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Research Article

Cubic Ideals in Near Subtraction Semigroups

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Abstract

Fuzzy set theory plays a significant role in mathematics. The study of algebra in fuzzy setting has always attracted researchers to a greater extend. Young Bae Jun made effort in defining a remarkable structure namely cubic structure and ideal theory in subtraction algebra. Concept of cubic sets encompasses interval-valued fuzzy set and fuzzy set. Interval-valued fuzzy set is another generalization of fuzzy sets that was introduced by Lotfi Asker Zadeh. Dheena introduced near-subtraction semigroups in fuzzy algebra. Motivated by the theory of cubic structure and near-subtraction semigroups. Our aim in this paper is to introduce the notion of cubic ideals of near-subtraction semigroups, homomorphism of near-subtraction semigroups and family of cubic ideals in intersection. We also provide some results, examples and study their related properties.

Keywords: Semigroups; Subtraction semigroups; Near-subtraction semigroups; Cubic ideal.

Introduction

The notion of subtraction algebra was introduced by Abbott [1] in 1969. Using this notion Schein [2] introduced the concept of sub-subtraction semigroups in 1992. The system of the form $(\varphi, \circ, \backslash)$. Here φ is a set of function closed under the composition "$\circ$" of function (and hence $(\varphi, \circ)$ is a function of semigroup) and the set theoretical subtraction "\" (*and hence* $(\varphi, \backslash)$ is subtraction algebra). Solved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. Zelinka [3] discussed a problem proposed by Schein concerning the structure of multiplication in a subtraction semigroup and discussed a special type of subtraction algebra denoted atomic subtraction algebra.

Jun et al [4] introduced the notion of on ideals in subtraction algebra and discussed characterization of ideals. Lee et al [5] provided some equations on fuzzifications of ideals in subtraction algebras. Zekiye Ciloglu [6] et al defined on fuzzy ideals of subtraction semigroups

Dheena et al [7] introduced the near-subtraction semigroups and strongly regular near-subtraction semigroups. Lekkoksung [8] introduced on fuzzy ideals in near –subtraction ordered semigroups. Jun et al [9] introduced the concept of cubic sets. This structure encompasses interval-valued fuzzy set and fuzzy set. Also Jun et al [10] introduced the notion of cubic subgroups. Vijayabalaji et al [11] introduced the notion of cubic linear space. Chinnadurai et al [12] introduced the notion of cubic ideals of Γ-semigroups. Also Chinnadurai et al [13] introduced the notion of cubic ring. The concept of fuzzy subset was introduced by Zadeh [14,15] in order to study mathematical vague situations. Many researchers who are involved in studying, applying, refining and teaching fuzzy sets have successfully applied this theory in many different fields. The purpose of this paper to introduce the notion of cubic ideals in near-subtraction semigroups and homomorphism in near-subtraction semigroups. We investigate some basic results, examples and properties.

Preliminaries

Now we recall some known concepts related to cubic ideal in near-subtraction semigroups from the literature, which will be needed in the sequel.

Definition 2.1. [1] A non-empty set X together with a binary operation "-" is said to be a subtraction algebra if it satisfies the following conditions:

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$i)\ x-(y-x)=x$
$ii)\ x-(x-y)=y-(y-x)$
$iii)\ (x-y)-z=(x-z)-y$
$\forall x,y,z\in X.$

Definition 2.2. [1] Let A be any non-empty set. Then (P (A), \) is a subtraction algebra, where P (A) denotes the power set of A and "\" denotes the set theoretic subtraction.

Definition 2.3. [1] A subset I of subtraction algebra X is called subalgebra of X if $x-y\in I$ for all $x,y\in I$.
In subtraction algebra the following holds: [1]
$S1)\ x-0=x$ and $0-x=0$
$S2)\ x-(x-y)\le y$
$S3)\ x\le y$ if and only if $x=y-w$ for some $w\in X$
$S4)\ x\le y$ implies $x-z\le y-z$ and $z-y\le z-x$ for all $z\in X$
$S5)\ x-(x-(x-y))=x-y$
$S6)\ (x-y)-x=0$
$S7)\ (x-y)-y=x-y$

Definition 2.4. [1] Anon-empty set X together with the binary operations "-" and "." is said to be a subtraction semigroup. If it satisfies the following conditions:
$i)(X,-)$ is subtraction algebra
$ii)(X,\bullet)$ is semigroup
$iii)\ x(y-z)=xy-xz$ and $(x-y)z=xz-yz\ \forall x,y,z\in X$.

