extrEMe–I: Novel Scalable Multi-Responses 3-D MT Inverse Solver

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Summary

We present novel open source 3-D magnetotelluric (MT) inverse solver extrEMe-I. As a forward modelling engine the modern scalable solvers *extrEMe* or *GIEM2G*, based on a contracting integral equation approach, are used. The (regularized) inversion exploits gradient-type (quasi-Newton) optimization algorithm. For fast computation of the misfit gradient the adjoint sources approach is invoked. The inverse solver is able to deal with highly detailed and contrasting models, allows for working (separately or jointly) with any type of single- or/and inter-site MT responses, including horizontal electric and magnetic tensors, and supports massive parallelization. Moreover, different parallelization strategies implemented in the code allow optimal usage of available computational resources for a given problem set up. To parametrize an inverse domain the so-called mask concept is implemented, which means that one can merge any subset of forward modelling cells in order to account for (usually) irregular distribution of observation sites. We report results of 3-D numerical experiments aimed at demonstrating the performance and scalability of the code, as well as at analyzing the resolution power of different MT responses and their combinations.

Keywords: 3-D inversion, magnetotellurics, contracting integral equation, adjoint sources approach

Introduction

Throughout the past two decades a number of 3-D MT inverse solvers (Mackie & Madden, 1993; Newman & Alumbaugh, 2000; Avdeev & Avdeeva, 2009; Egbert & Kelbert, 2012; Grayver, 2015; Usui, 2015; Kordy et al., 2015, among others) have been developed. With the goal of the 3-D EM inverse modeling being the same – namely, imaging the 3-D electrical conductivity distribution in the Earth's interiors – there are a number of differences between the specific inverse solvers. These differences lie, for example, in the way what method for solving 3-D forward problem is invoked, what optimization scheme is used to minimize penalty function, what parametrization is exploited, what parallelization scheme is implemented etc.

Despite the evident progress in solving 3-D MT inverse problems it remains very challenging from the computational point of view. Many of existing inverse MT codes are still too slow to tackle the problems of practical interest, i.e. the problems involving high levels of complexity and spatial detail. In addition, most of existing 3-D inverse solvers are not freely available for academia. ModEM 3D (Egbert and Kelbert, 2012) is currently the only open source 3-D MT inverse code. It is well-known that the inversion of real MT data is not straightforward, owing to the non-uniqueness of the inverse problem and the variety of parameter settings that can influence the inversion results. Therefore, it is important to use different inversion solvers in order to mutually test for consistency and robustness of the recovered inversion models. Thus, there is a pressing need for alternative freely available 3-D MT inverse code(s). In this paper we present such a code which exploits as a forward engine modern scalable solvers, based on a contracting integral equation (CIE) approach.

Main features of the new code

The key features of the presented solver are listed below:

- A possibility to invert different single- and inter-site MT-responses as impedance, tipper, horizontal magnetic tensor, and horizontal electric tensor either separately or jointly;
- Mask parametrization of the inversion domain, which means that one can merge any subset of forward modelling cells in order to account for (usually) irregular distribution of observation sites;
- Use of adjoint source approach to compute the misfit gradient;
- Use of two scalable CIE-based forward solvers, depending on the problem set up: extrEMe (Kruglyakov et al., 2016) for small-scale problems, and GIEM2G (Kruglyakov & Bloshanskaya, 2017) for large-scale ones;

- Use of parallelization scheme which allows effective exploitation up to $2N_f \times N_x$ nodes, where N_f is a number of frequencies, N_x is a number of cells in x direction.

