M-QAM OFDM and PCC-OFDM Performance in the Presence of Phase Noise

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***Abstract*— In this paper we address the issue of OFDM and PCC-OFDM systems impaired by phase noise (PN). PCC-OFDM is a variation of OFDM in which data is mapped onto adjacent subcarrier pairs. As a result intercarrier interference (ICI) in adjacent subcarriers is cancelled. Phase noise in OFDM causes both ICI and common phase error (CPE). PCC-OFDM reduces the ICI component, but the CPE is unchanged. Theoretical and simulations results for bit error rate (BER) are presented for M-QAM OFDM and PCC-OFDM systems over Rayleigh fading channels. The performance with and without CPE correction at the receiver is considered. It is shown that PCC-OFDM has a lower BER in all cases. The theoretical results agree closely with the computer simulations.**

***Index Terms*— Orthogonal frequency division multiplexing, phase noise, Intercarrier interference, Rayleigh fading.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier transmission technique, which divides the available spectrum into many carriers, each one being modulated by a low rate data stream. Practical implementations of OFDM technology use an inverse fast Fourier transform (IFFT) to generate a digitized version of the composite time domain signal. It is used in many wireless applications such as digital video broadcasting (DVB), ADSL (asymmetric digital subscriber loop), wireless local area networks (IEEE 802.11 a/g, HIPERLAN/2) and possibly future 4 G networks.

Polynomial cancellation coded OFDM (PCC-OFDM) maps data to be transmitted into adjacent weighted subcarriers. For example the simplest form of PCC-OFDM uses +1 and -1 as the weighted coefficients and maps data symbols into the IFFT as $d_{n+1} = -d_n$. Previous work [2] has shown that PCC-OFDM is less sensitive to frequency offset and multipath transmission.

Phase noise (PN) is the mismatch between the phase of the carrier and the phase of the local Oscillator. OFDM systems are highly sensitive to PN perturbations due to the compactness of the subcarriers.

The effects of PN have been analyzed in many papers [4],[5],[8]. PN in OFDM causes loss of orthogonality of the subcarriers resulting in ICI. It has been shown in previous literature such as [3] that, PN also introduces a common phase rotation among the demodulated subcarriers in the scatter diagram. The effect of PN on PCC-OFDM in an AWGN channel was analysed in [1]. This paper extends the work of [1] to include Rayleigh fading channels. The theoretical and simulated BER values for M-QAM modulated OFDM and PCC-OFDM systems in the presence of PN are calculated. The ICI term is modelled as Gaussian noise and expressions for the resulting conditional and averaged BER for Rayleigh fading channels have been obtained.

The rest of the paper is organized as follows. In section II we describe the effects of phase noise for OFDM and PCC-OFDM systems in brief. Section III includes an analysis of theoretical expressions for BER due to PN effects. Simulations have been performed to obtain the BER by varying parameters such as E_b/N_0, σ_θ^2 and results are presented in section IV. Finally section V concludes the main findings of this paper.

II. Phase Noise in OFDM and PCC-OFDM

A. OFDM

We will now consider the effect of PN in OFDM. Assuming that the length of the added cyclic prefix is greater than the channel memory we can write the received sampled baseband signal affected by multipath fading as,

$$y(n) = e^{j\Phi(n)}[x(n) * h(n)] + w(n) \tag{1}$$

Where $x(n) * h(n)$ denotes the circular convolution and $w(n)$ is the AWGN with single sided power spectral density (PSD) equal to N_0. $\Phi(n)$ is the PN. The channel impulse response can be modelled as a tapped delay line,

$$h(n) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l) \tag{2}$$

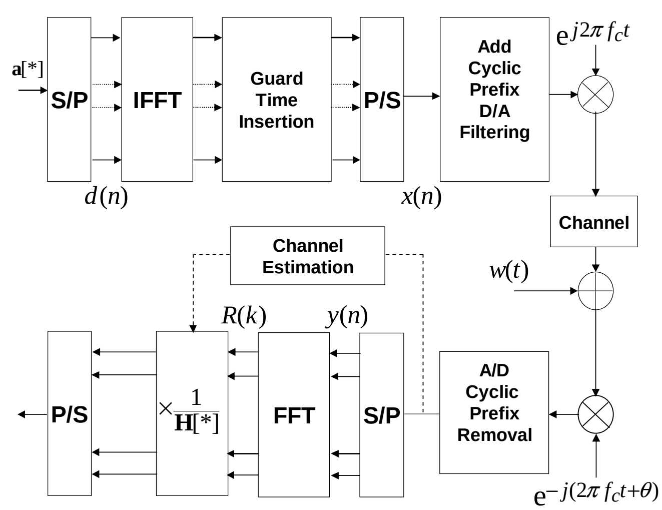

Fig. 1. Simplified block diagram of an OFDM system.

