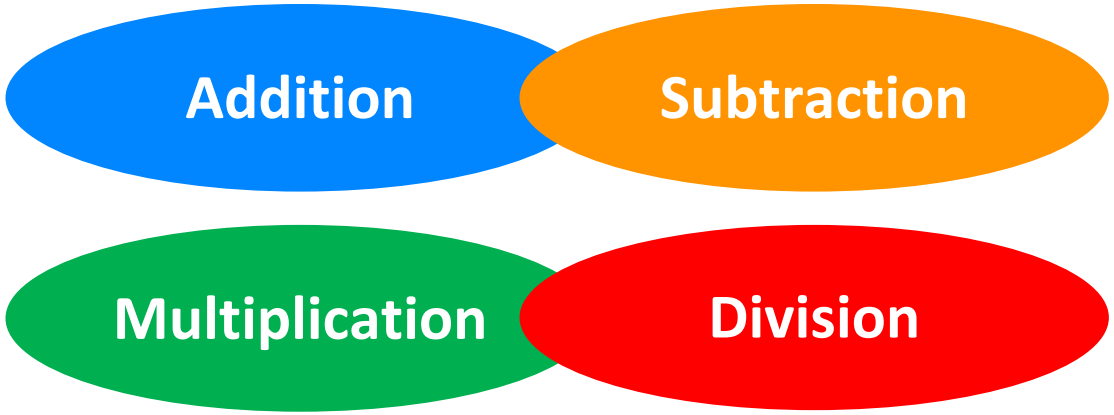
INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