Synthetic example

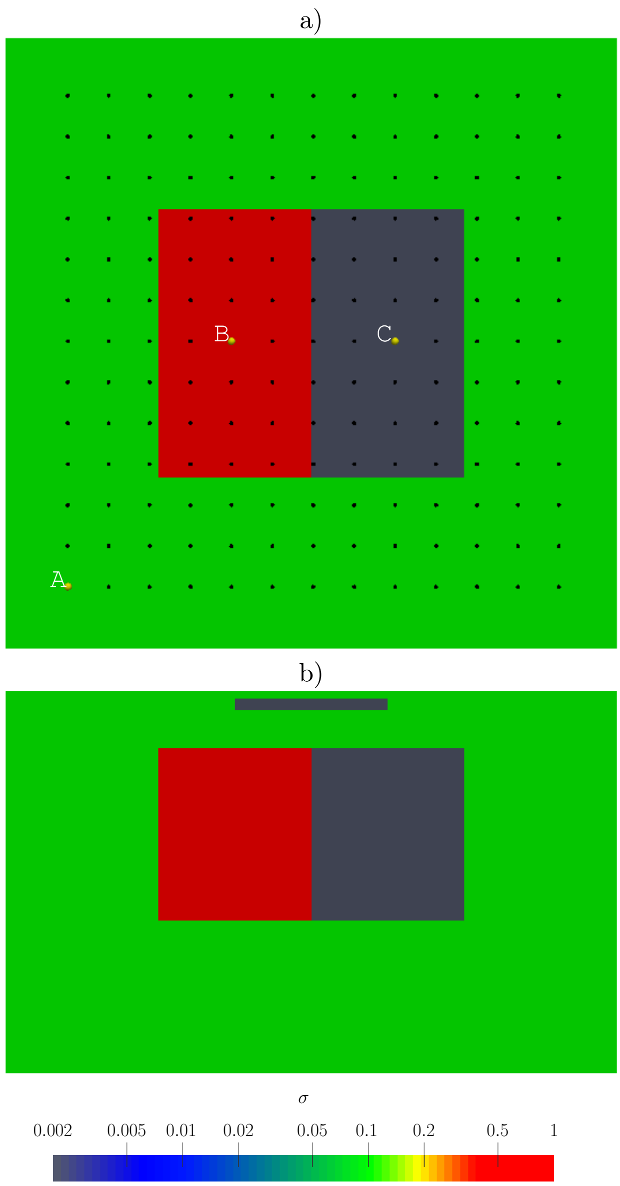

Figure 1: True model. a) - plane view, b) side view. Location of survey sites are depicted by black dots in Figure 1a. Yellow dots in the same figure (denoted as A, B, C) stand for the hypothetical locations of the reference site. Note, that the reference site is involved in the analysis, if (inter-site) horizontal electric or magnetic tensors are inverted.

To demonstrate the performance of the developed solver, inversions of the MT data from the model used by Grayver (2015) were performed (Figure 1). The data at 16 frequencies, from 10^{-4} to 10^{3} Hz, obtained from 169 recievers (black dots in Figure 1a) were used. 5% random noise was added to responses. To avoid an inverse crime, the grid used in the inversions was 4 times coarser than those used for the responses calculations. The results of inversions are shown in Figures 2–4. The (almost perfect) scalability of the code is demonstrated in Figure 5.

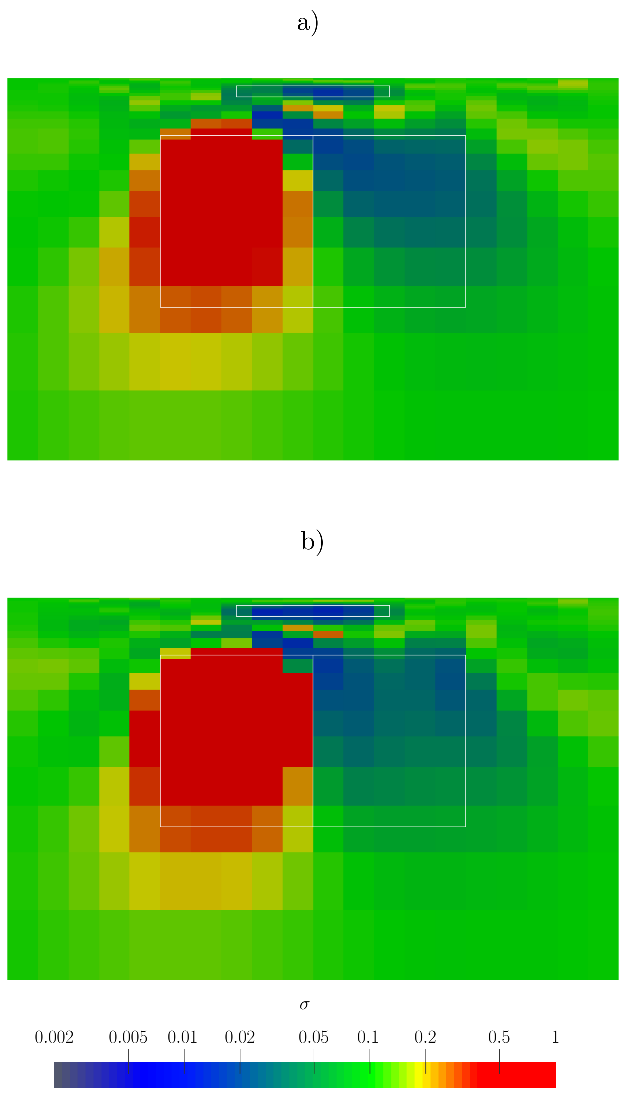

Figure 2: Model obtained by inversion of: a) horizontal electrical tensor T; b) simultaneous inversion of T and impedance Z b). In both cases the site A (see Figure 1a) was used as a reference site.

