Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

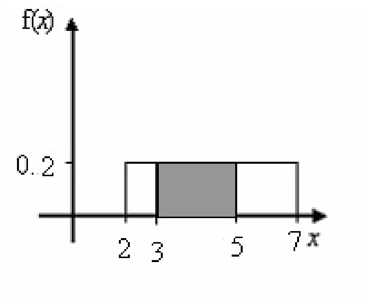
The continuous random variable $X \sim U[2, 7]$.
Find

a $P(3 < X < 5)$,

b $P(X > 4)$.

Solution:

a $\frac{1}{b-a} = \frac{1}{7-2} = 0.2$



$$P(3 < X < 5) = (5-3) \times 0.2$$
$$= 0.4$$

b $P(X > 4) = (7-4) \times 0.2$
$$= 0.6$$

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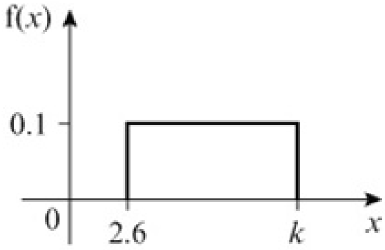
Exercise A, Question 2

Question:

The continuous random variable X has p.d.f. as shown in the diagram.



Find

a the value of k,

b $P(4 < X < 7.9)$.

Solution:

a
$$\text{Area} = 1$$
$$(k - 2.6) \times 0.1 = 1$$
$$(k - 2.6) = 10$$
$$k = 12.6$$

b $P(4 < X < 7.9) = (7.9 - 4) \times 0.1$
$= 0.39$

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Exercise A, Question 3

Question:

The continuous random variable X has p.d.f.

$$f(x) = \begin{cases} k, & -2 \le x \le 6, \\ 0, & \text{otherwise.} \end{cases}$$

Find

a the value of k,

b $P(-1.3 < X < 4.2)$.

Solution:

a

$$\text{Area} = 1$$
$$k \times (6 - (-2)) = 1$$
$$8k = 1$$
$$k = \frac{1}{8}$$

b

$$P(-1.3 < X < 4.2) = \frac{1}{8} \times (4.2 - (-1.3))$$
$$= 0.6875$$

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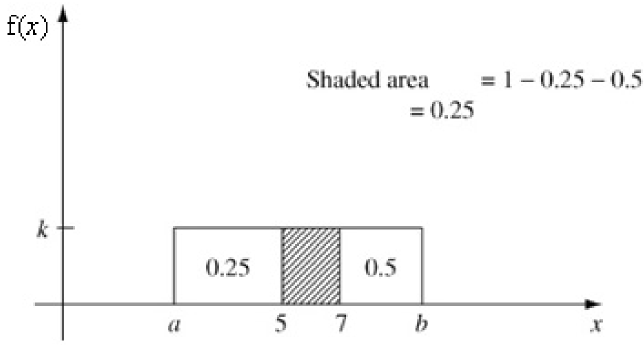
Exercise A, Question 4

Question:

The continuous random variable $Y \sim \text{U}[a, b]$. Given that $\text{P}(Y < 5) = \frac{1}{4}$ and $\text{P}(Y > 7) = \frac{1}{2}$, find the value of a and the value of b.

Solution:



$$\text{shaded area} = \frac{1}{4}$$
$$2 \times k = \frac{1}{4}$$
$$k = \frac{1}{8}$$
$$(b-7) \times \frac{1}{8} = 0.5$$
$$(b-7) = 4$$
$$b = 11$$
$$(5-a) \times \frac{1}{8} = 0.25$$
$$(5-a) = 2$$
$$a = 3$$

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Exercise A, Question 5

Question:

The continuous random variable $X \sim \mathrm{U}[2, 8]$.

a Write down the distribution of $Y = 2X + 5$.

b Find $\mathrm{P}(12 < Y < 20)$.

Solution:

a

$2 \times 2 + 5 = 9$

$2 \times 8 + 5 = 21$

$Y \sim \mathrm{U}[9, 21]$

b For Y, $\frac{1}{b-a} = \frac{1}{21-9} = \frac{1}{12}$

$P(12 < Y < 20) = (20 - 12) \times \frac{1}{12}$

$= \frac{2}{3}$

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Exercise B, Question 1

Question:

The continuous variable Y is uniformly distributed over the interval $[-3, 5]$.

