CHAPTER 3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

EXERCISE 3.1

1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be". (Isn't this interesting?) Represent this situation algebraically and graphically.

Sol :

Let the present age of Aftab be x years and his daughter's age be y years.

Seven years ago :

$$\text{Aftab's age} = (x-7) \text{ years}$$

$$\text{Daughter's age} = (y-7) \text{ years}$$

According to the question,

$$(x-7) = 7(y-7)$$

$$x-7 = 7y-49$$

$$x-7y+42 = 0 \qquad \text{...(i)}$$

After three years :

$$\text{Aftab's age} = (x+3) \text{ years}$$

$$\text{His daughter's age} = (y+3) \text{ years}$$

According to the question,

$$(x+3) = 3(y+3)$$

$$x+3 = 3y+9$$

$$x-3y-6 = 0 \qquad \text{...(ii)}$$

Graphical Representation :

From equation (i), we have,

$$l_1 : x-7y+42 = 0$$

$$y = \frac{x+42}{7}$$

x	0	7	35
y	6	7	11
(x, y)	(0, 6)	(7, 7)	(35, 11)

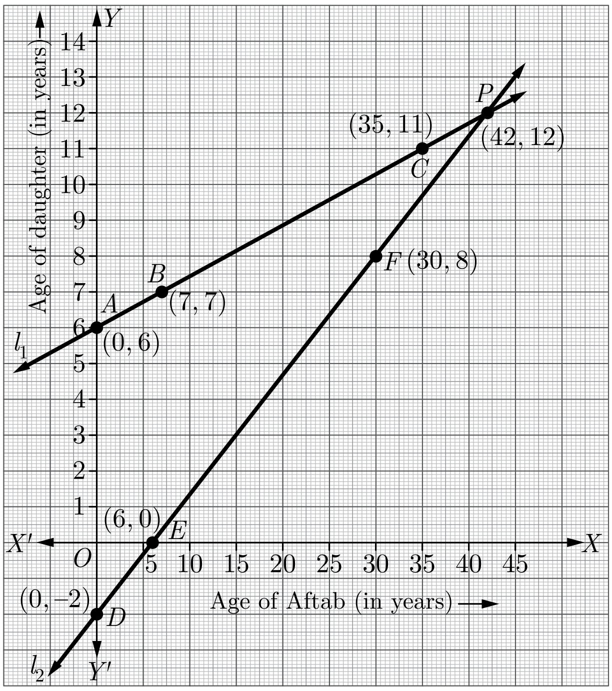
From equation (ii), we have

$$l_2 : x-3y-6 = 0$$

$$y = \frac{x-6}{3}$$

x	0	6	30
y	-2	0	8
(x, y)	$(0, -2)$	(6, 0)	(30, 8)

Scale:
Along X-axis : 10 small division = 5 years
Along Y-axis : 10 small division = 1 year



The lines l_1 and l_2 intersect at (42, 12).

PRACTICE :

1. Ramesh tells his daughter, "five years ago, I was five times as old as you were then. Also, ten years from now, I shall be twice times as old as you will be". (Isn't this interesting?) Represent this situation algebraically and graphically.
   Ans : $x-5y = -20,\ x-2y = 10$
2. Mahesh tells his daughter, "five years ago, I was three times as old as you were then. Also, ten years from now, I shall be three times as old as you will be". (Isn't this interesting?) Represent this situation algebraically and graphically.
   Ans : $x-3y = -10,\ x-2y = 10$

2. The coach of a cricket team buys 3 bats and 6 balls for ₹ 3900. Later, she buys another bat and 3 more balls of the same kind for ₹ 1300. Represent this situation algebraically and geometrically.

Sol :

Let the cost of a bat be x and the cost of a ball be y

Case I : $3x + 6y = 3900$

or $x + 2y = 1300$ ...(i)

Case II : $x + 3y = 1300$ ...(ii)

Thus, (i) and (ii) are the algebraic representations of the given situation.

Geometrical Representation:

We have for equation (i),

l_1 : $x + 2y = 1300$

$$y = \frac{1300 - x}{2}$$

x	0	1300	100
y	650	0	600

and for equation (ii),

l_2 : $x + 3y = 1300$

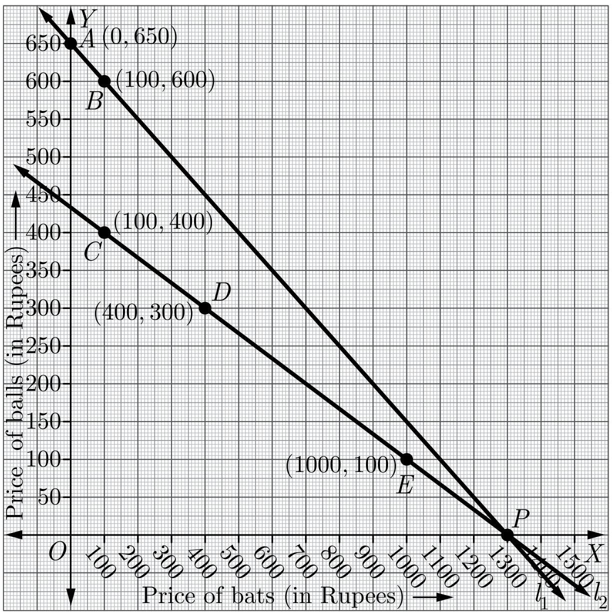
$$y = \frac{1300 - x}{3}$$

x	400	100	1000
y	300	400	100

Scale:

Along X-axis : 10 small division = Rs.100

Along Y-axis : 10 small division = Rs.50



We can also see from the obtained figure that the straight lines representing the two equations intersect at (1300, 0).

PRACTICE :

1. The coach of a cricket team buys 6 Pencil and 7 pen for ₹ 100. Later, she buys 7 pencil & 6 pen of the same kind for ₹ 95. Represent this situation algebraically and geometrically.

   Ans : $6x + 7y = 100,\ 7x + 6y = 95$

2. The coach of a cricket team buys 3 bags and 4 pens for ₹ 257. Later, she buys 4 bags & 3 pen of the same kind for ₹ 324. Represent this situation algebraically and geometrically.

   Ans : $3x + 4y = 257,\ 4x + 3y = 324$

3. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹ 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹ 300. Represent the situation algebraically and geometrically.

Sol :

Let the cost of 1 kg of apples be x and the cost of 1 kg of grapes be y.

Now $2x + y = 160$ ...(i)

and $4x + 2y = 300$

or $2x + y = 150$ ...(ii)

Geometrical Representation :

For equation (1) we have,

$l_1 : 2x + y = 160$

$y = 160 - 2x$

x	50	40	30
y	60	80	100

$y = 150 - 2x$

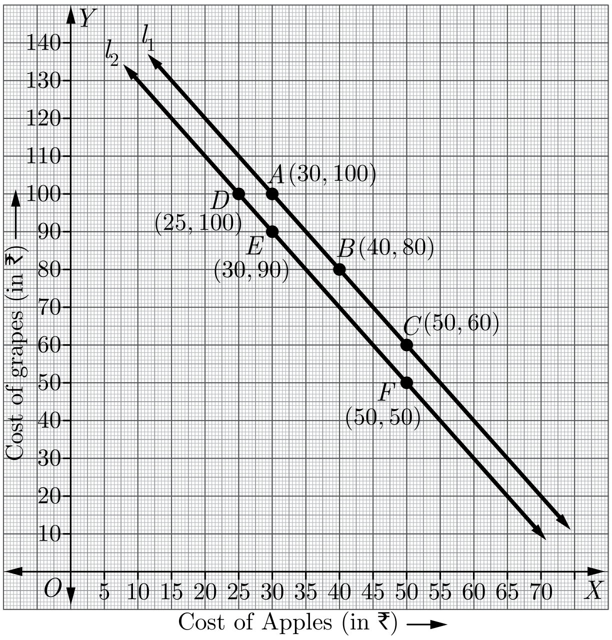
x	50	30	25
y	50	90	100

The straight lines l_1 and l_2 are the graphical representations of the equations (i) and (ii) respectively. The lines are parallel.

Scale:

Along X-axis : 10 small division = Rs.5

Along Y-axis : 10 small division = Rs.10



PRACTICE :

1. The sum of prices of an box and a chair is ₹ 2340 and their difference in price is ₹ 140. Represent the situation algebraically and geometrically.

   Ans : $x+y=2340,\ x-y=140$

2. The cost of 4 kg of apples and 2 kg of grapes on a day was found to be ₹ 320. After a month, the cost of 6 kg of apples and 4 kg of grapes is ₹ 600. Represent the situation algebraically and geometrically.

   Ans : $4x+2y=320,\ 6x+4y=600$

EXERCISE 3.2

1. Form the pair of linear equations in the following problems, and find their solutions graphically.
   (i) 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
   (ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencil and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

Sol :

(i) 10 students of class X took part in a mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Let the number of boys be x and number of girls be y.

Now $\quad x+y=10 \quad$ ...(i)

$$\text{Number of girls} = (\text{Number of boys}) + 4$$

$$y = x+4$$

$$x-y=-4 \quad \text{...(ii)}$$

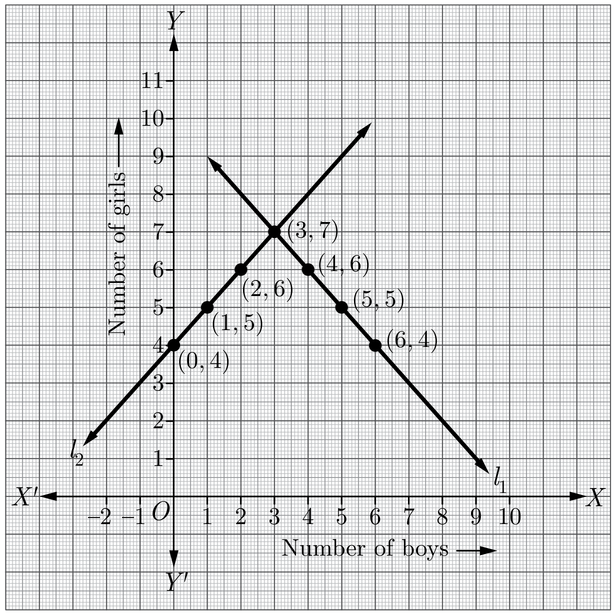
Now, from the equation (i), we have $l_1 : y = 10-x$

x	6	4	5
y	4	6	5

And from the equation (ii), we have $l_2 : y = x+4$

x	0	1	2
y	4	5	6

Scale:
Along X-axis : 10 small division = 1 boy
Along Y-axis : 10 small division = 1 girl



l_1 and l_2 intersects at the points (3, 7). The solution of the pair of linear equations is

$$x = 3 \text{ and } y = 7$$

Required number of boys $=3$ and required number of girls $=7$

(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencil and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

Let the cost of a each pencil be x and the cost of each pen be y.

Case I : $\quad 5x+7y = 50 \quad$ ...(i)

Case II : $\quad 7x+5y = 46 \quad$ ...(ii)

From equation (i), we have

$$5x+7y = 50$$

$$y = \frac{50-5x}{7}$$

Table of Equation (i)

For x	10	3	-4
For y	0	5	10
For (x, y)	(3, 5)	(10, 0)	$(-4, 10)$

From equation (ii), we have

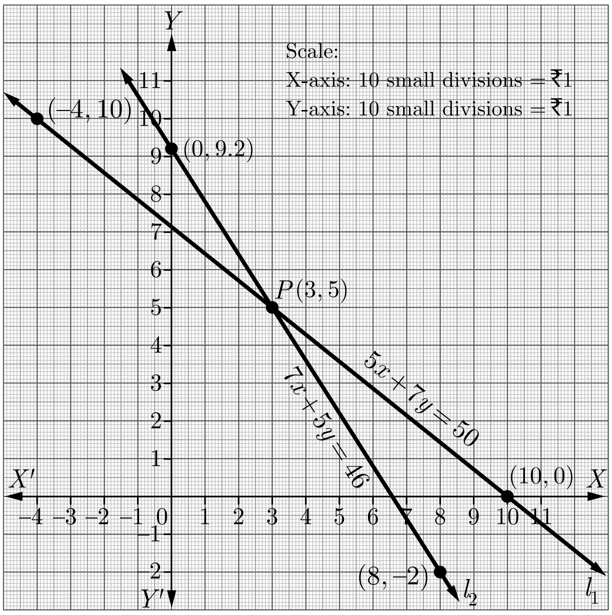
$$7x+5y = 46$$

$$y = \frac{46-7y}{5}$$

Table of Equation (ii)

For x	3	8	-2
For y	5	-2	12
For (x, y)	(3, 5)	$(8, -2)$	$(-2, 12)$

Now tracing the lines (i) and (ii) with the help of above table, we have



From graph paper, we observe that both lines (i) and (ii) intersect each other at point $P(3, 5)$, so that, $x = 3$ and $y = 5$.

Hence, the cost of a pencil $=$ ₹ 3

and cost of a pen $=$ ₹ 5

PRACTICE :

1. A and B each have a certain no. of mangoes. A says to B, "If you give me 30 of your mangoes, I will have twice as many as left with you." B replies, "If you give me 10, I will have thrice as many as left with you." How many mangoes does each have

 Ans : A = 34, B = 62

2. 4 tables and 3 chairs together cost ₹ 2600 and 3 tables and 4 chairs together cost ₹ 2300. Find the cost of 1 tables & 1 chairs.

 Ans : Tables = ₹ 500, Chairs = ₹ 200

2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident :

(i) $5x - 4y + 8 = 0,\ 7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0,\ 18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0,\ 2x - y + 9 = 0$

Sol :

Here we compare the given equations with,

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

(i) $5x - 4y + 8 = 0,\ 7x + 6y - 9 = 0$

We have $5x - 4y + 8 = 0$

$$7x + 6y - 9 = 0$$

Comparing with standard equation we have,

$$a_1 = 5,\ b_1 = -4,\ c_1 = 8$$
$$a_2 = 7,\ b_2 = 6,\ c_2 = -9$$
$$\frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$$
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the lines are intersecting, i.e., they intersect at a point.

(ii) $9x + 3y + 12 = 0,\ 18x + 6y + 24 = 0$

We have $9x + 3y + 12 = 0$

$$18x + 6y + 24 = 0$$

Comparing with standard equation we have,

$$a_1 = 9,\ b_1 = 3,\ c_1 = 12$$
$$a_2 = 18,\ b_2 = 6,\ c_2 = 24$$
$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$$
$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

and $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, the lines are coincident.

(iii) $6x - 3y + 10 = 0,\ 2x - y + 9 = 0$

We have $6x - 3y + 10 = 0$

$$2x - y + 9 = 0$$

Comparing with standard equation we have,

$$a_1 = 6,\ b_1 = -3,\ c_1 = 10$$
$$a_2 = 2,\ b_2 = -1,\ c_2 = 9$$
$$\frac{a_1}{a_2} = \frac{6}{2} = 3$$
$$\frac{b_1}{b_2} = \frac{-3}{-1} = 3$$
$$\frac{c_1}{c_2} = \frac{1}{9}$$
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the lines are parallel.