Definition 2.5. [7] A non-empty set X together with the binary operations "-" and "." is said to be near-subtraction semigroup if it satisfies the following conditions
$i)(X,-)$ is subtraction algebra
$ii)(X,\bullet)$ is semigroup
$iii)\ (x-y)z=xz-yz\ \forall x,y,z\in X$.
It is clear that $0x=0$ for all $x\in X$. Similarly we can define a near-subtraction semigroup (left). Hereafter a near-subtraction semigroup it is a near-subtraction semigroup (right) only.

Definition 2.6. [7] A near-subtraction semigroup X is said to be zero-symmetric if $x0=0$ $\forall x\in X$.

Definition 2.7. [7] A near-subtraction semigroup X is said have an identity if there exists an element.

$1\in X$ such that $1.x=x.1=x$ for every $x\in X$.
Definition 2.8. [17] Let $(X,-,\bullet)$ be a near-subtraction semigroup. A non-empty subset I of X is called (I1) a left ideal if I is a subalgebra of $(X,-)$ and $(xi-x(y-i)\in I$ for all $x,y\in X$ and $i\in I$. (I2) a right ideal if I is a subalgebra of $(X,-)$ and $Ix\subseteq I$. (I3) an ideal if I is both a left and right ideal.

Definition 2.9. [18] A mapping $\mu: X\to[0,1]$ is called a fuzzy subset of X.
Definition 2.10. [17] A fuzzy set μ in X is called a fuzzy ideal of X if it satisfies the following conditions:
(FI1) $\mu(x-y)\ge\min\{\mu(x),\mu(y)\}$
(FI2) $\mu(ax-a(b-x))\ge\mu(x)$
(FI3) $\mu(xy)\ge\mu(x)$ for all $x,y,a,b\in X$.
Note that μ is a fuzzy left ideal of X if it satisfies (FI1) and (FI2) and μ is a fuzzy right ideal of X if it satisfies (FI1) and (FI3).

Definition 2.11. [18] Let X be a non-empty set. A mapping $\bar{\mu}: X\to D[0,1]$ is called interval-valued fuzzy set, where D[0,1] denote the family of all closed sub intervals of [0,1] and $\bar{\mu}(x)=[\mu^-(x),\mu^+(x)]$ for all $x\in X$, where μ^- and μ^+ are fuzzy subsets of X such that $\mu^-(x)\le\mu^+(x)$ for all $x\in X$.
Definition 2.12. [10] Let X be a non-empty set. A cubic set $\mathcal{A}$ in X is a structure $\mathcal{A}=\{\langle x,\bar{\mu}_A(x),f_A(x)\rangle : x\in X\}$ which is briefly denoted by $\mathcal{A}=\langle\bar{\mu}_A,f_A\rangle$, where $\bar{\mu}_A=[\mu_A^-,\mu_A^+]$ is an interval-valued fuzzy set (briefly, IVF) in X and f is a fuzzy set in X. In this case, we will use

$$\mathcal{A}(x)=\langle\bar{\mu}_A(x),f_A(x)\rangle = \langle[\mu^-(x),\mu^+(x)],f_A(x)\rangle\quad\forall x\in X.$$

Definition 2.13. [7] Let X and Y be given classical sets. A mapping f: X→Y induces two mappings
C_f: $C(X)\to C(Y)$, $\mathcal{A}_1\to C_f(\mathcal{A}_1)$ and $C_f^{-1}: C(Y)\to C(X)$, $\mathcal{A}_2\to C_f^{-1}(\mathcal{A}_2)$. where the mapping C_f is called cubic transformation and C_f^{-1} is called inverse cubic transformation.

Definition 2.14. [16] A cubic set $\mathcal{A}=<\bar{\mu},\lambda>$ in X has the cubic property if for any subset T of X there exist $x_0\in T$ such that $\bar{\mu}(x_0)=\sup_{x\in T}\bar{\mu}(x)$ and $\lambda(x_0)=\inf_{x\in T}\lambda(x)$.

Definition 2.15. [10] Let f be a mapping from a set X to a set Y and $\mathcal{A}=<\bar{\mu},\lambda>$ be a cubic set of X then the image of P $C_f(\mathcal{A})=<C_f(\bar{\mu}),C_f(\lambda)>$ is a cubic set of Y is defined by

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$C_f(\mathcal{A})(y) = \begin{cases} C_f(\bar{\mu})(y) = \begin{cases} \sup_{y=f(x)} \bar{\mu}(x) & \text{if } f^{-1}(y) \neq \emptyset \\ [0,0] & \text{otherwise} \end{cases} \\ C_f(\lambda)(y) = \begin{cases} \inf_{y=f(x)} \lambda(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases} \end{cases}$

$\forall\, y \in Y$ and

Let f be a mapping from a set X to Y and $\mathcal{A} =< \bar{\mu}, \lambda >$ be a cubic set of Y then the pre image of Y $C_f^{-1}(\mathcal{A}) =< C_f^{-1}(\bar{\mu}), C_f^{-1}(\lambda) >$ is a cubic set of X is defined by $C_f^{-1}(\mathcal{A}) = \begin{cases} C_f^{-1}(\bar{\mu}(x)) = \bar{\mu}(f(x)) \\ C_f^{-1}(\lambda(x)) = \lambda(f(x)) \end{cases}$ for all $x \in X$.

