THE LINK BUDGET

Understanding Link Budget components and the effects of signal loss and noise on communication system performance.

The Link Budget

The "Link Budget" is a communication system designer's analysis tool for determining the ability of a communication system to detect (receive) a transmitted signal in the presence of noise. The link encompasses the entire communications path, from the transmitter output to the receiver input. The link analysis consists of the calculations and tabulations of the available signal power and interfering noise power present at the receiver. Therefore, the Link Budget is a balance sheet of gains and losses associated with each link component.

The Link Budget is an estimation technique for evaluating communication system error performance. Error probability can be related to Signal to Noise Ratio *SNR* for various types of modulation in the presence of Gaussian noise. The required system error performance dictates the *SNR* necessary at the receiver. The Link Budget calculates the system operating point to determine if it meets the required system error performance. Using the link budget, the designer can determine the specifications of the individual system components required to meet system performance. If incorporated with other analysis techniques, the link budget can help predict component cost, size, power consumption and overall technical risk. Tradeoffs can be made between link components and system performance to achieve desired system goals. For example, a low power battery operated system may achieve the required Effective Radiated Power *ERP* from the transmitter by using a larger antenna with more gain, instead of a higher power amplifier. Alternatively, the battery power requirement may result in a reduction of the system *ERP* requirement whether or not a tradeoff can be found.

In digital communications the term E_b/N_o, a normalized version of *SNR*, is used as a figure of merit. E_b is the bit energy and is described as the signal power S divided by the bit rate R_b in bits per second. N_o is the noise power spectral density and is described as the noise power N divided by the bandwidth W. Therefore E_b/N_o is S/N normalized by bandwidth and bit rate as follows:

$$\frac{E_b}{N_o} = \frac{S}{N}\left(\frac{W}{R_b}\right) \tag{1}$$

One of the most important metrics of performance in digital communication systems is the bit-error probability P_b. For every modulation format, specifications can be found relating received E_b/N_o to bit-error probability P_b [References 1,6]. Required E_b/N_o is the metric that characterizes the performance of one system versus another; the smaller the required E_b/N_o, the more efficient is the system detection process for a given P_b. Each digital communication system has a unique modulating, and detection, scheme and thus a unique requirement for P_b. E_b/N_o allows the system designer to compare one system versus another at the bit level. The analysis and derivation behind the P_b vs. E_b/N_o requirements are considered beyond the scope of this paper. Detailed information on the subject can be found in all listed references.

There are two primary causes of degradation of error performance. First is the loss in *SNR*, and second is signal distortion as in inter-symbol interference *ISI*. If *ISI* occurs, an increase in signal power and *SNR* will not improve the error performance. *ISI* is not included in the Link Budget calculations. Error performance based upon *SNR* or E_b/N_o is the focus of the Link Budget. The Link Budget is the tally sheet of gains and losses of both the signal and the noise, since *SNR* can be degraded by either signal loss or an increase in noise power. Signal losses occur when the transmitted signal is attenuated due to absorption, scattering, or reflection along its path to the receiver. Noise sources include thermal noise, atmospheric noise, and interfering signals from other sources. The net effect of loss and noise is *SNR* degradation.

SIGNAL PATH LOSS AND ANTENNA CHARACTERISTICS:

The Link Budget determines what the receiver E_b/N_o is and how much margin exists above what is required. The process begins by determining the signal path loss from the transmit antenna input to the receive antenna output. Understanding of path loss begins with the assumption of an ideal, omni-directional antenna, transmitting uniformly over 4π steradians. This ideal antenna is known as an isotropic radiator. An isotropic radiator can be thought of as a point source that transmits a plane wave having uniform power at any location at a fixed distance d from the source. The power density P_d at this location on a hypothetical sphere can be related to the transmitted power P_t by:

$$P_d = \frac{P_t}{4\pi d^2} \quad \frac{W}{m^2} \tag{2}$$

Since $4\pi d^2$ is the area of the sphere, the received power P_r is a function of the absorptive area of the receive antenna. The receive antenna absorptive area is known as the antenna effective area A_{er} , and is defined as:

$$Aer == \frac{PowerExtracted}{PowerAvailable} \tag{3}$$

The received power P_r is then

$$P_r = P_d A_{er} = \frac{P_t A_{er}}{4\pi d^2} \tag{4}$$

When discussing effective area of an antenna, the term A_e is commonly used. If the antenna being discussed is a receiving or transmitting antenna the effective area is typically referred to as A_{er} or A_{et} respectively.