PRACTICE :

1. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) $5x+3y=8,\ 2x-y+9=0$
(ii) $4x+5y=20,\ 2x+2.5y=10$
(iii) $6x-4y=8,\ 3x-2y=12$

Ans :

(i) Intersecting lines
(ii) Coincident
(iii) Parallel lines

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) $9x-4y=5,\ \frac{3x}{2}+7y=4$

(ii) $3x-5y=9,\ 6x-10y=18$

(iii) $8x+7y=10,\ 4x+3.5y-5=0$

Ans :

(i) Intersecting liens
(ii) Coincident
(iii) Coincident lines

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent or inconsistent.

(i) $3x+2y=5;\ 2x-3y=7$

(ii) $2x-3y=8;\ 4x-6y=9$

(iii) $\frac{3}{2}x+\frac{5}{3}y=7;\ 9x-10y=14$

(iv) $5x-3y=11;\ -10x+6y=-22$

(v) $\frac{4}{3}x+2y=8;\ 2x+3y=12$

Sol :

(i) $3x+2y=5;\ 2x-3y=7$

We have $3x+2y=5$

$2x-3y=7$

Comparing with standard equation we have,

$$a_1=3,\ b_1=2,\ c_1=-5$$

$$a_2=2,\ b_2=-3,\ c_2=-7$$

$$\frac{a_1}{a_2}=\frac{3}{2}$$

$$\frac{b_1}{b_2}=\frac{2}{-3}$$

and $$\frac{c_1}{c_2}=\frac{-5}{-7}=\frac{5}{7}$$

So, lines are intersecting i.e., they intersect at a point. It is consistent pair of equations.

(ii) $2x-3y=8;\ 4x-6y=9$

We have $2x-3y=8$

$4x-6y=9$

Comparing with standard equation we have,

$$a_1=2\ b_1=-3,\ c_1=-8$$

$$a_2=4,\ b_2=-6,\ c_2=-9$$

$$\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2}$$

$$\frac{b_1}{b_2}=\frac{-3}{-6}=\frac{1}{2}$$

and $$\frac{c_1}{c_2}=\frac{-8}{-9}=\frac{8}{9}$$

Here, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.

So, lines are parallel i.e., the given pair of linear equations has no solution.
It is in consistent pair of equations.

(iii) $\frac{3}{2}x+\frac{5}{3}y=7;\ 9x-10y=14$

We have $\frac{3}{2}x+\frac{5}{3}y=7$

$9x-10y=14$

Comparing with standard equation we have,

$$a_1=\frac{3}{2},\ b_1=\frac{5}{3},\ c_1=-7$$

$$a_2=9,\ b_2=-10,\ c_2=-14$$

$$\frac{a_1}{a_2}=\frac{3/2}{9}=\frac{3}{2}\times\frac{1}{9}=\frac{1}{6}$$

$$\frac{b_1}{b_2}=\frac{5/3}{-10}=\frac{5}{3}\times\frac{1}{-10}=-\frac{1}{6}$$

and $$\frac{c_1}{c_2}=\frac{-7}{-14}=\frac{1}{2}$$

Here, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

So lines are intersecting. The given pair of linear equations has one solution. It is a consistent pair of equations.

(iv) $5x-3y=11;\ -10x+6y=-22$

We have $5x-3y=11$

$-10x+6y=-22$

Comparing with standard equation we have,

$$a_1=5,\ b_1=-3,\ c_1=-11$$

$$a_2=-10,\ b_2=6,\ c_2=22$$

$$\frac{a_1}{a_2}=\frac{5}{-10}=-\frac{1}{2}$$

$$\frac{b_1}{b_2}=\frac{-3}{6}=-\frac{1}{2}$$

$$\frac{c_1}{c_2}=\frac{-11}{22}=-\frac{1}{2}$$

Here, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

So, lines are consistent. The given pair of linear equations has infinitely many solutions. Thus, they are consistent.

(v) $\frac{4}{3}x + 2y = 8;\ 2x + 3y = 12$

We have $\frac{4}{3}x + 2y = 8$

$$2x + 3y = 12$$

Comparing with standard equation we have,

$$a_1 = \frac{4}{3},\ b_1 = 2,\ c_1 = -8$$

$$a_2 = 2,\ b_2 = 3,\ c_2 = -12$$

$$\frac{a_1}{a_2} = \frac{4/3}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$$

$$\frac{b_1}{b_2} = \frac{2}{3}$$

$$\frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the lines are coincident i.e., they have infinitely many solutions. The given pair of linear equations are consistent.

PRACTICE :

1. On comparing the rations $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether following pair of linear equations are consistent, or inconsistent.

(i) $2y - x = 9,\ -3x + 6y = 21$
(ii) $3x + y - 8,\ 9x + 3y = 24$
(iii) $4x - 2y = 5,\ x + 4y = 9$
(iv) $x - 2y = 8,\ 2x - y = 5$
(v) $x - 2y = 6,\ 3x - 6y = 5$

Ans :

(i) Inconsistent (ii) Consistent
(iii) Consistent (iv) Consistent
(v) Inconsistent

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5,\ 2x + 2y = 10$

(ii) $x - y = 8;\ 3x - 3y = 16$

(iii) $2x + y - 6 = 0,\ 4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0,\ 4x - 4y - 5 = 0$

Sol :

(i) $x + y = 5,\ 2x + 2y = 10$

We have $x + y = 5$

$$2x + 2y = 10$$

Comparing with standard equation we have,

$$a_1 = 1,\ b_1 = 1,\ c_1 = -5$$

$$a_2 = 2,\ b_2 = 2,\ c_2 = -10$$

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{2}$$

and $\frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Their graph lines are coincident as shown.

l_1: $x + y = 5$

$$y = 5 - x$$

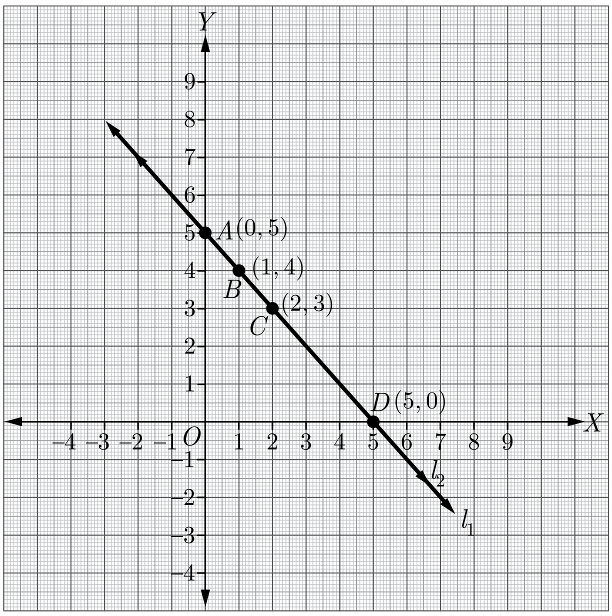
x	0	5	1
y	5	0	4

l_2: $2x + 2y = 10$

$$x + y = 5$$

$$y = 5 - x$$

x	2	0	5
y	3	5	0



So, lines l_1 and l_2 are coinciding. i.e., They have infinitely many solutions and are consistent.

(ii) $x - y = 8;\ 3x - 3y = 16$

We have $x - y = 8$

$$3x - 3y = 16$$

Comparing with standard equation we have,

$$a_1 = 1,\ b_1 = -1,\ c_1 = -8$$

$$a_2 = 3,\ b_2 = -3,\ c_2 = -16$$

$$\frac{a_1}{a_2} = \frac{1}{3}$$

$$\frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$$

and $\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The pair of linear equations is inconsistent and lines are parallel.

(iii) $2x + y - 6 = 0,\ 4x - 2y - 4 = 0$

We have $2x + y - 6 = 0$

$$4x - 2y - 4 = 0$$

Comparing with standard equation we have,

$$a_1 = 2,\ b_1 = 1,\ c_1 = -6$$

and $a_2 = 4,\ b_2 = -2,\ c_2 = -4$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{-2}$$

$$\frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, it is a consistent pair of linear,

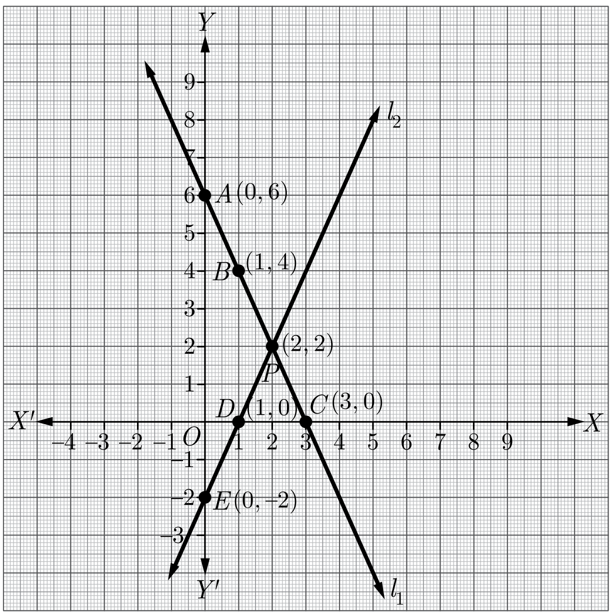
l_1: $y = 6 - 2x$

x	0	3	1
y	6	0	4
(x, y)	(0, 6)	(3, 0)	(1, 4)

$$2y = 4x - 4$$

l_2: $y = 2x - 2$

x	0	1	2
y	-2	0	2
(x, y)	$(0, -2)$	(1, 0)	(2, 2)



l_1 and l_2 intersects at (2, 2)

$$x = 2 \text{ and } y = 2$$

(iv) $2x - 2y - 2 = 0,\ 4x - 4y - 5 = 0$

We have $2x - 2y - 2 = 0$

$$4x - 4y - 5 = 0$$

Comparing with standard equation we have,

$$a_1 = 2,\ b_1 = -2,\ c_1 = -2$$

$$a_2 = 4,\ b_2 = -4,\ c_2 = -5$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the given pair of linear equations is inconsistent and lines are parallel.

PRACTICE :

1. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $2x + 3y = 6,\ 4x + 6y = 12$

(ii) $3x - 4y - 2 = 0,\ 2x\frac{-8y}{3} + 4 = 0$

(iii) $2x + y - 5 = 0,\ 3x - y - 5 = 0$

(iv) $5x - 4y = 11,\ 10x - 8y = 20$

Ans :

(i) Consistent $\left(x = \frac{6-3y}{2}\right)$ or $y = \frac{6-2x}{3}$

(ii) Inconsistent

(iii) Consistent $(x = 2, y = 1)$

(iv) Inconsistent

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Sol :

Let the width of the garden be x and the length of the garden be y.

According to question,

$$4 + \text{width} = \text{length}$$

$$4 + x = y$$

Also, $\frac{1}{2}\text{ perimeter} = 36$

or $\frac{1}{2}(2(x + y)) = 36$

$$y + x = 36$$

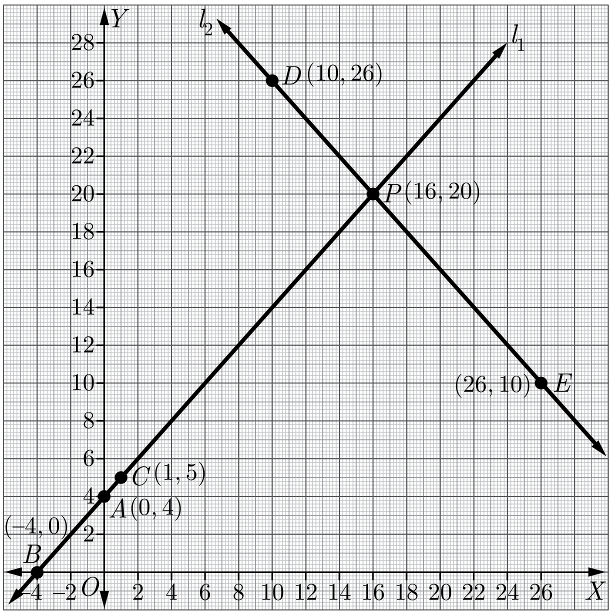
l_1: $y = x + 4$

x	0	-4	1
y	4	0	5

l_2 : $x + y = 36$

$$y = 36 - x$$

x	10	26	16
y	26	10	20



The lines l_1 and l_2 intersect at (16, 20).

$$x = 16$$

and $$y = 20$$

So, Length of the garden $= 20$ m

and Width of the garden $= 16$ m

PRACTICE :

1. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 20 m. Find the dimensions of the garden.

   Ans : Length 12 m, Breadth = 8 m

2. Half the perimeter of a rectangular garden, whose length is 20 m more than its width, is 50 m. Find the dimensions of the garden.

   Ans : Length = 35 m, Breadth = 15 m

6. Given the linear equation $12x+3y-8=0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
(i) intersecting lines
(ii) parallel lines
(iii) coincident lines

Sol :

(i) Intersecting lines

We have $$2x+3y-8 = 0$$

Here, $a_1 = 2,\ b_1 = 3,\ c_1 = -8$

and another line,

$$a_2x+b_2y+c_2 = 0$$

For intersecting lines,

We have, $$\frac{2}{a_2} \neq \frac{3}{b_2} \neq \frac{-8}{c_2}$$

We can have, $a_2 = 3,\ b_2 = 2,\ c_2 = -7$

The required equation is,

$$3x+2y-7 = 0$$

(ii) Parallel lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Any line parallel to $2x+3y-8 = 0$ can be taken as,

$$2x+3y-12 = 0$$

(iii) Coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Any line parallel to $2x+3y-8 = 0$ can be

$$2(2x+3y-8 = 0)$$

$$4x+6y-16 = 0$$

PRACTICE :

1. Given the linear equation $4x+6y-9=0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
   (i) intersecting lines
   (ii) parallel lines
   (iii) coincident lines

   Ans :

   (i) $4x-6y-8=0$
   (ii) $8x+12y-15=0$
   (iii) $12x+18y-27=0$

2. Given the linear equation $5x+4y-8=0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
   (i) intersecting lines
   (ii) parallel lines
   (iii) coincident lines

   Ans :

   (i) $6x-2y-7=0$ (Answer may vary)
   (ii) $10x+8y-10=0$ (Answer may vary)
   (iii) $10x+8y-16=0$ (Answer may vary)

7. Draw the graphs of the equations $x-y+1=0$ and $3x+2y-12=0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Sol :

We have $$x-y+1 = 0$$

and $$3x+2y-12 = 0$$

$$l_1 : x-y+1 = 0$$

$$y = x+1$$

$$x = -1+y$$

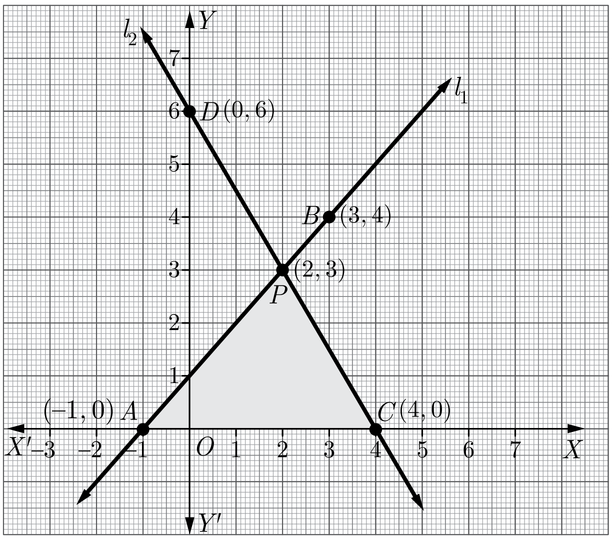
x	2	-1	3
y	3	0	4

Plotting, the points (2, 3), $(-1, 0)$ and (3, 4), we get a straight line l_1.