Main results

In this section we introduced the new concept of cubic ideals of near-subtraction semigroups and discuss some of its properties. Throughout this paper S denote near-subtraction semigroup, unless otherwise mentioned.

Definition 3.1. Let S be a near subtraction semigroup, $(S, \bar{\mu})$ be an interval-valued fuzzy ideal and (S, ω) be a fuzzy ideal. A cubic set $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is called a cubic ideal of S. if it satisfies the following conditions:

$(i)\, \bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and $\omega(x - y) \leq \max\{\omega(x), \omega(y)\}$,

$(ii)\, \bar{\mu}(ax - a(b - x)) \geq \bar{\mu}(x)$ and $\omega(ax - a(b - x)) \leq \omega(x)$,

$(iii)\, \bar{\mu}(xy) \geq \bar{\mu}(x)$ and $\omega(xy) \leq \omega(x)\ \forall\, x, y, a, b \in S$.

If $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic left ideal of S if it satisfies $(i), (ii)$ and if $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic right ideal of S if it satisfies $(i), (iii)$.

Example 3.2. Let $S = \{0, a, b, c\}$ in which " − " and " • " are defined by

•	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	0	0	0	c

-	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Then $(S, -, \bullet)$ is a near-subtraction semigroup. Define an interval-valued fuzzy set $\bar{\mu}$:S→D[0,1] by $\bar{\mu}$(0)=[0.9,1], $\bar{\mu}$(a)=[0.6,0.7], $\bar{\mu}$(b)=[0.4,0.5] and $\bar{\mu}$(c)=[0,0.1] is an interval-valued fuzzy ideal of near subtraction semigroup. Define a fuzzy set ω:S→[0,1] by ω(0)=0, ω(a)=0.5, ω(b)=0.7 and ω(c)=1 is a fuzzy ideal of near subtraction semigroup. Thus $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic ideal of near-subtraction semigroup.

Theorem 3.3. If is a cubic ideal of S, then the set $S_{\mathcal{A}} = \{x \in S \mid \mathcal{A}$ $(i.e.,)\, S_{\mathcal{A}} = \{x \in S \mid \bar{\mu}(x) = \bar{\mu}(0)$ and $\omega(x) = \omega(0)$ is a cubic ideal of S.

Proof: Let $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a cubic ideal of S and x, y $\in S$, then $\mathcal{A}(x) = \mathcal{A}(0)$ and $\mathcal{A}(y) = \mathcal{A}(0)$.

Suppose x , y $\in S_{\mathcal{A}}$ then $\bar{\mu}(x) = \bar{\mu}(0), \bar{\mu}(y) = \bar{\mu}(0)$ and $\omega(x) = \omega(0),\ \omega(y) = \omega(0)$.

Since $\bar{\mu}$ be an i-v fuzzy ideal of S, then $\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$

$= \min\{\bar{\mu}(0), \bar{\mu}(0)\} = \bar{\mu}(0)$ and

ω be a fuzzy ideal of S, then $\omega(x - y) \leq \max\{\omega(x), \omega(y)\}$

$= \max\{\omega(0), \omega(0)\} = \omega(0)$

Thus $x - y \in S_{\mathcal{A}}$.

For every $a, b \in S_{\mathcal{A}}$ and $x \in S_{\mathcal{A}}$ then $\bar{\mu}(x) = \bar{\mu}(0)$ and $\omega(x) = \omega(0)$, then $\bar{\mu}(ax - a(b - x)) \geq \bar{\mu}(x) = \bar{\mu}(0)$ and $\omega(ax - a(b - x)) \leq \omega(x) = \omega(0)$

Thus $ax - a(b - x) \in S_{\mathcal{A}}$.

Suppose x , y $\in S_{\mathcal{A}}$ then $\bar{\mu}(x) = \bar{\mu}(0), \bar{\mu}(y) = \bar{\mu}(0)$ and $\omega(x) = \omega(0),\ \omega(y) = \omega(0)$.

$\bar{\mu}(xy) \geq \bar{\mu}(x) = \bar{\mu}(0)$ and $\omega(xy) \leq \omega(x) = \omega(0)$

Thus $xy \in S_{\mathcal{A}}$.

Hence $S_{\mathcal{A}} = \{x \in S \mid \mathcal{A}(x) = \mathcal{A}(0)\}$ is a cubic ideal of S.

