

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise A, Question 1

Question:

Describe the dimensions of these matrices.

a $\begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$

b $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

c $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

d (1 2 3)

e (3 -1)

f $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Solution:

a $\begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$ is 2×2

b $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is 2×1

c $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & -1 \end{pmatrix}$ is 2×3

d (1 2 3) is 1×3

e (3 -1) is 1×2

f $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 3×3

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Matrix algebra

Exercise A, Question 2

Question:

For the matrices

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix},$$

find

a $\mathbf{A} + \mathbf{C}$

b $\mathbf{B} - \mathbf{A}$

c $\mathbf{A} + \mathbf{B} - \mathbf{C}$.

Solution:

a $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 8 & -1 \\ 1 & 4 \end{pmatrix}$

b $\begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ -2 & -5 \end{pmatrix}$

c $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

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Exercise A, Question 3

Question:

For the matrices

$$A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, B = (1 \ -1), C = (-1 \ 1 \ 0),$$

$$D = (0 \ 1 \ -1), E = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, F = (2 \ 1 \ 3),$$

find where possible:

a $A+B$

b $A-E$

c $F-D+C$

d $B+C$

e $F-(D+C)$

f $A-F$

g $C-(F-D)$.

Solution:

a $A+B$ is $(2 \times 1) + (1 \times 2)$ Not possible

b $A-E = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}.$

c

$$\begin{aligned} F-D+C &= (2 \ 1 \ 3) - (0 \ 1 \ -1) + (-1 \ 1 \ 0) \\ &= (1 \ 1 \ 4) \end{aligned}$$

d $B+C$ is $(1 \times 2) + (1 \times 3)$ Not possible

e

$$\begin{aligned} F-(D+C) &= (2 \ 1 \ 3) - [(0 \ 1 \ -1) + (-1 \ 1 \ 0)] \\ &= (2 \ 1 \ 3) - (-1 \ 2 \ -1) \\ &= (3 \ -1 \ 4) \end{aligned}$$

f $A-F = (2 \times 1) - (1 \times 3)$ Not possible.

g

$$\begin{aligned} C-(F-D) &= (-1 \ 1 \ 0) - [(2 \ 1 \ 3) - (0 \ 1 \ -1)] \\ &= (-1 \ 1 \ 0) - (2 \ 0 \ 4) \\ &= (-3 \ 1 \ -4) \end{aligned}$$

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Exercise A, Question 4

Question:

Given that $\begin{pmatrix} a & 2 \\ -1 & b \end{pmatrix} - \begin{pmatrix} 1 & c \\ d & -2 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$, find the values of the constants a , b , c and d .

Solution:

$$a - 1 = 5 \Rightarrow a = 6$$

$$2 - c = 0 \Rightarrow c = 2$$

$$-1 - d = 0 \Rightarrow d = -1$$

$$b - (-2) = 5 \Rightarrow b = 3$$

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Exercise A, Question 5

Question:

Given that $\begin{pmatrix} 1 & 2 & 0 \\ a & b & c \end{pmatrix} + \begin{pmatrix} a & b & c \\ 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} c & 5 & c \\ c & c & c \end{pmatrix}$, find the values of a , b and c .

Solution:

$$\begin{aligned} 1+a &= c & \textcircled{1} \\ 2+b &= 5 & \Rightarrow b=3 \\ 0+c &= c \\ a+1 &= c \\ b+2 &= c & \textcircled{2} \\ c+0 &= c \end{aligned}$$

$$\text{Use } b=3 \text{ in } \textcircled{2} \Rightarrow c=5$$

$$\text{Use } c=5 \text{ in } \textcircled{1} \Rightarrow a=4$$

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Exercise A, Question 6

Question:

Given that $\begin{pmatrix} 5 & 3 \\ 0 & -1 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} 7 & 1 \\ 2 & 0 \\ 1 & 4 \end{pmatrix}$, find the values of a , b , c , d , e and f .

Solution:

$$\begin{aligned} 5+a &= 7 &\Rightarrow a=2 \\ 3+b &= 1 &\Rightarrow b=-2 \\ 0+c &= 2 &\Rightarrow c=2 \\ -1+d &= 0 &\Rightarrow d=1 \\ 2+e &= 1 &\Rightarrow e=-1 \\ 1+f &= 4 &\Rightarrow f=3 \end{aligned}$$

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Exercise B, Question 1

Question:

For the matrices $A = \begin{pmatrix} 2 & 0 \\ 4 & -6 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, find

a $3A$

b $\frac{1}{2}A$

c $2B$.

Solution:

a $3 \begin{pmatrix} 2 & 0 \\ 4 & -6 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 12 & -18 \end{pmatrix}$

b $\frac{1}{2} \begin{pmatrix} 2 & 0 \\ 4 & -6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & -3 \end{pmatrix}$

c $2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

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Exercise B, Question 2

Question:

Find the value of k and the value of x so that $\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} + k\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 7 \\ x & 0 \end{pmatrix}$.

Solution:

$$\begin{aligned} 1 + 2k &= 7 \\ \Rightarrow 2k &= 6 \\ \Rightarrow k &= 3 \\ 2 - k &= x \\ \Rightarrow 2 - 3 &= x \\ \therefore x &= -1 \end{aligned}$$

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Exercise B, Question 3

Question:

Find the values of a , b , c and d so that $2\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix} - 3\begin{pmatrix} 1 & c \\ d & -1 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ -4 & -4 \end{pmatrix}$.

Solution:

$$\begin{aligned} 2a - 3 &= 3 &\Rightarrow 2a &= 6 \\ &&\Rightarrow a &= 3 \\ 0 - 3c &= 3 &\Rightarrow c &= -1 \\ 2 - 3d &= -4 &\Rightarrow -3d &= -6 \\ &&\Rightarrow d &= 2 \\ 2b + 3 &= -4 &\Rightarrow 2b &= -7 \\ &&\Rightarrow b &= -3.5 \end{aligned}$$

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Exercise B, Question 4

Question:

Find the values of a , b , c and d so that $\begin{pmatrix} 5 & a \\ b & 0 \end{pmatrix} - 2\begin{pmatrix} c & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 9 & 1 \\ 3 & d \end{pmatrix}$.

Solution:

$$\begin{aligned} & 5 - 2c = 9 \\ \Rightarrow & -4 = 2c \\ \Rightarrow & c = -2 \\ & a - 4 = 1 \\ \Rightarrow & a = 5 \\ & b - 2 = 3 \\ \Rightarrow & b = 5 \\ & 0 + 2 = d \\ \Rightarrow & d = 2 \end{aligned}$$

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Exercise B, Question 5

Question:

Find the value of k so that $\begin{pmatrix} -3 \\ k \end{pmatrix} + k \begin{pmatrix} 2k \\ 2k \end{pmatrix} = \begin{pmatrix} k \\ 6 \end{pmatrix}$.

Solution:

$$\begin{aligned} & -3 + 2k^2 = k \\ \implies & 2k^2 - k - 3 = 0 \\ & (2k - 3)(k + 1) = 0 \\ \therefore & k = \frac{3}{2} \text{ or } -1 \end{aligned}$$

$$\begin{aligned} \text{AND } & k + 2k^2 = 6 \\ \implies & 2k^2 + k - 6 = 0 \\ & (2k - 3)(k + 2) = 0 \\ \therefore & k = \frac{3}{2} \text{ or } -2 \end{aligned}$$

So common value is $k = \frac{3}{2}$

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Matrix algebra

Exercise C, Question 1

Question:

Given the dimensions of the following matrices:

Matrix	A	B	C	D	E
Dimension	2×2	1×2	1×3	3×2	2×3

Give the dimensions of these matrix products.

a BA

b DE

c CD

d ED

e AE

f DA

Solution:

a $(1 \times 2) \cdot (2 \times 2) = 1 \times 2$

b $(3 \times 2) \cdot (2 \times 3) = 3 \times 3$

c $(1 \times 3) \cdot (3 \times 2) = 1 \times 2$

d $(2 \times 3) \cdot (3 \times 2) = 2 \times 2$

e $(2 \times 2) \cdot (2 \times 3) = 2 \times 3$

f $(3 \times 2) \cdot (2 \times 2) = 3 \times 2$

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Matrix algebra

Exercise C, Question 2

Question:

Find these products.

a $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

b $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 0 & 5 \\ -1 & -2 \end{pmatrix}$

Solution:

a $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

b $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 0 & 5 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -4 & 7 \end{pmatrix}$

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Exercise C, Question 3

Question:

The matrix $A = \begin{pmatrix} -1 & -2 \\ 0 & 3 \end{pmatrix}$ and the matrix $B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

Find

A^2 means $A \times A$

a AB

b A^2

Solution:

a $\begin{pmatrix} -1 & -2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & -2 & -1 \\ 3 & 3 & 0 \end{pmatrix}$

b $\begin{pmatrix} -1 & -2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} -1 & -2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 0 & 9 \end{pmatrix}$

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Matrix algebra

Exercise C, Question 4

Question:

The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}, \quad \mathbf{C} = (-3 \ -2)$$

Determine whether or not the following products are possible and find the products of those that are.

a AB

b AC

c BC

d BA

e CA

f CB

Solution:

a AB is $(2 \times 1) \cdot (2 \times 2)$ Not possible

$$\mathbf{b AC} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}(-3 \ -2) = \begin{pmatrix} -6 & -4 \\ -3 & -2 \end{pmatrix}$$

c BC is $(2 \times 2) \cdot (1 \times 2)$ Not possible

$$\mathbf{d BA} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$$

$$\mathbf{e CA} = (-3 \ -2) \begin{pmatrix} 2 \\ 1 \end{pmatrix} = (-8).$$

$$\mathbf{f CB} = (-3 \ -2) \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = (-7 \ -7)$$

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Matrix algebra
Exercise C, Question 5

Question:

Find in terms of a $\begin{pmatrix} 2 & a \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 0 \\ 0 & -1 & 2 \end{pmatrix}$.