Typically, as shown above, an antenna is not a perfect transducer in either converting the electromagnetic energy into an electrical signal (receiving) or in converting an electrical signal into electromagnetic energy (transmitting). The relationship between an antennas effective area and its physical area A_p is given by an efficiency parameter η, such that

$$A_e = \eta A_p \tag{5}$$

Typical values of η range from 0.55 for a parabolic dish antenna, to approximately 0.75 for a horn antenna. The parabolic dish antenna is highly directive with respect to an isotropic radiator, and is considered to have directional gain. This directional gain G can be thought of as the additional energy above what would be present if the antenna were isotropic.

A transmitter is commonly specified by its effective radiated power with respect to an isotropic radiator or effective isotropic radiated power *EIRP*. *EIRP* is the product of the power transmitted P_t and the gain of the transmitting antenna G_t. The *EIRP* and thus the received power are then

$$EIRP = P_t G_t \tag{6}$$

$$P_r = EIRP \frac{A_{er}}{4\pi d^2} = \frac{P_t G_t A_{er}}{4\pi d^2} \tag{7}$$

The relationship between antenna gain and A_e is given to be:

$$G = \frac{4\pi A_e}{\lambda^2} \tag{8}$$

Where λ is the wavelength of the carrier and is related to the frequency *f* by λ=c/*f*, where c is the speed of light ~ 3×10^8 m/s. Setting $G = 1$ in the above equation yields

$$A_e = \frac{\lambda^2}{4\pi} \tag{9}$$

Thus, the power received is commonly expressed without the use of effective area terms as

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2} \tag{10}$$

$\lambda^2 / (4\pi d)^2$ is known as the path loss or the free space path loss equation. The path loss is truly a function of geometry of the propagating electromagnetic wave. The flux density is reduced at the receive antenna by $4\pi d^2$, due to the inverse-square law. Since the A_e is a function of wavelength, the net path loss is greater at higher frequencies or at smaller wavelengths due to smaller antenna effective areas.

As the name implies, the free space path loss equation does not consider atmospheric loss factors or other environmental considerations such as multi-path fading. Atmospheric and other transmission losses can be treated as a separate entry in the link calculations and usually require more system margin to accommodate these losses.

THERMAL NOISE POWER:

Thermal noise in communications receivers is modeled as an additive white Gaussian noise *AWGN* process. Thermal noise is a result of electron movement in any conductor or semi-conductor. The most sensitive areas for thermal noise generation are in the coupling between the antenna and receiver and in the initial amplifier stages of the receiver. The physical model for thermal noise is a noise generator with an open circuit voltage

$$V_n = \sqrt{4kT^\circ BR} \tag{11}$$

Where:

k = Botlzmann's constant = 1.38×10^{-23} J/K or W/K-Hz

= -228.6dBW/K-Hz

T° = temperature in degrees Kelvin

B = bandwidth in Hz

R = resistance in ohms

Maximum power transfer from a source to a load occurs when the load resistance and the source resistance are equal. Thus, the maximum thermal noise power N_p generated at an amplifier input (load) due to the noise generator is V_i^2/R. Where

$$V_i = \frac{V_n}{2} = \frac{\sqrt{4kT^\circ BR}}{2} \tag{12}$$

$$N_p = \frac{\left(\frac{\sqrt{4kT^\circ BR}}{2}\right)^2}{R} = kT^\circ B \quad \text{Watts} \tag{13}$$

The Link Budget commonly references noise power spectral density N_o, which is the noise power in a 1-Hz bandwidth (B =1). Thus the maximum thermal noise power spectral density that could be coupled from the noise generator into the front end of an amplifier is dependent upon the ambient temperature of the source

$$N_o = kT^\circ \qquad \text{Watts/Hertz} \tag{14}$$

NOISE FIGURE AND NOISE FACTOR:

In order to compare noise performance of devices or systems, we commonly use the term noise factor F. The noise factor of a device or system is defined as the ratio of SNR at the input to the SNR at the output.

$$F = \frac{SNR_{in}}{SNR_{out}} \tag{15}$$

F, when expressed in linear terms is referred to as noise factor. Expressing F in dB is referred to as Noise Figure NF where

$$NF = 10 * Log_{10}(F) \tag{16}$$

Noise factor is a comparison of noise performance among devices at a specific reference temperature of $T_o{}^\circ$ = 290K. At this temperature the maximum thermal noise power spectral density that could be coupled into the front end of an amplifier using the noise generator model is

$$N_o = kT_o{}^\circ = 4x10^{-21}\ W/Hz \tag{17}$$

The expression of the noise power spectral density is most conveniently expressed in link budget calculations in terms of *dBW* (dB referenced to 1 W) or *dBm* (dB referenced to 1 mW), where *dBm* = *dBw* + 30. N_o in terms of *dBw* and *dBm* are

$$N_o = -204\ dBW / Hz \tag{18}$$

$$N_o = -174\ dBm / Hz \tag{19}$$

Noise factor expresses the noisiness of a component or system relative to an input noise source. Noise figure does not express the total noise density, but the additional noise density above a reference input noise source. Noise factor is convenient since it is defined for a reference input noise source at a temperature of $T_o°$ = 290K. Most terrestrial system applications are at 290K. A component or system that does not contribute additional noise density to the defined input reference has a noise factor of 1, or *NF* = 0dB. The total noise power spectral density of a component or system, accounting for the additional noise above the defined reference, when the noise factor is not 1 is

$$N_o = k T_o° F \quad W / Hz \tag{20}$$

$$N_o = 10 * Log_{10}(k T_o°) \quad + \quad NF \quad dBW / Hz \tag{21}$$

NOISE TEMPERATURE

Noise temperature expresses the noisiness of a component or system as an additional noise source operating at an effective noise temperature $T_R°$. Here, again, noise temperature expresses the additive noise of a component or system above an input reference source. However, the input reference source temperature is not defined to be $T_o°$ = 290K. For many terrestrial applications, $T_o°$ = 290K is used as the reference source temperature. For space applications, a reference temperature of 290K is not appropriate. In general, applications involving very low noise devices favor effective temperature measurements over noise factor. $T_R°$ can be thought of as the effective change in noise power spectral density as a result of an additional noise source at an effective noise temperature $T_R°$, added to a reference noise source. When using effective temperature, it is necessary to know the reference temperature to find the total noise power spectral density. The total noise power spectral density for the additional noise source at an effective noise temperature $T_R°$ above a reference noise source at $T_o°$ = 290K is

$$N_o = k(T_o° + T_R°) \quad W/Hz \tag{22}$$

The noise factor and effective temperature are related by the following where $T_o°$ = 290K.

$$F = 1 + \frac{T_R°}{T_O°} \tag{23}$$

$$T_R° = (F-1)T_O° \tag{24}$$

LOSSY LINES AND COMPOSITE NOISE

Almost all receiver hardware configurations include the antenna coupled to a low noise amplifier *LNA* stage through lossy components such as waveguides, diplexers, switches and transmission lines. All losses before an amplifier gain stage contribute directly to the noise factor or effective temperature of the amplifier. Applying the noise factor and effective temperature equations to the lossy components or transmission line with loss factor L, the noise factor F_L and the effective noise temperature $T_L°$ can be determined by:

$$F_L = 1 + \frac{T_L°}{290K} = L \tag{25}$$

$$T_L° = (L-1)290K \tag{26}$$

The composite noise factor of the transmission line loss combined with an *LNA* can be found from the following multistage composite noise factor equation.

$$F_{comp} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_n - 1}{G_1 G_2 \cdots G_{n-1}} \qquad (27)$$