Theorem 3.4. Let H be a non-empty subset of S. If $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a cubic set of S defined by

$$\mathcal{A}(x) = \begin{cases} \bar{\mu}(x) = \begin{cases} [p_1, p_2] & \text{if } x \in H \\ [q_1, q_2] & \text{otherwise} \end{cases} \\ \omega(x) = \begin{cases} 1 - p & \text{if } x \in H \\ 1 - q & \text{otherwise} \end{cases} \end{cases}$$

for all $x \in S, [p_1, p_2], [q_1, q_2] \in D[0,1]$ $p, q \in [0,1]$ with $[p_1, p_2] > [q_1, q_2]$ and $p > q$.

Then H is an ideal of S if and only if $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic ideal of S.

Proof: Let $\mathcal{A} = < \bar{\mu}, \omega >$ be a cubic ideal of S and let $x, y \in H$.

Since $\bar{\mu}$ be an i-v fuzzy ideal of S, then $\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$

$= \min\{[p_1, p_2], [p_1, p_2]\}$

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$= [p_1, p_2]$

and ω be a fuzzy ideal of S, then $\omega(x-y) \leq \max\{\omega(x), \omega(y)\}$
$= \max\{1-p, 1-p\}$
$= 1-p$

Thus $x-y \in H$.

For every $a, b \in S$ and $x \in H$, we have $\bar{\mu}(ax-a(b-x)) \geq \bar{\mu}(x) = [p_1, p_2]$ and $\omega(ax-a(b-x)) \leq \omega(x) = 1-p$.

Thus $ax-a(b-x) \in H$.

For all $x, y \in H$. Then $\bar{\mu}(xy) \geq \bar{\mu}(x) = [p_1, p_2]$ and $\omega(xy) \leq \omega(x) = 1-p$.

Thus $xy \in H$.

Hence H is an ideal of S.

Conversly, assume that H is an ideal of X. Let $x, y \in S$. If at least one of S does not belong to H, then $x-y \notin H$, we have $\bar{\mu}(x-y) \geq [q_1, q_2] = \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ $\omega(x-y) \leq 1-q = \max\{\omega(x), \omega(y)\}$

If $x, y \in H$ then $x-y \in H$, we have $\bar{\mu}(x-y) \geq [p_1, p_2] = \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ $\omega(x-y) \leq 1-p = \max\{\omega(x), \omega(y)\}$

Let $a, b, x \in S$ and if $x \in H$, such that $ax-a(b-x) \in H$, we have $\bar{\mu}(ax-a(b-x)) \geq \bar{\mu}(x) = [p_1, p_2]$ and $\omega(ax-a(b-x)) \leq \omega(x) = 1-p$

If $x \notin H$ such that $ax-a(b-x) \notin H$, we have $\bar{\mu}(ax-a(b-x)) \geq \bar{\mu}(x) = [q_1, q_2]$ and $\omega(ax-a(b-x)) \leq \omega(x) = 1-q$

If $x \in H$ and $y \in H$, then $xy \in H$, we have $\bar{\mu}(xy) \geq [p_1, p_2] = \bar{\mu}(x)$ and $\omega(xy) \leq 1-p = \omega(x)$

Suppose $x \notin H$ we have $xy \notin H$, we have $\bar{\mu}(xy) \geq [q_1, q_2] = \bar{\mu}(x)$ and $\omega(xy) \leq 1-q = \omega(x)$

Hence $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is cubic ideal of S.

Theorem 3.5. If $\{\mathcal{A}_i\}_{i\in\wedge} = \langle \bar{\mu}_i, \omega_i | i \in \wedge \rangle$ is a family of cubic ideals of S, then $\sqcap_{i\in\wedge}\mathcal{A}_i = \langle \cap_{i\in\wedge}\bar{\mu}_i, \cup_{i\in\wedge}\omega_i \rangle$ is a cubic ideal of S.

Proof: Let $\{\mathcal{A}_i\}_{i\in\wedge}$ be a family of cubic ideals of S.

let $\cap\bar{\mu}_i(x) = (\inf \bar{\mu}_i)(x) = \inf \bar{\mu}_i(x)$ and $\cup\,\omega_i(x) = (\sup \omega_i)(x) = \sup \omega_i(x)$

i) For all $x, y \in S$, we have

$(\cap_{i\in\wedge}\bar{\mu}_i)(x-y) = \inf\{\bar{\mu}_i(x-y) | i \in \wedge\}\}$
$\geq \inf\{\min\{\bar{\mu}_i(x), \bar{\mu}_i(y)\} | i \in \wedge\}$
$= \min\{\inf\{\bar{\mu}_i(x) | i \in \wedge\}, \inf\{\bar{\mu}_i(y) | i \in \wedge\}\}$
$= \min\{(\cap_{i\in\wedge}\bar{\mu}_i)(x), (\cap_{i\in\wedge}\bar{\mu}_i)(y)\}$ and