Solution:

$$\begin{pmatrix} 2 & a \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 0 \\ 0 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 6-a & 2a \\ 1 & 4 & -2 \end{pmatrix}$$

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Matrix algebra

Exercise C, Question 6

Question:

Find in terms of x $\begin{pmatrix} 3 & 2 \\ -1 & x \end{pmatrix} \begin{pmatrix} x & -2 \\ 1 & 3 \end{pmatrix}$.

Solution:

$$\begin{pmatrix} 3 & 2 \\ -1 & x \end{pmatrix} \begin{pmatrix} x & -2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 3x+2 & 0 \\ 0 & 3x+2 \end{pmatrix}$$

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Exercise C, Question 7

Question:

The matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

Find

a A^2

b A^3

c Suggest a form for A^k .

You might be asked to prove this formula for A^k in FP1 using induction from Chapter 6.

Solution:

a $A^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

b $A^3 = AA^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$

Note $A^2 = \begin{pmatrix} 1 & 2 \times 2 \\ 0 & 1 \end{pmatrix}$

$$A^3 = \begin{pmatrix} 1 & 2 \times 3 \\ 0 & 1 \end{pmatrix}$$

Suggests $A^k = \begin{pmatrix} 1 & 2 \times k \\ 0 & 1 \end{pmatrix}$

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Matrix algebra

Exercise C, Question 8

Question:

The matrix $\mathbf{A} = \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}$.

a Find, in terms of a and b , the matrix \mathbf{A}^2 .

Given that $\mathbf{A}^2 = 3\mathbf{A}$

b find the value of a .

Solution:

$$\mathbf{a} \quad \mathbf{A}^2 = \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} = \begin{pmatrix} a^2 & 0 \\ ab & 0 \end{pmatrix}$$

$$\mathbf{b} \quad \mathbf{A}^2 = 3\mathbf{A} \Rightarrow \begin{pmatrix} a^2 & 0 \\ ab & 0 \end{pmatrix} = \begin{pmatrix} 3a & 0 \\ 3b & 0 \end{pmatrix}$$

$$\Rightarrow a^2 = 3a \quad \Rightarrow a = 3 \quad (\text{or } 0)$$

$$\text{and } ab = 3b \quad \Rightarrow a = 3$$
$$\therefore a = 3$$

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Matrix algebra

Exercise C, Question 9

Question:

$$A = \begin{pmatrix} -1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 4 & -2 \\ 0 & -3 \end{pmatrix}.$$

Find **a** BAC

b AC^2

Solution:

a

$$\begin{aligned} BAC &= \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -1 & 3 \end{pmatrix} \begin{pmatrix} 4 & -2 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -4 & -7 \end{pmatrix} \\ &= \begin{pmatrix} -8 & -14 \\ -4 & -7 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

b

$$\begin{aligned} AC^2 &= \begin{pmatrix} -1 & 3 \end{pmatrix} \begin{pmatrix} 4 & -2 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 4 & -2 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} -4 & -7 \end{pmatrix} \begin{pmatrix} 4 & -2 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} -16 & 29 \end{pmatrix} \end{aligned}$$

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Exercise C, Question 10

Question:

$$A = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad B = (3 \quad -2 \quad -3).$$

Find **a** ABA

b BAB

Solution:

a

$$\begin{aligned} ABA &= \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} (3 \quad -2 \quad -3) \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} (-1) \\ &= \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} \end{aligned}$$

b

$$\begin{aligned} BAB &= (3 \quad -2 \quad -3) \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} (3 \quad -2 \quad -3) \\ &= (-1)(3 \quad -2 \quad -3) \\ &= (-3 \quad 2 \quad 3) \end{aligned}$$

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Matrix algebra

Exercise D, Question 1

Question:

Which of the following are not linear transformations?

a P: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2x \\ y+1 \end{pmatrix}$

b Q: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x^2 \\ y \end{pmatrix}$

c R: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2x+y \\ x+xy \end{pmatrix}$

d S: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 3y \\ -x \end{pmatrix}$

e T: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y+3 \\ x+3 \end{pmatrix}$

f U: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2x \\ 3y-2x \end{pmatrix}$

Solution:

a P is not $\because (0,0) \rightarrow (0,1)$

b Q is not $\because x \rightarrow x^2$ is not linear

c R is not $\because y \rightarrow x+xy$ is not linear

d S is linear

e T is not $\because (0,0) \rightarrow (3,3)$

f U is linear.

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Matrix algebra

Exercise D, Question 2

Question:

Identify which of these are linear transformations and give their matrix representations. Give reasons to explain why the other transformations are not linear.

a S: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2x - y \\ 3x \end{pmatrix}$

b T: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2y + 1 \\ x - 1 \end{pmatrix}$

c U: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} xy \\ 0 \end{pmatrix}$

d V: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2y \\ -x \end{pmatrix}$

e W: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ x \end{pmatrix}$

Solution:

a S is represented by $\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$

b T is not linear $\because (0,0) \rightarrow (1, -1)$

c U is not linear $\because x \rightarrow xy$ is not linear

d V is represented by $\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$

e W is represented by $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

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Matrix algebra

Exercise D, Question 3

Question:

Identify which of these are linear transformations and give their matrix representations. Give reasons to explain why the other transformations are not linear.

a S: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x^2 \\ y^2 \end{pmatrix}$

b T: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$

c U: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x-y \\ x-y \end{pmatrix}$

d V: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

e W: $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \end{pmatrix}$

Solution:

a S is not linear $\because x \rightarrow x^2$ and $y \rightarrow y^2$ are not linear

b T is represented by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

c U is represented by $\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$

d V is represented by $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

e W is represented by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

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Matrix algebra

Exercise D, Question 4

Question:

Find matrix representations for these linear transformations.

a $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y + 2x \\ -y \end{pmatrix}$

b $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x + 2y \end{pmatrix}$

Solution:

a $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y + 2x \\ -y \end{pmatrix} = \begin{pmatrix} 2x + y \\ 0x - y \end{pmatrix}$ is represented by $\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$

b $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 0 - y \\ x + 2y \end{pmatrix}$ is represented by $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$

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Matrix algebra

Exercise D, Question 5

Question:

The triangle T has vertices at $(-1, 1)$, $(2, 3)$ and $(5, 1)$.

Find the vertices of the image of T under the transformations represented by these matrices.

a $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

b $\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}$

c $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

Solution:

a $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 & 5 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -5 \\ 1 & 3 & 1 \end{pmatrix}$

\therefore vertices of image of T are at $(1,1)$; $(-1,3)$; $(-5,1)$

b $\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} -1 & 2 & 5 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 14 & 9 \\ -2 & -6 & -2 \end{pmatrix}$

\therefore vertices of image of T are at $(3, -2)$; $(14, -6)$; $(9, -2)$

c $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} -1 & 2 & 5 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -6 & -2 \\ -2 & 4 & 10 \end{pmatrix}$

\therefore vertices of image of T are at $(-2, -2)$; $(-6,4)$; $(-2,10)$

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Matrix algebra

Exercise D, Question 6

Question:

The square S has vertices at $(-1, 0)$, $(0, 1)$, $(1, 0)$ and $(0, -1)$.

Find the vertices of the image of S under the transformations represented by these matrices.

a $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

b $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

c $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Solution:

a $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 2 & 0 \\ 0 & 3 & 0 & -3 \end{pmatrix}$

\therefore vertices of the image of S are $(-2,0)$; $(0,3)$; $(2,0)$; $(0,-3)$

b $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 \end{pmatrix}$

\therefore vertices of the image of S are $(-1,-1)$; $(-1,1)$; $(1,1)$; $(1,-1)$

c $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \end{pmatrix}$

\therefore vertices of the image of S are $(-1,-1)$; $(1,-1)$; $(1,1)$; $(-1,1)$

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Matrix algebra

Exercise E, Question 1

Question:

Describe fully the geometrical transformations represented by these matrices.

a $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

b $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

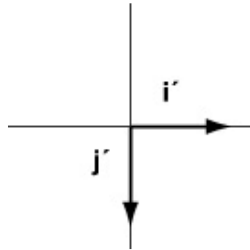
c $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Solution:

a

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

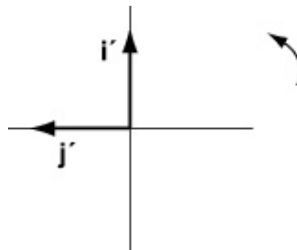


Reflection in x -axis (or line $y = 0$)

b

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

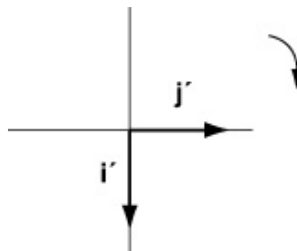


Rotation 90° anticlockwise about $(0,0)$

c

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



Rotation 90° clockwise (or 270° anticlockwise) about $(0,0)$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise E, Question 2

Question:

Describe fully the geometrical transformations represented by these matrices.

$$\mathbf{a} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$\mathbf{b} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

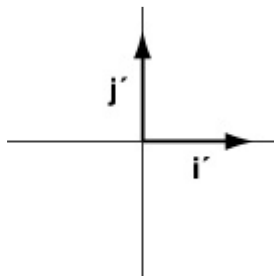
$$\mathbf{c} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Solution:

a

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$$

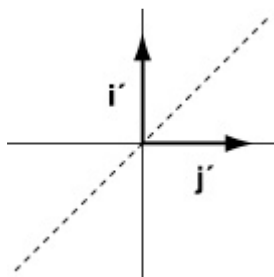


Enlargement - scale factor $\frac{1}{2}$ centre (0,0)

b

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

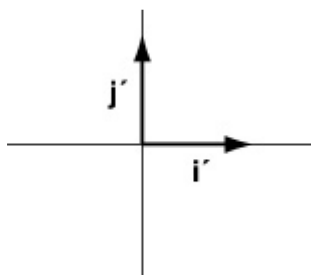


Reflection in line $y=x$

c

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



No change so this is the Identity matrix.