Applying equation 26 the transmission line and *LNA* coupling results in $F_1 = L$, $G_1 = 1/L$, and $F_2 = F$ of the *LNA*. Thus $F_{comp} = LF$, and the net effect on the *LNA* noise factor can be seen. The composite effective temperature of the transmission line loss combined with an *LNA* can be found from the following multistage composite effective temperature equation.

$$T°_{comp} = T_1° + \frac{T_2°}{G_1} + \frac{T_3°}{G_1 G_2} + \cdots + \frac{T_n°}{G_1 G_2 \cdots G_{n-1}} \qquad (28)$$

Here, $T_1° = T_L°$, $T_2° = T_R°$, $G_1 = 1/L$, and $T°_{comp} = T_L° + L*T_R°$.

Communications system design requires careful attention to the composite noise factor of the receiver front end *RFE*. It can be seen from the composite equations that the noise factor of subsequent stages is reduced by the gains of previous stages. Achieving a low noise factor requires minimizing the transmission line loss and selecting a first stage amplifier with suitable noise factor and gain to achieve system performance goals.

ANTENNA TEMPERATURE AND SYSTEM TEMPERATURE:

To a transmission line, an antenna looks like a 2 terminal circuit element with an input impedance *Z* (complex) and a radiation resistance R_r. The radiation resistance is not an actual resistance associated with the antenna but is a resistance coupled from the antenna and its environment to the input terminals and thus affecting the input impedance. The radiation resistance accounts for the energy radiated by the antenna when driven by a source. When an antenna is used for receiving, a thermal noise is associated with the radiation resistance. This noise is due to RF thermal radiation received from the environment the antenna is “looking at.” This noise is characterized by an antenna effective temperature $T_A°$. The noise can be due to electrical storms, black body radiation from nearby objects, manmade interference, or galactic radiation from sources in space. The $T_A°$ is dependent upon the objects or environment observed by the antenna, the antenna design, and the frequency of operation. An antenna looking into space has the lowest $T_A°$ between 1 and 10 GHz, the desired band for space (satellite) communications. Below 100 MHz, noise from electrical storms and man-made sources result in high antenna temperatures. At 10 GHz, atmospheric absorption becomes significant and black body radiation from the atmosphere causes high antenna temperatures.

Antenna temperature information can be part of the Link Budget calculations and can be used to find the total system temperature $T°_{sys}$. Once the composite effective temperature $T°_{comp}$ from the transmission line forward is known, $T°_{sys}$ is found from

$$T_{sys} = T_A + T_{comp} \tag{29}$$

$T_A°$ represents a noise source from the outside world arriving at the antenna. $T°_{comp}$ represents thermal noise generated by electron motion in all conductors and semiconductors. $T°_A$ is added to $T°_{comp}$ without the use of gain reduction factors as found in equation (28). The noise an antenna sees is presented to the receiver unaltered as a source noise or source temperature. Therefore, when an antenna temperature is specified, it is considered to be the source temperature and it is added to the composite temperature. No other source temperature is used to calculate the system noise degradation.

LINK BUDGET TERMS AND DEFINITIONS

Effective Isotropic Radiated Power *EIRP*:

$$EIRP(dBW) = P_T(dBW) + G_T(dB) - Losses\ (dB)$$
(30)

Pathloss P_L:

$$P_L(dB) = 20 * Log_{10}(\frac{\lambda}{4\pi d}) \tag{31}$$

The Receiver Figure of Merit $G/T°$:

Since the *SNR* at the receiver is dependent upon the receive antenna gain and the effective system temperature of the receiver, both characteristics can be defined in one term as the receiver $G/T°$ and is defined by:

$$\frac{G}{T°}(dB/K) = G_R(dB) - T°_{sys}(dB) \tag{32}$$

Where G_R (dB) = 10* LOG$_{10}$ (G_R), and $T°_{sys}$ (dB) = 10*LOG$_{10}$($T°_{sys}$).

Margin M:

Margin M is defined as the amount of extra signal power required to accommodate expected and unexpected link losses due to atmospheric, galactic or man-made interference.