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Where L is the total number of paths, τ_l is the delay associated with the lth path and $\delta(\cdot)$ is the Dirac function,

$$h_l = h_r + jh_i$$

The real and imaginary complex coefficients are chosen from Gaussian distributions for Rayleigh fading. Without loss of generality we assume that each path is delayed by an integer multiple of the sampling time. Hence the channel attenuation factor for the mth subcarrier is given by,

$$H(m) = \sum_{l=0}^{L-1} h_l \exp\left(\frac{-j2\pi ml}{N}\right) \tag{3}$$

The received signal is demodulated using a FFT. Thus the kth demodulated subcarrier is given by,

$$R(k) = \frac{1}{N}\sum_{n=0}^{N-1} y(n)\exp\left(\frac{j2\pi nk}{N}\right) + W(k) \tag{4}$$

$$R(k) = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{i=0}^{N-1} X_i H_i \exp\left(\frac{j2\pi(i-k)}{N}\right) .\exp\left(j\theta_n\right) + W(k) \tag{5}$$

Where $W(k)$ are the frequency domain samples of channel AWGN. Interchanging the inner and outer summation we obtain the following mathematical expression for the mth subcarrier reported in [1] as,

$$R(k) = \frac{1}{N}\sum_{n=0}^{N-1} X_i H_i \times \sum_{i=0}^{N-1} \exp\left(\frac{j2\pi(i-k)}{N}\right) .\exp\left(j\theta_n\right) + W(k) \tag{6}$$

Let

$$I(q) = \frac{1}{N}\sum_{n=0}^{N-1} \exp\left(j\theta_n\right)\exp\left(\frac{j2\pi nq}{N}\right)$$

$$R(k) = X(k)H(k)\underbrace{I(0)}_{CPE} + \underbrace{\sum_{\substack{i=0\\ i\neq k}}^{N-1} X(i)H(i)I(i-k)}_{ICI} + W(k) \tag{7}$$

Note from (7) that all N *useful* subcarrier components are multiplied with a common phase factor which is the average of all the phase error terms. The CPE results in an overall rotation of the received constellation for that symbol. It can be eliminated by several techniques outlined in previous literature. One method is to measure the phase variation of a pilot subcarrier and subtract the rotated angle from all subcarriers [1].

The second summation expression in (7) is the ICI which causes loss of orthogonality among the received subcarriers. It is obtained by multiplying $N-1$ channel attenuated subcarriers with discrete Fourier transform (DFT) PN coefficients evaluated at q/N other than the evaluated kth subcarrier. This also results in a complex number which is added to the useful part of the signal and has characteristics of Gaussian distribution for large N (because of the central limit theorem).

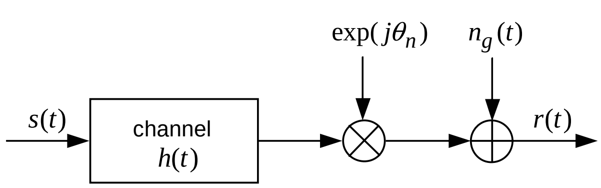

Fig. 2. PN model.

B. PCC-OFDM

In this sub section we discuss the characteristics of CPE and ICI of PCC-OFDM. Assuming that the adjacent channel transfer function coefficients are almost equal:($H_{k+1} \approx H_k$) we can write the following expression for the mth demodulated subcarrier as [1],

$$R(k) = \frac{1}{N}\sum_{n=0}^{N/2-1} X_i H_i \times \sum_{i=0}^{N-1}\left\{1 - \exp\left(\frac{j2\pi n}{N}\right)\right\}. \exp\left(\frac{j2\pi(i-k)}{N}\right)\exp\left(j\theta_n\right) + W(k) \tag{8}$$

Hence,

$$\begin{aligned} R(k) &= X(k)H(k)I(0)\left\{I(0) - I(-1)\right\} \\ &+ \sum_{\substack{i=0\\ i\neq k}}^{N/2-1} H(i)X(i)\left[I(i-k) - I(i-k-1)\right] \\ &+ W(k) \end{aligned} \tag{9}$$