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Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise E, Question 3

Question:

Describe fully the geometrical transformations represented by these matrices.

$$\mathbf{a} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\mathbf{b} \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

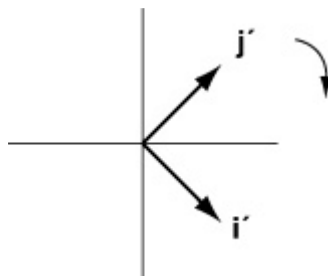
$$\mathbf{c} \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$$

Solution:

a

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

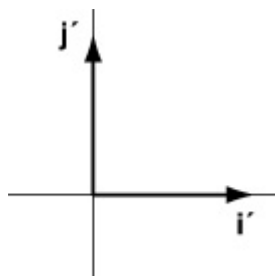


Rotation 45° clockwise about $(0,0)$

b

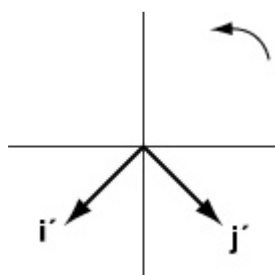
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$



Enlargement Scale factor 4 centre $(0,0)$

c



$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Rotation 225° anti-clockwise about (0,0) or 135° clockwise

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Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise E, Question 4

Question:

Find the matrix that represents these transformations.

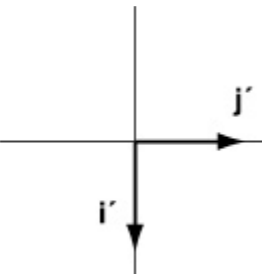
a Rotation of 90° clockwise about $(0, 0)$.

b Reflection in the x -axis.

c Enlargement centre $(0, 0)$ scale factor 2.

Solution:

a

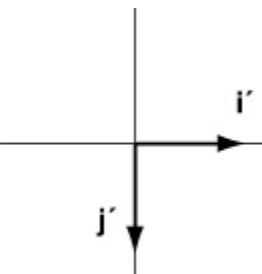


$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

\therefore Matrix is $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

b

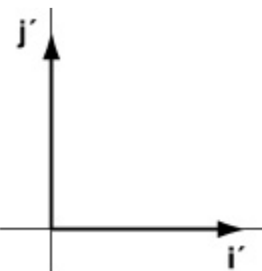


$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

\therefore Matrix is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

c



$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

\therefore Matrix is $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise E, Question 5

Question:

Find the matrix that represents these transformations.

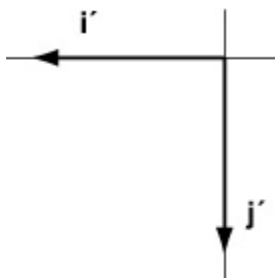
a Enlargement scale factor -4 centre $(0, 0)$.

b Reflection in the line $y = x$.

c Rotation about $(0, 0)$ of 135° anticlockwise.

Solution:

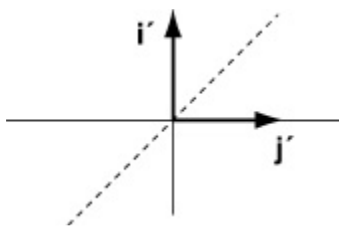
a



$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -4 \end{pmatrix} \quad \therefore \text{Matrix is } \begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix}$$

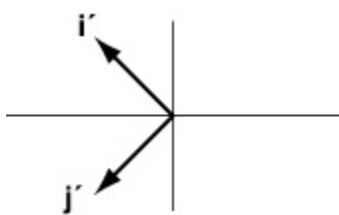
b



$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \therefore \text{Matrix is } \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

c



$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \quad \therefore \text{Matrix is } \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise F, Question 1

Question:

$$A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, C = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Find these matrix products and describe the single transformation represented by the product.

a AB

b BA

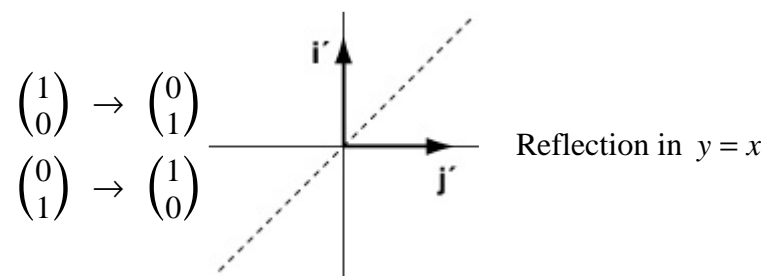
c AC

d A²

e C²

Solution:

$$\mathbf{a} \quad AB = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



$$\mathbf{b} \quad BA = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{Reflection in } y = x$$

$$\mathbf{c} \quad AC = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \quad \text{Enlargement scale factor } -2 \text{ centre } (0,0)$$

$$\mathbf{d} \quad A^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{Identity (No transformation)}$$

[This can be thought of as a rotation of $180^\circ + 180^\circ = 360^\circ$]

$$\mathbf{e} \quad C^2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

Enlargement scale factor 4 centre (0,0)

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise F, Question 2

Question:

A = rotation of 90° anticlockwise about $(0, 0)$

B = rotation of 180° about $(0, 0)$

C = reflection in the x -axis

D = reflection in the y -axis

a Find matrix representations of each of the four transformations A , B , C and D .

b Use matrix products to identify the single geometric transformation represented by each of these combinations.

i Reflection in the x -axis followed by a rotation of 180° about $(0, 0)$.

ii Rotation of 180° about $(0, 0)$ followed by a reflection in the x -axis.

iii Reflection in the y -axis followed by reflection in the x -axis.

iv Reflection in the y -axis followed by rotation of 90° about $(0, 0)$.

v Rotation of 180° about $(0, 0)$ followed by a second rotation of 180° about $(0, 0)$.

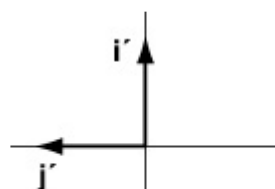
vi Reflection in the x -axis followed by rotation of 90° about $(0, 0)$ followed by a reflection in the y -axis.

vii Reflection in the y -axis followed by rotation of 180° about $(0, 0)$ followed by a reflection in the x -axis.

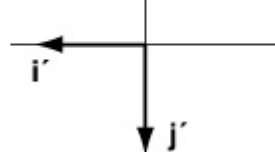
Solution:

a

Rotation of 90° anticlockwise $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$



Rotation of 180° about $(0,0)$ $B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$




Reflection in x -axis

$C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$



Reflection in y-axis

$$D = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$


b

$$\text{i } BC = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (=D)$$

Reflection in y-axis

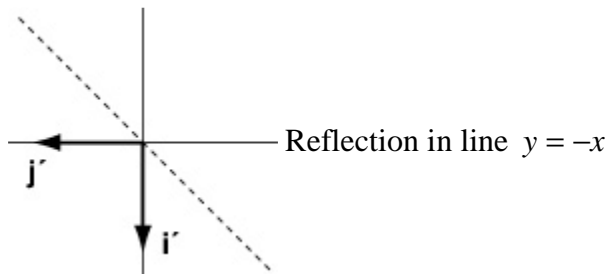
$$\text{ii } CB = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (=D)$$

Reflection in y-axis

$$\text{iii } CD = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad (=B)$$

Rotation of 180° about $(0,0)$

$$\text{iv } AD = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$



$$\text{v } BB = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Rotation of 360° about $(0,0)$ or Identity**vi**

$$\begin{aligned} DAC &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (=A) \end{aligned}$$

Rotation of 90° anticlockwise about $(0,0)$ **vii**

$$\begin{aligned} \text{CBD} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

Identity - no transformation

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Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

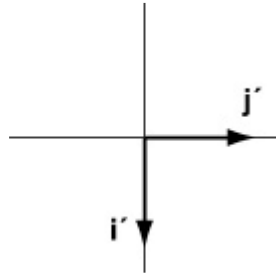
Exercise F, Question 3

Question:

Use a matrix product to find the single geometric transformation represented by a rotation of 270° anticlockwise about $(0, 0)$ followed by a reflection in the x -axis.