E_b/N_o Required:

The E_b/N_o will be given as the minimum *SNR* required to achieve the desired system error probability P_e.

Botlzmann's Constant k:

$$k = -228.6 \frac{dBW}{K - Hz}$$

Bit Rate R_b:

$$R_b(dB) = 10 * Log_{10}(BitRate) \tag{33}$$

LINK BUDGET SUMMARY

The Link Budget tells the communication system designer if the specified system error performance can be met and if so by how much. The "how much" is the system margin *M*. It is useful to add *M* directly to the E_b/N_o required to find the E_b/N_o desired.

$$\frac{E_b}{N_o} Desired = \frac{E_b}{N_o} \mathrm{Re}\, quired + M \tag{34}$$

Obviously a Link will close, or meet specification, with 0 dB margin. However, a typical communications link design will incorporate 5-10 dB of margin depending upon the environment and geometry of the link.

There are obviously two approaches the system designer can take when applying Link Budget calculations. First, the achieved *Eb/No* can be found by calculating the Link from the transmitter to the receiver. Thus for a fixed transmitter specification, what receiver parameters much change to achieve the required system error performance and Link margin. The result is the first Link Budget equation based upon the material presented thus far

$$achieved\,\frac{E_b}{N_o}(dB-Hz) = EIRP - P_L + \frac{G}{T^\circ} - k - R_b - M \tag{35}$$

Alternatively, equation (35) can be modified to find the required *EIRP* necessary to achieve the specified system error performance and Link margin.

$$EIRP(dBW) = \frac{E_b}{N_o}\mathrm{Re}\,q + M + R_b + P_L - \frac{G}{T^\circ} + k \tag{36}$$

LINK BUDGET EXAMPLES:

Examples 1 through 3 are Link Budget calculations for determining the required *EIRP* of a mobile earth transmitter to a Geosynchronous Earth Orbit (GEO) satellite, a Medium Earth Orbit (MEO) satellite and a Low Earth Orbit (LEO) satellite respectively. The GEO application is typical of a NASA Tracking and Data Relay (TDRS) satellite Link, the MEO application is typical of a TRW Odessey satellite phone/data Link and the LEO application is typical of a Global Star or Iridium satellite phone/data link. From the *EIRP* requirement the designer can then determine the antenna requirements, transmitter output power requirements and DC power requirements. All examples will assume Binary Phase Shift Keying (BPSK) modulation, with a required E_b/N_o of 11 dB for a system error performance of 1 part per million (ppm) or 1e-6. The Bit Rate R_b of 9600 bits per second will be used for all examples. Required *EIRP* will be found using equation (36).

Example 4 will find the system margin for a satellite downlink to a fixed earth station receiver. The same assumptions for modulation and data rate will apply. E_b/N_o at the earth station will be found by using equation (35).

EXAMPLE 1:

Find the *EIRP* required for a mobile transmitter to a GEO satellite given the following specifications:

Frequency, f = 2.2 GHz.

Bit Rate, R_b = 9600 bps.

Distance, d = 40,000 km.

Receive antenna gain, G_R = 30 dB.

Receiver effective temperature $T°_{sys}$ = 640 K (total system).

E_b/N_o Required = 11 dB-Hz

Margin, M = 6 dB.

Link Budget Calculations Example 1:

PARAMETER	**VALUE**	**Link Budget Entry**
Required E_b/N_o	11 dB	11 dB-Hz
M	6 dB	+ 6 dB
R_b	39.8 dB	+ 39.8 dB
λ	0.136 m	
d	4 x 10^7 m	
P_L	191.3 dB	+ 191.3 dB
G_R	30 dB	
$T°_{sys}$	640 K	
$G/T°$	1.94 dB	- 1.94 dB/K
k	-228.6 dBW/K-Hz	-228.6 dBW/K-Hz
EIRP Required		**17.56 dBW**
EIRP Required		**57 W**

EXAMPLE 2:

Find the *EIRP* required for a mobile transmitter to a MEO satellite given the following specifications:

Frequency, f = 1.6 GHz.