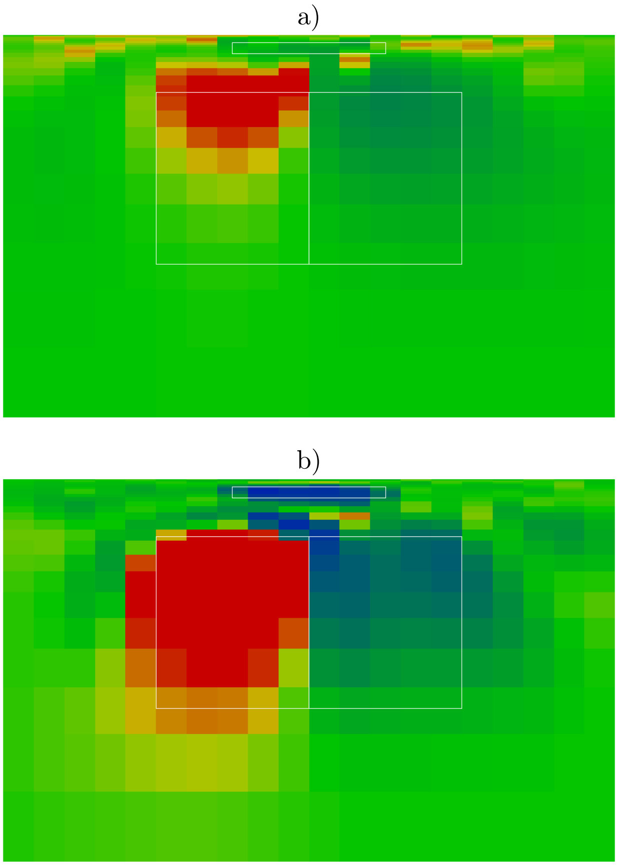

Figure 3: Model obtained by inversion of: a) horizontal magnetic tensor M; b) simultaneous inversion of M and impedance Z. In both cases the site A (see Figure 1a) was used as a reference site.

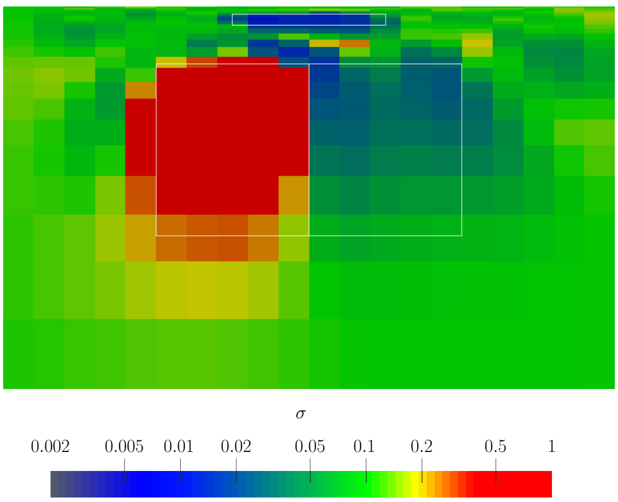

Figure 4: Model obtained by inversion of impedance Z

Conclusions and discussion

The developed code demonstrates remarkable scalability and allows to invert any type of single- or/and inter-site MT responses, including horizontal electric and magnetic tensors. From our model study presented in previous section we conclude that: a) inversion of either T or Z responses delivers comparable models. This, in particular, raises the question: whether we need magnetic field measurements at all in the course of MT surveys? b) the recovery of true structure is much worse if only M responses are inverted; c) simultaneous inversion of either M and Z, or T and Z does not improve the results; d) additional runs of inversion (not shown in the abstract) indicate that the choice of reference site (either A, B or C) has only marginal effect on the inversion results.

In spite of remarkable performance of the solver there is a room for further improvements. One of the option could be an advancing the forward problem engine by using high-order polynomial approximation of the fields inside the cells (Kruglyakov & Kuvshinov, 2017) instead of piece-wise representation used in *extrEMe* and *GIEM2G*.

Final comment of this section is that the code is available on a request from Mikhail Kruglyakov.

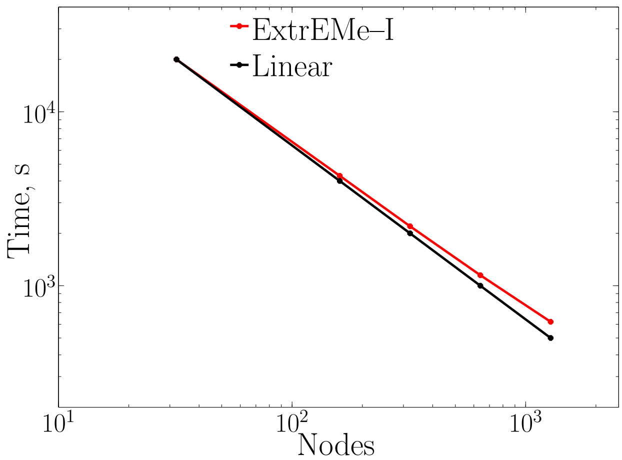

Figure 5: The inversion time with respect to the number of used nodes. The figure demonstrates perfect scalability of the code.

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