$(\cap_{i\in\wedge}\omega_i)(x-y)$
$= \sup\{\omega_i(x-y) | i \in \wedge\}$
$\leq \sup\{\max\{\omega_i(x), \omega_i(y)\} | i \in \wedge\}$
$= \max\{\sup\{\omega_i(x) | i \in \wedge\}, \sup\{\omega_i(y) | i \in \wedge\}\}$
$= \max\{(\cup_{i\in\wedge}\omega_i)(x), (\cup_{i\in\wedge}\omega_i)(y)\}$

ii) For all $a, b, x \in S$, we have

$(\cap_{i\in\wedge}\bar{\mu}_i)(ax-a(b-x))$
$= \inf\{\bar{\mu}_i(ax-a(b-x)) | i \in \wedge\}$
$\geq \inf\{\bar{\mu}_i(x) | i \in \wedge\}$
$= (\cap_{i\in\wedge}\bar{\mu}_i)(x)$ and

$(\cap_{i\in\wedge}\omega_i)(ax-a(b-x))$
$= \sup\{\omega_i(ax-a(b-x)) | i \in \wedge\}$
$\leq \sup\{\omega_i(x) | i \in \wedge\}$
$= ((\cup_{i\in\wedge}\omega_i))(x)$

iii) For all $x, y \in S$, we have

$(\cap_{i\in\wedge}\bar{\mu}_i)(xy) = \inf\{\bar{\mu}_i(xy) | i \in \wedge\}$
$\geq \inf\{\bar{\mu}_i(x) | i \in \wedge\}$
$= (\cap_{i\in\wedge}\bar{\mu}_i)(x)$ and

$(\cap_{i\in\wedge}\omega_i)(xy) = \sup\{\omega_i(xy) | i \in \wedge\}$
$\leq \sup\{\omega_i(x) | i \in \wedge\}$
$= ((\cup_{i\in\wedge}\omega_i))(x)$

Hence $\sqcap_{i\in\wedge}\mathcal{A}_i = \langle \cap_{i\in\wedge}\bar{\mu}_i, \cup_{i\in\wedge}\omega_i \rangle$ is a cubic ideal of S.

Theorem 3.6. Let $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a cubic subset of S. Then $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic ideal of S $\Leftrightarrow$ each non-empty level subset of $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is an ideal of S.

Proof: Assume that $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic ideal of S. Let $x, y \in U(\mathcal{A}; \tilde{t}, n)$ for all $\tilde{t} \in D[0,1]$ and $n \in [0,1]$. Then $\bar{\mu}(x) = \bar{\mu}(y) \geq \tilde{t}$ and $\omega(x) = \omega(y) \leq n$. By the definition of cubic ideal

$\bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\} \geq \tilde{t}$ and $\omega(x-y) \leq \max\{\omega(x), \omega(y)\} \leq n$

Hence $x-y \in U(\mathcal{A}; \tilde{t}, n)$.

Let $a, b, x \in U(\mathcal{A}; \tilde{t}, n)$. Then $\bar{\mu}(x) \geq \tilde{t}$ and $\omega(x) \leq n$. We know that $\bar{\mu}(ax-a(b-x)) \geq \bar{\mu}(x) = \tilde{t}$ and $\omega(ax-a(b-x)) \leq \omega(x) = n$.

This implies that $ax-a(b-x) \in U(\mathcal{A}; \tilde{t}, n)$.

Let $x, y \in U(\mathcal{A}; \tilde{t}, n)$ then $\bar{\mu}(x) = \bar{\mu}(y) \geq \tilde{t}$ and $\omega(x) = \omega(y) \leq n$ according the cubic ideal of S $\bar{\mu}(xy) \geq \bar{\mu}(x) = \tilde{t}$ and $\omega(xy) \leq \omega(x) = n$.

Thus $xy \in U(\mathcal{A}; \tilde{t}, n)$.

Hence $U(\mathcal{A}; \tilde{t}, n)$ is an ideal of S.