Bit Rate, R_b = 9600 bps.

Distance, d = 10,000 km.

Receive antenna gain, G_R = 20 dB.

Receiver effective temperature $T°_{sys}$ = 720 K (total system).

E_b/N_o Required = 11 dB-Hz

Margin, M = 9 dB.

Link Budget Calculations Example 2:

PARAMETER	**VALUE**	**Link Budget Entry**
Required E_b/N_o	11 dB	11 dB-Hz
M	9 dB	+ 9 dB
R_b	39.8 dB	+ 39.8 dB
λ	0.187 m	
d	1 x 10^7 m	
P_L	176.5 dB	+ 176.5 dB
G_R	20 dB	
$T°_{sys}$	720 K	
$G/T°$	-8.6 dB	+ 8.6 dB/K
k	-228.6 dBW/K-Hz	-228.6 dBW/K-Hz
EIRP Required		**16.3 dBW**
EIRP Required		**42.66 W**

EXAMPLE 3:

Find the *EIRP* required for a mobile transmitter to a LEO satellite given the following specifications:

Frequency, f = 1.6 GHz.

Bit Rate, R_b = 9600 bps.

Distance, d = 700 km.

Receive antenna gain, G_R = 10 dB.

Receiver effective temperature $T°_{sys}$ = 800 K (total system).

E_b/N_o Required = 11 dB-Hz

Margin, M = 18 dB.

Link Budget Calculations Example 3:

PARAMETER	**VALUE**	**Link Budget Entry**
Required E_b/N_o	11 dB	11 dB-Hz
M	18 dB	+ 18 dB
R_b	39.8 dB	+ 39.8 dB
λ	0.187 m	
d	7 x 10^5 m	
P_L	153.4 dB	+ 153.4 dB
G_R	10 dB	
$T°_{sys}$	800 K	
$G/T°$	-19 dB	+ 19 dB/K
k	-228.6 dBW/K-Hz	-228.6 dBW/K-Hz
EIRP Required		**12.6 dBW**
EIRP Required		**18.2 W**

EXAMPLE 4:

Find the system margin available for a satellite downlink to a fixed earth station receiver given the following specifications:

Frequency, f = 4.0 GHz.

Bit Rate, R_b = 9600 bps.

Transmitter Output Power, P_T = 10 W

Transmit Antenna Gain, G_T = 6 dB

Distance, d = 4 x 10^7 km.

Receive antenna = 3 meter dish.

Receive Antenna efficiency, n = 0.55.

Antenna Temperature, T_A° = 50 K.

Receiver effective temperature, T_R° = 290 K.

E_b/N_o Required = 11 dB-Hz

Link Budget Calculations Example 4:

PARAMETER	**VALUE**	**Link Budget Entry**
P_T	10 W	
G_T	6 dB	
EIRP	16 dBW	16 dBW
λ	0.075 m	
d	4 x 10^7 m	
P_L	196.5 dB	- 196.5 dB
Receive antenna physical area $A_{pr} = \pi r^2$	7.07 m^2	
Receive antenna effective area $A_{er} = n A_{pr}$	3.9 m^2	
G_R	39.4 dB	
$T°_{sys}$	340 K	

G/T°	14.1 dB/K	+ 14.1 dB/K
k	-228.6 dBW/K-Hz	+ 228.6 dBW/K-Hz
R_b	39.8 dB	- 39.8 dB
***Eb/No* Achieved**		**22.4 dB-Hz**
M		**11.4 dB**

CONCLUSION:

The basic theory and calculations behind the communications system Link Budget has been presented and examples provided. The Link Budget determines if a communications system error performance specification can be met and what the system margin is. However, the analysis behind the system error performance requirements is beyond the scope of this paper. Additional information on the relationship between E_b/N_o, the system bit error probability and the modulation/demodulation formats can be found in all references.

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Link Budget Clarification:

Noise Density (No) required for C/No determination.