Solution:

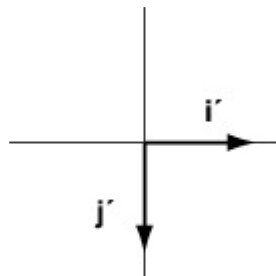
Rotation of 270° about $(0,0)$



$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \therefore \text{Matrix is } \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Reflection in x -axis



$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

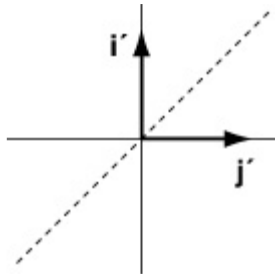
$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad \therefore \text{Matrix is } \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Rotation of 270° followed by reflection in x -axis is:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



Reflection is $y = x$

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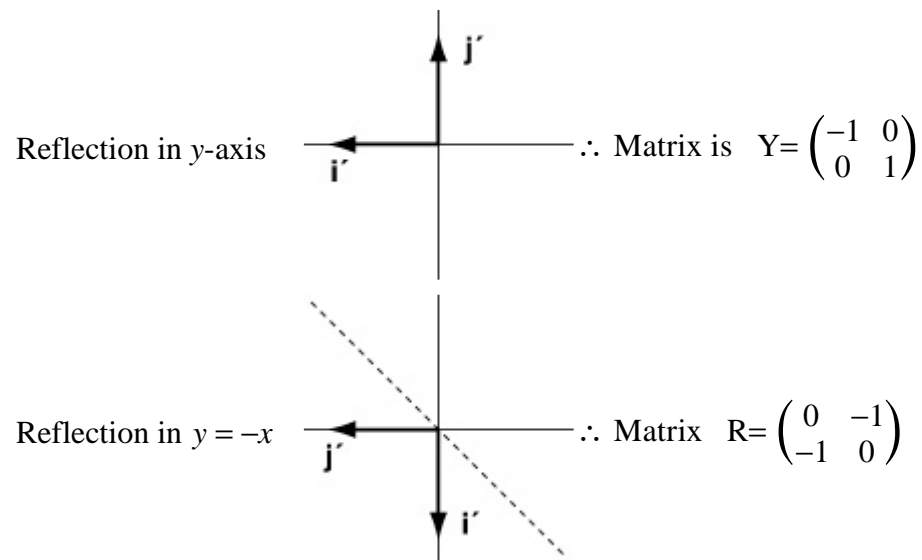
Matrix algebra

Exercise F, Question 4

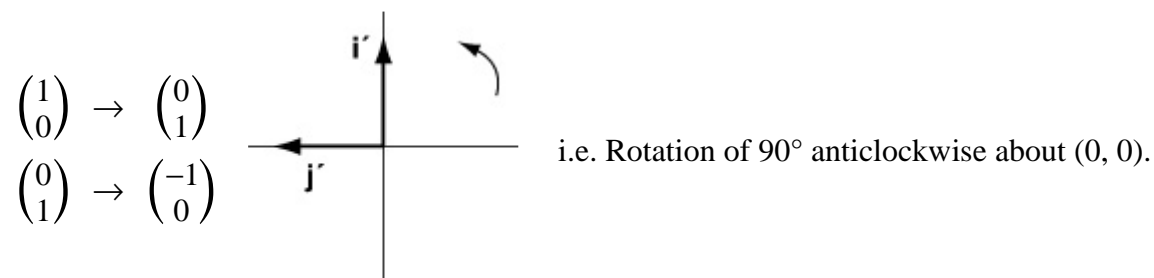
Question:

Use matrices to show that a reflection in the y -axis followed by a reflection in the line $y = -x$ is equivalent to a rotation of 90° anticlockwise about $(0, 0)$.

Solution:



$$RY = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise F, Question 5

Question:

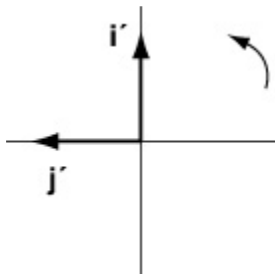
The matrix \mathbf{R} is given by $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$.

- a** Find \mathbf{R}^2 .
- b** Describe the geometric transformation represented by \mathbf{R}^2 .
- c** Hence describe the geometric transformation represented by \mathbf{R} .
- d** Write down \mathbf{R}^8 .

Solution:

$$\mathbf{a} \quad \mathbf{R}^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

b



i.e. \mathbf{R}^2 represents rotation of 90° anticlockwise about $(0, 0)$

c \mathbf{R} represents a rotation of 45° anticlockwise about $(0, 0)$

d \mathbf{R}^8 will represent rotation of $8 \times 45^\circ = 360^\circ$

This is equivalent to no transformation

$$\therefore \mathbf{R}^8 = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise F, Question 6

Question:

$$\mathbf{P} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} -1 & -2 \\ 3 & 5 \end{pmatrix}$$

The transformation represented by the matrix \mathbf{R} is the result of the transformation represented by the matrix \mathbf{P} followed by the transformation represented by the matrix \mathbf{Q} .

a Find \mathbf{R} .

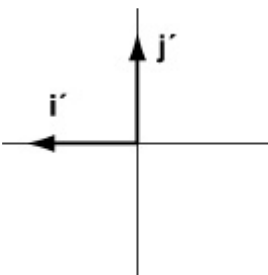
b Give a geometrical interpretation of the transformation represented by \mathbf{R} .

Solution:

$$\mathbf{a} \quad \mathbf{R} = \mathbf{QP} = \begin{pmatrix} -1 & -2 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

b

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$


Reflection in y-axis

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise F, Question 7

Question:

$$\mathbf{A} = \begin{pmatrix} 5 & -7 \\ 7 & -10 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 & 1 \\ -1 & 1 \end{pmatrix}$$

Matrices **A**, **B** and **C** represent three transformations. By combining the three transformations in the order **B**, followed by **A**, followed by **C** a single transformation is obtained.

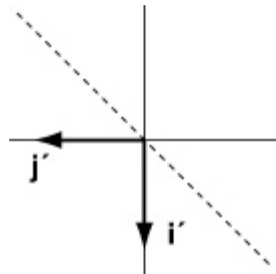
Find a matrix representation of this transformation and interpret it geometrically.

Solution:

$$\begin{aligned} \mathbf{CAB} &= \begin{pmatrix} -2 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 5 & -7 \\ 7 & -10 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix} \\ &= \begin{pmatrix} -3 & 4 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$



Reflection in the line $y = -x$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise F, Question 8

Question:

$$\mathbf{P} = \begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix}$$

Matrices \mathbf{P} , \mathbf{Q} and \mathbf{R} represent three transformations. By combining the three transformations in the order \mathbf{R} , followed by \mathbf{Q} , followed by \mathbf{P} a single transformation is obtained.

Find a matrix representation of this transformation and interpret it geometrically.

Solution:

$$\begin{aligned} \mathbf{PQR} &= \begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 2 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} \end{aligned}$$

Enlargement scale factor 8

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Matrix algebra

Exercise G, Question 1

Question:

Determine which of these matrices are singular and which are non-singular. For those that are non-singular find the inverse matrix.

a $\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$

b $\begin{pmatrix} 3 & 3 \\ -1 & -1 \end{pmatrix}$

c $\begin{pmatrix} 2 & 5 \\ 0 & 0 \end{pmatrix}$

d $\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$

e $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}$

f $\begin{pmatrix} 4 & 3 \\ 6 & 2 \end{pmatrix}$

Solution:

a

$$\begin{aligned} \det \begin{vmatrix} 3 & -1 \\ -4 & 2 \end{vmatrix} &= 6 - (-4) \times (-1) \\ &= 6 - 4 \\ &= 2 \neq 0 \end{aligned}$$

\therefore the Matrix is non-singular

So inverse is $\frac{1}{2} \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

or $\begin{pmatrix} 1 & 0.5 \\ 2 & 1.5 \end{pmatrix}$

b

$$\begin{aligned} \det \begin{vmatrix} 3 & 3 \\ -1 & -1 \end{vmatrix} &= -3 - (-1) \times 3 \\ &= -3 + 3 \\ &= 0 \end{aligned}$$

\therefore Matrix is singular.

c

$$\begin{aligned} \det \begin{vmatrix} 2 & 5 \\ 0 & 0 \end{vmatrix} &= 0 - 0 \\ &= 0 \end{aligned}$$

∴ Matrix is singular

d

$$\begin{aligned}\det \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} &= 5 - 6 \\ &= -1 \neq 0\end{aligned}$$

∴ Matrix is non-singular

$$\text{Inverse is } \frac{1}{-1} \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$$

e

$$\begin{aligned}\det \begin{vmatrix} 6 & 3 \\ 4 & 2 \end{vmatrix} &= 12 - 12 \\ &= 0\end{aligned}$$

∴ Matrix is singular

f

$$\begin{aligned}\det \begin{vmatrix} 4 & 3 \\ 6 & 2 \end{vmatrix} &= 8 - 18 \\ &= -10 \neq 0\end{aligned}$$

∴ Matrix is non-singular

$$\begin{aligned}\text{Inverse is } \frac{1}{-10} \begin{pmatrix} 2 & -3 \\ -6 & 4 \end{pmatrix} \\ &= \begin{pmatrix} -0.2 & 0.3 \\ 0.6 & -0.4 \end{pmatrix}\end{aligned}$$

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Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise G, Question 2

Question:

Find the value of a for which these matrices are singular.