Conversly, assume that $U(\mathcal{A}; \tilde{t}, n)$ is an ideal of S. Suppose assume that

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$\bar{\mu}(x-y) < \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and $\omega(x-y) > \max\{\omega(x), \omega(y)\}$ for some $x, y \in U(\mathcal{A}; \tilde{t}, n)$ then by taking $\tilde{t}_1 = \frac{1}{2}\{\bar{\mu}(x-y) + \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and $n_1 = \frac{1}{2}\{\omega(x-y) + \max\{\omega(x), \omega(y)\}$ for all $\tilde{t}_1 \in D[0,1]$ and $n_1 \in [0,1]$ we have $\bar{\mu}(x-y) > \tilde{t}_1$ for $\bar{\mu}(x) \geq \tilde{t}_1, \bar{\mu}(y) \geq \tilde{t}_1$ and $\omega(x-y) < n_1$ for $\omega(x) \leq n_1, \omega(x) \leq n_1$ thus $x - y \notin U(\mathcal{A}; \tilde{t}, n)$ for some $x, y \in U(\mathcal{A}; \tilde{t}, n)$ This is a contradiction. So, $\bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and $\omega(x-y) \leq \max\{\omega(x), \omega(y)\}$.

Suppose, that $\bar{\mu}(ax - a(b-x)) < \bar{\mu}(x)$ and $\omega(ax - a(b-x)) > \omega(x)$ for some $x \in U(\mathcal{A}; \tilde{t}, n)$ and for all $a, b \in S$, then by taking $\tilde{t}_1 = \frac{1}{2}\{\bar{\mu}(ax - a(b-x)) + \bar{\mu}(x)\}$ and $n_1 = \frac{1}{2}\{\omega(ax - a(b-x)) + \omega(x)\}$ for all $\tilde{t}_1 \in D[0,1]$ and $n_1 \in [0,1]$ we have $\bar{\mu}(ax - a(b-x)) > \tilde{t}_1$ for $\bar{\mu}(x) \geq \tilde{t}_1$ and $\omega(ax - a(b-x)) < n_1$ for $\omega(x) \leq n_1$ thus $ax - a(b-x) \notin U(\mathcal{A}; \tilde{t}, n)$. This is a contradiction. Hence $\bar{\mu}(ax - a(b-x)) \geq \bar{\mu}(x)$ and $\omega(ax - a(b-x)) \leq \omega(x)$.

Suppose $\bar{\mu}(xy) \geq \bar{\mu}(x)$ and $\omega(xy) \leq \omega(x)$ for some $x, y \in U(\mathcal{A}; \tilde{t}, n)$ then by taking $\tilde{t}_1 = \frac{1}{2}\{\bar{\mu}(xy) + \bar{\mu}(x)\}$ and $n_1 = \frac{1}{2}\{\omega(xy) + \omega(x)\}$ for all $\tilde{t}_1 \in D[0,1]$ and $n_1 \in [0,1]$ we have $\bar{\mu}(xy) > \tilde{t}_1$ for $\bar{\mu}(x) = \bar{\mu}(y) \geq \tilde{t}$ and $\omega(xy) < n_1$ for $\omega(x) = \omega(y) \leq n$ thus $xy \notin U(\mathcal{A}; \tilde{t}, n)$. Hence $\bar{\mu}(xy) \geq \bar{\mu}(x)$ and $\omega(xy) \leq \omega(x)$. Therefore $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic ideal of S.

Theorem 3.7. Let f: S→ S_1 be a homomorphism of near-subtraction semigroups and $C_f^{-1}: C(S_1) \to C(S)$ be the inverse cubic transformation induced by f. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic left (right) ideal of S_1 by the cubic property then $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic ideal of S.

Proof: Let $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic ideal of S_1. For all $x, y \in S$ then

i) $C_f^{-1}(\bar{\mu}(x-y)) = \bar{\mu}(f(x-y))$

$= \bar{\mu}(f(x) - f(y))$

$\geq \min\{\bar{\mu}(f(x)), \bar{\mu}(f(y))\}$

$C_f^{-1}(\bar{\mu}(x-y)) \geq \min\{C_f^{-1}(\bar{\mu}(x)), C_f^{-1}(\bar{\mu}(y))\}$

$C_f^{-1}(\omega(x-y)) = \omega(f(x-y))$

$= \omega(f(x) - f(y))$

$\leq \max\{\omega(f(x)), \omega(f(y))\}$

$C_f^{-1}(\omega(x-y)) \leq \max\{C_f^{-1}(\omega(x)), C_f^{-1}(\omega(y))\}$

ii) For all $a, b, x \in S$ we have

$C_f^{-1}(\bar{\mu}(ax - a(b-x))$

$= \bar{\mu}(f(ax - a(b-x))$

$= \bar{\mu}(f(ax) - f(a(b-x)))$

$= \bar{\mu}(f(a)f(x) - f(a)(f(b) - f(x)))$

$\geq \bar{\mu}(f(x))$

$= C_f^{-1}(\bar{\mu}(x))$

$C_f^{-1}(\omega(ax - a(b-x))$

$= \omega(f(ax - a(b-x))$

$= \omega(f(ax) - f(a(b-x)))$

$= \omega(f(a)f(x) - f(a)(f(b) - f(x)))$

$\leq \omega(f(x))$

$= C_f^{-1}(\omega(x))$

iii) $C_f^{-1}(\bar{\mu}(xy)) = \bar{\mu}(f(xy))$

$= \bar{\mu}(f(x)f(y))$

$\geq \bar{\mu}(f(x))$

$= C_f^{-1}(\bar{\mu}(x))$

$C_f^{-1}(\omega(xy)) = \omega(f(xy))$

$= \omega(f(x)f(y))$

$\leq \omega(f(x))$

$= C_f^{-1}(\omega(x))$

Hence $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic ideal of S.