Also, $l_2 : 3x + 2y - 12 = 0$

$$y = \frac{12 - 3x}{2}$$

x	2	4	0
y	3	0	6



The lines l_1 and l_2 intersect at (2, 3). Thus, co-ordinaxtes of the vertices of the shaded triangular region are $(4, 0)$, $(-1, 0)$ and (2, 3).

PRACTICE :

1. Draw the graphs of the equations $x - y = 1$ and $2x + y = 8$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, find its area.

Ans : $(0, -1)$, $(3, 2)$, $(0, 8)$ Area = 13.5 sq. units

EXERCISE 3.3

1. Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14,\ x - y = 4$

(ii) $s - t = 3,\ \frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3,\ 9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3,\ 0 - 4x + 0.5y = 2.3$

(v) $\sqrt{2}\,x + \sqrt{3}\,y = 0,\ \sqrt{3}\,x - \sqrt{8}\,y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2,\ \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Sol :

(i) $x + y = 14,\ x - y = 4$

We have $x + y = 14$ ...(i)

$x - y = 4$ ...(ii)

From, the equation (i), we get

$$x = (14 - y)$$

Substituting this value of x in equation (ii), we have

$$(14 - y) - y = 4$$
$$14 - 2y = 4$$
$$-2y = -10$$
$$y = 5$$

Now, substituting $y = 5$ in equation (i), we have

$$x + 5 = 14$$
$$x = 9$$

Hence, $x = 9,\ y = 5$

(ii) $s - t = 3,\ \frac{s}{3} + \frac{t}{2} = 6$

We have $s - t = 3$ ...(i)

$\frac{s}{3} + \frac{t}{2} = 6$ ...(ii)

From, equation (i), we have

, $s = (3 + t)$ (iii)

Substituting this value of s in equation (ii) we get

$$\frac{(3 + t)}{3} + \frac{t}{2} = 6$$
$$2(3 + t) + 3(t) = 6 \times 6$$
$$6 + 2t + 3t = 36$$
$$5t = 30$$
$$t = \frac{30}{5} = 6$$

Substituting, $t = 6$ in equation (iii) we get,

$$s = 3 + 6 = 9$$

Thus, $s = 9$ and $t = 6$

(iii) $3x - y = 3,\ 9x - 3y = 9$

We have $3x - y = 3$ ...(i)

$9x - 3y = 9$ ...(ii)

From, equation (i),

$$y = (3x - 3)$$

Substituting this value of y in equation (ii), we get

$$9x - 3(3x - 3) = 9$$
$$9x - 9x + 9 = 9$$
$$9 = 9 \text{ Which is true,}$$

The equations (i) and (ii) have infinitely many solutions.

(iv) $0.2x + 0.3y = 1.3$

We have $\frac{2}{10}x + \frac{3y}{10} = \frac{13}{10}$

$2x + 3y = 13$ ...(i)

and $0.4x + 0.5y = 2.3$

$$\frac{4x}{10}+\frac{5y}{10} = \frac{23}{10}$$

$$4x+5y = 23 \qquad ...(ii)$$

From eq (i), we get

$$x = \frac{13-3y}{2}$$

Substituting $x = \frac{13-3y}{2}$ in eq (ii) we have

$$4\left(\frac{13-3y}{2}\right)+5y = 23$$

$$26-6y+5y = 23$$

$$-y = -3$$

$$y = 3$$

Substituting $y = 3$ in eq (i), we have

$$2x+9 = 13$$

$$2x = 4 \Rightarrow x = 2$$

Thus $x = 2$ and $y = 3$

(v) $\sqrt{2}\,x+\sqrt{3}\,y = 0,\ \sqrt{3}\,x-\sqrt{8}\,y = 0$

$$\sqrt{2}\,x+\sqrt{3}\,y = 0 \qquad ...(i)$$

$$\sqrt{3}\,x-\sqrt{8}\,y = 0 \qquad ...(ii)$$

From, equation (ii), we have

$$\sqrt{3}\,x = \sqrt{8}\,y$$

$$x = \left[\frac{\sqrt{8}}{\sqrt{3}}y\right] \qquad ...(iii)$$

From, equation (iii) and (i), we have

$$\sqrt{2}\left[\frac{\sqrt{8}}{\sqrt{3}}y\right]+\sqrt{3}\,y = 0$$

$$\frac{\sqrt{16}}{\sqrt{3}}y+\sqrt{3}\,y = 0$$

$$\frac{4}{\sqrt{3}}y+\sqrt{3}\,y = 0$$

$$\left[\frac{4}{\sqrt{3}}+\sqrt{3}\right]y = 0$$

$$y = 0$$

Substituting $y = 0$ in equation (iii), we have

$$x = \frac{\sqrt{8}}{\sqrt{3}}(0) = 0$$

Thus, $x = 0$ and $y = 0$

(vi) $\frac{3x}{2}-\frac{5y}{3} = -2,\ \frac{x}{3}+\frac{y}{2} = \frac{13}{6}$

$$\frac{3x}{2}-\frac{5y}{3} = -2 \qquad ...(i)$$

$$\frac{x}{3}+\frac{y}{2} = \frac{13}{6} \qquad ...(ii)$$

From, equation (ii), we have

$$\frac{x}{3} = \frac{13}{6}-\frac{y}{2}$$

$$x = 3\times\left(\frac{13}{6}-\frac{y}{2}\right)$$

or

$$x = \left[\frac{13}{2}-\frac{3}{2}y\right] \qquad ...(iii)$$

Substituting the value of x in equation (i), we have

$$\frac{3}{2}\left[\frac{13}{2}-\frac{3}{2}y\right]-\frac{5y}{3} = -2$$

$$\frac{39}{4}-\frac{9y}{4}-\frac{5y}{3} = -2$$

$$\frac{117-47y}{12} = -2$$

$$117-47y = -24$$

$$-47y = -24-117 = -141$$

$$y = \frac{-141}{-47} = 3$$

Now, substituting $y = 3$ in equation (iii), we have

$$x = \frac{13}{2}-\frac{3}{2}(3) = \frac{13}{2}-\frac{9}{2} = \frac{4}{2} = 2$$

Thus, $x = 2$ and $y = 3$.

PRACTICE :

1. Solve the following pair of linear equations by the substitution method.

(i) $3x-7y+10 = 0,\ y-2x-3 = 0$
(ii) $0.5x+0.7y = 0.74,\ 0.7x+0.5y = 0.70$
(iii) $5x+8y = 9,\ 3x-2y = 19$
(iv) $x+\frac{y}{2} = 4,\ 2x+y = 8$
(v) $x+y = 7,\ 2x-3y = 11$

Ans :

(i) $x = -1,\ y = 1$ (ii) $x = 0.5,\ y = 0.7$
(iii) $x = 5,\ y = -2$ (iv) $x = 3,\ y = 2$
(v) $x = \frac{32}{5},\ y = \frac{3}{5}$

2. Solve the following pair of linear equations by the substitution method.

(i) $y = 3x,\ 5x-2y = -15$
(ii) $x+y = 10,\ 4x-5y = -14$
(iii) $y-x = 3,\ x+y = 15$
(iv) $15x+60y = 1,\ 24x+48y = 1$
(v) $125x+110y = 15200, 110x+125y = 153500$

Ans :

(i) $x = 15,\ y = 45$ (ii) $x = 4,\ y = 6$
(iii) $x = 6,\ y = 9$ (iv) $x = \frac{1}{60},\ y = \frac{1}{80}$
(v) $x = 600,\ y = 700$

2. Solve $2x+3y = 11$ and $2x-4y = -24$ and hence find the value of m for which $y = mx+3$.

Sol :

We have $\quad 2x+3y = 11 \qquad ...(i)$

and $\quad 2x-4y = -24 \qquad ...(ii)$

From, equation (i), we have

$$2x = 11-3y$$

$$x = \left[\frac{11-3y}{2}\right] \quad ...(iii)$$

Substituting this value of x in (ii), we have

$$2\left[\frac{11-3y}{2}\right] - 4y = -24$$

$$11 - 3y - 4y = -24$$

$$-7y = -24 - 11 = -35$$

$$y = \frac{-35}{-7} = 5$$

Substituting $y = 5$ in equation (iii), we get

$$x = \frac{11-3(5)}{2} = \frac{11-15}{2} = \frac{-4}{2} = -2$$

Thus, $x = -2$ and $y = 5$

Now, $y = mx + 3$

$$5 = m(-2) + 3$$

$$-2m = 5 - 3 = 2$$

$$m = \frac{-2}{2} = -1$$

Thus, $m = -1$

PRACTICE :

1. Solve $4x - 3y = 6$, and $2x + y = 8$ and hence find the value of m for which $2y = mx + 5$

Ans : $m = -\frac{1}{3}$

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge fort he distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes 9/11, if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes 5/6. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Sol :

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

Let the two numbers be x and y such that $x > y$

Difference between two numbers = 26

$$x - y = 26 \quad ...(i)$$

Again one number = 3 [the other number]

$$x = 3y \quad ...(ii)$$

Substituting $x = 3y$ in equation (i), we get

$$3y - y = 26$$

$$2y = 26$$

$$y = \frac{26}{2} = 13$$

Now, substituting $y = 13$ in equation (ii), we have

$$x = 3(13) = 39$$

Thus, Two numbers are 39 and 13.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Let the two angles be x and y such that $x > y$

The larger angle exceeds the smaller by 18°

$$x = y + 18^\circ \quad ...(i)$$

Also, sum of two supplementary angles = 180°

$$x + y = 180^\circ \quad ...(ii)$$

Substituting the value of equation (i) in equation (ii), we get

$$(18^\circ + y) + y = 180^\circ$$

$$2y = 180^\circ - 18^\circ = 162^\circ$$

$$y = \frac{162^\circ}{2} = 81^\circ$$

Substituting, $y = 81^\circ$ in equation (i), we get

$$x = 18^\circ + 81^\circ = 99^\circ$$

Thus $x = 99^\circ$ and $y = 81^\circ$

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

Let the cost of a bat=₹ x and the cost of a ball =₹ y

[cost of 7 bats]+[cost of 6 balls] =₹ 3800

$$7x + 6y = 3800 \quad ...(i)$$

Again, [cost of 3 bats]+[cost of 5 balls] =₹ 1750

$$3x + 5y = 1750 \quad ...(ii)$$

From, equation (ii), we have,

$$y = \left[\frac{1750-3x}{5}\right] \quad ...(iii)$$

Substituting this value of y in equation (i), we have

$$7x + 6\left[\frac{1750-3x}{5}\right] = 3800$$

$$35x + 10500 - 18x = 19000$$

$$17x = 19000 - 10500$$

$$x = \frac{8500}{17} = 500$$

Substituting $x = 500$ in equation (iii), we have

$$y = \frac{1750-3(500)}{5}$$

$$y = \frac{1750-1500}{5} = \frac{250}{5} = 50$$

Thus $x = 500$ and $y = 50$

Cost of a bat $=$ ₹ 500 and cost of a ball $=$ ₹ 50

(iv) The taxi charges in a city consist of a fixed charge together with the charge fort he distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

Let fixed charges be x and charges per km be y.

Charges for the journey of 10 km $=$ ₹ 105

$$x + 10y = 105 \quad \text{...(i)}$$

and charges for the journey of 15 km $=$ ₹ 155

$$x + 15y = 155 \quad \text{...(ii)}$$

From, equation (i) we have

$$x = 105 - 10y \quad \text{...(iii)}$$

Substituting the value of x in equation (ii), we get

$$(105-10y) + 15y = 155$$

$$5y = 155 - 105 = 50$$

$$y = 10$$

Substituting $y = 10$ in equation (iii), we get

$$x = (105-10y) + 15y = 155$$

$$5y = 155 - 105 = 50$$

$$y = 10$$

Substituting $y = 10$ in equation (iii), we get

$$x = 105 - 10(10) = 5$$

Thus, $x = 5$ and $y = 10$

Fixed charges $=$ ₹ 5 and Charges per km $=$ ₹ 10

Now,

Charges for 25 km $= x + 25y$

$= 5 + 25(10)$

$= 5 + 250 =$ ₹ 255

The charges for 25 km journey $=$ ₹ 255

(v) A fraction becomes 9/11, if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes 5/6. Find the fraction.

Let, the numerator be x and denominator be y.

$$\text{Fraction} = \frac{x}{y}$$

Case I : $$\frac{x+2}{y+2} = \frac{9}{11}$$

$$11(x+2) = 9(y+2)$$

$$11x + 22 = 9y + 18$$

$$11x - 9y + 4 = 0 \quad \text{...(i)}$$

Case II: $$\frac{x+3}{y+3} = \frac{5}{6}$$

$$6(x+3) = 5(y+3)$$

$$6x + 18 = 5y + 15$$

$$6x - 5y + 3 = 0 \quad \text{...(ii)}$$

Now, from equation (ii), we get

$$x = \left[\frac{5y-3}{6}\right]$$

Substituting this value of x in equation (i), we get

$$11\left[\frac{5y-3}{6}\right] - 9y + 4 = 0$$

$$55y - 33 - 54y + 24 = 0$$

$$y - 9 = 0$$

$$y = 9$$

Now, substituting $y = 9$ in equation (iii), we get

$$x = \frac{5(9)-3}{6} = \frac{45-3}{6} = \frac{42}{6} = 7$$

Thus $x = 7$ and $y = 9$ and fraction $= \frac{7}{9}$

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Let, the present age of Jacob be x years and the present age of his son be y years 5 years hence,

$$\text{Age of Jacob} = (x+5) \text{ years}$$

Now,

$$[\text{Age of Jacob}] = 3[\text{Age of his son}]$$

$$x + 5 = 3(y+5)$$

$$x + 5 = 3y + 15$$

$$x - 3y - 10 = 0$$

5 years ago :

$$\text{Age of Jacob} = (x-5) \text{ years}$$

$$\text{Age of his son} = (y-5)$$

Also,

$$[\text{Age of Jacob}] = 7\ [\text{Age of his son}]$$

$$(x-5) = 7(y-5)$$

$$x - 5 = 7y - 35$$

$$x - 7y + 30 = 0$$

From, equation (i), we have

$$x = [10 + 3y]$$

Substituting this value of x in equation (ii), we have

$$(10+3y) - 7y + 30 = 0$$

$$-4y = -40$$

$$y = 10$$

Now, substituting $y = 10$ in equation (iii), we have

$$x = 10 + 3(10) = 10 + 30 = 40$$

Thus $x = 40$ and $y = 10$

Present age of Jacob $= 40$ years and

Present age his son $= 10$ years.

PRACTICE :

1. Form the pair of linear equations for the following problems and find their solution by substitution method.
   (i) The difference between two numbers is 30 and one number is four times the other. Find them.
   (ii) The larger of two supplementary angles exceeds the smaller by 20 degrees. Find them.
   (iii) The coach of a cricket team buys 6 bats and 5 balls for ₹ 6100. Later she buys 4 bats and 3 balls for ₹ 4060. Find the cost of each bat and each ball.
   (iv) The taxi charges in a city consist of a fixed charge together with the charge fort he distance covered. For a distance of 12 cm, the charge paid is ₹ 190 and for a journey of 20 km, the charge paid is ₹ 310. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 30 km?
   (v) A fraction becomes 1/3, if 2 is added to both the numerator and the denominator. If 1 is added to both the numerator and the denominator it becomes 1/2. Find the fraction.
   (vi) Ten years hence, the age of Jacob will be twice times that of his son. Five years ago, Jacob's age was five times that of his son. What are their present ages?