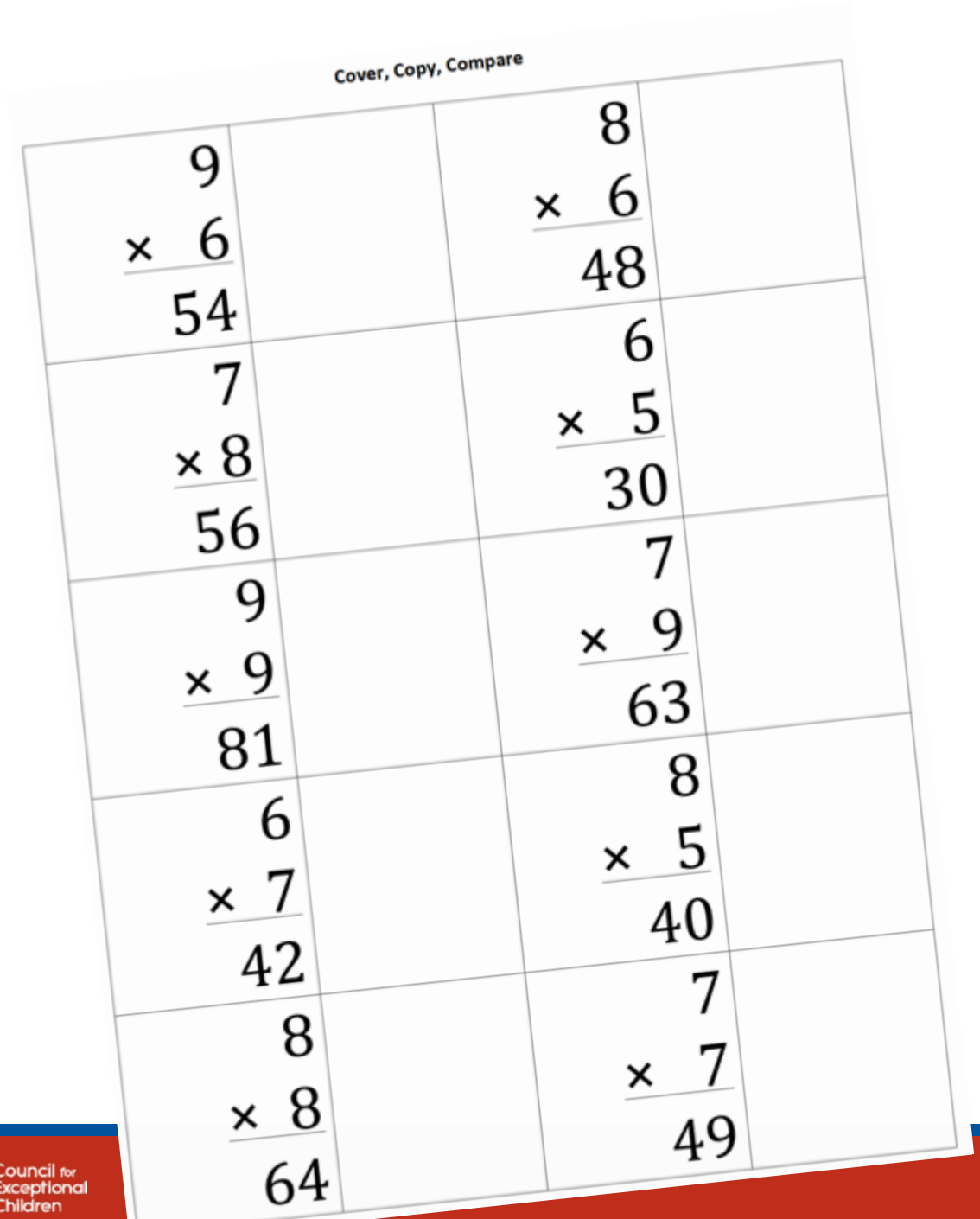
- Fluency building


Addition
Subtraction
Multiplication
Division



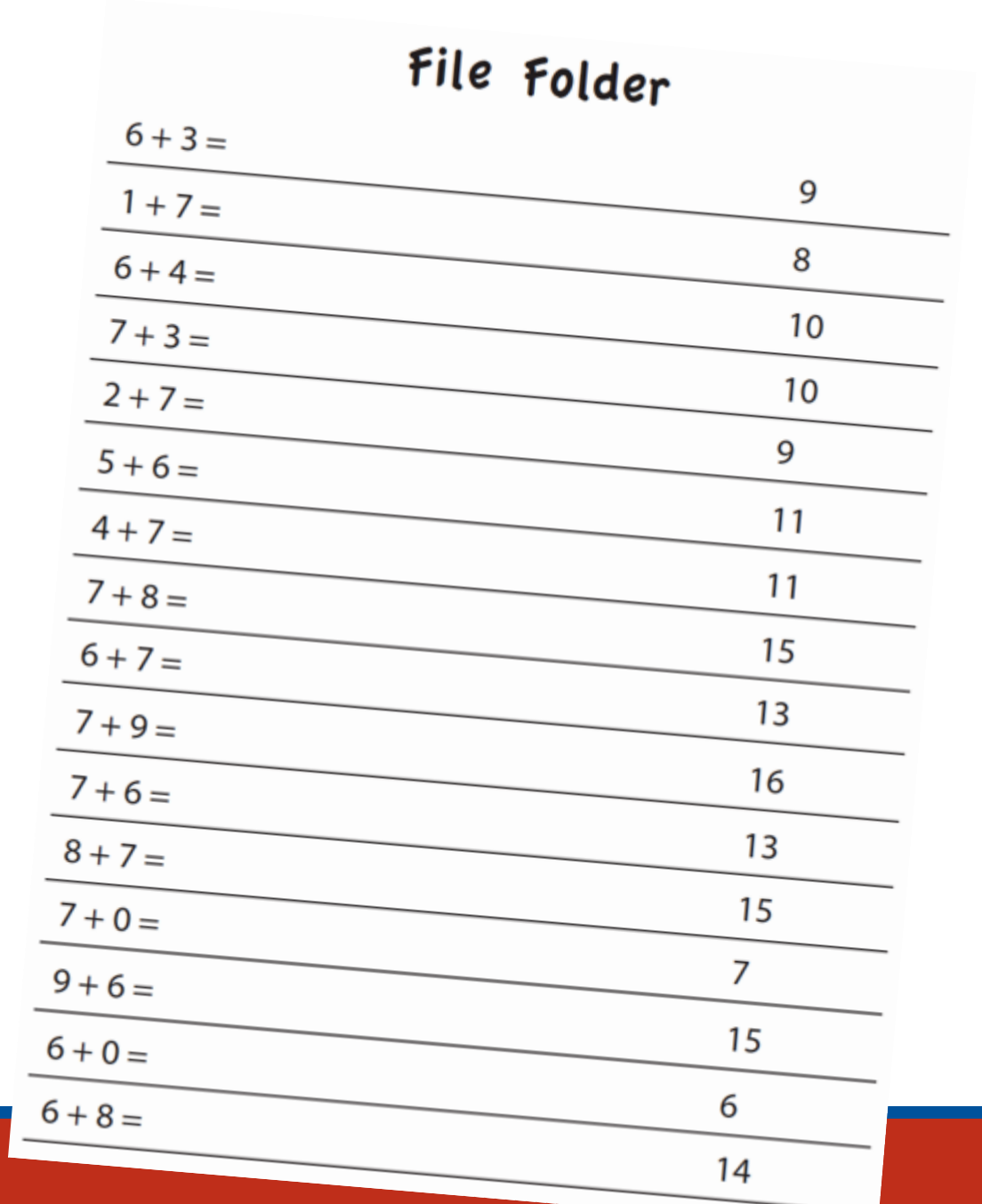
BRIEF
(1-2 min)

DAILY
(everyday)