Find:

a $\mathrm{E}(X)$,

b $\mathrm{Var}(X)$,

c $\mathrm{E}(X^2)$,

d the cumulative distribution function of X, for all x.

Solution:

a $\mathrm{E}(X) = \frac{5+(-3)}{2}$

$= 1$

b $\mathrm{Var}(X) = \frac{(5-(-3))^2}{12}$

$= 5\frac{1}{3}$

c $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

$5\frac{1}{3} = \mathrm{E}(X^2) - 1$

$\mathrm{E}(X^2) = 6\frac{1}{3}$

d $\mathrm{F}(x) = \int_{-3}^{x} \frac{1}{5-(-3)}\, \mathrm{d}t$

$= \left[\frac{t}{8}\right]_{-3}^{x}$

$= \frac{x+3}{8}$

$$\mathrm{F}(x) = \begin{cases} 0 & x < -3 \\ \frac{x+3}{8} & -3 \le x \le 5 \\ 1 & x > 5 \end{cases}$$

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Exercise B, Question 2

Question:

Find E(X) and Var(X) for the following probability density functions.

a $f(x) = \begin{cases} \frac{1}{4}, & 1 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$

b $f(x) = \begin{cases} \frac{1}{8}, & -2 \le x \le 6, \\ 0, & \text{otherwise.} \end{cases}$

Solution:

a $E(X) = \frac{5+1}{2}$

$= 3$

$\text{Var}(X) = \frac{(5-1)^2}{12}$

$= 1\frac{1}{3}$

b $E(X) = \frac{6+(-2)}{2}$

$= 2$

$\text{Var}(X) = \frac{(6-(-2))^2}{12}$

$= 5\frac{1}{3}$

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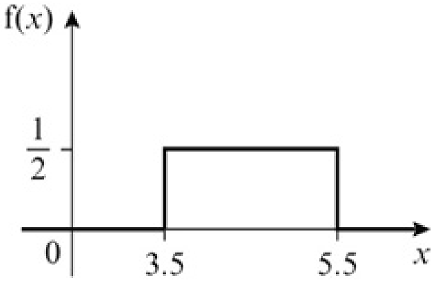
Exercise B, Question 3

Question:

The continuous random variable X has p.d.f as shown in the diagram.



Find:

a $E(X)$,

b $Var(X)$,

c $E(X^2)$,

d the cumulative distribution function of X, for all x.

Solution:

a $E(X) = \frac{5.5+3.5}{2}$

$= 4.5$

b $\text{Var}(X) = \frac{(5.5-3.5)^2}{12}$

$= \frac{1}{3}$

c $\text{Var}(X) = E(X^2) - (E(X))^2$

$\frac{1}{3} = E(X^2) - 20.25$

$E(X^2) = 20\frac{7}{12} = 20.6 \text{ (3 s.f.)}$

d $F(x) = \int_{3.5}^{x} \frac{1}{5.5-3.5} \, dt$

$= \left[\frac{t}{2}\right]_{3.5}^{x}$

$= \frac{x}{2} - \frac{3.5}{2}$

$= \frac{x}{2} - 1.75$

$$F(x) = \begin{cases} 0 & x < 3.5 \\ \frac{x}{2} - 1.75 & 3.5 \le x \le 5.5 \\ 1 & x > 5.5 \end{cases}$$

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Exercise B, Question 4

Question:

The continuous random variable $Y \sim \mathrm{U}[a, b]$. Given $\mathrm{E}(Y) = 1$ and $\mathrm{Var}(Y) = \frac{4}{3}$, find the value of a and the value of b.

Solution:

$\mathrm{E}(Y)$

$\frac{a+b}{2} = 1$

$a + b = 2 \qquad (1)$

$\mathrm{Var}\ (Y)$

$\frac{(b-a)^2}{12} = \frac{4}{3}$

$(b-a)^2 = 16 \qquad (2)$

Solving equations **(1)** and **(2)** simultaneously

$b = 2 - a$

$(2 - a - a)^2 = 16$

$(2 - 2a) = \pm 4$

$2 - 2a = 4$	$2 - 2a = -4$
$a = -1$	$a = 3$
$b = 2 - (-1)$	$b = 2 - 3$
$= 3$	$= -1$

Since $a < b$ $a = -1$ and $b = 3$

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Exercise B, Question 5

Question:

The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{6}, & -1 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

Given that $Y = 4X - 6$, find $E(Y)$ and $Var(Y)$.