Ans :

(i) 40, 10
(ii) 100, 80
(iii) Bat = ₹ 1000, Ball = ₹ 20
(iv) ₹ 460
(v) 3/7
(vi) Jacob = 30 years, Son's age = 10 years

EXERCISE 3.4

1. Solve the following pair of linear equations by the elimination method and the substitution method.
   (i) $x+y=5$ and $2x-3y=4$
   (ii) $3x+4y=10$ and $2x-2y=2$
   (iii) $3x-5y-4=0$ and $9x=2y+7$
   (iv) $\frac{x}{2}+\frac{2y}{3}=-1$ and $x-\frac{y}{3}=3$

Sol :

(i) $x+y=5$ and $2x-3y=4$

Elimination Method :

We have
$$x+y=5 \quad \text{...(i)}$$
$$2x-3y=4 \quad \text{...(ii)}$$

Multiplying equation (i) by 3, we get
$$3x+3y=15 \quad \text{...(iii)}$$

Adding equation (ii) and (iii), we get
$$5x=19$$
$$x=\frac{19}{5}$$

Now, Substituting $x=\frac{19}{5}$ in (i), we get
$$\frac{19}{5}+y=5$$
$$y=5-\frac{19}{5}=\frac{25-19}{5}=\frac{6}{5}$$

Thus, $x=\frac{19}{5}$ and $y=\frac{6}{5}$

Substitution Method :
$$x+y=5$$
$$y=5-x \quad \text{...(i)}$$
$$2x-3y=4 \quad \text{...(ii)}$$

Substituting $y=5-x$ in equation (ii), we have
$$2x-3(5-x)=4$$
$$2x-15+3x=4$$
$$5x=19$$
$$x=\frac{19}{5}$$

From, equation (i),
$$y=5-\frac{19}{5}=\frac{25-19}{5}=\frac{6}{5}$$

Hence, $x=\frac{19}{5}$ and $y=\frac{6}{5}$

(ii) $3x+4y=10$ and $2x-2y=2$

Elimination Method:

We have
$$3x+4y=10 \quad \text{...(i)}$$
$$2x-2y=2 \quad \text{...(ii)}$$

Multiplying equation (ii) by 2, we have
$$4x-4y=4$$

Adding equation (i) and (iii), we get
$$7x=14$$
$$x=\frac{14}{7}=2$$

Substituting, $x=2$ in equation (i), we get
$$3(2)+4y=10$$
$$4y=10-6$$
$$y=\frac{4}{4}=1$$

Thus $x=2$ and $y=1$

Substitution Method :
$$3x+4y=10$$
$$y=\frac{10-3x}{4} \quad \text{...(i)}$$
$$2x-2y=2$$

$$x - y = 1 \quad ...(ii)$$

Substituting $y = \frac{10-3x}{4}$ in equation (ii) we have

$$x - \left(\frac{10-3x}{4}\right) = 1$$

$$4x - 10 + 3x = 4$$

$$7x = 14$$

$$x = \frac{14}{7} = 2$$

Substituting $x = 2$ in (i), we have

$$y = \frac{10 - 3\times 2}{4} = \frac{10-6}{4} = 1$$

Hence, $x = 2$ and $y = 1$

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

Elimination Method :

We have $3x - 5y - 4 = 0 \quad ...(i)$

$$9x = 2y + 7$$

or $\quad 9x - 2y - 7 = 0 \quad ...(ii)$

Multiplying equation (i) by (iii), we get

$$9x - 15y - 12 = 0 \quad ...(iii)$$

Subtracting equation (ii) from (iii), we have

$$-13y - 5 = 0$$

$$y = \left(\frac{-5}{13}\right)$$

Substituting the value of y in equation (i), we get

$$3x - 5\left(\frac{-5}{13}\right) - 4 = 0$$

$$3x + \frac{25}{13} - 4 = 0$$

$$3x = \frac{-25+52}{13} = \frac{27}{13}$$

$$x = \frac{27}{13}\times\frac{1}{3} = \frac{9}{13}$$

Thus, $x = \frac{9}{13}$ and $y = -\frac{5}{13}$

Substitution Method:

$$3x - 5y - 4 = 0$$

$$y = \frac{3x-4}{5} \quad ...(i)$$

$$9x - 2y - 7 = 0 \quad ...(ii)$$

Substituting $y = \frac{3x-4}{5}$ in equation (ii), we get

$$9x - 2\left(\frac{3x-4}{5}\right) - 7 = 0$$

$$45x - 6x + 8 - 35 = 0$$

$$39x = 27$$

$$x = \frac{27}{39} = \frac{9}{13}$$

Substituting $x = \frac{9}{13}$ in equation (i), we get

$$y = \frac{3\times\frac{9}{13} - 4}{5} = \frac{27-52}{65} = \frac{-25}{65} = \frac{-5}{13}$$

Hence, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Elimination Method:

We have $\quad \frac{x}{2} + \frac{2y}{3} = -1 \quad ...(i)$

$$x - \frac{y}{3} = 3 \quad ...(ii)$$

Multiplying equation (ii) by 2, we get

$$2x - \frac{2y}{3} = 6 \quad ...(iii)$$

Adding equation (i) and (iii), we get

$$\frac{x}{2} + 2x = 5$$

$$\frac{5}{2}x = 5$$

$$x = 5\times\frac{2}{5} = 2$$

Substituting, $x = 2$ in (i), we get

$$\frac{2}{2} + \frac{2y}{3} = -1$$

$$\frac{2y}{3} = -1 - 1 = -2$$

$$y = -2\times\frac{3}{2} = -3$$

Thus $x = 2$ and $y = -3$

Substitution Method:

$$\frac{x}{2} + \frac{2y}{3} = -1 \quad ...(i)$$

$$x - \frac{y}{3} = 3$$

$$y = 3(x-3) \quad ...(ii)$$

Substituting $y = 3(x-3)$ in equation (i), we have

$$\frac{x}{2} + \frac{2}{3}\times 3(x-3) = -1$$

$$\frac{x}{2} + 2x - 6 = -1$$

$$\frac{5x}{2} = 5$$

$$x = 5\times\frac{2}{5} = 2$$

Substituting $x = 2$ in equation (ii), we have

$$y = 3(2-3) = 3(-1) = -3$$

Hence, $x = 2$ and $y = -3$

PRACTICE :

1. Solve the following pair of linear equations by the elimination method and the substitution method.
(i) $x-y=7,\ x+y=11$
(ii) $y-2x=3,\ 2y-x=30$
(iii) $\frac{7x-2}{3}-y=2,\ x-\frac{3}{7}y=\frac{8}{7}$
(iv) $2x+3y=795,\ 3x+5y=1300$
Ans :
(i) $x=9,\ y=2$
(ii) $x=8,\ y=19$
(iii) $x=2,\ y=2$
(iv) $x=75,\ y=215$

2. Solve the following pair of linear equations by the elimination method and the substitution method.
(i) $3x-y=40,\ 4x-2y=50$
(ii) $2x-3y+1=0,\ 3x+4y-5=0$
(iii) $-9a+8b=6,\ 6a+10b=19$
(iv) $y-4x=1,\ 6x-5y=9$
Ans :
(i) $x=15,\ y=5$
(ii) $x=\frac{11}{17},\ y=\frac{13}{17}$
(iii) $a=\frac{2}{3},\ b=\frac{3}{2}$
(iv) $x=-1,\ y=-3$

2. Form the pair of linear equations in the following problems and find their solutions if they exist by the elimination method.
(i) If we add 1 to the numerator and subtract 1 from the denominator a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?
(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?
(iii) The sum of the digits of a two-digit number is 9. Also nine times this number is twice the number obtained by reversing the order of the digits. Find the number.
(iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.
(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Sol :

(i) If we add 1 to the numerator and subtract 1 from the denominator a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

Let, the numerator be x and the denominator be y.

Now Fraction $=\frac{x}{y}$

Case I : $\frac{x+1}{y-1}=1$

$$x+1=y-1$$

$$x-y=-2 \quad \text{...(i)}$$

Case II: $\frac{x}{y+1}=\frac{1}{2}$

$$x=\frac{1}{2}(y+1)$$

$$x-\frac{y}{2}=\frac{1}{2} \quad \text{...(ii)}$$

Subtracting equation (ii) from equation (i), we have

$$-y+\frac{y}{2}=-2-\frac{1}{2}$$

$$-\frac{1}{2}y=-\frac{5}{2}$$

$$y=5$$

Now, Substituting $y=5$ in equation (ii), we have

$$x-\frac{5}{2}=\frac{1}{2}$$

$$x=\frac{1}{2}+\frac{5}{2}=\frac{6}{2}=3$$

Thus, $x=3$ and $y=5$

Hence, the required fraction $=\frac{3}{5}$

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Let, the present age of Nuri be x years and the present age of Sonu be y years.

5 years ago :

Age of Nuri $=(x-5)$ years,

Age of Sonu $=(y-5)$ years

According to question,

$$x-5=3[y-5]$$

$$x-5=3y-15$$

$$x-3y+10=0 \quad \text{...(i)}$$

10 years later :

Age of Nuri $=(x+10)$ years

Age of Sonu $=(y+10)$ years

According to question,

$$x+10=2(y+10)$$

$$x+10=2y+20$$

$$x-2y-10=0 \quad \text{...(ii)}$$

Subtracting equation (i) from (ii), we have

$$y-20=0$$

$$y=20$$

Substituting, $y=20$ in equation (i), we get

$$x-3(20)+10=0$$

$$x-50=0$$

$$x = 50$$

Thus $x = 50$ and $y = 20$

$$\text{Age of Nuri} = 50 \text{ years and}$$

$$\text{Age of Sonu} = 20 \text{ years}$$

(iii) The sum of the digits of a two-digit number is 9. Also nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Let, the digit at unit's place be x and the digit at ten's place be y.

$$\text{The number} = 10y + x$$

The number obtained by reversing the digits

$$= 10x + y$$

According to the question,

$$9[\text{The number}] = 2 \quad [\text{Number obtained by reversing the digits}]$$

$$9[10y + x] = 2[10x + y]$$

$$90y + 9x = 20x + 2y$$

$$x - 8y = 0 \qquad \text{...(i)}$$

Also,sum of the digit $= 9$

$$x + y = 9 \qquad \text{...(ii)}$$

Subtracting equation (i) from (ii), we have

$$9y = 9$$

$$y = 1$$

Substituting $y = 1$ in equation (ii), we have

$$x + 1 = 9$$

$$x = 8$$

Thus, $x = 8$ and $y = 1$

The required number $= (10 \times 1) + 8 = 10 + 8 = 18$

(iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

Let the number of 50 rupees notes be x and the number of 100 rupees notes be y.
According to the question,
Total number of notes $= 25$

$$x + y = 25 \qquad \text{...(i)}$$

The value of all the notes =₹ 2000

$$50x + 100y = 2000$$

$$x + 2y = 40 \qquad \text{...(ii)}$$

Subtracting equation (i) from (ii), we get

$$y = 15$$

Substituting $y = 15$ in equation (i), we have

$$x + 15 = 25$$

$$x = 25 - 15 = 10$$

Thus, $x = 10$ and $y = 15$
Number of 50 rupees notes $= 10$
and number of 100 rupees notes $= 15$

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Let the fixed charge (for the three days) be x and the additional charge for each extra day be y.
First condition,

Charge for 7 days =₹ 27

$$x + 4y = 27 \qquad \text{...(i)}$$

[Extra days $= 7 - 3 = 4$]

Charge for 5 days =₹ 21

$$x + 2y = 21 \qquad \text{...(ii)}$$

[Extra days $= 5 - 3 = 2$]

Subtracting equation (ii) from (i), we get

$$2y = 6$$

$$y = \frac{6}{2} = 3$$

Substituting, $y = 3$ in equation (ii) we have

$$x + 2(3) = 21$$

$$x = 21 - 6 = 15$$

Thus $x = 15$ and $y = 3$

Fixed charge=₹ 15 and Additional charge per day ₹ 3

PRACTICE :

1. Form the pair of linear equations in the following problems and find their solutions if they exist by the elimination method.

(i) If 2 is added to the numerator of fraction, it reduces to $\frac{1}{2}$ and if 1 is subtracted from the denominator, it reduces to $\frac{1}{3}$. Find the fraction

(ii) Ravi is three time as old as sonu. Five years later, Ravi will be two and a half times as old as sonu. Find their present ages

(iii) The sum of a two digit number and the number obtained by reversing the order of its digit is 121 and the two digit differ by 3. find the number

(iv) Sandeep went to a bank to withdraw ₹ 1000. He asked the cashier to give her ₹ 10 and ₹ 20 notes only. He got 80 notes in all. Find how many notes of ₹ 20 ₹ 10 he received

(v) The total expenditure per month of a house hold consist of a fixed rent and men charge depending upon the number of people sharing the house. The total monthly expenditure is ₹ 3500 for 3 people and ₹ 5500 for 7 people. Find the rent of the house and mess charge per head per month

Ans :

(i) $\frac{3}{10}$

(ii) Ravi = 45 years, Sonu =15 years

(iii) 47 or 74

(iv) ₹ 20 = 20 notes, ₹ 10 = 60 notes

(v) Monthly rent = ₹ 2000
Men charge = ₹ 500 per head per month

PRACTICE :

2. Form the pair of linear equations in the following problems and find their solutions if they exist by the elimination method.

(i) The sum of the numerator and denominator of a fraction is 8. If 3 is added to both the Name-rotor and denominator, the fraction be comes $\frac{3}{4}$. find the fraction

(ii) Six years hence a man's age will be three times his son;s age and three years ago, he was nine times as old as his son. Find their present ages.

(iii) A two digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 then adding 3. Find the number

(iv) The cost of 4 pens and 4 pencil is ₹ 100. Three times the cost of a pen is ₹ 15 more then the cost of a pencil box. Find the cost of a pen and a pencil box

Ans :

(i) $\frac{3}{5}$

(ii) Father = 30 years , Son = 6 years

(iii) 83

(iv) Pen = ₹ 10, Pencil box = ₹ 15

EXERCISE 3.5

1. Which of the following pairs of linear equations has unique solution, no solution or infinitely many solution. In case there is a unique solution, find it by using cross multiplication method.

(i) $x-3y-3=0,\ 3x-9y-2=0$

(ii) $2x+y=5,\ 3x+2y=8$

(iii) $3x-5y=20,\ 6x-10y=40$

(iv) $x-3y-7=0,\ 3x-3y-15=0$

Sol :

(i) $x-3y-3=0,\ 3x-9y-2=0$

We have $x-3y-3=0$

$3x-9y-2=0$

Comparing with standard equation we have,

$$a_1=1,\ b_1=-3,\ c_1=-3$$

$$a_2=3,\ b_2=-9,\ c_2=-2$$

$$\frac{a_1}{a_2}=\frac{1}{3}$$

$$\frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3}$$

$$\frac{c_1}{c_2}=\frac{-3}{-2}=\frac{3}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The given system has no solution.