Cover, Copy, Compare

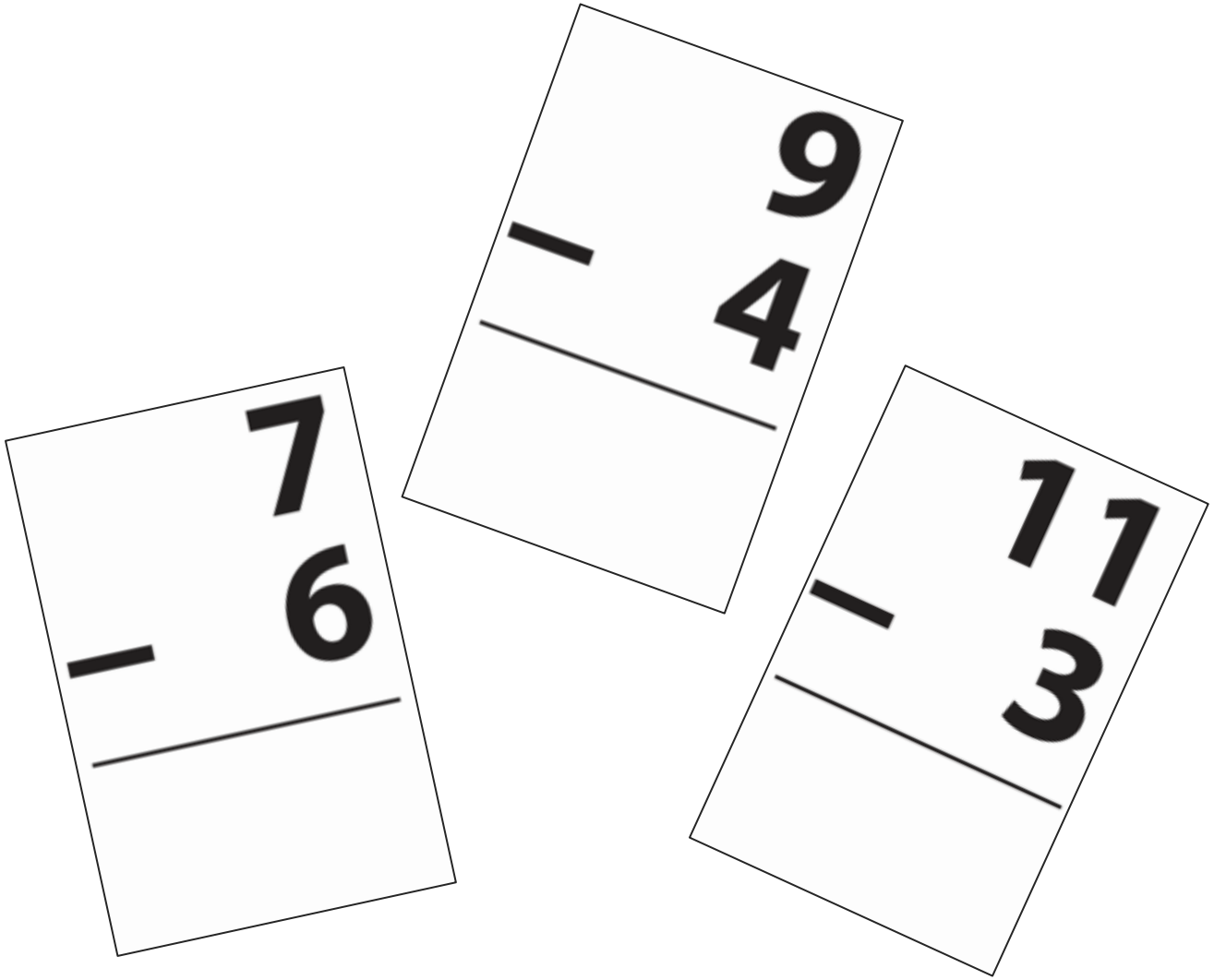
9 × 6 = 54		8 × 6 = 48	
7 × 8 = 56		6 × 5 = 30	
9 × 9 = 81		7 × 9 = 63	
6 × 7 = 42		8 × 5 = 40	
8 × 8 = 64		7 × 7 = 49	



File Folder

6 + 3 =	9
1 + 7 =	8
6 + 4 =	10
7 + 3 =	10
2 + 7 =	9
5 + 6 =	11
4 + 7 =	11
7 + 8 =	15
6 + 7 =	13
7 + 9 =	16
7 + 6 =	13
8 + 7 =	15
7 + 0 =	7
9 + 6 =	15
6 + 0 =	6
6 + 8 =	14


Taped Problems
6 × 5
8 × 6
7 × 9
6 × 8
9 × 8
8 × 5
7 × 8
6 × 6
7 × 7
6 × 9
5 × 9
8 × 4
9 × 4
6 × 9
9 × 5
8 × 7
6 × 7
8 × 8
4 × 8
5 × 7



Name: _____

Flash Card Graph

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10	Day 11	Day 12
38												
37												
36												
35												
34												
33												
32												
31												
30												
29												
28												
27												
26												
25												
24												
23												
22												
21												
20												
19												
18												
17												
16												
15												
14												
13												
12												
11												
10												
9												
8												
7												
6												
5												
4												
3												
2												
1												

		(place sum or product from baggie here)