The corresponding output of the subcarrier pair from the weighing and adding block will result in,

$$Z(k) = \frac{R(k) - R(k+1)}{2} \tag{10}$$

$$= \frac{1}{2}\left[\begin{array}{l} X(k)H(k)\left\{-I(-1) + 2I(0) - I(1)\right\} \\ \\ +\sum_{\substack{i=0\\ i\neq k}}^{N/2-1} H(i)X(i)\left\{I(i-k) - I(i-k-1)\right\} \\ \\ +\left\{W(k) - W(k+1)\right\} \end{array}\right]$$

It is evident from the above mathematical analysis that all $Z(k)$'s for PCC-OFDM too are subjected to a CPE. However the ICI term depends on the difference between adjacent terms. If adjacent terms are highly correlated so that $I(k) \approx I(k+1)$ then the ICI term will be less than for conventional OFDM.

Fig. 3 shows the results of the same PN on one received symbol in OFDM and PCC-OFDM. While the rotation of the constellation is identical the noise like ICI term is much less in PCC-OFDM.

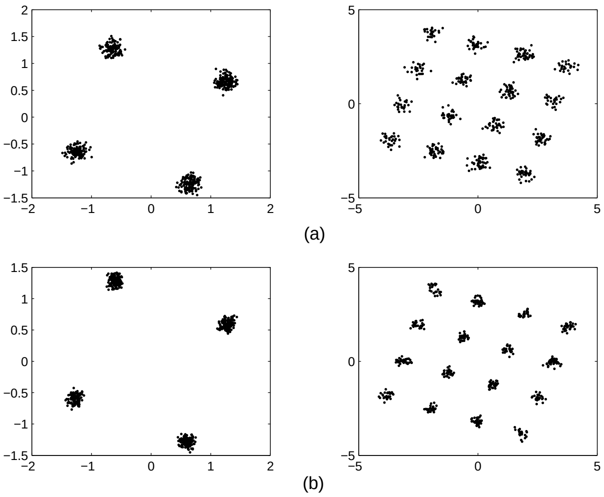

Fig. 3. PN effects (a) OFDM (b) PCC-OFDM.

III. BER Analysis in Fading Channels

A. M-QAM OFDM

In this section we present a theoretical analysis of BER for OFDM and PCC-OFDM systems modulated with Gray coded M-QAM symbols in Rayleigh fading channels. The analysis assumes that the CPE is corrected. The average attenuated signal power disturbed by the CPE can be written as,

$$P_K = \frac{E}{N^2}\left|X_k H_k \sum_{n=0}^{N-1} \exp\left(j\theta_N\right)\right|^2 \tag{11}$$

Where $E(\cdot)$ denotes the expectation operator. Assuming the phase terms are small and using the approximation $\exp(j\theta_n \approx 1 + j\theta_n$ the above expression can be written as [1],

$$P_k = E_s E\left|H_k\left\{1 + \frac{j}{N}\sum_{n=0}^{N-1}\theta_n\right\}\right|^2 \tag{12}$$

The average total ICI power can be expressed as,

$$\sigma_u^2 = E\left|\sum_{\substack{i=0\\ i\neq k}}^{N-1} X(i)H(i)I(I-k)\right|^2 \tag{13}$$

With perfect channel state information (CSI) the estimated symbol $\hat{R}(k)$ obtained with zero forcing equalization (ZF) is given by,

$$\hat{R}(k) = R(k)I(0) + \frac{1}{H(k)} \times \sum_{\substack{i=0\\ i\neq k}}^{N-1} X(i)H(i)I(I-k) + \frac{W(k)}{H(k)} \tag{14}$$

Now signal to noise ratio (SNR) with total knowledge of $H(k)$ becomes,

$$SNR_k = \frac{E_s\left|I(0)\right|^2}{\frac{\sigma_u^2}{|H(k)|} + \frac{\sigma_g^2}{|H(k)|}} \tag{15}$$

Instantaneous SNR γ per symbol of the mth channel path in Rayleigh fading is expressed as [11],

$$P_\gamma(\gamma;\bar{\gamma}) = \frac{1}{\gamma}\exp\left(\frac{-\gamma}{\bar{\gamma}}\right), \gamma \geq 0$$

Where $\bar{\gamma}$ is the average SNR. The unconditional BER can be derived by averaging the conditional BER $P_{bit}(\gamma)$ i.e.,

$$P_{bit}(E) = \int_0^\infty P_{bit}(E|\gamma)P_\gamma(\gamma;\bar{\gamma})d\gamma \tag{16}$$