(ii) $2x + y = 5,\ 3x + 2y = 8$

We have $2x + y - 5 = 0,$

$$3x + 2y - 8 = 0$$

Comparing with standard equation we have,

$$a_1 = 2,\ b_1 = 1,\ c_1 = -5$$

$$a_2 = 3,\ b_2 = 2,\ c_2 = -8$$

We find that $\frac{a_1}{a_2} = \frac{2}{3}$

$$\frac{b_1}{b_2} = \frac{1}{2}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The given system has a unique solution. To solve the equation, we have

$$\frac{x}{\begin{array}{cc}1 & -5\\ 2 & -8\end{array}} = \frac{y}{\begin{array}{cc}-5 & 2\\ -8 & 3\end{array}} = \frac{1}{\begin{array}{cc}2 & 1\\ 3 & 2\end{array}}$$

$$\frac{x}{(-8) - (10)} = \frac{y}{(-15) - (-16)} = \frac{1}{4 - 3}$$

$$\frac{x}{2} = \frac{y}{1} = 1$$

$$\frac{x}{2} = 1 \text{ or } x = 2$$

and $\frac{y}{1} = 1 \text{ or } y = 1$

(iii) $3x - 5y = 20,\ 6x - 10y = 40$

We have $3x - 5y - 20 = 0$

$$6x - 10y - 40 = 0$$

Comparing with standard equation we have,

$$a_1 = 3,\ b_1 = -5,\ c_1 = -20$$

$$a_2 = 6,\ b_2 = -10,\ c_2 = -40$$

Since, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$

$$\frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

The given system of linear equations has infinitely many solutions.

(iv) $x - 3y - 7 = 0,\ 3x - 3y - 15 = 0$

$$x - 3y - 7 = 0$$

$$3x - 3y - 15 = 0$$

Comparing with standard equation we have,

$$a_1 = 1,\ b_1 = -3,\ c_1 = -7$$

$$a_2 = 3,\ b_2 = -3,\ c_2 = -15$$

$$\frac{a_1}{a_2} = \frac{1}{3},\ \frac{b_1}{b_2} = \frac{-3}{-3} = 1$$

and $\frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given system has unique solution.
Now, using cross multiplication method, we have

$$\frac{x}{\begin{array}{cc}-3 & -7\\ -3 & -15\end{array}} = \frac{y}{\begin{array}{cc}-7 & 1\\ -15 & 3\end{array}} = \frac{1}{\begin{array}{cc}1 & -3\\ 3 & -3\end{array}}$$

$$\frac{x}{45 - 21} = \frac{y}{-21 + 15} = \frac{1}{-3 + 9}$$

$$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$$x = \frac{24}{6} = 4$$

$$y = \frac{-6}{6} = -1$$

Thus, $x = 4$ and $y = -1$

PRACTICE :

1. Which of the following pairs of linear of linear equations has unique solution, no solution, or infinitely many solution. In case there is a unique solution, find it by using cross multiplication method.

(i) $2x + 5y = -1,\ 3x - 2y = 8$

(ii) $4x + 6y = 9,\ 2x + 3y = 10$

(iii) $x - 2y = 5,\ 2x - 4y = 10$

(iv) $2x - y = 4,\ x + 3y = 9$

Ans :

(i) $x = 2,\ y = -1$

(ii) No solution

(iii) Inefficiently many solution

(iv) $x = 3,\ y = 2$

2. Which of the following pairs of linear of linear equations has unique solution, no solution, or infinitely many solution. In case there is a unique solution, find it by using cross multiplication method.

(i) $x + y = 8,\ x - y = 2$

(ii) $3x + 2y = 5,\ 6x + 4y - 10 = 0$

(iii) $47x + 31y = 63,\ 31x + 47y = 15$

(iv) $ax + by = a^2,\ 2ax + 2by = 2b^2$

Ans :

(i) $x = 5,\ y = 3$

(ii) Infinitely Many Solution

(iii) $x = 2,\ y = -1$

(iv) No Solution

2. (i) For which value of a and b does the following pair of linear equations have an infinite number of solutions? $2x + 3y = 7,\quad (a - b)x + (a + b)y = 3a + b - 2$

(ii) For which value of k will the following pair of linear equations have no solution? $3x+y=1$, $(2k-1)x+(k-1)y=2k+1$

Sol :

(i) For which value of a and b does the following pair of linear equations have an infinite number of solutions? $2x+3y=7$, $(a-b)x+(a+b)y=3a+b-2$

We have $\quad 2x+3y=7$

and $\quad (a-b)x+(a+b)y=(3a+b-2)$

Comparing with standard equation we have,

$$a_1=2,\ b_1=3,\ c_1=-7$$

$$a_2=(a-b),\ b_2=(a+b)$$

$$c_2=-(3a+b-2)$$

For an infinite number of solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{(a-b)}=\frac{3}{(a+b)}=\frac{-7}{-(3a+b-2)}$$

From the first two fraction, we get

$$\frac{2}{a-b}=\frac{3}{a+b}$$

$$2a+2b=3a-3b$$

$$a-5b=0 \qquad \text{...(i)}$$

From last two fractions, we get

$$\frac{3}{a+b}=\frac{-7}{-(3a+b-2)}$$

$$9a+3b-6=7a+7b$$

$$2a-4b=6$$

$$a-2b-3=0 \qquad \text{...(ii)}$$

Now, to solve by cross multiplication method, we have

$$\frac{a}{\begin{array}{cc}-5 & 0\\ -2 & -3\end{array}}=\frac{b}{\begin{array}{cc}0 & 1\\ -3 & 1\end{array}}=\frac{1}{\begin{array}{cc}1 & -5\\ 1 & -2\end{array}}$$

$$\frac{a}{15-0}=\frac{b}{0+3}=\frac{1}{-2+5}$$

$$\frac{a}{15}=\frac{b}{3}=\frac{1}{3}$$

$$a=\frac{1}{3}\times 15=5$$

$$b=\frac{1}{3}\times 3=1$$

Thus, $a=5$ and $b=1$

(ii) For which value of k will the following pair of linear equations have no solution? $3x+y=1$, $(2k-1)x+(k-1)y=2k+1$

We have $\quad 3x+y-1=0$

$$(2k-1)x+(k-1)y-(2k+1)=0$$

Comparing with standard equation we have,

$$a_1=3,\ b_1=1,\ c_1=-1$$

$$a_2=2k-1,\ b_2=k-1$$

$$c_2=-(2k+1)$$

For no solution

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{3}{2k-1}=\frac{1}{k-1}\neq\frac{-1}{-(2k+1)}$$

Taking first two, we get

$$3(k-1)=2k-1$$

$$3k-3=2k-1$$

$$3k-2k=-1+3$$

$$k=2$$

PRACTICE :

1. (i) For what value of k, the following system of equations has infinite solutions $x+(k+1)y=5$; $(k+1)x+9y=8.k-1$
   (ii) For what value of p will the following systems of linear equations have no solutions $3x+y=1$; $(2k-1)x(k-1)y=2k+1$

   Ans :

   (i) $k=2$ (ii) $k=2$
2. (i) For what value of a and b, the following system of equations will have infinite number of solutions $(a+b)x-2by=5a+2b+1$, $3x-y=14$
   (ii) Find the value of k if the following system of equation have no solution $x-ky=2$, $3x+2y=-5$

   Ans :

   (i) $a=5,\ b=1$ (ii) $k=\frac{-2}{3}$

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods: $8x+5y=9$, $3x+2y=4$

Sol :

Substitution Method :

$$8x+5y=9 \qquad \text{...(i)}$$

$$3x+2y=4 \qquad \text{...(ii)}$$

From equation (ii), we have

$$y=\frac{4-3x}{2} \qquad \text{...(iii)}$$

Substituting this value of y in equation (i), we have

$$8x+5\left[\frac{4-3x}{2}\right]=9$$

$$16x+20-15x=18$$

$$x=18-20=-2$$

Now, Substituting $x=-2$ in equation (iii), we have

$$y=\frac{4-3(-2)}{2}=\frac{4+6}{2}=5$$

Thus $x=-2$ and $y=5$

Cross Multiplication Method :

We have $8x+5y-9=0$

$$3x+2y-4=0$$

By cross multiplication, we get

$$\frac{x}{\begin{array}{cc}5 & -9\\ 2 & -4\end{array}}=\frac{y}{\begin{array}{cc}-9 & 8\\ -4 & 3\end{array}}=\frac{1}{\begin{array}{cc}8 & 5\\ 3 & 2\end{array}}$$

$$\frac{x}{-20+18}=\frac{y}{-27+32}=\frac{1}{16-15}$$

$$x=1\times-2=-2$$

and $$y=1\times 5=5$$

Thus, $x=-2$ and $y=5$

PRACTICE :

1. Solve the following pair of linear equations by the substitution and cross-multiplication methods:
   $2x-3y+1=0,\ 3x+4y-5=0$
   Ans : $x=\frac{11}{17},\ y=\frac{13}{17}$

2. Solve the following pair of linear equations by the substitution and cross-multiplication methods:
   $x-y=0.9,\ x+y=5.5$
   Ans : $x=3.2,\ y=2.3$

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method :
   (i) A part of monthly hostel charges is fixed and the remaining depends on the mess. When a student A takes food for 20 days she has to pay ₹ 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. Find the fixed charges and the cost of food per day.
   (ii) A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes 1/4 when 8 is added to its denominator. Find the fraction.
   (iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks 100 for incorrect answer then Yash would have scored 50 marks. How many questions were there in the test?
   (iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speed of the two cars?
   (v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units the area increases by 67 square units. Find the dimensions of the rectangle.

Sol :

(i) A part of monthly hostel charges is fixed and the remaining depends on the mess. When a student A takes food for 20 days she has to pay ₹ 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. Find the fixed charges and the cost of food per day.

Let the fixed charges be x and charges of food per day be y.

For Student A :

$$\text{Number of days}=20$$

$$\text{Cost of food for 20 days}=₹\,20y$$

According to the question,

$$x+20y=1000$$

$$x+20y-1000=0 \qquad ...(i)$$

For Student B :

$$\text{Number of days}=26$$

$$\text{Cost of food for 26 days}=₹\,26y$$

According to the question we have

$$x+26y=1180$$

$$x+26y-1180=0 \qquad ...(ii)$$

Solving these by cross multiplication, we get

$$\frac{x}{\begin{array}{cc}20 & -1000\\ 26 & -1180\end{array}}=\frac{y}{\begin{array}{cc}-1000 & 1\\ -1180 & 1\end{array}}=\frac{1}{\begin{array}{cc}1 & 20\\ 1 & 26\end{array}}$$

$$\frac{x}{-23600+2600}=\frac{y}{-1000+1180}=\frac{1}{26-20}$$

$$\frac{x}{2400}=\frac{y}{180}=\frac{1}{6}$$

$$x=\frac{1}{6}\times 2400=400$$

$$y=\frac{1}{6}\times 180=30$$

Thus $x=400$ and $y=30$

Fixed charges=₹ 400 and cost of food per day =₹ 30

(ii) A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes 1/4 when 8 is added to its denominator. Find the fraction.

Let the numerator be x and the denominator be y.

$$\text{Fraction}=\frac{x}{y}$$

Case-I : $$\frac{x-1}{y}=\frac{1}{3}$$

$$3x-3=y$$

$$3x-y-3=0 \qquad ...(i)$$

Case-II : $$\frac{x}{y+8}=\frac{1}{4}$$

$$4x=y+8$$

$$4x-y-8=0 \qquad ...(ii)$$

Solving them by cross multiplication, we get

$$\frac{x}{\begin{array}{cc}-1 & -3\\ -1 & -8\end{array}}=\frac{y}{\begin{array}{cc}-3 & 3\\ -8 & 4\end{array}}=\frac{1}{\begin{array}{cc}3 & -1\\ 4 & -1\end{array}}$$

$$\frac{x}{8-3}=\frac{y}{-12+24}=\frac{1}{-3+4}$$

$$\frac{x}{5}=\frac{y}{12}=\frac{1}{1}$$

$$x=1\times 5$$

and $$y=1\times 12=12$$

Thus, $x=5$ and $y=12$,

$$\text{Fraction}=\frac{5}{12}$$

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks 100 for incorrect answer then Yash would have scored 50 marks. How many questions were there in the test?

Let the number of correct answers be x and the number of wrong answers be y.

Case-I :

Marks for all correct answers $(3\times x)=3x$

Mark for all wrong answers $=(1\times y)=y$

According to the question, we have

$$3x-y=40 \qquad ...(i)$$

Case-II :

Mark for all correct answers $=(4\times x)=4x$

Marks for all wrong answers $=(2\times y)=2y$

According to the question we have

$$4x-2y=50 \qquad ...(ii)$$

From equation (i) and (ii), we have

$$a_1=3,\ b_1=-1,\ c_1=-40,\ a_2=2,\ b_2=-1$$

$$c_2=-25$$

By cross-multiplication, we get

$$\frac{x}{\begin{array}{cc}-1 & -40\\ -1 & -25\end{array}}=\frac{y}{\begin{array}{cc}-40 & 3\\ -25 & 2\end{array}}=\frac{1}{\begin{array}{cc}3 & -1\\ 2 & -1\end{array}}$$

$$\frac{x}{25-40}=\frac{y}{-80+75}=\frac{1}{-3+2}$$

$$\frac{x}{-15}=\frac{y}{-5}=\frac{1}{-1}$$

$$x=\frac{1}{-1}\times(-15)=15$$

$$y=\frac{1}{-1}\times(-5)=5$$

Thus $x=15$ and $y=5$

Now, total number of questions =[Number of correct answer]+[Number of wrong answers]

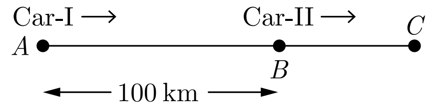
$$=15+5=20$$

Thus, required number of questions $=20$

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speed of the two cars?

Let the speed of car-I be x km/hr. and the speed of car-II be y km/hr.

Case-I :



Distance travelled by car-I $=AC$

Speed $\times$ time $=5\times x$ km

$$AC=5x$$

Distance travelled by car-II

$$BC=5y$$

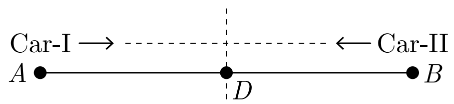
Since, $$AB=AC-BC$$

$$100=5x-5y$$

$$5x-5y-100=0$$

$$x-y-20=0 \qquad ...(i)$$

Case-II :



Distance travelled by car-I $=AD$

$$AD=1\times x=x$$

Distance travelled by car-I in 1 hour $=BD$

$$BD=1\times y=y$$

Now, $$AB=AD+DB$$

$$100=x+y$$

$$x+y=100 \qquad ...(ii)$$

Using cross-multiplication we get

$$x-y-20=0$$

$$x+y-100=0$$

where, $$a_1=1,\ b_1=-1,\ c_1=-20$$

$$a_2=1,\ b_2=1,\ c_2=-100$$

By cross-multiplication, we get

$$\frac{x}{\begin{array}{cc}-1 & -20\\ 1 & -100\end{array}}=\frac{y}{\begin{array}{cc}-20 & 1\\ -100 & 1\end{array}}=\frac{1}{\begin{array}{cc}1 & -1\\ 1 & 1\end{array}}$$

$$\frac{x}{100+20}=\frac{y}{-20+100}=\frac{1}{1+1}$$

$$\frac{x}{120}=\frac{y}{80}=\frac{1}{2}$$

$$x=\frac{1}{2}\times 120=60$$

$$y=\frac{1}{2}\times 80=40$$

Thus, speed of car-I $=60$ km/hr

Speed of car-II $= 40$ km/hr

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units the area increases by 67 square units. Find the dimensions of the rectangle.