Solution:

$$\begin{aligned} E(X) &= \frac{5+(-1)}{2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} Var(X) &= \frac{(5-(-1))^2}{12} \\ &= 3 \end{aligned}$$

$$\begin{aligned} E(Y) &= 4E(X) - 6 \\ &= 8 - 6 \\ &= 2 \end{aligned}$$

$$\begin{aligned} Var(Y) &= 16\ Var(X) \\ &= 48 \end{aligned}$$

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Exercise B, Question 6

Question:

The random variable X is the length of a side of a square. $X \sim U[4.5, 5.5]$.
The random variable Y is the area of the square.
Find $E(Y)$.

Solution:

$E(Y) = E(X^2)$

$E(X) = \frac{4.5+5.5}{2}$

$= 5$

$\text{Var}(X) = \frac{(5.5-4.5)^2}{12}$

$= \frac{1}{12}$

$\text{Var}(X) = E(X^2) - (E(X))^2$

$\frac{1}{12} = E(X^2) - 25$

$E(X^2) = 25\frac{1}{12}$

or $\int_{4.5}^{5.5} x^2 dx = \left[\frac{x^3}{3}\right]_{4.5}^{5.5}$

$= \left[\frac{5.5^3}{3}\right] - \left[\frac{4.5^3}{3}\right]$

$= 25\frac{1}{12}$

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Exercise B, Question 7

Question:

In a computer game an alien appears every 2 seconds. The player stops the alien by pressing a key. The object of the game is to stop the alien as soon as it appears. Given that the player actually presses the key T s after the alien first appears, a simple model of the game assumes that T is a continuous uniform random variable defined over the interval [0, 1].

a Write down $P(T < 0.2)$.

b Write down $E(T)$.

c Use integration to find Var (T).

Solution:

$$\frac{1}{b-a} = \frac{1}{1-0} = 1$$

a $P(T < 0.2) = (0.2 - 0) \times 1$
$= 0.2$

b $E(T) = 0.5$

c $\text{Var}(T) = \int_0^1 t^2 \, dt - 0.5^2$
$= \left[\frac{t^3}{3}\right]_0^1 - 0.25$
$= \frac{1}{12}$

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

The continuous random variable X is uniformly distributed over the interval $[-2, 5]$.

a Sketch the probability density function $f(x)$ of X.

Find

b $E(X)$,

c $Var(X)$,

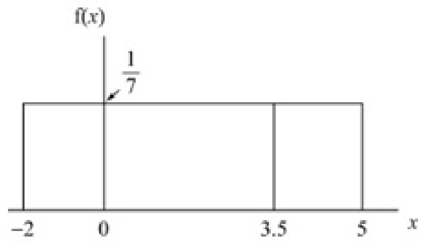
d the cumulative distribution function of X, for all x,

e $P(3.5 < X < 5.5)$,

f $P(X = 4)$.

Solution:

a



b $E(X) = \frac{5+(-2)}{2} = 1.5$

c $\text{Var}(X) = \frac{(5-(-2))^2}{12} = 4\frac{1}{12}$

d $F(x) = \int_{-2}^{x} \frac{1}{5-(-2)}\, dt$

$= \left[\frac{t}{7}\right]_{-2}^{x}$

$= \frac{x+2}{7}$

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{x+2}{7} & -2 \le x \le 5 \\ 1 & x > 5 \end{cases}$$

e $P(3.5 < X < 5.5) = P(3.5 < X < 5)$

$= ((5 - 3.5) \times \frac{1}{7})$

$= \frac{3}{14}$

f $P(X = 4) = 0$

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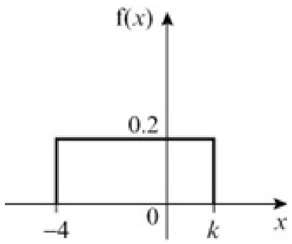
Exercise C, Question 2

Question:

The continuous random variable X has p.d.f. as shown in the diagram.



Find

a the value of k,

b $P(-2 < X < -1)$,

c $E(X)$,

d $Var(X)$,

e the cumulative distribution function of X, for all x.