Since the overall noise at the receiver in (7) consists of two independent Gaussian distributions we are able to use expressions for M-QAM $P_{bit}(E|\gamma)$ derived in [10] such as, (for 4-QAM)

$$\left[P_{bit}(E|\gamma)\right] = \text{erfc}(\sqrt{\gamma}) \tag{17}$$

and for 16-QAM,

$$\left[P_{bit}(E|\gamma)\right] = \frac{3}{4}\text{erfc}\left(\sqrt{\frac{1}{5}\gamma}\right) + \frac{1}{2}\text{erfc}\left(\sqrt{\frac{9}{5}\gamma}\right) - \frac{1}{4}\text{erfc}\left(\sqrt{5\gamma}\right) \tag{18}$$

to obtain the theoretical BER in (17). $\text{erfc}(x)$ is defined as $1/\sqrt{2\pi}\int_x^\infty \exp^{\frac{-y^2}{2}} dy$.

B. M-QAM PCC-OFDM

The average power of the signal corresponding to the kth subcarrier is [1],

$$P_k = \frac{1}{2}E_s E\left|H_k - I(-1) + 2I(0) - I(1)\right|^2 \tag{19}$$

$$P_k = \frac{1}{2}E_s E\left|H_k\left\{\frac{4}{N}\sum_{n=0}^{N-1}\sin^2\left(\frac{\pi n}{N}\right)\exp(j\theta_n)\right\}\right|^2 \tag{20}$$

By assuming the PN coefficients are small we simplify the above expression as,

$$P_k = \frac{1}{2}E_s E\left|H_k\left\{2 + \sum_{n=0}^{N-1}\theta_n \sin^2\left(\frac{\pi n}{N}\right)\right\}\right|^2 \tag{21}$$

The average ICI power corresponding to the kth subcarrier is [1],

$$\sigma_v^2 = E_s\, E\left|H_k \sum_{\substack{i=0\\ i\neq k}}^{N/2-1}\left\{\sum_{n=0}^{N-1}\sin^2\left(\frac{\pi n}{N}\right)\right.\right. . \left.\left.\exp\left(\frac{j\theta_n\left\{2\pi n(i-k)\right\}}{N}\right)\right|^2 \tag{22}$$

A significant difference for PCC-OFDM compared to conventional OFDM which is obvious from the noise power terms (21-22) is that the inclusion of the squared *sin* value in the PN summation. Since the squared *sin* term is always equal or less than unity it acts as an attenuator for the PN terms making the overall contribution less than the case for the OFDM. Hence we can speculate a reduction in both CPE and ICI terms for PCC-OFDM and this attribute contributes towards better BER performance. It can be proven that in (22) the summation term inside the magnitude is approximately $3N/2$ less than the corresponding value for OFDM. The expression for BER for PCC-OFDM can also be obtained with a similar approach as above and we define SNR with ZF for this case as,

$$SNR_k = \frac{E_s \left|2I(0) - I(1) - I(-1)\right|^2}{\frac{\sigma_v^2}{|H(k)|} + \frac{\sigma_g^2}{|H(k)|}} \qquad (23)$$

IV. Simulations and Results

A. System Parameters for the Fading Channel

In all of the simulations we assumed the total number of subcarriers $N = 128$. Cyclic prefix length was assumed to be 10% of the total symbol period for all M-QAM modulation schemes. Two multipath channels were used: a 4-tap static channel described in [12] and a Rayleigh fading channel. Rayleigh channel coefficients were generated by independent stationary complex zero mean Gaussian processes with unit variance and having an exponentially decaying power delay profile. The total number of paths was assumed to be $L = 10$. The phase of the channel coefficients are uniformly distributed in the range 0 to 2π. The Channel has static characteristics during one OFDM period. Finally path delays were assumed to be integer multiples of the sampling intervals of the OFDM and PCC-OFDM signals. The static 4-tap channel has fractional power values of $(0.15, 0.65, 0.15, 0.05)$.