Let the length of the rectangle be x units and the breadth of the rectangle y units

$$\text{Area of the rectangle} = x \times y = xy$$

Condition-I :

$$(x-5)(y+3) = xy-9$$

$$3x-5y-15 = -9$$

$$3x-5y-6 = 0 \qquad \text{...(i)}$$

Condition II :

$$(x+3)(y+2) = xy+67$$

$$2x+3y+6 = 67$$

$$2x+3y-61 = 0 \qquad \text{...(ii)}$$

Now, using cross multiplication method in equation (i) and (ii), where

$$a_1 = 3,\ b_1 = -5,\ c_1 = -6$$

$$a_2 = 2,\ b_2 = 3,\ c_2 = -61$$

By cross-multiplication, we get

$$\frac{x}{\begin{array}{cc}-5 & -6\\ 3 & -61\end{array}} = \frac{y}{\begin{array}{cc}-6 & 3\\ -61 & 2\end{array}} = \frac{1}{\begin{array}{cc}3 & -5\\ 2 & 3\end{array}}$$

$$\frac{x}{305+18} = \frac{y}{-12+183} = \frac{1}{9+10}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$$x = \frac{1}{19} \times 323 = 17$$

$$y = \frac{1}{19} \times 171 = 9$$

Thus, length of the rectangle $= 17$ units and breadth of the rectangle $= 9$ units.

PRACTICE :

1. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method :

(i) A part of monthly hostel charges is fixed and the remaining depends on the mess. When a student A takes food for 15 days she has to pay ₹ 1250 as hostel charges whereas a student B, who takes food for 20 days, pays ₹ 1500 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes 7/9 when 3 is subtracted from the numerator and it becomes 10/11 when 2 is added to its denominator. Find the fraction.

(iii) Yash scored 84 marks in a test, getting 5 marks for each right answer and losing 2 mark for each wrong answer. Had 6 marks been awarded for each correct answer and 3 marks been deducted for each in correct answer, then Yash would have scored 96 marks. Find total number of the questions in the test?

(iv) Places A and B are 120 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 6 hours. If they travel towards each other, they meet in 6/7 hour. What are the speed of the two cars?

(v) The area of a rectangle gets reduced by 26 square units, if its length is reduced by 2 units and breadth is reduced by 3 units. If we increase the length by 2 units and breadth by 3 units the area increases by 58 square units. Find the dimensions of the rectangle.

Ans :

(i) Fixed charge = ₹ 500
Food charge = ₹ 50 per day

(ii) $\frac{10}{9}$

(iii) 28

(iv) A' s speed 80 km/hr
B's speed 60 km/hr

(v) length =12 unit
breadth = 8 units

PRACTICE :

2. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method :

(i) A part of monthly hostel charges is fixed and the remaining depends on the mess. When a student A takes food for 22 days she has to pay ₹ 1680 as hostel charges whereas a student B, who takes food for 30 days, pays ₹ 2000 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes 1/3 if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator, It be common $\frac{2}{5}$. Find the fraction.

(iii) Yash scored 70 marks in a test, getting 4 marks for each right answer and losing 2 mark for each wrong answer. Had 5 marks been awarded for each incorrect answer, then Yash would have scored 80 marks. How many questions were there in the test?

(iv) Places A and B are 80km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 4 hours. If they travel towards each other, they meet in $\frac{2}{3}$ hour. What are the speed of the two cars?

(v) The area of a rectangle gets reduced by 6 square units, if its length is reduced by 3 units and breadth is increased by 2 units. If we increase the length by 3 units and breadth by 2 units the area increases by 66 square units. Find the dimensions of the rectangle.

Ans :

(i) Fixed = ₹ 800, Food = ₹ 40 per day

(ii) $\frac{1}{7}$

(iii) 40

(iv) A = 70 km/hr, B = 50 km/hr

(v) Length = 15 unit, breadth = 10 unit

EXERCISE 3.6

1. Solve the following pairs of equations by reducing them to a pair of linear equations.

(i) $\frac{1}{2x}+\frac{1}{3y}=2,\ \frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2,\ \frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14,\ \frac{3}{x}-4y=23$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2,\ \frac{6}{x-1}-\frac{3}{y-2}=1$

(v) $\frac{7x-2y}{xy}=5,\ \frac{8x+7y}{xy}=15$

(vi) $6x+3y=6xy,\ 2x+4y=5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4,\ \frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4},\ \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Sol :

(i) $\frac{1}{2x}+\frac{1}{3y}=2,\ \frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Substituting $\frac{1}{x}=u$ and $\frac{1}{y}=v$ we have

$$\frac{1}{2x}+\frac{1}{3y}=2$$

$$\frac{u}{2}+\frac{v}{3}=2 \qquad \text{...(i)}$$

and
$$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$$

$$\frac{u}{3}+\frac{v}{2}=\frac{13}{6} \qquad \text{...(ii)}$$

Multiplying equation (i) by $\frac{1}{3}$ and (ii) by $\frac{1}{2}$, we get

$$\frac{u}{6}+\frac{v}{9}=\frac{2}{3} \qquad \text{(iii)}$$

$$\frac{u}{6}+\frac{v}{4}=\frac{13}{12} \qquad \text{(iv)}$$

Subtracting equation (iii) from equation (iv), we get

$$\frac{v}{4}-\frac{v}{9}=\frac{13}{12}-\frac{2}{3}$$

$$\frac{9v-4v}{36}=\frac{13-8}{12}$$

$$\frac{5}{36}v=\frac{5}{12}\Rightarrow v=\frac{5}{12}\times\frac{36}{5}=3$$

Substituting, $v=3$ in equation (iii), we get

$$\frac{u}{6}+\frac{3}{9}=\frac{2}{3}$$

$$\frac{u}{6}=\frac{2}{3}-\frac{3}{9}=\frac{6-3}{9}=\frac{3}{9}$$

$$u=\frac{3}{9}\times 6=2$$

Thus $v=3$ and $u=2$

But
$$u=\frac{1}{x}$$

$$\frac{1}{x}=2\Rightarrow x=\frac{1}{2}$$

and
$$v=\frac{1}{y}$$

$$\frac{1}{y}=3\Rightarrow y=\frac{1}{3}$$

The required solution is,

$$x=\frac{1}{2} \text{ and } y=\frac{1}{3}$$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2,\ \frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

We have,
$$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2 \qquad \text{...(i)}$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1 \qquad ...(ii)$$

Let $\frac{1}{\sqrt{x}} = u$ and $\frac{1}{\sqrt{y}} = u$

Equation (i) and (ii) can be written as

$$2u + 3v = 2$$

or $$2u + 3v - 2 = 0 \qquad ...(iii)$$

$$4u - 9v = -1$$

$$4u - 9v + 1 = 0 \qquad ...(iv)$$

Here, $a_1 = 2,\ b_1 = 3,\ c_1 = -2$

$a_2 = 4,\ b_2 = -9,\ c_2 = 1$

By cross multiplication, we get

$$\frac{u}{\begin{matrix} 3 & -2 \\ -9 & 1 \end{matrix}} = \frac{v}{\begin{matrix} -2 & 2 \\ 1 & 4 \end{matrix}} = \frac{1}{\begin{matrix} 2 & 3 \\ 4 & -9 \end{matrix}}$$

$$\frac{u}{3-18} = \frac{v}{-8-2} = \frac{1}{-18-12}$$

$$\frac{u}{-15} = \frac{v}{-10} = \frac{1}{-30}$$

$$u = \frac{1}{-30} \times (-15) = \frac{1}{2}$$

$$v = \frac{1}{-30} \times (-10) = \frac{1}{3}$$

Since, $$u = \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow x = 4$$

and $$\frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow y = 9$$

The required solution is $x = 4$ and $y = 9$

(iii) $\frac{4}{x} + 3y = 14,\ \frac{3}{x} - 4y = 23$

We have, $$\frac{4}{x} + 3y = 14 \qquad ...(i)$$

$$\frac{3}{x} - 4y = 23 \qquad ...(ii)$$

Let $\frac{1}{x} = p$, then equations (i) and (ii) becomes,

$$4p + 3y = 14$$

or $$4p + 3y - 14 = 0 \qquad ...(iii)$$

$$3p - 4y = 23$$

or $$3p - 4y - 23 = 0 \qquad ...(iv)$$

Here, $a_1 = 4,\ b_1 = 3,\ c_1 = -14$

$a_2 = 3,\ b_2 = -4,\ c_2 = -23$

Solving equation (iii) and (iv) by cross multiplication method, we get

$$\frac{p}{\begin{matrix} 3 & -14 \\ -4 & -23 \end{matrix}} = \frac{y}{\begin{matrix} -14 & 4 \\ -23 & 3 \end{matrix}} = \frac{1}{\begin{matrix} 4 & 3 \\ 3 & -4 \end{matrix}}$$

$$\frac{p}{-69-56} = \frac{y}{-42+92} = \frac{1}{-16-9}$$

$$\frac{p}{-125} = \frac{y}{50} = \frac{1}{-25}$$

$$p = \frac{1}{-25} \times -125 = 5$$

and $$y = \frac{1}{-25} \times 50 = -2$$

Since, $$p = \frac{1}{x} = 5 \Rightarrow x = \frac{1}{5}$$

Thus, the required solution is

$$x = \frac{1}{5} \text{ and } y = -2$$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2,\ \frac{6}{x-1} - \frac{3}{y-2} = 1$

We have, $$\frac{5}{x-1} + \frac{1}{y-2} = 2 \qquad ...(i)$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1 \qquad ...(ii)$$

Let $\frac{1}{x-1} = u$ and $\frac{1}{y-2} = v$

Equations (i) and (ii) can be expressed as,

$$5u + v = 2$$

or $$5u + v - 2 = 0 \qquad ...(iii)$$

$$6u - 3v = 1$$

or $$6u - 3v - 1 = 0 \qquad ...(iv)$$

Here, $a_1 = 5,\ b_1 = 1,\ c_1 = -2$

and $a_2 = 6,\ b_2 = -3,\ c_2 = -1$

Solving equation (iii) and (iv) by cross multiplication method

$$\frac{u}{\begin{matrix} 1 & -2 \\ -3 & -1 \end{matrix}} = \frac{v}{\begin{matrix} -2 & 5 \\ -1 & 6 \end{matrix}} = \frac{1}{\begin{matrix} 5 & 1 \\ 6 & -3 \end{matrix}}$$

$$\frac{u}{-1-6} = \frac{v}{-12+5} = \frac{1}{-15-6}$$

$$\frac{u}{-7} = \frac{v}{-7} = \frac{1}{-21}$$

$$v = \frac{1}{(-21)} \times (-7) = \frac{1}{3}$$

But, $$u = \frac{1}{x-1} = \frac{1}{3}$$

$$3 = x - 1 \Rightarrow x = 4$$

and $$v = \frac{1}{y-2} = \frac{1}{3}$$

$$3 = y - 2 \Rightarrow y = 5$$

Thus, the required solution is $x = 4,\ y = 5$

(v) $\frac{7x-2y}{xy} = 5,\ \frac{8x+7y}{xy} = 15$

We have, $$\frac{7x-2y}{xy} = 5 \qquad ...(i)$$

$$\frac{8x+7y}{xy} = 15 \qquad ...(ii)$$

From equation (i),

$$\frac{7x}{xy} - \frac{2y}{xy} = 5$$

$$\frac{7}{y}-\frac{2}{x} = 5 \qquad \text{...(iii)}$$

From equation (ii),

$$\frac{8x}{xy}+\frac{7y}{xy} = 15$$

$$\frac{8}{y}+\frac{7}{x} = 15 \qquad \text{...(iv)}$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

Equation (iii) and (iv) can be expressed as

$$7q-2p-5 = 0 \qquad \text{...(v)}$$

$$8q+7p-15 = 0 \qquad \text{(vi)}$$

Here, $a_1 = 7,\ b_1 = -2,\ c_1 = -5$

$a_2 = 8,\ b_2 = 7,\ c_2 = -15$

Using cross multiplication method to solve (v) and (vi), we get

$$\frac{q}{\begin{matrix}-2 & -5\\ 7 & -15\end{matrix}} = \frac{p}{\begin{matrix}-5 & 7\\ -15 & 8\end{matrix}} = \frac{1}{\begin{matrix}7 & -2\\ 8 & 7\end{matrix}}$$

$$\frac{q}{30+35} = \frac{p}{-40+105} = \frac{1}{49+16}$$

$$\frac{q}{65} = \frac{p}{65} = \frac{1}{65}$$

$$p = \frac{1}{65}\times 65 = 1$$

$$q = \frac{1}{65}\times 65 = 1$$

Since, $p = \frac{1}{x} = 1 \Rightarrow x = 1$

and $q = \frac{1}{y} = 1 \Rightarrow y = 1$

Thus, the required solution is $x=1,\ y=1$

(vi) $6x+3y=6xy,\ 2x+4y=5xy$

We have, $6x+3y = 6xy$ ...(i)

$2x+4y = 5xy$ ...(ii)

From equation (i), we get

$$\frac{6x}{xy}+\frac{3y}{xy} = \frac{6xy}{xy}$$

$$\frac{6}{y}+\frac{3}{x} = 6 \qquad \text{...(iii)}$$

From equation (ii), we get

$$\frac{2x}{xy}+\frac{4y}{xy} = \frac{5xy}{xy}$$

$$\frac{2}{y}+\frac{4}{x} = 5 \qquad \text{...(iv)}$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

Equation (iii) and (iv) can be expressed as

$$6q+3p = 6$$

$$2q+4p = 5 \qquad \text{...(vi)}$$

Multiply equation (vi) by 3, we get

$$6q+12p = 15 \qquad \text{(vii)}$$

Subtracting equation (v) from (vii), we get

$$9p = 9 \Rightarrow p = 1$$

Substituting $p = 1$ in equation (v), we get

$$6q+3(1) = 6$$

$$6q = 6-3 = 3$$

$$q = \frac{3}{6} = \frac{1}{2}$$

Since, $p = \frac{1}{x} = 1 \Rightarrow x = 1$

and $q = \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$

Thus, the required solution is $x = 1$ and $y = 2$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4,\ \frac{15}{x+y}-\frac{5}{x-y}=-2$

We have, $\frac{10}{x+y}+\frac{2}{x-y} = 4$ ...(i)

$\frac{15}{x+y}-\frac{5}{x-y} = -2$ ...(ii)

Let, $\frac{1}{x+y} = p,\ \frac{1}{x-y} = q$

Equations (i) and (ii) can be expressed as

$$10p+2q = 4$$

or $10p+2q-4 = 0$ ...(iii)

$$15p-5q = -2$$

or $15p-5q+2 = 0$ ...(iv)