For Amplifier Electronics with a Noise Factor (F) an equivalent Noise Temperature (T_R) can be found.

$$T_R{}^\circ = (F-1)T_O{}^\circ$$

Where T_o = Source Temperature. No can be found to be

$$N_o = k(T_o{}^\circ + T_R{}^\circ) \quad W/Hz$$

For Amplifier Electronics connected to an Antenna Temperature (T_A) via a cable with Loss (L) a total system temperature is found

$$T_{sys} = T_A + (FL-1)T_o$$

And the No found to be

$$N_o = k(T_{SYS}{}^\circ) \quad W/Hz$$

Link Budget Clarification:

The Link Budget can be computed for a required Eb/No, or a required coherent SNR (gamma). When computing for required coherent SNR, the non-coherent sequences should not be included in the total SNR computations and thus the system Margin. The non-coherent sequences are accounted for in the gamma calculation.

Detection Performance of the Non-Coherent Integration of a Signal in AWGN

A baseband complex signal x is coherently integrated for a period T or filtered to a bandwidth B = 1/T:

$$x(n) = s(n) + z(n)$$

where s(n) has signal energy $s(n)s^*(n) = C\,T$, and $z(n) = z_I(n) + i\,z_Q(n)$ is complex AWGN with $E[z_I(n)] = E[z_Q(n)] = 0$ and $E[z_I^2(n)] = E[z_Q^2(n)] = N_o/2$.

N of these are noncoherently summed:

$$y = \sum_{n=1}^{N} x(n)\,x^*(n)$$

Then the result, y, is distributed as Chi-square with 2N degrees of freedom (Proakis, "Digital Communication", McGraw Hill, 1995). When the signal is absent the distribution is central Chi-square with its first two moments:

$$E[y|H_0] = N\,N_o$$
$$\mathrm{var}[y|H_0] = N\,N_o^2$$

When the signal is present the distribution is non-central Chi-square, also with 2N degrees of freedom, but with a noncentral parameter of $2\,N\,C\,T/N_o$. Its first two moments are:

$$E[y|H_1] = N(N_o + C\,T) = N\,N_o\,(1 + \gamma)$$
$$\mathrm{var}[y|H_1] = N(N_o^2 + 2\,N_o\,C\,T) = N\,N_o^2\,(1 + 2\,\gamma)$$

where $\gamma = C\,T/N_o$ is the pre-detection or coherent SNR.

To facilitate computation with tables and Matlab functions it is useful to scale x so that it has unit variance on each quadrature, i.e. $x' = x/\sqrt{N_o/2}$. The resulting noncoherent sum y' has statistics

$$E[y'|H_0] = 2\,N$$
$$\mathrm{var}[y'|H_0] = 4\,N$$
$$E[y'|H_1] = 2\,N\,(1 + \gamma)$$
$$\mathrm{var}[y'|H_1] = 4\,N\,(1 + 2\,\gamma)$$

The non-central distribution has a non-central parameter of $2\,N\,C\,T/N_o$.

Computation of P_d and P_f

P_d and P_f are evaluated with Matlab's Statistics Toolbox functions. A threshold providing the specified P_f is given by

threshold = chi2inv(1-P_f, df)

where df = 2 N (degrees of freedom). The resulting P_d is given by

Pd = 1 - ncx2cdf(threshold, 2 N, df*gamma)

where gamma is the coherent SNR = C T/N_o.

Example:

C/No = 34 dB-Hz, T = 10 ms $\Rightarrow$ γ = 14 dB
N = 1, Pf = 1e-6 $\Rightarrow$ Pd = .972

C/No = 34 dB-Hz, T = 1 ms $\Rightarrow$ γ = 4 dB
N = 10, Pf = 1e-6 $\Rightarrow$ Pd = .6

C/No = 36 dB-Hz, T = 1 ms $\Rightarrow$ γ = 6 dB
N = 10, Pf = 1e-6 $\Rightarrow$ Pd = .975

Comparing the first and last examples shows that in going from a single coherent integration of 10 ms to a non-coherent integration of ten 1 ms coherent segments, the SNR needed to be increased by 2 dB to maintain the same Pd and Pf. Thus the squaring loss for this case is 2 dB.