a $\begin{pmatrix} a & 1+a \\ 3 & 2 \end{pmatrix}$

b $\begin{pmatrix} 1+a & 3-a \\ a+2 & 1-a \end{pmatrix}$

c $\begin{pmatrix} 2+a & 1-a \\ 1-a & a \end{pmatrix}$

Solution:

a

$$\begin{aligned} \det \begin{vmatrix} a & 1+a \\ 3 & 2 \end{vmatrix} &= 2a - 3(1+a) \\ &= 2a - 3 - 3a \\ &= -3 - a \end{aligned}$$

Matrix is singular for $a = -3$

b

$$\begin{aligned} \text{Let } A &= \begin{pmatrix} 1+a & 3-a \\ a+2 & 1-a \end{pmatrix} \\ \det A &= (1+a)(1-a) - (3-a)(a+2) \\ &= 1 - a^2 - (-a^2 + a + 6) \\ &= 1 - a^2 + a^2 - a - 6 \\ &= -a - 5 \\ \det A &= 0 \Rightarrow a = -5 \end{aligned}$$

c

$$\begin{aligned} \text{Let } B &= \begin{pmatrix} 2+a & 1-a \\ 1-a & a \end{pmatrix} \\ \det B &= 2a + a^2 - (1-a)^2 \\ &= 2a + a^2 - 1 + 2a - a^2 \\ &= 4a - 1 \\ \det B &= 0 \Rightarrow a = \frac{1}{4} \end{aligned}$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise G, Question 3

Question:

Find inverses of these matrices.

$$\mathbf{a} \begin{pmatrix} a & 1+a \\ 1+a & 2+a \end{pmatrix}$$

$$\mathbf{b} \begin{pmatrix} 2a & 3b \\ -a & -b \end{pmatrix}$$

Solution:

a

$$\text{Let } A = \begin{pmatrix} a & 1+a \\ 1+a & 2+a \end{pmatrix}$$

$$\begin{aligned} \det A &= 2a + a^2 - (1+a)^2 \\ &= 2a + a^2 - 1 - 2a - a^2 \\ &= -1 \end{aligned}$$

$$A^{-1} = \frac{1}{-1} \begin{pmatrix} 2+a & -(1+a) \\ -(1+a) & a \end{pmatrix} = \begin{pmatrix} -[2+a] & (1+a) \\ (1+a) & -a \end{pmatrix}$$

b

$$\text{Let } B = \begin{pmatrix} 2a & 3b \\ -a & -b \end{pmatrix}$$

$$\begin{aligned} \det B &= -2ab - (-a) \times 3b \\ &= -2ab + 3ab \\ &= ab \end{aligned}$$

$$\begin{aligned} B^{-1} &= \frac{1}{ab} \begin{pmatrix} -b & -3b \\ a & 2a \end{pmatrix} \\ &= \begin{pmatrix} -\frac{1}{a} & -\frac{3}{a} \\ \frac{1}{b} & \frac{2}{b} \end{pmatrix} \text{ provided that } ab \neq 0 \end{aligned}$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise G, Question 4

Question:

a Given that $ABC = I$, prove that $B^{-1} = CA$.

b Given that $A = \begin{pmatrix} 0 & 1 \\ -1 & -6 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}$, find B .

Solution:

a

$$\begin{aligned} ABC &= I \\ \Rightarrow A^{-1}ABC &= A^{-1}I \\ \Rightarrow BC &= A^{-1} \\ \Rightarrow BCC^{-1} &= A^{-1}C^{-1} \\ \Rightarrow B &= A^{-1}C^{-1} = (CA)^{-1} \\ \therefore B^{-1} &= CA \end{aligned}$$

b

$$\begin{aligned} CA &= \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & -6 \end{pmatrix} \\ &= \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix} \\ \therefore (CA)^{-1} &= \frac{1}{-3+4} \begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix} \\ \therefore B &= \begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix} \end{aligned}$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise G, Question 5

Question:

a Given that $\mathbf{AB} = \mathbf{C}$, find an expression for \mathbf{B} .

b Given further that $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 6 \\ 1 & 22 \end{pmatrix}$, find \mathbf{B} .

Solution:

a

$$\begin{aligned} \mathbf{AB} &= \mathbf{C} \\ \Rightarrow \mathbf{A}^{-1}\mathbf{AB} &= \mathbf{A}^{-1}\mathbf{C} \\ \Rightarrow \mathbf{B} &= \mathbf{A}^{-1}\mathbf{C} \end{aligned}$$

b

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix} \Rightarrow \det \mathbf{A} = 6 - -4 = 10 \\ \therefore \mathbf{A}^{-1} &= \frac{1}{10} \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix} \\ \therefore \mathbf{B} &= \mathbf{A}^{-1}\mathbf{C} \\ &= \frac{1}{10} \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} 3 & 6 \\ 1 & 22 \end{pmatrix} \\ &= \frac{1}{10} \begin{pmatrix} 10 & 40 \\ -10 & 20 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 4 \\ -1 & 2 \end{pmatrix} \end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise G, Question 6

Question:

a Given that $\mathbf{BAC} = \mathbf{B}$, where \mathbf{B} is a non-singular matrix, find an expression for \mathbf{A} .

b When $\mathbf{C} = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$, find \mathbf{A} .

Solution:

a

$$\begin{aligned} \mathbf{BAC} &= \mathbf{B} \\ \Rightarrow \mathbf{B}^{-1}\mathbf{BAC} &= \mathbf{B}^{-1}\mathbf{B} \\ \Rightarrow \mathbf{AC} &= \mathbf{I} \\ \Rightarrow \mathbf{A} &= \mathbf{C}^{-1} \end{aligned}$$

b

$$\begin{aligned} \mathbf{C} &= \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix} \\ \det \mathbf{C} &= 10 - 9 = 1 \\ \therefore \mathbf{C}^{-1} &= \frac{1}{1} \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix} \\ \therefore \mathbf{A} &= \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix} \end{aligned}$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise G, Question 7

Question:

The matrix $A = \begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}$ and $AB = \begin{pmatrix} 4 & 7 & -8 \\ -8 & -13 & 18 \end{pmatrix}$. Find the matrix B .

Solution:

$$A = \begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix} \Rightarrow \det A = 6 - (-4) \times (-1) = 2$$

$$\therefore A^{-1} = \frac{1}{2} \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 4 & 7 & -8 \\ -8 & -13 & 18 \end{pmatrix} \quad (\times \text{ on left by } A^{-1})$$

$$\Rightarrow B = \frac{1}{2} \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 4 & 7 & -8 \\ -8 & -13 & 18 \end{pmatrix}$$

$$B = \frac{1}{2} \begin{pmatrix} 4 & 8 & -6 \\ 0 & 2 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 4 & -3 \\ 0 & 1 & 2 \end{pmatrix}$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise G, Question 8

Question:

The matrix $\mathbf{B} = \begin{pmatrix} 5 & -4 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} 11 & -1 \\ -8 & 9 \\ -2 & -1 \end{pmatrix}$. Find the matrix \mathbf{A} .

Solution:

$$\mathbf{B} = \begin{pmatrix} 5 & -4 \\ 2 & 1 \end{pmatrix} \Rightarrow \det \mathbf{B} = 5 + 8 = 13$$

$$\mathbf{B}^{-1} = \frac{1}{13} \begin{pmatrix} 1 & 4 \\ -2 & 5 \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} 11 & -1 \\ -8 & 9 \\ -2 & -1 \end{pmatrix} \quad (\times \text{ on right by } \mathbf{B}^{-1})$$

$$\Rightarrow \mathbf{ABB}^{-1} = \begin{pmatrix} 11 & -1 \\ -8 & 9 \\ -2 & -1 \end{pmatrix} \mathbf{B}^{-1}$$

$$\begin{aligned} \therefore \mathbf{A} &= \frac{1}{13} \begin{pmatrix} 11 & -1 \\ -8 & 9 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ -2 & 5 \end{pmatrix} \\ &= \frac{1}{13} \begin{pmatrix} 13 & 39 \\ -26 & 13 \\ 0 & -13 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 3 \\ -2 & 1 \\ 0 & -1 \end{pmatrix} \end{aligned}$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise G, Question 9

Question:

The matrix $A = \begin{pmatrix} 3a & b \\ 4a & 2b \end{pmatrix}$, where a and b are non-zero constants.

a Find A^{-1} .

The matrix $B = \begin{pmatrix} -a & b \\ 3a & 2b \end{pmatrix}$ and the matrix X is given by $B = XA$.

b Find X .

Solution:

a

$$A = \begin{pmatrix} 3a & b \\ 4a & 2b \end{pmatrix} \Rightarrow \det A = 6ab - 4ab = 2ab$$

$$\therefore A^{-1} = \frac{1}{2ab} \begin{pmatrix} 2b & -b \\ -4a & 3a \end{pmatrix}$$

b

$$B = XA$$

$$\Rightarrow BA^{-1} = XAA^{-1}$$

$$\therefore X = BA^{-1}$$

$$\text{So } X = \begin{pmatrix} -a & b \\ 3a & 2b \end{pmatrix} \begin{pmatrix} 2b & -b \\ -4a & 3a \end{pmatrix} \times \frac{1}{2ab}$$

$$= \frac{1}{2ab} \begin{pmatrix} -6ab & 4ab \\ -2ab & 3ab \end{pmatrix}$$

$$\therefore X = \begin{pmatrix} -3 & 2 \\ -1 & 3/2 \end{pmatrix}$$

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Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise G, Question 10

Question:

The matrix $\mathbf{A} = \begin{pmatrix} a & 2a \\ b & 2b \end{pmatrix}$ and the matrix $\mathbf{B} = \begin{pmatrix} 2b & -2a \\ -b & a \end{pmatrix}$.

a Find $\det(\mathbf{A})$ and $\det(\mathbf{B})$.

b Find \mathbf{AB} .