Theorem 3.8. Let f: S→ S_1 be an onto homomorphism of near-subtraction semigroups S and S_1. Let $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic subset of S_1 by the cubic property if $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic ideal of S then $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic ideal of S_1.

Proof: Let $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic subset of S_1

i) Let $x', y' \in S_1$. Then $f(x) = x', f(y) = y'$

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for some $x, y \in S$. It follows that

$$\bar{\mu}(x' - y') = \bar{\mu}(f(x) - f(y))$$
$$= \bar{\mu}(f(x - y))$$
$$= (\mathcal{C}_f^{-1}(\bar{\mu})(x - y)$$
$$\geq \min\{\mathcal{C}_f^{-1}(\bar{\mu})(x), \mathcal{C}_f^{-1}(\bar{\mu})(y)\}$$
$$= \min\{\bar{\mu}(f(x)), \bar{\mu}(f(x))\}$$
$$= \min\{\bar{\mu}(x'), \bar{\mu}(y')\}$$

and

$$\omega(x' - y') = \omega(f(x) - f(y))$$
$$= \omega(f(x - y))$$
$$= (\mathcal{C}_f^{-1}(\omega)(x - y)$$
$$\leq \max\{\mathcal{C}_f^{-1}(\omega)(x), \mathcal{C}_f^{-1}(\omega)(y)\}$$
$$= \max\{\omega(f(x)), \omega(f(x))\}$$
$$= \max\{\omega(x'), (y')\}$$

ii) Let $a', b', x' \in S_1$ there exist $a, b, x \in S$ such that $f(a) = a', f(b) = b'$ *and* $f(x) = x'$, we have $\bar{\mu}\left(a'x' - a'(b' - x')\right)$

$$= \bar{\mu}\left(f(a)f(x) - f(a)(f(b) - (x))\right)$$
$$= \bar{\mu}\left(f(ax) - f(a)(f(b - x))\right)$$
$$= \bar{\mu}\left(f(ax) - f(a(b - x))\right)$$
$$= \bar{\mu}\left(f(ax - a(b - x))\right)$$
$$= \mathcal{C}_f^{-1}(\bar{\mu})((ax - a(b - x)))$$
$$\geq \mathcal{C}_f^{-1}(\bar{\mu})(x)$$
$$= \bar{\mu}(f(x))$$
$$= \bar{\mu}(x')$$

$$\omega\left(a'x' - a'(b' - x')\right)$$
$$= \omega(f(a)f(x) - f(a)(f(b) - (x)))$$
$$= \omega\left(f(ax) - f(a)(f(b - x))\right)$$
$$= \omega\left(f(ax) - f(a(b - x))\right)$$
$$= \omega\left(f(ax - a(b - x))\right)$$
$$= \mathcal{C}_f^{-1}(\omega)((ax - a(b - x)))$$
$$\leq \mathcal{C}_f^{-1}(\omega)(x)$$
$$= \omega(f(x))$$
$$= \omega(x')$$

iii) Let $x', y' \in S_1$. Then $f(x) = x', f(y) = y'$ for some $x, y \in S$. It follows that

$$\bar{\mu}(x'y') = \bar{\mu}(f(x)f(y))$$
$$= \bar{\mu}(f(xy))$$
$$= (\mathcal{C}_f^{-1}(\bar{\mu})(xy)$$
$$\geq \mathcal{C}_f^{-1}(\bar{\mu})(y)$$
$$= \bar{\mu}(f(x)$$
$$= \bar{\mu}(y')$$

and

$$\omega(x'y') = \omega(f(x)f(y))$$
$$= \omega(f(xy)$$
$$= (\mathcal{C}_f^{-1}(\omega)(xy)$$
$$\leq \mathcal{C}_f^{-1}(\omega)(y)$$
$$= \omega(f(y)$$
$$= \omega(y')$$

Hence $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic ideal of S_1.

Theorem 3.9. For a homomorphism f: S→ S_1 of near subtraction semigroups, let $\mathcal{C}_f$: $\mathcal{C}(S) \rightarrow \mathcal{C}(S_1)$ be the cubic transformation respectively induced by f. If $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic ideal of S which has the cubic property, then $\mathcal{C}_f(\mathcal{A})$ is a cubic ideal of S_1.