5

4

12

8

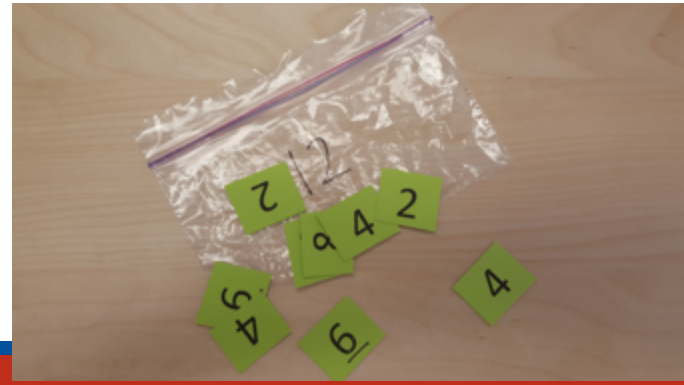
6

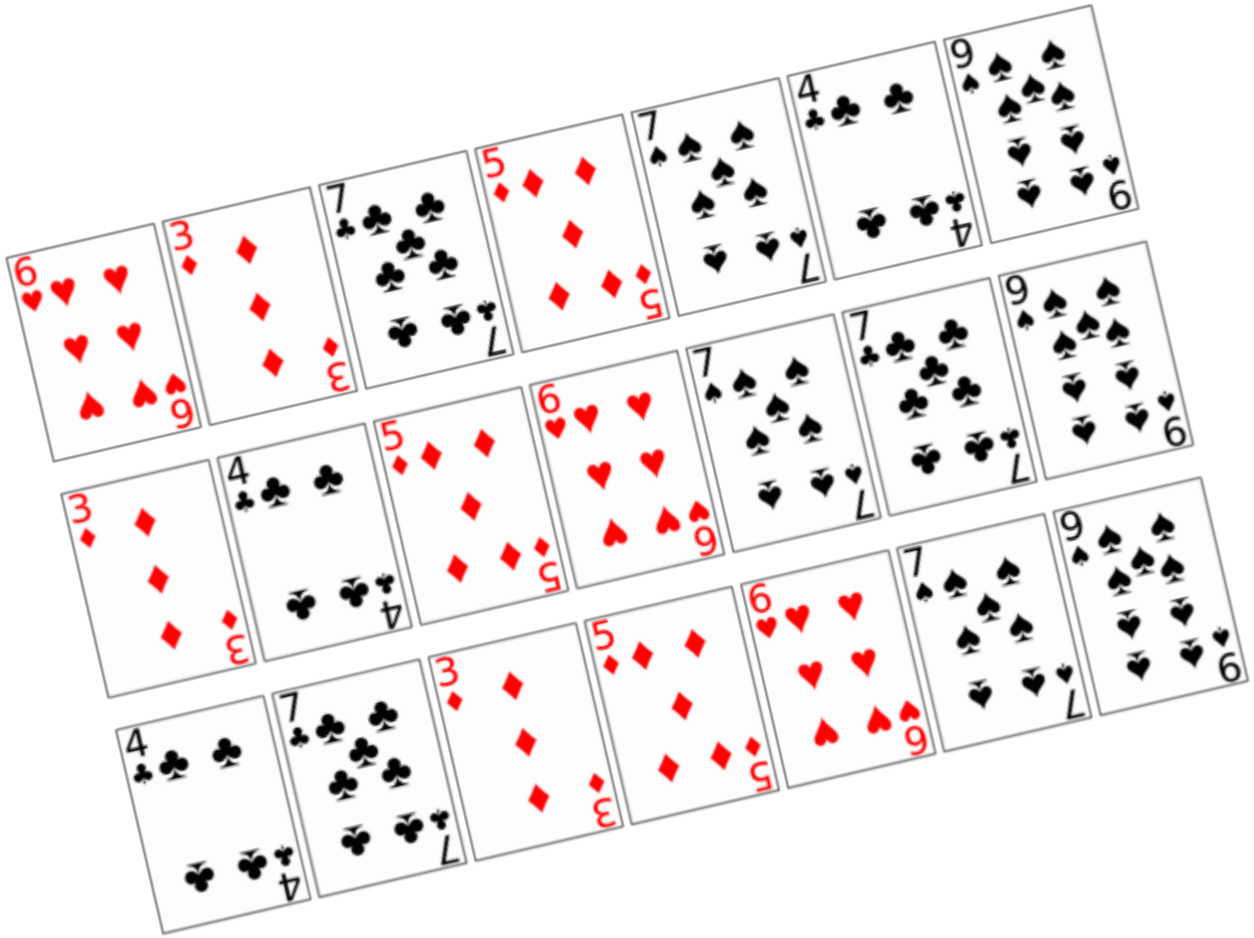
1

3

3

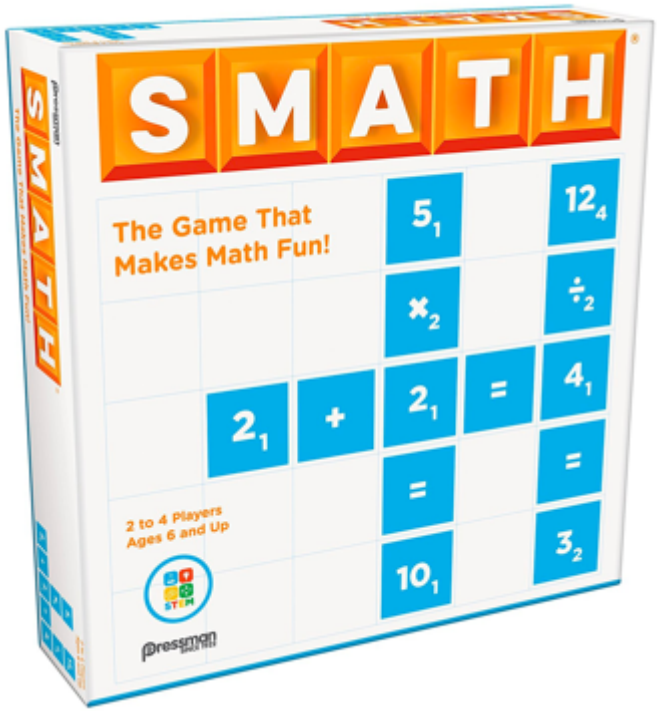
6

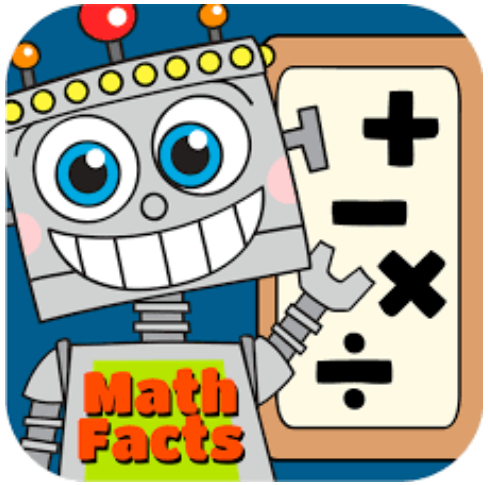





Wrap-Ups

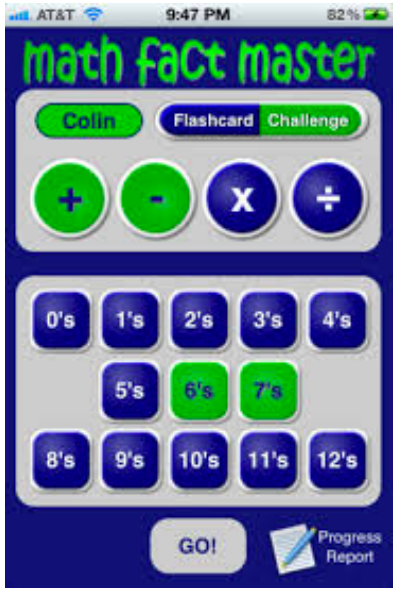

möbi

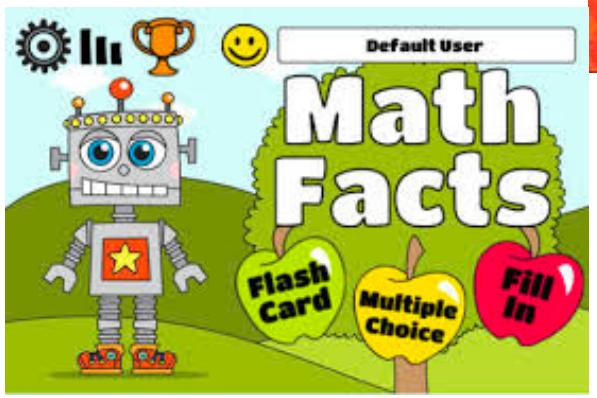

SMATH
The Game That Makes Math Fun!
2 to 4 Players
Ages 6 and Up
Pressman


Math
Facts


MATH
BINGO
PLAY