B. PN Model

In general PN is characterized by its frequency domain PSD, $S_\theta(f)$ where f is the frequency [6],[7]. PN was generated from by a method described in previous literature [9]. It generates PN samples from a coloured noise process. This method models PN as an identically independently distributed (i.i.d), zero mean unit variance Gaussian process which are filtered by a linear time invariant filter having the following transfer function or the PN mask.

$$H(z) = \sum_k h(k) z^{-k}$$

For the simulations we use the PN mask described in [9] with $\alpha = 0.9999$ and $\beta = 0.0316$.

$$H(z) = \frac{\beta}{1 - \alpha z^{-1}}$$

C. Performance Comparisons

Simulation results show that the performance of both OFDM and PCC-OFDM is reduced in fading channels when compared with AWGN, while the Rayleigh channel exhibits the worst. The CPE corrected systems make less bit errors than systems with no CPE correction. It is clear that for all cases considered PCC-OFDM out performs OFDM with less BER. Fig. 4 shows the graph of BER versus E_b/N_0 over AWGN and PN variance $\sigma_\theta^2 = 0.2, 0.1, 0.05$ and 0.01 rad^2. As expected when the PN variance is high BER exhibits a high value. For $\sigma_\theta^2 = 0.2$ the BER shows an error floor of approximately 0.02. When the PN variance is 0.01 the BER curve does not show signs of *flattening* and reduces to well below 10^{-3} at $E_b/N_0 = 8$dB. Figs. 5-6 show the BER performance of the OFDM and PCC-OFDM systems over the Rayleigh and static 4-tap channels for several PN variances. CPE was not corrected. BER degradation for Rayleigh channel is higher compared with the 4-tap channel. Even with $\sigma_\theta^2 = 0.01$ rad^2 BER in the Rayleigh channel is well beyond the value of 10^{-4} and the system performance reduces significantly due to PN effects as well as fading. In Fig. 5 for a PN variance of 0.02 rad^2 BER for OFDM is approximately 10^{-3} while for the same PN power BER for OFDM in the 4-tap channel is 10^{-4} at high E_b/N_0 ratios. We present the BER performance over the 4-tap channel with CPE correction in Fig. 7. In this case the improvement in PCC-OFDM is even more pronounced. For PCC-OFDM BER for $\sigma_\theta^2 = 0.1, 0.05$ and 0.02 rad^2 are below 10^{-4}. The performance of OFDM trails and even its BER for $\sigma_\theta^2 = 0.02$ rad^2 is higher than the same for PCC-OFDM for $\sigma_\theta^2 = 0.1$ rad^2. The performance improvement of OFDM with CPE correction is not high as PCC-OFDM. As

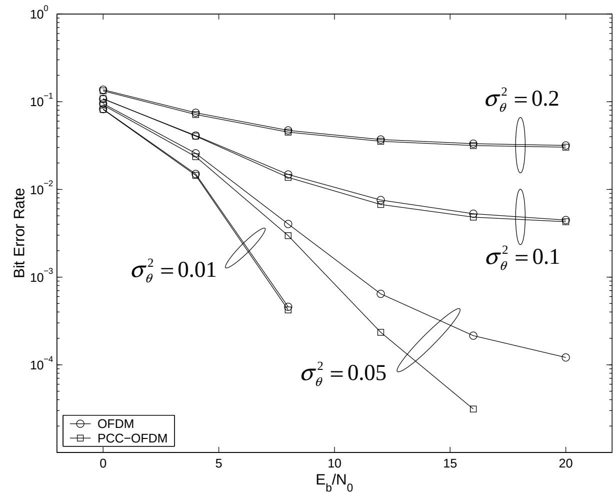

Fig. 4. BER versus E_b/N_0 for OFDM and PCC-OFDM over AWGN channel without correcting CPE. $N = 128$, 4-QAM.

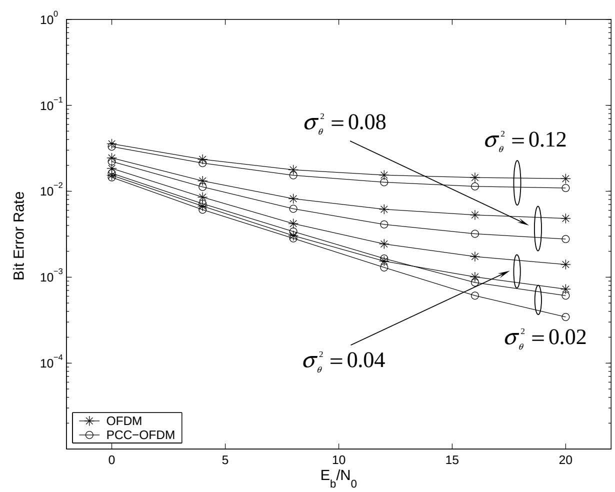

Fig. 5. BER versus E_b/N_0 for OFDM and PCC-OFDM over Rayleigh channel. CPE not corrected. N = 128