Solution:

a Area = 1 so

$$(k+4)\times 0.2 = 1$$
$$0.2k + 0.8 = 1$$
$$k = 1$$

b $P(-2 < X < -1) = 1 \times 0.2 = 0.2$

c $E(X) = \frac{-4+1}{2} = -1.5$

d $Var(X) = \frac{(b-a)^2}{12} = \frac{(1-(-4))^2}{12} = 2\frac{1}{12}$

e

$$F(x) = \int_{-4}^{x} \frac{1}{1-(-4)}\,dt$$
$$= \left[\frac{t}{5}\right]_{-4}^{x}$$
$$= \frac{x+4}{5}$$

$$F(x) = \begin{cases} 0 & x < -4 \\ \frac{x+4}{5} & -4 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

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Exercise C, Question 3

Question:

The continuous random variable Y is uniformly distributed on the interval $a \le Y \le b$.
Given $E(Y) = 2$ and $Var(Y) = 3$.
Find

a the value of a and the value of b,

b $P(X > 1.8)$.

Solution:

a $E(Y) = \dfrac{a+b}{2} = 2$ so $a + b = 4$ so $a = 4 - b$

$Var(Y) = \dfrac{(b-a)^2}{12} = 3$

Substituting for a gives $(2b-4)^2 = 36$

$(2b-4) = \pm 6$

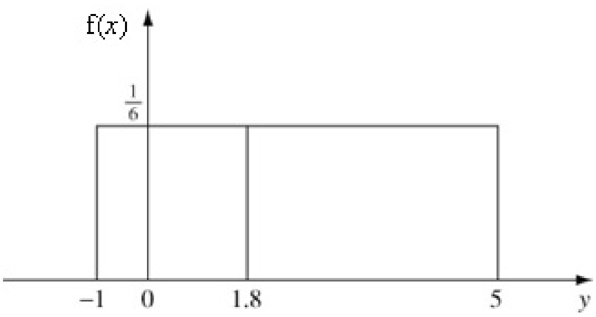
$b = 5$ or $b = -1$

$a = -1$ $a = 5$

but $b > a$

$b = 5$ $a = -1$

b



$P(X > 1.8) = (5 - 1.8) \times \frac{1}{6} = 0.533$ (3 s.f.)

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Exercise C, Question 4

Question:

A child has a pair of scissors and a piece of string 20 cm long, which has a mark on one end. The child cuts the string, at a randomly chosen point, into two pieces. Let X represent the length of the piece of string with the mark on it.

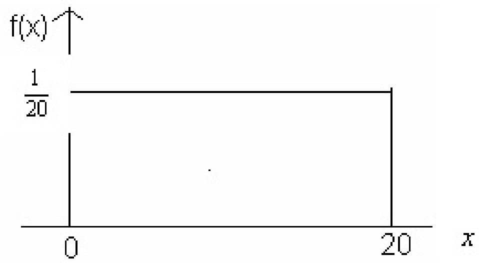
a Write down the name of the probability distribution of X and sketch the graph of its probability density function.

b Find the values of E(X) and Var(X).

c Using your model, calculate the probability that the shorter piece of string is at least 8 cm long.

Solution:

a $X \sim \text{U}(0, 20)$



b

$$\text{E}(X) = \frac{20+0}{2} = 10$$

$$\text{Var}(X) = \frac{(20-0)^2}{12} = \frac{400}{12} = 33\frac{1}{3}$$

c $\text{P}(8 \le X \le 12) = (12-8) \times \frac{1}{20} = 0.2$

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Exercise C, Question 5

Question:

Joan records the temperature every day. The highest temperature she recorded was 29 °C to the nearest degree. Let X represent the error in the measured temperature.

a Suggest a suitable model for the distribution of X.

b Using your model calculate the probability that the error will be less than 0.2 °C.

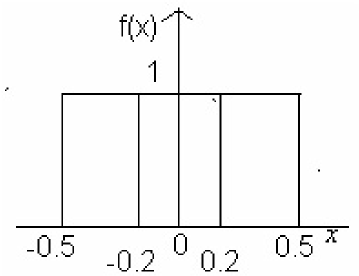
c Find the variance of the error in the measured temperature.