Here, $a_1 = 10,\ b_1 = 2,\ c_1 = -4$

$a_2 = 15,\ b_2 = -5,\ c_2 = 2$

By cross-multiplication, we get

$$\frac{p}{\begin{matrix}2 & -4\\ -5 & 2\end{matrix}} = \frac{q}{\begin{matrix}-4 & 10\\ 2 & 15\end{matrix}} = \frac{1}{\begin{matrix}10 & 2\\ 15 & -5\end{matrix}}$$

$$\frac{p}{4-20} = \frac{q}{-60-20} = \frac{1}{-50-30}$$

$$\frac{p}{-16} = \frac{q}{-80} = \frac{1}{-80}$$

$$p = \frac{1}{(-80)}\times(-16) = \frac{1}{5}$$

and $q = \frac{1}{(-80)}\times(-80) = 1$

But, $p = \frac{1}{x+y} = \frac{1}{5}$

$$x+y = 5 \qquad \text{...(v)}$$

and $q = \frac{1}{x-y} = \frac{1}{1}$

$$x-y = 1 \qquad \text{(vi)}$$

Adding equation (v) and (vi), we have

$$2x = 6 \Rightarrow x = 3$$

From equation (v),

$$3+y = 5 \Rightarrow y = 2$$

Thus, the required solution is $x=3,\ y=2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}\ \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

We have, $\frac{1}{(3x+y)}+\frac{1}{(3x-y)}=\frac{3}{4}$ ...(i)

$$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8} \quad ...(ii)$$

Let, $\frac{1}{(3x+y)}=p$

and $\frac{1}{(3x-y)}=q$

Equation (i) and (ii) can be expressed as

$$p+q=\frac{3}{4} \quad ...(iii)$$

$$\frac{p}{2}-\frac{q}{2}=-\frac{1}{8} \quad ...(iv)$$

Multiplying equation (iii) by $\frac{1}{2}$, we get

$$\frac{p}{2}+\frac{q}{2}=\frac{3}{8} \quad ...(v)$$

Adding equation (iv) and (v), we get

$$\left(\frac{p}{2}+\frac{p}{2}\right)=\left(\frac{3}{8}-\frac{1}{8}\right)$$

$$p=\frac{2}{8}=\frac{1}{4}$$

From equation (iii),

$$\frac{1}{4}+q=\frac{3}{4}$$

$$q=\frac{3}{4}-\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$$

But, $p=\frac{1}{3x+y}=\frac{1}{4}$

$$3x+y=4 \quad ...(vi)$$

And $q=\frac{1}{3x-y}=\frac{1}{2}$

$$3x-y=2 \quad (vii)$$

Adding equation (vi) and (vii) we get

$$6x=6\Rightarrow x=1$$

Subtracting equation (vii) from (vi)

$$2y=2\Rightarrow y=1$$

Thus, the required solution is $x=1,\ y=1$

PRACTICE :

1. Solve the following pairs of equations by reducing them to a pair of linear equations.

(i) $\frac{21}{x-y}+\frac{38}{x+y}=9,\ \frac{6}{x-y}+\frac{57}{x+y}=5$

(ii) $\frac{2}{x}+\frac{3}{y}=13,\ \frac{5}{x}-\frac{4}{y}=-2$

(iii) $4x+\frac{6}{y}=15,\ 6x-\frac{8}{y}=14$

(iv) $8x-9y=6xy,\ 10x+6y=19xy$

(v) $\frac{4}{\sqrt{x}}+\frac{3}{\sqrt{y}}=3,\ \frac{6}{\sqrt{x}}-\frac{9}{\sqrt{y}}=0$

(vi) $\frac{14}{x+y}+\frac{3}{x-y}=5,\ \frac{21}{x+y}-\frac{1}{x-y}=2$

(vii) $\frac{1}{2(x+2y)}+\frac{5}{3(x-2y)}=\frac{-3}{2}$

$\frac{5}{4(x+2y)}-\frac{3}{(3x-2y)}=\frac{61}{60}$

(viii) $\frac{x+y}{xy}=2,\ \frac{x-y}{xy}=6$

Ans :

(i) $x=11,\ y=8$ (ii) $x=\frac{1}{2},\ y=\frac{1}{3}$

(iii) $x=3,\ y=2$ (iv) $x=\frac{3}{2},\ y=\frac{2}{3}$

(v) $x=4,\ y=a$ (vi) $x=4,\ y=3$

(vii) $x=\frac{1}{2},\ y=\frac{5}{4}$ (viii) $x=\frac{-1}{2},\ y=\frac{1}{4}$

PRACTICE :

2. Solve the following pairs of equations by reducing them to a pair of linear equations.

(i) $\frac{3}{x}-\frac{1}{y}=-9,\ \frac{1}{x}+\frac{3}{y}=5$

(ii) $\frac{5}{x+1}-\frac{2}{y-1}=\frac{1}{2},\ \frac{10}{x+1}+\frac{2}{y-1}=\frac{5}{2}$

(iii) $\frac{1}{5x}+\frac{y}{9}=5,\ \frac{1}{3x}+\frac{y}{2}=12$

(iv) $\frac{6}{x+y}=\frac{7}{x-y}+3,\ \frac{1}{2(x+y)}-\frac{1}{3(x-y)}=0$

(v) $x+y=5xy,\ 3x+2y=13xy$

(vi) $2(3u-v)=5uv,\ 2(u+3v)=5uv$

(vii) $\frac{2}{3x+2y}+\frac{3}{3x-2y}-\frac{17}{5}$

$\frac{5}{3x+2y}+\frac{1}{3x-2y}=2$

(viii) $4x+\frac{6}{y}=15,\ 3x-\frac{4}{y}=7$

Ans :

(i) $x=\frac{-1}{2},\ y=\frac{1}{3}$

(ii) $x=4,\ y=5$

(iii) $x=\frac{1}{15},\ y=18$

(iv) $x=\frac{-5}{y},\ y=\frac{-1}{4}$

(v) $x=\frac{1}{2},\ y=\frac{1}{3}$

(vi) $u=2,\ v=1$

(vii) $x=1,\ y=1$

(viii) $x=3,\ y=2$

2. Formulate the following problems as a pair of equations, and hence find their solutions.

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus If she travels 100 km by train and the remaining by bus she takes 10 minutes longer. Find the speed of the train and the bus separately.

Sol :

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

Let the speed of Ritu in still water be x and the speed of the water current y.

$$\text{Downstream speed} = (x+y)\text{ km/hr}$$

$$\text{Upstream speed} = (x-y)\text{ km/hr}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$2 = \frac{20}{x+y}$$

$$x+y = 10 \qquad ...(i)$$

and

$$2 = \frac{4}{x-y}$$

$$x-y = 2 \qquad ...(ii)$$

Adding equation (i) and (ii), we get

$$2x = 12$$

$$x = \frac{12}{2} = 6$$

From equation (i),

$$6+y = 10$$

$$y = 10-6 = 4$$

Thus, speed of rowing in still water = 6 km/hr, speed of water current = 4 km/hr

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken that taken by 1 man alone.

Let the time taken to finish the task by one woman alone be x days and by one man alone y days

$$\text{1 woman's 1 day work} = \frac{1}{x}$$

$$\text{1 man's 1 day work} = \frac{1}{y}$$

Since, [2 women + 5 men] finish the task in 4 days

So,
$$4\times\left[\frac{2}{x}+\frac{5}{y}\right] = 1$$

$$\frac{2}{x}+\frac{5}{y} = \frac{1}{4}$$

Again [3 women + 6 men], finish the task in 3 days

$$3\times\left[\frac{3}{x}+\frac{6}{y}\right] = 1$$

$$\frac{3}{x}+\frac{6}{y} = \frac{1}{3}$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

Equation (i) and (ii) are expressed as

$$2p+5q = \frac{1}{4}$$

$$8p+20q-1 = 0$$

$$3p+6q = \frac{1}{3}$$

$$9p+18q-1 = 0$$

Using cross multiplication method, we get

$$a_1 = 8,\ b_1 = 20 \text{ and } c_1 = -1$$

$$a_2 = 9,\ b_2 = 18 \text{ and } c_2 = -1$$

$$\frac{p}{\begin{matrix}20 & -1\\ 18 & -1\end{matrix}} = \frac{q}{\begin{matrix}-1 & 8\\ -1 & 9\end{matrix}} = \frac{1}{\begin{matrix}8 & 20\\ 9 & 18\end{matrix}}$$

$$\frac{p}{-20+18} = \frac{q}{-9+8} = \frac{1}{144-180}$$

$$\frac{p}{-2} = \frac{q}{-1} = \frac{1}{-36}$$

$$p = \frac{1}{-36} \times (-2) = \frac{1}{18}$$

$$q = \frac{1}{-36} \times (-1) = \frac{1}{36}$$

Since, $p = \frac{1}{x} = \frac{1}{18} \Rightarrow x = 18$

and $q = \frac{1}{y} = \frac{1}{36} \Rightarrow y = 36$

1 man can finish the work in 36 days and 1 woman can finish the work in 18 days.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus If she travels 100 km by train and the remaining by bus she takes 10 minutes longer. Find the speed of the train and the bus separately.

Let the speed of the train be x km/hr and the speed of the bus y km/hr

$$\text{Time} = \frac{\text{Distance}}{\text{speed}}$$

Case I :

$$\text{Total journey} = 300\,\text{km}$$

$$\text{Journey travelled by train} = 60\,\text{km}$$

$$\text{Journey travelled by bus} = (300 - 60)\text{ km}$$

$$= 240\text{ km}$$

$$\text{Total time taken} = 4\text{ hours}$$

$$\frac{60}{x} + \frac{240}{y} = 4$$

$$\frac{1}{x} + \frac{4}{y} = \frac{1}{15} \quad \text{...(i)}$$

Case II :

$$\text{Distance travelled by train} = 100\text{ km}$$

$$\text{Distance travelled by bus} = (300 - 100)\,\text{km}$$

$$= 200\text{ km}$$

$$\text{Total time} = 4\text{ hrs }10\text{ mins}$$

$$= \left(4 + \frac{10}{60}\right)\text{hrs} = \frac{25}{6}\text{ hrs}$$

$$\frac{100}{x} + \frac{200}{y} = \frac{25}{6}$$

$$\frac{4}{x} + \frac{8}{y} = \frac{1}{6} \quad \text{...(ii)}$$

Multiplying equation (i) by 4, we get

$$\frac{4}{x} + \frac{16}{y} = \frac{4}{15} \quad \text{...(iii)}$$

Subtracting equation (iii) from (ii), we get

$$\frac{8}{y} - \frac{16}{y} = \frac{1}{6} - \frac{4}{15}$$

$$\frac{-8}{y} = \frac{5-8}{30} = \frac{-3}{30}$$

$$\frac{1}{y} = \frac{-3}{30} \times \frac{1}{(-8)} = \frac{1}{80}$$

$$y = 80$$

From equation (i),

$$\frac{1}{x} + \frac{4}{80} = \frac{1}{15}$$

$$\frac{1}{x} = \frac{1}{15} - \frac{4}{80} = \frac{1}{15} - \frac{1}{20} = \frac{1}{60}$$

$$x = 60$$

Thus, speed of the train = 60 km/hr and speed of the bus = 80 km/hr

PRACTICE :

1. Formulate the following problems as a pair of equations, and hence find their solutions.

(i) Ritu can row downstream 36 km in 3 hours, and upstream 6 km in 3 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 8 women and 12 men can together finish an embroidery work in 10 days, while 6 women and 8 men can finish it in 14 days. Find the time taken that taken by 1 man alone.

(iii) Roohi travels 600 km to her home partly by train and partly by bus. She takes $6\frac{1}{2}$ hours if she travels 400 km by train and the remaining by bus If she travels 200 km by train and the remaining by bus she takes 30 minutes longer. Find the speed of the train and the bus separately.

Ans :

(i) 7 km/hr, 5 km/hr

(ii) 100 km/hr, 80 km/hr

2. Formulate the following problems as a pair of equations, and hence find their solutions.

(i) Ritu can row downstream 54 km in 3 hours, and upstream 8 km in 4 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 7 men can together finish an embroidery work in 4 days, while 4 women and 4 men can finish it in 3 days. Find the time taken that taken by 1 man alone.

(iii) Roohi travels 370 km to her home partly by train and partly by bus. She takes 4 hours if she travels 250 km by train and the remaining by bus If she travels 130 km by train and the remaining by bus. Find the by bus, she takes 18 minutes longer. Find the speed of the train and the bus separately.

Ans :

(i) 10 km/hr, 8 km/hr

(ii) 15 days, 60 days

(iii) 100 km/hr, 80 km/hr

EXERCISE 3.7

1. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Sol :

Let the age of Ani be x years and the age of Biju's y years

Case I : $y > x$

According to 1st condition :

$$y - x = 3 \quad \text{...(i)}$$

$$[\text{Age of Ani's father}] = 2\,[\text{Age of Anil}] = 2x \text{ years}$$

Also, $$[\text{Age of Biju's sister}] = \frac{1}{2}[\text{Age of Biju}] = \frac{1}{2}y$$

According to IInd condition :

$$2x - \frac{1}{2}y = 30$$

$$4x - y = 60 \quad \text{...(ii)}$$

Adding equation (i) and (ii), we get

$$3x = 63$$

$$x = \frac{63}{3} = 21$$

From equation (i), $y - 21 = 3$

$$y = 3 + 21 = 24$$

Age of Ani = 21 years

Age of Biju = 24 years

Case II : $x > y$

$$x - y = 3 \quad \text{...(i)}$$

According to the condition :

$$2x - \frac{1}{2}y = 30$$

$$4x - y = 60 \quad \text{...(ii)}$$

Subtracting equation (i) from (ii), we get

$$3x = 57$$

$$x = \frac{57}{3} = 19$$

From equation (i), $19 - y = 3$

$$y = 16$$

Age of Ani = 19 years

and Age of Biju = 16 years

PRACTICE :

1. Father's is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of the father.

Ans : 45 years

2. Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. Find their present years.

Ans : Salim = 38 years, Daughter = 14 years

2. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II]
[Hint: $x + 100 = 2(y - 100)$, $y + 10 = 6(x - 10)$]

Sol :

Let the capital of 1st friend be x and the capital of 2nd friend be y.

According to the question,

$$x + 100 = 2(y - 100)$$

$$x + 100 - 2y + 200 = 0$$

$$x - 2y + 300 = 0 \quad \text{...(i)}$$

and $$6(x - 10) = y + 10$$

$$6x - y - 70 = 0 \quad \text{...(ii)}$$

From equation (i),

$$x = -300 + 2y$$

From equation (ii),

$$6x - y - 70 = 0$$

$$6[-300 + 2y] - y - 70 = 0$$

$$-1870 + 11y = 0$$

$$y = \frac{1870}{11} = 170$$

Now, $$x = -300 + 2y = -300 + 2(170) = -300 + 340 = 40$$

Thus, 1st friend has ₹ 40 and the 2nd friend has ₹ 170

PRACTICE :

1. Amit and Sumit each have some money. If Amit gives ₹ 30 to sumit, then sumit will have twice the money left with Amit. But, if Sumit gives ₹ 10 to Amit, then Amit will have thrice as much as is left with Sumit. How much money does each have?