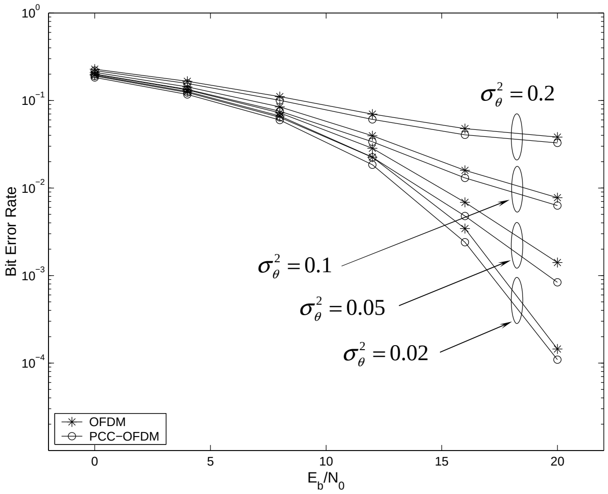

Fig. 6. BER versus E_b/N_0 for 4 QAM OFDM and PCC-OFDM over 4-tap channel. CPE not corrected and N=128.

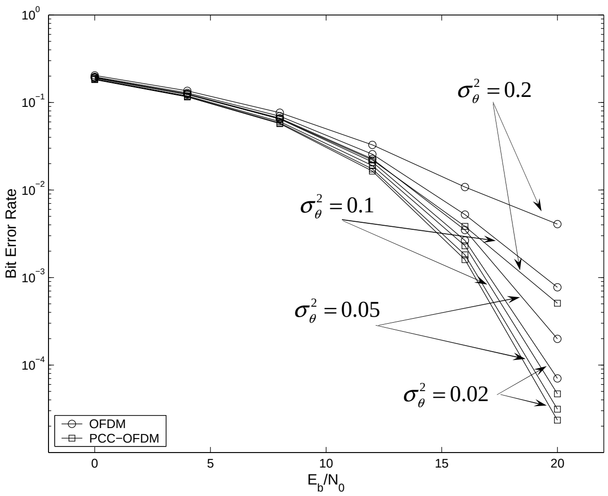

Fig. 7. BER versus E_b/N_0 for OFDM and PCC-OFDM over 4-tap channel with CPE corrected. $N = 128$.

seen from Figs. 6-7 some cases show that the results are better for even CPE not corrected PCC-OFDM than CPE corrected OFDM. In most of the practical applications CPE will be corrected before final demodulation therefore the ability of PCC-OFDM to reduce ICI will increase its system performance even more. A further advantage of PCC-OFDM is that the reduced ICI will result in more accurate CPE estimates. We have evaluated the validity of the theoretical BER analysis in Fig. 8 for OFDM in the 4-tap channel. It can be clearly seen that the practical values are consistant with the simulations. This agreement between theory and simulations is due to several reasons. When the interference is the sum of many variables the central limit theorem is applicable and the approximation of the true BER by a single Gaussian random variable gets more accurate.

V. Conclusions

In this paper we addressed the PN problem for conventional OFDM and PCC-OFDM systems over multipath fading channels. PN effects have been discussed in terms of the CPE and

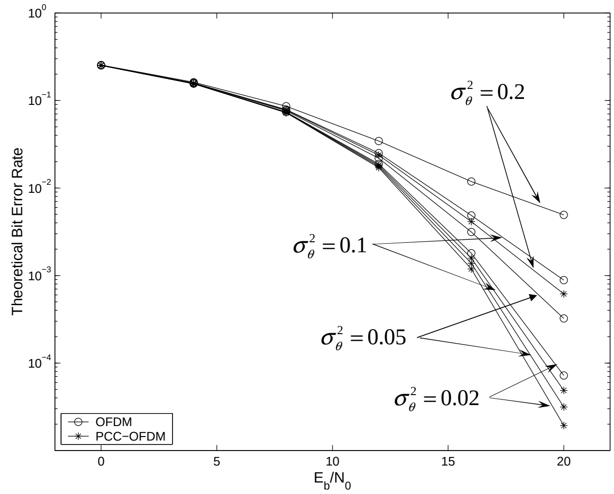

Fig. 8. Theoretical BER versus E_b/N_0 for OFDM and PCC-OFDM over 4-tap channel. CPE corrected and $N = 128$.

ICI and it has been shown that while the ICI is reduced in PCC-OFDM the CPE is unchanged. The BER performances of the two systems have been analysed. Simulation results have been presented for two different channel models and for both CPE correction and no correction in the receiver. The theoretical results agree closely with the simulation results. PCC-OFDM performs better in PN perturbations.

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