Proof: Given $f(x), f(y) \in f(S)$, let $x_0 \in f^{-1}(f(x))$ and $y_0 \in f^{-1}(f(y))$ be such that

$\bar{\mu}(x_0) = \sup_{p \in f^{-1}(f(x))} \bar{\mu}(p)$,

$\omega(x_0) = \inf_{p \in f^{-1}(f(x))} \omega(p)$ and

$\bar{\mu}(y_0) = \sup_{q \in f^{-1}(f(y))} \bar{\mu}(q)$,

$\omega(y_0) = \inf_{q \in f^{-1}(f(y))} \omega(q)$ respectively. Then

(i) $\mathcal{C}_f(\bar{\mu})(f(x) - f(y))$

$$= \sup_{z \in f^{-1}(f(x) - f(y))} \bar{\mu}(z)$$
$$\geq \bar{\mu}(x_0 - y_0)$$
$$\geq \min\{\bar{\mu}(x_0), \bar{\mu}(y_0)\}$$
$$= \min\left\{\sup_{p \in f^{-1}(f(x))} \bar{\mu}(p), \sup_{q \in f^{-1}(f(y))} \bar{\mu}(q)\right\}$$
$$= \min\{\mathcal{C}_f(\bar{\mu})(f(x)), \mathcal{C}_f(\bar{\mu})(f(y))\}$$

$$\mathcal{C}_f(\omega)(f(x) - f(y))$$
$$= \inf_{z \in f^{-1}(f(x) - f(y))} \omega(z)$$
$$\leq \omega(x_0 - y_0)$$
$$\leq \max\{\omega(x_0), \omega(y_0)\}$$
$$= \max\{\inf_{p \in f^{-1}(f(x))} \omega(p), \inf_{q \in f^{-1}(f(y))} \omega(q)\} = \max\{\mathcal{C}_f(\omega)(f(x)), \mathcal{C}_f(\omega)(f(y))\}$$

(ii) Let $f(a), f(b), f(x) \in f(S)$ *then*

$$\mathcal{C}_f(\bar{\mu})\left(f(a)f(x) - f(a)(f(b) - f(x))\right)$$
$$= \sup_{z \in f^{-1}(f(a)f(x) - f(a)(f(b) - f(x)))} \bar{\mu}(z)$$
$$\geq \bar{\mu}(x_0)$$
$$= \sup_{p \in f^{-1}(f(x))} \bar{\mu}(p)$$
$$= \mathcal{C}_f(\bar{\mu})(f(x))$$

$$\mathcal{C}_f(\omega)\left(f(a)f(x) - f(a)(f(b) - f(x))\right)$$
$$= \inf_{z \in f^{-1}(f(a)f(x) - f(a)(f(b) - f(x)))} \omega(z)$$
$$\leq \omega(x_0)$$
$$= \inf_{p \in f^{-1}(f(x))} \bar{\mu}(p)$$
$$= \mathcal{C}_f(\omega)(f(x))$$

(iii) Let $f(x), f(y) \in f(S)$ *then*

$$\mathcal{C}_f(\bar{\mu})(f(x)f(y)) = \sup_{z \in f^{-1}(f(x)f(y))} \bar{\mu}(z)$$
$$\geq \bar{\mu}(x_0)$$
$$= \sup_{p \in f^{-1}(f(x))} \bar{\mu}(p)$$
$$= \mathcal{C}_f(\bar{\mu})(f(x))$$

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$$\mathcal{C}_f(\omega)(f(x)f(y)) = \inf_{z\in f^{-1}(f(x)f(y))} \omega(z)$$
$$\leq \omega(x_0)$$
$$\leq \inf_{p\in f^{-1}(f(x))} \omega(p)$$
$$= \mathcal{C}_f(\omega)(f(x))$$

Hence $\mathcal{C}_f(\mathcal{A}) = < \mathcal{C}_f(\bar{\mu}), \mathcal{C}_f(\gamma) >$ is a cubic ideal of S_1.

Conclusion

In the structural theory of fuzzy algebraic systems, fuzzy ideals with special properties always play in important role. In this paper, we have presented some properties of cubic ideals of near-subtraction semigroups. We applied the interval-valued fuzzy set theory and fuzzy set theory to left almost semigroups, subtraction semigroups and near-subtraction semigroups by their cubic ideals. The obtained results probably can be applied in various fields, such as robotics, computer networks and neural networks. In our future study we try to extend this concept to cubic bi-ideals in near-subtraction semigroups.

Conflict Interest

The authors declare that there is no conflict of interest regarding the publication of our paper.

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