SCORE BOARD
HOW TO PLAY

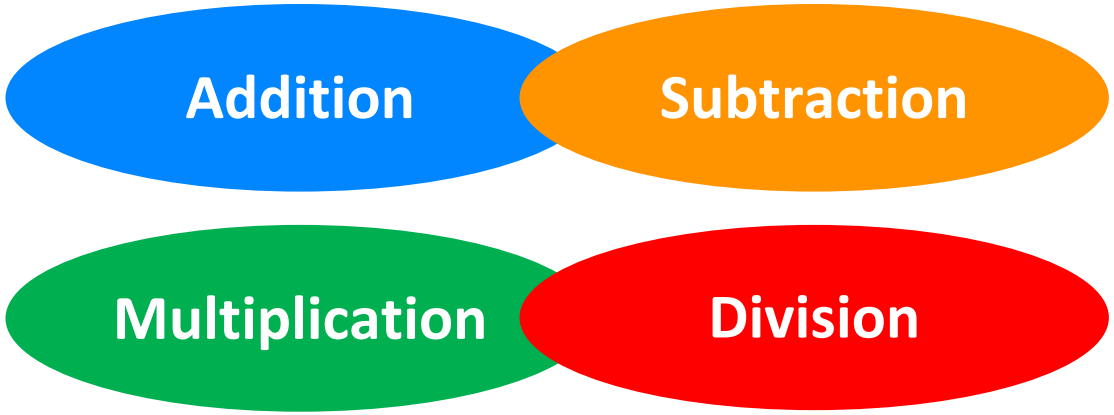

AT&T
10:44 AM
9 + 8
19
16
17
15
Rnd


AT&T
9:47 PM
82%
math fact master
Colin
Flashcard
Challenge
+
-
x
÷
0's
1's
2's
3's
4's
5's
6's
7's
8's
9's
10's
11's
12's
GO!
Progress
Report


Default User
Math
Facts
Flash
Card
Multiple
Choice
Fill
In


5+4=
2?3=6
27÷3?11

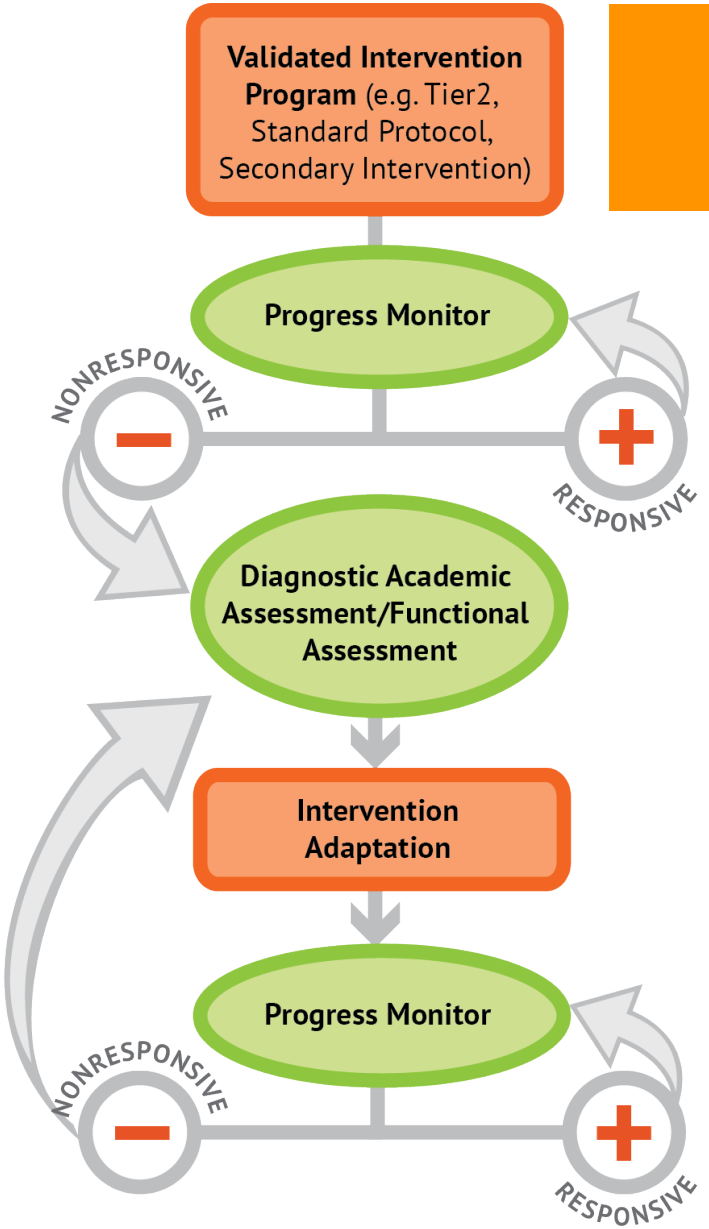

PRACTICE
MATH



BRIEF
(1-2 min)