Solution:

a

$$\mathbf{A} = \begin{pmatrix} a & 2a \\ b & 2b \end{pmatrix} \Rightarrow \det \mathbf{A} = 2ab - 2ab = 0$$

$$\mathbf{B} = \begin{pmatrix} 2b & -2a \\ -b & a \end{pmatrix} \Rightarrow \det \mathbf{B} = 2ab - 2ab = 0$$

b

$$\begin{aligned} \mathbf{AB} &= \begin{pmatrix} a & 2a \\ b & 2b \end{pmatrix} \begin{pmatrix} 2b & -2a \\ -b & a \end{pmatrix} \\ &= \begin{pmatrix} 2ab - 2ab & -2a^2 + 2a^2 \\ 2b^2 - 2b^2 & -2ab + 2ab \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise G, Question 11

Question:

The non-singular matrices \mathbf{A} and \mathbf{B} are commutative (i.e. $\mathbf{AB} = \mathbf{BA}$) and $\mathbf{ABA} = \mathbf{B}$.

a Prove that $\mathbf{A}^2 = \mathbf{I}$.

Given that $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, by considering a matrix \mathbf{B} of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

b show that $a = d$ and $b = c$.

Solution:

a

$$\begin{aligned} & \text{Given } \mathbf{AB} = \mathbf{BA} \\ & \text{and } \mathbf{ABA} = \mathbf{B} \\ \Rightarrow & \mathbf{A}(\mathbf{AB}) = \mathbf{B} \\ \Rightarrow & \mathbf{A}^2 \mathbf{B} = \mathbf{B} \\ \Rightarrow & \mathbf{A}^2 \mathbf{B} \mathbf{B}^{-1} = \mathbf{B} \mathbf{B}^{-1} \\ \Rightarrow & \mathbf{A}^2 = \mathbf{I} \end{aligned}$$

b

$$\mathbf{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} c & d \\ a & b \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} b & a \\ d & c \end{pmatrix}$$

$$\mathbf{AB} = \mathbf{BA} \Rightarrow \begin{aligned} b &= c \\ d &= a \end{aligned}$$

i.e. $a = d$ and $b = c$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise H, Question 1

Question:

The matrix $\mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

a Give a geometrical interpretation of the transformation represented by \mathbf{R} .

b Find \mathbf{R}^{-1} .

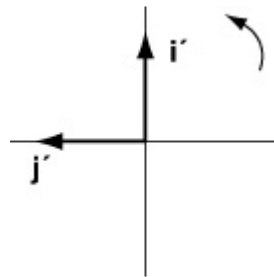
c Give a geometrical interpretation of the transformation represented by \mathbf{R}^{-1} .

Solution:

a

$$(1,0) \rightarrow (0,1)$$

$$(0,1) \rightarrow (-1,0)$$



\mathbf{R} represents a rotation of 90° anticlockwise about $(0, 0)$

b

$$\det \mathbf{R} = 0 - (-1) = 1$$

$$\therefore \mathbf{R}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

c \mathbf{R}^{-1} represents a rotation of -90° anticlockwise about $(0,0)$

(or ... 90° clockwise ... or ... 270° anticlockwise ...)

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise H, Question 2

Question:

a The matrix $\mathbf{S} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

i Give a geometrical interpretation of the transformation represented by \mathbf{S} .

ii Show that $\mathbf{S}^2 = \mathbf{I}$.

iii Give a geometrical interpretation of the transformation represented by \mathbf{S}^{-1} .

b The matrix $\mathbf{T} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

i Give a geometrical interpretation of the transformation represented by \mathbf{T} .

ii Show that $\mathbf{T}^2 = \mathbf{I}$.

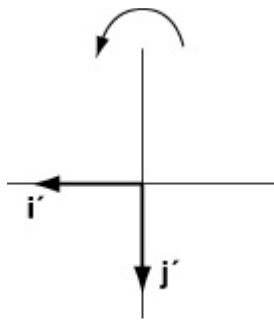
iii Give a geometrical interpretation of the transformation represented by \mathbf{T}^{-1} .

c Calculate $\det(\mathbf{S})$ and $\det(\mathbf{T})$ and comment on their values in the light of the transformations they represent.

Solution:

a i

$$\begin{aligned} (1,0) &\rightarrow (-1,0) \\ (0,1) &\rightarrow (0,-1) \end{aligned}$$



\mathbf{S} represents a rotation of 180° about $(0,0)$

ii \mathbf{S}^2 will be a rotation of $180 + 180 = 360^\circ$ about $(0,0)$ $\therefore \mathbf{S}^2 = \mathbf{I}$

or $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$

iii $\mathbf{S}^{-1} = \mathbf{S}$ = rotation of 180° about $(0,0)$

b i

$$(1,0) \rightarrow (0,-1)$$

$$(0,1) \rightarrow (-1,0)$$

\mathbf{T} represents a reflection in the line $y = -x$

$$\text{ii } \mathbf{T}^2 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$$

$$\text{iii } \mathbf{T}^{-1} = \mathbf{T} = \text{reflection in the line } y = -x$$

c

$$\det \mathbf{S} = 1 - 0 = 1$$

$$\det \mathbf{T} = 0 - 1 = -1$$

For both \mathbf{S} and \mathbf{T} , area is unaltered

\mathbf{T} represents a reflection and \therefore has a negative determinant. Orientation is reversed

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Matrix algebra

Exercise H, Question 3

Question:

The matrix **A** represents a reflection in the line $y = x$ and the matrix **B** represents a rotation of 270° about $(0, 0)$.

a Find the matrix $\mathbf{C} = \mathbf{BA}$ and interpret it geometrically.

b Find \mathbf{C}^{-1} and give a geometrical interpretation of the transformation represented by \mathbf{C}^{-1} .

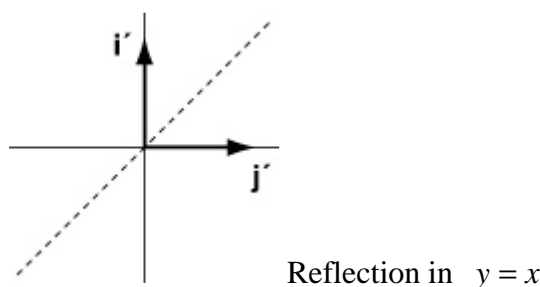
c Find the matrix $\mathbf{D} = \mathbf{AB}$ and interpret it geometrically.

d Find \mathbf{D}^{-1} and give a geometrical interpretation of the transformation represented by \mathbf{D}^{-1} .

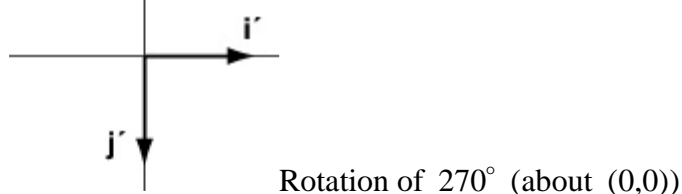
Solution:

a

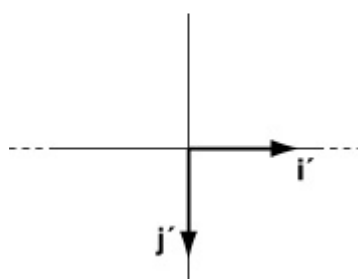
$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$



$$\mathbf{C} = \mathbf{BA} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

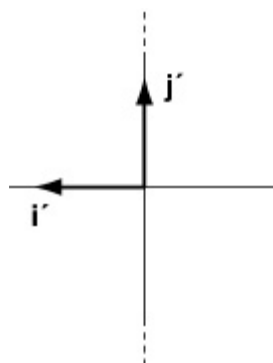


C represents a reflection in the line $y = 0$ (or the x -axis)

b $\mathbf{C}^{-1} = \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is a reflection in the line $y = 0$

c

$$\mathbf{D} = \mathbf{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$



\mathbf{D} represents a reflection in the line $x = 0$ (or the y -axis)

$\mathbf{d} \mathbf{D}^{-1} = \mathbf{D} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ is a reflection in the line $x = 0$

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Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise I, Question 1

Question:

The matrix $A = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$ is used to transform the rectangle R with vertices at the points $(0, 0)$, $(0, 1)$, $(4, 1)$ and $(4, 0)$.

a Find the coordinates of the vertices of the image of R .

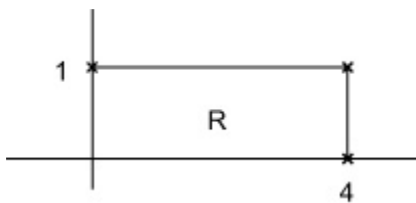
b Calculate the area of the image of R .

Solution:

$$\mathbf{a} \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 4 & 4 \\ 0 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 7 & 8 \\ 0 & 3 & 19 & 16 \end{pmatrix}$$

Coordinates of image are: $(0,0)$; $(-1,3)$; $(7,19)$; $(8,16)$

b



$$\text{Area of } R = 4 \times 1 = 4$$

$$\det \mathbf{A} = 6 - (-4) = 10$$

$$\begin{aligned} \therefore \text{Area of image} &= 10 \times 4 \\ &= 40. \end{aligned}$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise I, Question 2

Question:

The triangle T has vertices at the points $(-3.5, 2.5)$, $(-16, 10)$ and $(-7, 4)$.

a Find the coordinates of the vertices of T under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} -1 & -1 \\ 3 & 5 \end{pmatrix}$.

b Show that the area of the image of T is 7.5.