   Ans : Amit = ₹ 62, Sumit = ₹ 34

2. A & B have some money. A gives ₹ 70 to B, then B will have one and half times the money left with A. But, If B gives ₹ 30 to A, then A will have four times as much is left with B - How much money does each have

   Ans : A = ₹ 170, B = ₹ 80

3. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Sol :

Let, the actual speed of the train be x km/hr and the actual time taken y hours.

$$\text{Distance} = \text{speed} \times \text{time}$$

1st **Condition :**

$$(x+10)\times(y-2) = xy$$

$$xy-2x+10y-20 = xy$$

$$2x-10y+20 = 0 \qquad ...(i)$$

2nd **Condition :**

$$(x-10)\times(y+3) = xy$$

$$xy+3x-10y-30 = xy$$

$$3x-10y-30 = 0 \qquad ...(ii)$$

Using cross multiplication for solving equation (i) and (ii) we have

$$a_1 = 2,\ b_1 = -10 \text{ and } c_1 = 20;$$

$$a_2 = 3,\ b_2 = -10 \text{ and } c_2 = -30$$

$$\frac{x}{\begin{matrix}-10 & 20\\ -10 & -30\end{matrix}} = \frac{y}{\begin{matrix}20 & 2\\ -30 & 3\end{matrix}} = \frac{1}{\begin{matrix}2 & -10\\ 3 & -10\end{matrix}}$$

$$\frac{x}{300+200} = \frac{y}{60+60} = \frac{1}{-20+30}$$

$$\frac{x}{500} = \frac{y}{120} = \frac{1}{10}$$

$$x = \frac{1}{10}\times 500 = 50$$

$$y = \frac{1}{10}\times 120 = 12$$

Thus, the distance covered by the train

$$= 50\times 12\,\text{km} = 600 \text{ km}.$$

PRACTICE :

1. A train covered a certain distance at a uniform speed. If the train would have been 20 km/h faster, it would have taken 1 hours less than the scheduled time. And, if the train were slower by 30 km/h; it would have taken 3 hours more than the scheduled. time. Find the distance covered by the train.

   Ans : Distance = 400 km

2. A train covered a certain distance at a uniform speed. If the train would have been 20 km/h faster, it would have taken 1 hours less than the scheduled time. And, if the train were slower by 20 km/h; it would have taken 1.5 hours more than the scheduled. time. Find the distance covered by the train.

   Ans : Distance = 600 km

4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of student in the class.

Sol :

Let no. of students in each row be x and no. of rows be y.

Now $\qquad \text{Total students} = xy$

Case I : $\qquad (x+3)(y-1) = xy$

$$xy+3y-x-3 = xy$$

$$3y-x = 3 \qquad ...(i)$$

Case II: $\qquad (x-3)(y+2) = xy$

$$xy-3y+2x-6 = xy$$

$$-3y+2x = 6 \qquad ...(ii)$$

Solving equation (i) and (ii), we get

$$-x+2x = 3+6$$

$$x = 9$$

Substituting value of $x=9$ in equation (i), we get

$$3y-9 = 3$$

$$y = 4$$

So, $\qquad \text{total no. of students} = xy = 9\times 4$

$$= 36 \text{ Students}$$

PRACTICE :

1. The students of a class are made to stand in rows. If 5 students are extra in a row, there would be 5 row less. If 1 students are less in a row, there would be 2 rows more. Find the number of student in the class.

Ans : 84 students

2. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 4 row less. If students are less in a row, there would be 2 rows more. Find the number of student in the class.

Ans : 144 students

5. In a ΔABC, $\angle C = 3\angle B = 2(\angle A + B)$. Find the three angles.

Sol :

We know using ASP

$$LA + CB + CC = 180^\circ \quad \text{...(i)}$$

From the given $\angle C = 3\angle B = 2(\angle A + \angle B)$ we get

$$\angle C = 2(\angle A + \angle B)$$

$$\angle A + \angle B = \frac{\angle C}{2} \quad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{\angle C}{2} + \angle C = 180$$

$$\frac{3\angle C}{2} = 180$$

$$\angle C = 120^\circ$$

And $\angle C = 3\angle B$

$$\angle B = \frac{120}{3} = 40^\circ$$

$$\angle A = 20^\circ$$

PRACTICE :

1. In a ΔABC, $\angle A = x^\circ$, $\angle B = (3x - 2)$ $\angle C = y^\circ$ and $\angle C - \angle B = 9^\circ$. Find the three angles.

Ans : $A = 25^\circ$, $B = 73^\circ$, $C = 82^\circ$

2. In a ΔABC, $\angle A = x^\circ$, $\angle B = 3x^\circ$, $\angle C = y^\circ$ and $3C - 5A = 30^\circ$ Find the three angles.

Ans : $\angle A = 30^\circ$, $\angle B = 90^\circ$, $\angle C = 60^\circ$

6. Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y-axis.

Sol :

We have $5x - y = 5 \Rightarrow \quad l_1: \quad y = 5x - 5$

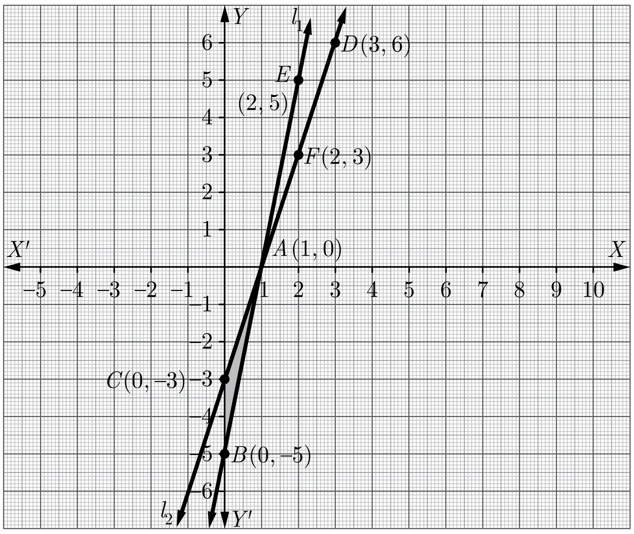
x	1	2	0
y	0	5	-5

and for equation,

$$3x - y = 3 \Rightarrow \quad l_2: \quad y = 3x - 3$$

x	2	3	0
y	3	6	-3

Plotting the points (1, 0), (2, 5) and (0, -5), we get a straight line l_1. Plotting the points (2, 3), (3, 6) and (0, -3), we get a straight line l_2.



From the figure, obviously, the vertices of the triangle formed are $A(1, 0)$, $B(0, -5)$ and $C(0, -3)$

7. Solve the following pair of linear equations

(i) $px + qy = p - q$, $qx - py = p + q$

(ii) $ax + by = c$, $bx + ay = 1 + c$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$, $ax + by = a^2 + b^2$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$,

$(a + b)(x + y) = a^2 + b^2$

(v) $152x - 378y = -74$, $-378x + 152y = -604$

Sol :

(i) $px + qy = p - q$, $qx - py = p + q$

We have, $px + qy = p - q \quad \text{...(i)}$

$$qx - py = p + q \quad \text{...(ii)}$$

Multiply equation (i) by p and (ii) by q, we get

$$p^2x + qpy = p^2 - pq \quad \text{...(iii)}$$

$$q^2x - pqy = q^2 + pq \quad \text{...(iv)}$$

Adding equation (iii) and (iv), we get

$$p^2x + q^2x = p^2 + q^2$$

$$(p^2 + q^2)x = p^2 + q^2$$

$$x = \frac{p^2 + q^2}{p^2 + q^2} = 1$$

From equation (i),

$$p + qy = p - q$$

$$y = -1$$

Thus, the required solution is $x = 1$, $y = -1$

(ii) $ax+by=c$, $bx+ay=1+c$

We have, ` $ax+by = c$ ...(i)

$$bx+ay = 1+c \quad ...(ii)$$

By cross multiplication, we have

$$\frac{x}{\begin{matrix} b & -c \\ a & -(1+c) \end{matrix}} = \frac{y}{\begin{matrix} -c & a \\ -(1+c) & b \end{matrix}} = \frac{1}{\begin{matrix} a & b \\ b & a \end{matrix}}$$

$$\frac{x}{-b-bc+ac} = \frac{y}{-bc+a+ac} = \frac{1}{a^2-b^2}$$

$$x = \frac{-b-bc+ac}{a^2-b^2}$$

$$y = \frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a}-\frac{y}{b}=0$, $ax+by=a^2+b^2$

We have, $\frac{x}{a}-\frac{y}{b} = 0$ ...(i)

$$ax+by = a^2+b^2 \quad ...(ii)$$

From equation (i), we have

$$\frac{x}{a} = \frac{y}{b}$$

$$y = \left(\frac{x}{a}\times b\right) \quad ...(iii)$$

Substituting $y = \left(\frac{b}{a}x\right)$ in equation (ii), we have

$$ax+b\left(\frac{b}{a}x\right) = a^2+b^2$$

$$x\left[\frac{a^2+b^2}{a}\right] = a^2+b^2$$

$$x = \frac{a^2+b^2}{a^2+b^2}\times a = a$$

Substituting $x = a$ in equation (iii), we get

$$y = \frac{a}{a}\times b = b$$

Thus, the required solution is $x=a$ $y=b$.

(iv) $(a-b)x+(a+b)y=a^2-2ab-b^2$,

$(a+b)(x+y)=a^2+b^2$

We have,

$$(a-b)x+(a+b)y = a^2-2ab-b^2 \quad ...(i)$$

$$(a+b)(x+y) = a^2+b^2 \quad ...(ii)$$

From equation (ii),

$$(a+b)x+(a+b)y = a^2+b^2 \quad ...(iii)$$

Subtracting equation (iii) from (i), we get

$$x[(a-b)-(a+b)] = a^2-2ab-b^2-a^2-b^2$$

$$x[a-b-a-b] = -2ab-2b^2$$

$$x(-2b) = -2b(a+b)$$

$$x = \frac{-2b(a+b)}{-2b} = a+b$$

Substituting $x = (a+b)$ in equation (i), we get

$$(a-b)(a+b)+(a+b)y = a^2-2ab-b^2$$

$$(a+b)y = a^2-2ab-b^2-a^2+b^2$$

$$(a+b)y = -2ab$$

$$y = \frac{-2ab}{(a+b)}$$

Thus we have $x = a+b$ and $y = -\frac{2ab}{a+b}$

(v) $152x-378y=-74$, $-378x+152y=-604$

We have, $152x-378y = -74$ ...(i)

$$-378x+152y = -604 \quad ...(ii)$$

Adding equation (i) and (ii), we have

$$-226x-226y = -678$$

$$x+y = 3 \quad ...(iii)$$

Subtracting equation (i) from (ii), we get

$$-530x+530y = -530$$

$$-x+y = -1$$

$$x-y = 1 \quad ...(iv)$$

Adding equation (iii) and (iv), we get

$$2x = 4 \Rightarrow x = 2$$

Subtracting equation (iii) from (iv), we get

$$-2y = -2 \Rightarrow y = 1$$

Thus, the required solution is $x=2$ and $y=1$

PRACTICE :

1. Solve the following pair of linear equations

(i) $99x+101y=499$, $101x+99y=501$

(ii) $x+y=a+b$, $ax-by=a^2-b^2$

(iii) $a(x+y)+b(x-y)=b^2+a^2-ab$

$-b(x-y)+a(x+y)=a^2+ab+b^2$

(iv) $\frac{a^2}{x}-\frac{b^2}{y}=0$, $\frac{a^2b}{x}+\frac{b^2a}{y}=a+b$

(v) $2(ax-by)+a+4b=0$

$2(bx+ay)+b-4a=0$

Ans :

(i) $x=3$, $y=2$ (ii) $x=a$, $y=b$

(iii) $x=\frac{b^2}{2a}, y=\frac{2a^2+b^2}{2a}$ (iv) $x=a^2$, $y=b^2$

(v) $x=\frac{-1}{2}, y=2$

2. Solve the following pair of linear equations

(i) $23x-29y=98$, $29x-23y=110$

(ii) $ax-by=a^2-b^2$, $\frac{x}{a}+\frac{y}{10}=2$

(iii) $\frac{a}{x}-\frac{b}{y}=0$, $\frac{ab^2}{x}+\frac{a^2b}{y}=a^2+b^2$

(iv) $mx-ny=m^2+n^2$, $x+y=2m$

(v) $ax+by=\frac{a+b}{2}$, $3x+5y-4$

Ans :

(i) $x=3$, $y=-1$

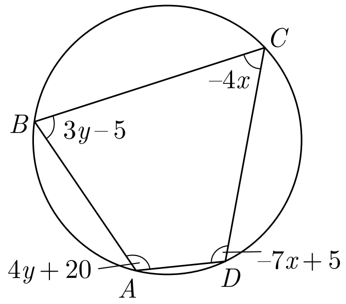
(ii) $x=a$, $y=b$

(iii) $x=a$, $y=b$

(iv) $x=m+n$, $y=m-n$

(v) $x=\frac{1}{2}$, $y=\frac{1}{2}$

8. $ABCD$ is a cyclic quadrilateral (see figure.) Find the angles of the cyclic quadrilateral.



Sol :

Here $ABCD$ is a cyclic quadrilateral.

$$\angle A + \angle C = 180^\circ$$

and $$\angle B + \angle D = 180^\circ$$

$$[4y + 20] + [-4x] = 180^\circ$$

$$4y - 4x - 160^\circ = 0$$

$$y - x - 40^\circ = 0 \quad \text{...(i)}$$

and $$[3y - 5] + [-7x + 5] = 180^\circ$$

$$3y - 7x - 180^\circ = 0 \quad \text{...(ii)}$$

Multiplying equation (i) by 7, we get

$$7y - 7x - 280^\circ = 0 \quad \text{...(iii)}$$

Subtracting equation (iii) from (ii), we get

$$-4y + 100^\circ = 0$$

$$y = \frac{-100^\circ}{-4} = 25^\circ$$

Now, substituting, $y = 25^\circ$ in equation (i), we get

$$-x = 40^\circ - 25^\circ = 15^\circ$$

$$x = -15^\circ$$

$$\angle A = 4y + 20^\circ = 4(25^\circ) + 20^\circ$$

$$= 120^\circ$$

$$\angle B = 3y - 5^\circ = 3(25^\circ) - 5^\circ = 70^\circ$$

$$\angle C = -4x = -4(-15^\circ) = 60^\circ$$

$$\angle D = -7x + 5^\circ = -7(-15^\circ) + 5^\circ$$

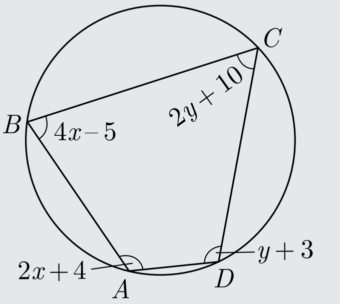
$$= 110^\circ$$

Thus, $$\angle A = 120^\circ, \angle B = 70^\circ$$

$$\angle C = 60^\circ, \angle D = 110^\circ$$

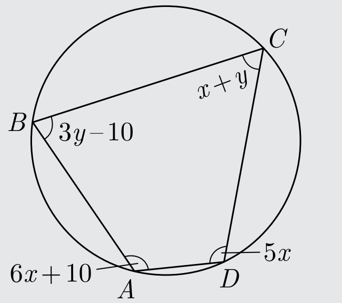
PRACTICE :

1. $ABCD$ is a cyclic quadrilateral (see figure.) Find the angles of the cyclic quadrilateral.



Ans : $A = 70^\circ$, $B = 53^\circ$, $C = 110^\circ$, $D = 127^\circ$

2. $ABCD$ is a cyclic quadrilateral (see figure.) Find the angles of the cyclic quadrilateral.



Ans : $\angle A = 130^\circ$, $\angle B = 80^\circ$, $\angle C = 50^\circ$, $\angle D = 100^\circ$

www.ncertpdf.in