DAILY
(everyday)

Instructional Platform



INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

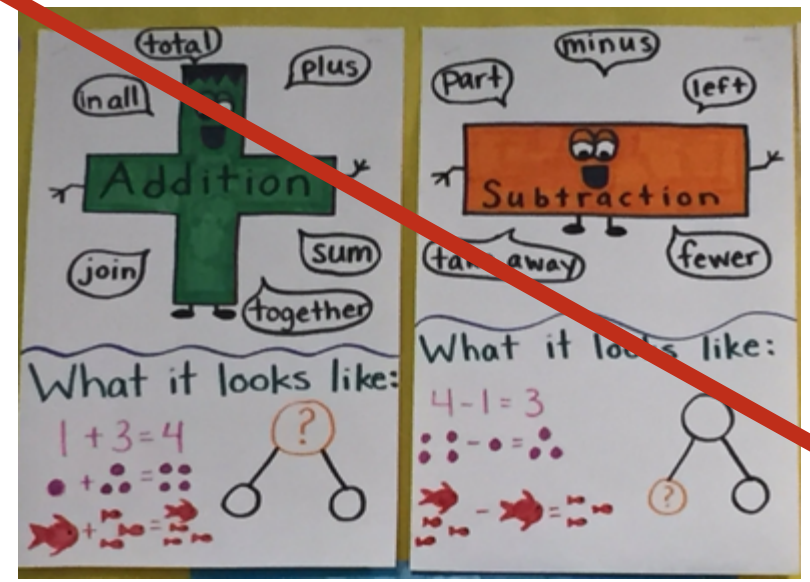
INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

Don’t tie key words to operations

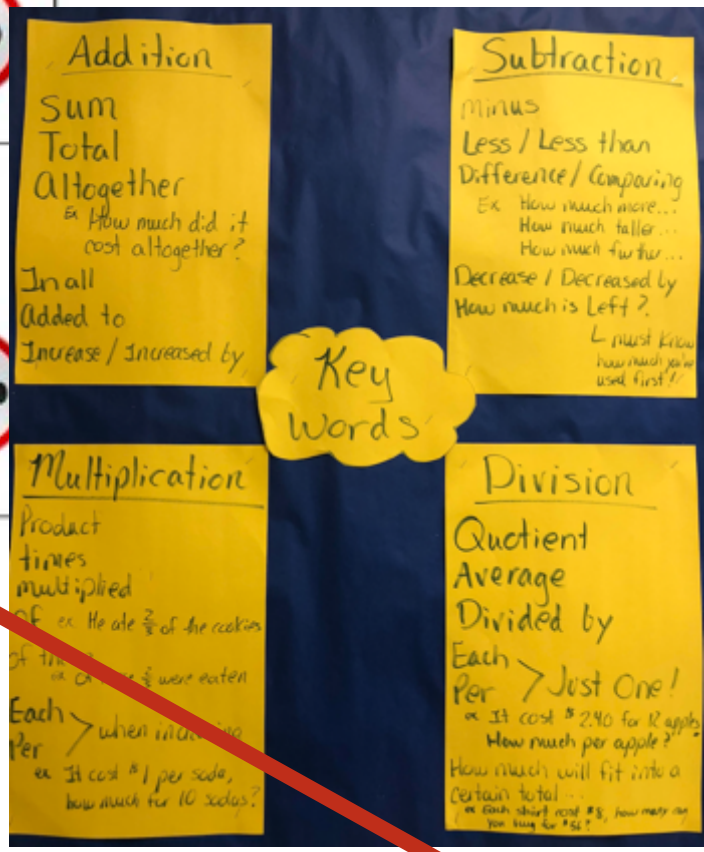
Do have an attack strategy

Do teach word-problem schemas


total
plus
in all
Addition
join
sum
together
What it looks like:
1 + 3 = 4
minus
Part
left
Subtraction
fewer
What it looks like:
4 - 1 = 3


Key Words Used in Math Word Problems
Addition Words
+ add
+ all together or altogether
+ and
+ both
+ combined
+ how many in all
+ how much
+ in all
+ increased by
+ plus
+ sum
+ together
+ total
Subtraction Words
- change
- decreased by
- difference
- fewer or fewer than
- how many are left (or have left)
- how many did not have
- how many (or much) more
- how much longer (shorter, taller, heavier, etc.)
- less or less than
- lost
- minus
- need to
- reduce
- remain
- subtract
- take away
Multiplication Words
x by (dimension)
x double
x each group
x every
x factor of
x increased by
x multiply by
x of
x product
x times
x triple
Division Words
÷ as much
÷ cut up
÷ each group has
÷ equal sharing
÷ half (or other fractions)
÷ how many in each
÷ parts
÷ per
÷ percent
÷ quotient of
÷ ratio of
÷ separated
÷ share something equally


combined
addition
in all
total
sum
together
plus
both
perimeter
add
more than
triple
twice
factor
product
multiply
each
per
in all
area
times
double
key words
average
division
split
quotient
equal groups
divide
half
shared equally
each
distribute
subtract
decrease
fewer
remain
away
minus
less
how many more...
©TheResourceRoomTeacher


Addition
Sum
Total
Altogether
Ex How much did it cost altogether?
In all
Added to
Increase / Increased by
Subtraction
Minus
Less / Less than
Difference / Comparing
Ex How much more...
How much taller...
How much further...
Decrease / Decreased by
How much is Left?
I must know how much you've used first!
Key Words
Multiplication
Product
times
multiplied
of ex He ate 2/3 of the cookies
were eaten
Each > when
Per
ex It cost $1 per soda, how much for 10 sodas?
Division
Quotient
Average
Divided by
Each
Per
> Just One!
ex It cost $240 for 12 apples
How much per apple?
How much will fit into a certain total...
ex Each shirt cost $8, how many can you buy for $56?