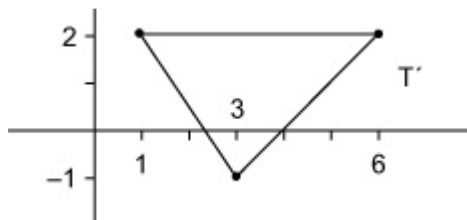
c Hence find the area of T .

Solution:

$$\mathbf{a} \begin{pmatrix} -1 & -1 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} -3.5 & -16 & -7 \\ 2.5 & 10 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 6 & 3 \\ 2 & 2 & -1 \end{pmatrix}$$

Coordinates of T' are $(1,2)$; $(6,2)$; $(3,-1)$

b



$$\text{Area of } T' = \frac{1}{2} \times 5 \times 3 = 7.5$$

c

$$\det \mathbf{M} = -5 + 3 = -2$$

$$\therefore \text{Area of } T \times |-2| = \text{Area of } T'$$

$$\Rightarrow \text{Area of } T = \frac{7.5}{2} \\ = 3.75$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise I, Question 3

Question:

The rectangle R has vertices at the points $(-1, 0)$, $(0, -3)$, $(4, 0)$ and $(3, 3)$.

The matrix $A = \begin{pmatrix} -2 & 3-a \\ 1 & a \end{pmatrix}$, where a is a constant.

a Find, in terms of a , the coordinates of the vertices of the image of R under the transformation given by A .

b Find $\det(A)$, leaving your answer in terms of a .

Given that the area of the image of R is 75

c find the positive value of a .

Solution:

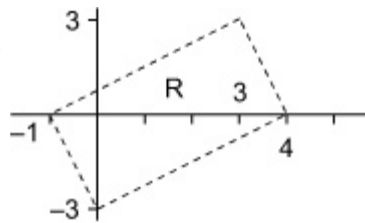
$$\mathbf{a} \begin{pmatrix} -2 & 3-a \\ 1 & a \end{pmatrix} \begin{pmatrix} -1 & 0 & 4 & 3 \\ 0 & -3 & 0 & 3 \end{pmatrix} = \begin{pmatrix} +2 & 3a-9 & -8 & 3-3a \\ -1 & -3a & 4 & 3+3a \end{pmatrix}$$

Image of R is : $(+2, -1); (3a-9, -3a); (-8, 4); (3-3a, 3+3a)$

b

$$\begin{aligned} \det A &= -2a - 3 + a \\ &= -a - 3 \end{aligned}$$

$$\begin{aligned} \text{Area of } R &= \left(\frac{1}{2} \times 5 \times 3\right) \times 2 \\ &= 15 \end{aligned}$$



c

$$\text{Area of } R \times |\det A| = 75$$

$$\therefore |\det A| = \frac{75}{15} = 5$$

$$\text{So } |-a-3| = 5$$

$$\Rightarrow -a-3 = 5 \text{ or } a+3 = 5$$

\therefore positive value of $a = 2$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise I, Question 4

Question:

$$\mathbf{P} = \begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$$

A rectangle of area 5 cm^2 is transformed by the matrix \mathbf{X} . Find the area of the image of the rectangle when \mathbf{X} is:

a \mathbf{P}

b \mathbf{Q}

c \mathbf{R}

d \mathbf{RQ}

e \mathbf{QR}

f \mathbf{RP}

Solution:

a $\det \mathbf{P} = 2 + 12 = 14 \quad \therefore \text{area of image is } 70 \text{ cm}^2$

b $\det \mathbf{Q} = 4 + 2 = 6 \quad \therefore \text{area of image is } 30 \text{ cm}^2$

c $\det \mathbf{R} = 1 - 4 = -3 \quad \therefore \text{area of image is } 15 \text{ cm}^2$

d $\det \mathbf{RQ} = \det \mathbf{R} \times \det \mathbf{Q} = -18 \quad \therefore \text{area of image is } 90 \text{ cm}^2$

e $\det \mathbf{QR} = \det \mathbf{Q} \times \det \mathbf{R} = -18 \quad \therefore \text{area of image is } 90 \text{ cm}^2$

f $\det \mathbf{RP} = \det \mathbf{R} \times \det \mathbf{P} = -42 \quad \therefore \text{area of image is } 210 \text{ cm}^2$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise I, Question 5

Question:

The triangle T has area 6 cm^2 and is transformed by the matrix $\begin{pmatrix} a & 3 \\ 3 & a+2 \end{pmatrix}$, where a is a constant, into triangle T' .

a Find $\det(\mathbf{A})$ in terms of a .

Given that the area of T' is 36 cm^2

b find the possible values of a .

Solution:

a

$$\begin{aligned} \det \mathbf{A} &= a(a+2) - 9 \\ &= a^2 + 2a - 9 \end{aligned}$$

b

$$\text{Area of } T \times |\det \mathbf{A}| = \text{Area of } T'$$

$$\therefore 6 \times |\det \mathbf{A}| = 36$$

$$\therefore \det \mathbf{A} = \pm 6$$

$$\Rightarrow a^2 + 2a - 9 = 6$$

$$a^2 + 2a - 15 = 0$$

$$(a+5)(a-3) = 0$$

$$\therefore a = 3 \text{ or } -5$$

or

$$\Rightarrow a^2 + 2a - 9 = -6$$

$$a^2 + 2a - 3 = 0$$

$$(a+3)(a-1) = 0$$

$$a = 1 \text{ or } -3$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise J, Question 1

Question:

Use inverse matrices to solve the following simultaneous equations

a $7x + 3y = 6$

$$-5x - 2y = -5$$

b $4x - y = -1$

$$-2x + 3y = 8$$

Solution:

$$\mathbf{a} \begin{pmatrix} 7 & 3 \\ -5 & -2 \end{pmatrix} = \mathbf{A} \quad \Rightarrow \quad \det \mathbf{A} = -14 + 15 = 1$$

$$\therefore \mathbf{A}^{-1} = \frac{1}{1} \begin{pmatrix} -2 & -3 \\ 5 & 7 \end{pmatrix}$$

$$\therefore \mathbf{A} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ -5 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} 6 \\ -5 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & -3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 6 \\ -5 \end{pmatrix}$$

$$= \begin{pmatrix} -12 + 15 \\ 30 - 35 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

$$\therefore x = 3, y = -5$$

$$\mathbf{b} \mathbf{B} = \begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix} \Rightarrow \det \mathbf{B} = 12 - (-2)(-1) = 10$$

$$\therefore \mathbf{B}^{-1} = \frac{1}{10} \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$$

$$\therefore \mathbf{B} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \end{pmatrix} \Rightarrow \mathbf{B}^{-1} \begin{pmatrix} -1 \\ 8 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\text{So } \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ 8 \end{pmatrix}$$

$$= \frac{1}{10} \begin{pmatrix} -3 + 8 \\ -2 + 32 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 3 \end{pmatrix}$$

$$\therefore x = 0.5, y = 3$$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise J, Question 2

Question:

Use inverse matrices to solve the following simultaneous equations

a $4x - y = 11$

$$3x + 2y = 0$$

b $5x + 2y = 3$

$$3x + 4y = 13$$

Solution:

a $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix} \Rightarrow \det \mathbf{A} = 8 + 3 = 11$

$$\therefore \mathbf{A}^{-1} = \frac{1}{11} \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$$

So $\mathbf{A} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} 11 \\ 0 \end{pmatrix}$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 11 \\ 0 \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 22 \\ -33 \end{pmatrix}$$

$$\therefore x = 2, y = -3$$

b $\mathbf{B} = \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix} \Rightarrow \det \mathbf{B} = 20 - 6 = 14$

$$\therefore \mathbf{B}^{-1} = \frac{1}{14} \begin{pmatrix} 4 & -2 \\ -3 & 5 \end{pmatrix}$$

So $\mathbf{B} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{B}^{-1} \begin{pmatrix} 3 \\ 13 \end{pmatrix}$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 4 & -2 \\ -3 & 5 \end{pmatrix} \begin{pmatrix} 3 \\ 13 \end{pmatrix}$$

$$= \frac{1}{14} \begin{pmatrix} 12 - 26 \\ -9 + 65 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} -14 \\ 56 \end{pmatrix}$$

$$\therefore x = -1, y = 4$$

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Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise K, Question 1

Question:

The matrix $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$ transforms the triangle PQR into the triangle with coordinates $(6, -2)$, $(4, 4)$, $(0, 8)$.

Find the coordinates of P , Q and R .

Solution:

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} \Rightarrow \det \mathbf{A} = 6 - 4 = 2.$$

$$\therefore \mathbf{A}^{-1} = \frac{1}{2} \begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}$$

$$\mathbf{A}(\Delta PQR) = \begin{pmatrix} 6 & 4 & 0 \\ -2 & 4 & 8 \end{pmatrix}$$

$$\begin{aligned} \therefore \Delta PQR \text{ given by } & \frac{1}{2} \begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 6 & 4 & 0 \\ -2 & 4 & 8 \end{pmatrix} \\ & = \frac{1}{2} \begin{pmatrix} 14 & 4 & -8 \\ -30 & -4 & 24 \end{pmatrix} \\ & = \begin{pmatrix} 7 & 2 & -4 \\ -15 & -2 & 12 \end{pmatrix} \end{aligned}$$

$\therefore P$ is $(7, -15)$, Q is $(2, -2)$, R is $(-4, 12)$

Solutionbank FP1

Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise K, Question 2

Question:

The matrix $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} 4 & 1 & 9 \\ 1 & 9 & 4 \end{pmatrix}$.