Solution:

a

$X \sim \text{U}(-0.5, 0.5)$

b



$\text{P}(-0.2 < X < 0.2) = 0.4 \times 1 = 0.4$

c $\text{Var}(X) = \dfrac{(b-a)^2}{12} = \dfrac{(0.5-(-0.5))^2}{12} = \dfrac{1}{12}$

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Exercise C, Question 6

Question:

Jameil catches a bus to work every morning. According to the timetable the bus is due at 9 a.m., but Jameil knows that the bus can arrive at a random time between three minutes early and ten minutes late. The random variable X represents the time, in minutes, after 9 a.m. when the bus arrives.

a Suggest a suitable model for the distribution of X and specify it fully.

b Calculate the mean value of X.

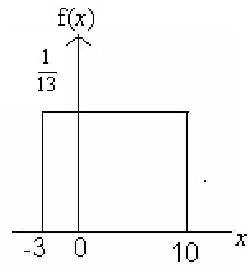
c Find the cumulative distribution function of X.

Jameil will be late for work if the bus arrives after 9.05 a.m.

d Find the probability that Jameil is late for work.

Solution:

a



$X \sim U(-3, 10)$

$$f(x) = \begin{cases} \frac{1}{13} & -3 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

b Mean $= E(X) = \frac{-3+10}{2} = 3.5$ minutes

c

$$F(x) = \int_{-3}^{x} \frac{1}{13}\, dt$$

$$= \left[\frac{t}{13}\right]_{-3}^{x}$$

$$= \frac{x+3}{13}$$

$$F(x) = \begin{cases} 0 & x < -3 \\ \frac{x+3}{13} & -3 \le x \le 10 \\ 1 & x > 10 \end{cases}$$

d $P(5 < X < 10) = (10 - 5) \times \frac{1}{13} = \frac{5}{13}$

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Exercise C, Question 7

Question:

A plumber measures, to the nearest cm, the lengths of pipes.

a Suggest a suitable model to represent the difference between the true lengths and the measured lengths.

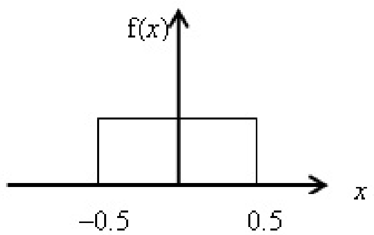
b Find the probability that for a randomly chosen rod the measured length will be within 0.2 cm of the true length.

c Three pipes are selected at random. Find the probability that all three pipes will be within 0.2 cm of the true length.

Solution:

a U(−0.5, 0.5)

b



$P(-0.2 \le X \le 0.2) = (0.2 - (-0.2)) \times 1 = 0.4$

c $P(3 \text{ pipes between } -0.2 \text{ and } 0.2) = 0.4^3 = 0.064$

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Exercise C, Question 8

Question:

A coffee machine dispenses coffee into cups. It is electronically controlled to cut off the flow of coffee randomly between 190 ml and 210 ml. The random variable X is the volume of coffee dispensed into a cup.

a Specify the probability density function of X and sketch its graph.

b Find the probability that the machine dispenses

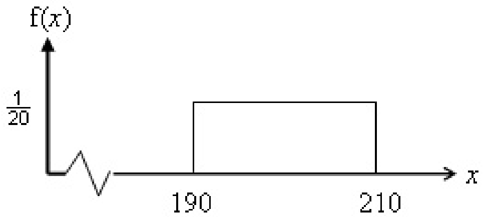
i less than 198 ml,

ii exactly 198 ml.

c Calculate the inter-quartile range of X.

Solution:

a



$$f(x) = \begin{cases} \frac{1}{20} & 190 \le x \le 210 \\ 0 & \text{otherwise} \end{cases}$$

b i $P(X < 198) = (198 - 190) \times \frac{1}{20} = \frac{2}{5}$

ii 0

c $\frac{210-190}{4} = 5$ is one quarter of the range

$Q_1 = 190 + 5 = 195 \qquad Q_3 = 210 - 5 = 205$

$\text{IQR} = 205 - 195 = 10$

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Exercise C, Question 9

Question:

Write down the name of the distribution you would recommend as a suitable model for each of the following situations.

a the difference between the true height and the height measured, to the nearest cm, of randomly chosen people.

b the heights of randomly selected 18-year-old females.

Solution:

a Uniform

b Normal