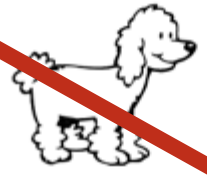
Word Problems: Subtraction

Name _____ Date _____

Subtraction

Read each problem. Write a number sentence and solve.

1. Mrs. Smith has 33 poodles and 18 boxers. How many more poodles does Mrs. Smith have?

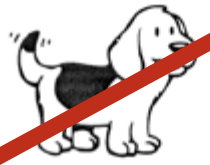


_____

2. The kennel holds 91 dogs. Mr. Glass has 67 dogs in the kennel now. How many spaces does he have left?

_____

3. Mr. Kelly has 44 beagles. 26 of them are puppies. How many adult beagles does Mr. Kelly have?



_____

4. Mr. Green has 60 terriers. 25 of them are boys. How many terriers are girls?

_____

5. There were 58 kittens at the pet shop on Friday. 29 of them were sold on Saturday. How many kittens were left?

_____

6. Pat counted 22 lizards in the tank at the pet shop. 8 were sold later that day. How many lizards were left in the tank?

_____

RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development.

Solve the equatio

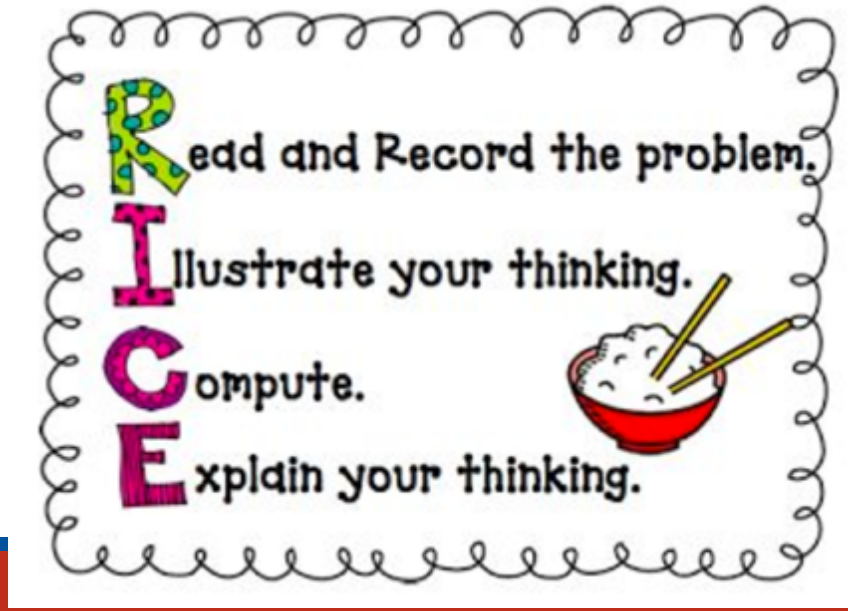
RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.



Attack Strategy

UPS✓

Understand

Read the problem.

Ask yourself:

- What information do I know?
- What is the question asking me to find?

Plan

Choose a strategy, a tool or an approach.

Solve

Show the math used to solve the problem.

Check

Check your math.

Ask yourself:

- Did I answer the question asked?
- Is my answer reasonable?