Find the matrix \mathbf{B} .

Solution:

$$\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix} \Rightarrow \det \mathbf{A} = 1 + 6 = 7$$

$$\therefore \mathbf{A}^{-1} = \frac{1}{7} \begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}$$

$$\mathbf{A}^{-1}(\mathbf{AB}) = \frac{1}{7} \begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 1 & 9 \\ 1 & 9 & 4 \end{pmatrix}$$

$$\therefore \mathbf{B} = \frac{1}{7} \begin{pmatrix} 7 & 28 & 21 \\ -7 & 7 & -14 \end{pmatrix}$$

$$\therefore \mathbf{B} = \begin{pmatrix} 1 & 4 & 3 \\ -1 & 1 & -2 \end{pmatrix}$$

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Matrix algebra

Exercise K, Question 3

Question:

$$A = \begin{pmatrix} -2 & 1 \\ 7 & -3 \end{pmatrix}, B = \begin{pmatrix} 4 & 1 \\ -5 & -1 \end{pmatrix}, C = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}.$$

The matrices **A**, **B** and **C** represent three transformations. By combining the three transformations in the order **A**, followed by **B**, followed by **C**, a simple single transformation is obtained which is represented by the matrix **R**.

a Find **R**.

b Give a geometrical interpretation of the transformation represented by **R**.

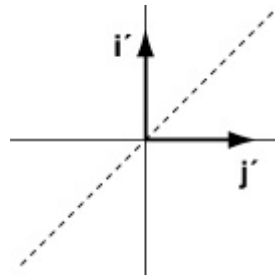
c Write down the matrix \mathbf{R}^2 .

Solution:

a

$$\begin{aligned} \mathbf{R} &= \mathbf{C} \mathbf{B} \mathbf{A} \\ &= \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ -5 & -1 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 7 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix} \end{aligned}$$

$$\mathbf{R} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



b **R** represents a reflection in the line $y = x$

c $\mathbf{R}^2 = \mathbf{I}$

Since repeating a reflection twice returns an object to its original position.

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Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise K, Question 4

Question:

The matrix \mathbf{Y} represents a rotation of 90° about $(0, 0)$.

a Find \mathbf{Y} .

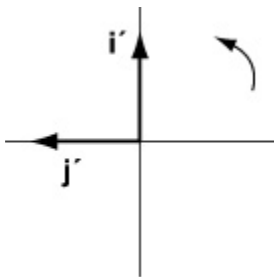
The matrices \mathbf{A} and \mathbf{B} are such that $\mathbf{AB} = \mathbf{Y}$. Given that $\mathbf{B} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$

b find \mathbf{A} .

c Simplify $\mathbf{ABABABAB}$.

Solution:

a



$$\mathbf{Y} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

b $\mathbf{AB} = \mathbf{Y} \Rightarrow \mathbf{A} = \mathbf{YB}^{-1}$.

$$\mathbf{B} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \Rightarrow \det \mathbf{B} = 3 - 4 = -1$$

$$\therefore \mathbf{B}^{-1} = \frac{1}{-1} \begin{pmatrix} 1 & -2 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}$$

$$\begin{aligned} \therefore \mathbf{A} &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix} \\ &= \begin{pmatrix} -2 & 3 \\ -1 & 2 \end{pmatrix} \end{aligned}$$

c $\mathbf{ABABABAB} = \mathbf{Y}^4$

$$\begin{aligned} &= \text{rotation of } 4 \times 90 = 360^\circ \text{ about } (0, 0) \\ &= \mathbf{I} \end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise K, Question 5

Question:

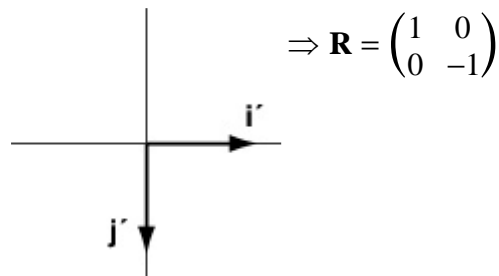
The matrix \mathbf{R} represents a reflection in the x -axis and the matrix \mathbf{E} represents an enlargement of scale factor 2 centre $(0, 0)$.

a Find the matrix $\mathbf{C} = \mathbf{ER}$ and interpret it geometrically.

b Find \mathbf{C}^{-1} and give a geometrical interpretation of the transformation represented by \mathbf{C}^{-1} .

Solution:

Reflection in x -axis



Enlargement S.F. 2 centre $(0, 0)$

$$\Rightarrow \mathbf{E} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

a

$$\begin{aligned} \mathbf{C} = \mathbf{ER} &= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \end{aligned}$$

Reflection in the x -axis and enlargement SF 2. Centre $(0, 0)$

$$\mathbf{b} \quad \mathbf{C}^{-1} = \frac{1}{-4} \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

Reflection in the x -axis and enlargement scale factor $\frac{1}{2}$. Centre $(0, 0)$

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Edexcel AS and A Level Modular Mathematics

Matrix algebra

Exercise K, Question 6

Question:

The quadrilateral R of area 4cm^2 is transformed to R' by the matrix $\mathbf{P} = \begin{pmatrix} 1+p & p \\ 2-p & p \end{pmatrix}$, where p is a constant.

a Find $\det(\mathbf{P})$ in terms of p .

Given that the area of $R' = 12\text{cm}^2$

b find the possible values of p .

Solution:

a

$$\begin{aligned} \mathbf{P} = \begin{pmatrix} 1+p & p \\ 2-p & p \end{pmatrix} &\Rightarrow \det \mathbf{P} = p(1+p) - p(2-p) \\ &= p + p^2 - 2p + p^2 \\ &= 2p^2 - p. \end{aligned}$$

b

Area of $R \times |\det p| = \text{Area of } R'$

$$\therefore 4 \times |\det p| = 12$$

$$\therefore \det p = \pm 3$$

$$\text{So } 2p^2 - p = 3$$

$$\Rightarrow 2p^2 - p - 3 = 0$$

$$(2p-3)(p+1) = 0$$

$$p = -1 \text{ or } \frac{3}{2}$$

$$\text{or } 2p^2 - p = -3$$

$$\Rightarrow 2p^2 - p + 3 = 0$$

Discriminant is $(-1)^2 - 4 \times 3 \times 2 = -23$

$$< 0$$

\therefore no solutions

so $p = -1$ or $\frac{3}{2}$ are the only solutions

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Matrix algebra

Exercise K, Question 7

Question:

The matrix $\mathbf{A} = \begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix}$, where a and b are non-zero constants.

a Find \mathbf{A}^{-1} .

The matrix $\mathbf{Y} = \begin{pmatrix} a & 2b \\ 2a & b \end{pmatrix}$ and the matrix \mathbf{X} is given by $\mathbf{XA} = \mathbf{Y}$.

b Find \mathbf{X} .

Solution:

a

$$\mathbf{A} = \begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix} \Rightarrow \det \mathbf{A} = 3ab - 2ab = ab$$

$$\therefore \mathbf{A}^{-1} = \frac{1}{ab} \begin{pmatrix} 3b & -b \\ -2a & a \end{pmatrix} = \begin{pmatrix} \frac{3}{a} & -\frac{1}{a} \\ -\frac{2}{b} & \frac{1}{b} \end{pmatrix}$$

b

$$\mathbf{XA} = \mathbf{Y} \Rightarrow \mathbf{X} = \mathbf{YA}^{-1}$$

$$\begin{aligned} \therefore \mathbf{X} &= \begin{pmatrix} a & 2b \\ 2a & b \end{pmatrix} \begin{pmatrix} \frac{3}{a} & -\frac{1}{a} \\ -\frac{2}{b} & \frac{1}{b} \end{pmatrix} \\ &= \begin{pmatrix} -1 & 1 \\ 4 & -1 \end{pmatrix} \end{aligned}$$

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Matrix algebra

Exercise K, Question 8

Question:

The 2×2 , non-singular matrices, \mathbf{A} , \mathbf{B} and \mathbf{X} satisfy $\mathbf{XB} = \mathbf{BA}$.

a Find an expression for \mathbf{X} .

b Given that $\mathbf{A} = \begin{pmatrix} 5 & 3 \\ 0 & -2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}$, find \mathbf{X} .

Solution:

a

$$\mathbf{XB} = \mathbf{BA}$$

$$\therefore (\mathbf{XB})\mathbf{B}^{-1} = \mathbf{BAB}^{-1}$$

$$\text{i.e. } \mathbf{X} = \mathbf{BAB}^{-1} \quad (\because \mathbf{BB}^{-1} = \mathbf{I})$$

b

$$\mathbf{B} = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix} \Rightarrow \det \mathbf{B} = -2 - (-1) = -1$$

$$\therefore \mathbf{B}^{-1} = \frac{1}{-1} \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$$

$$\therefore \mathbf{X} = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 5 & 3 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 2 & 4 \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} 6 & 2 \\ -4 & -3 \end{pmatrix}$$