Total

Difference

Change

Equal Groups

Comparison

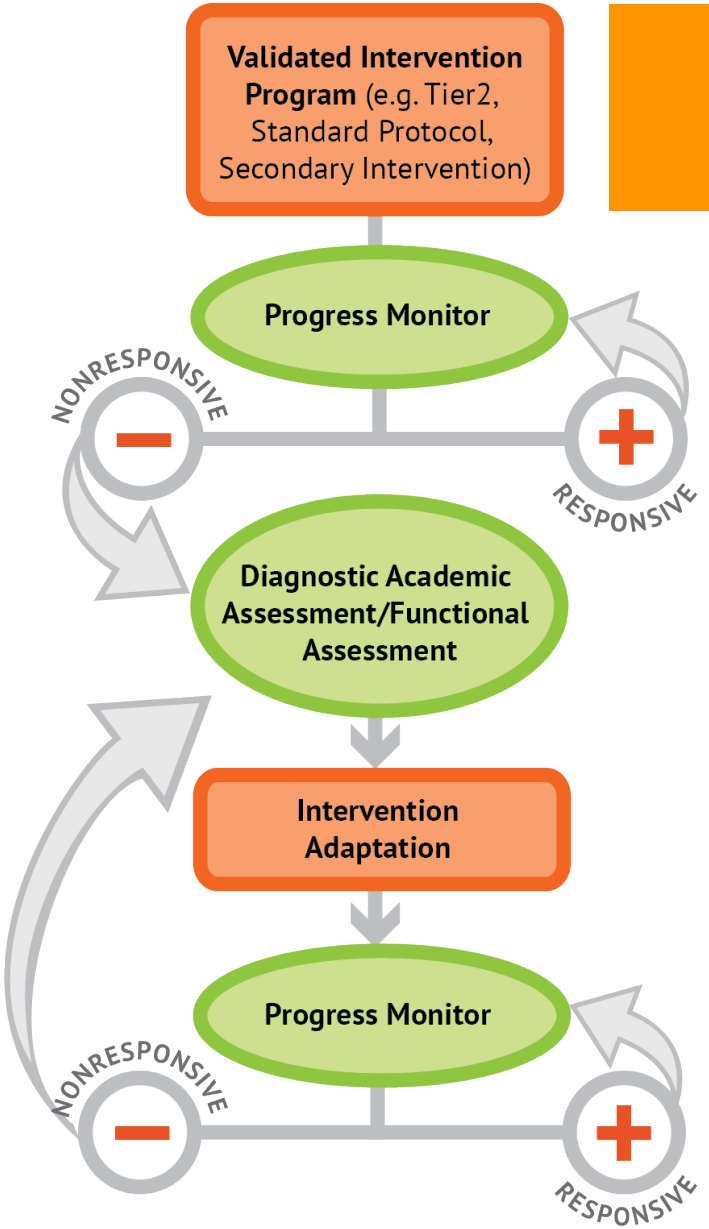
Ratios/Proportions

Don’t tie key words to operations

Do have an attack strategy

Do teach word-problem schemas

Instructional Platform

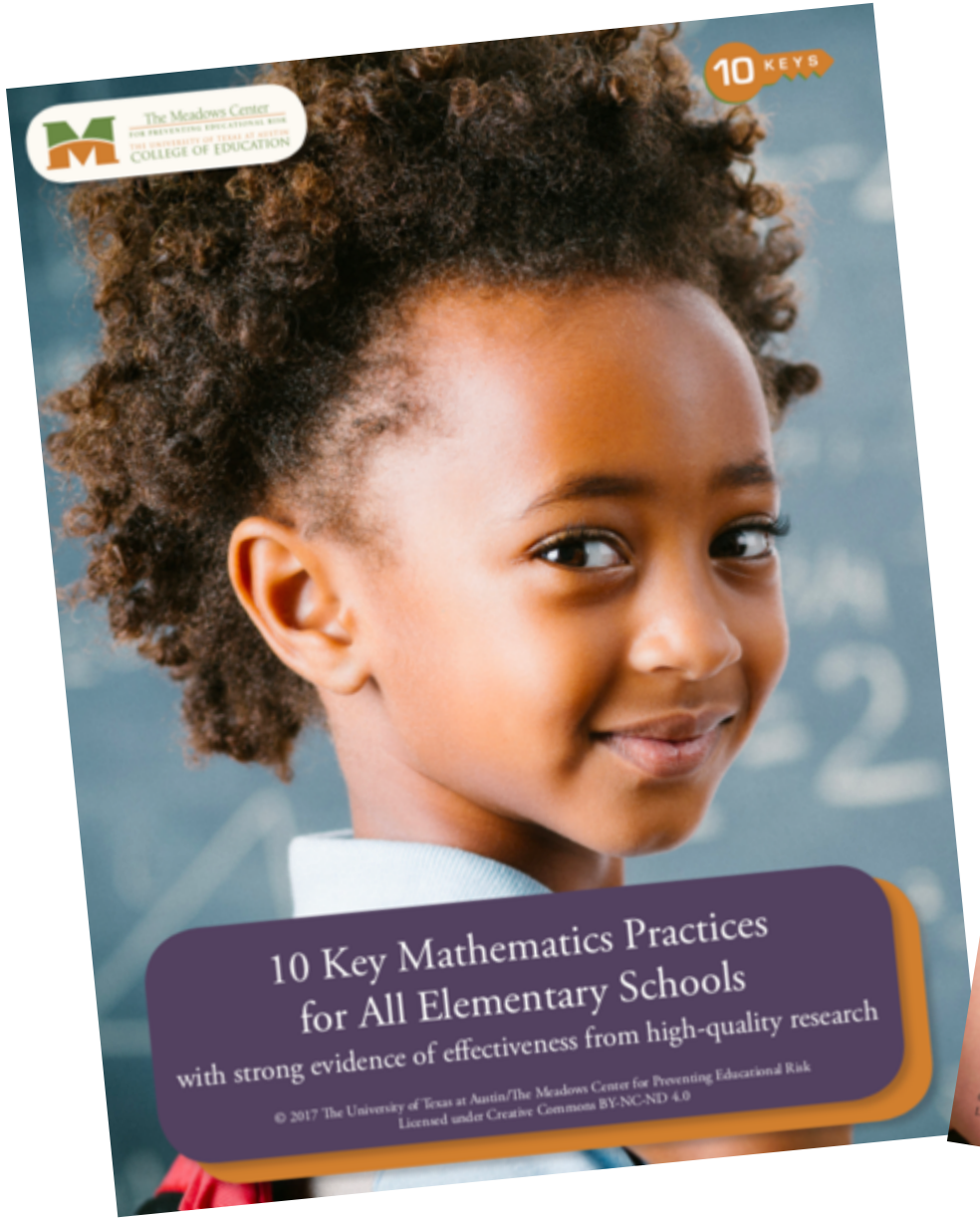


INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction



https://www.meadowscenter.org/files/resources/10Keys_ElemMath_Web.pdf

https://www.meadowscenter.org/files/resources/10Keys_SecMath_Web.pdf

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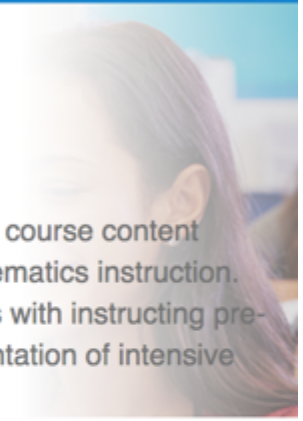

Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a high-leverage practice in special education, and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEEDAR Center, developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.

https://intensiveintervention.org/intensive-intervention-math-course


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