Forecast:
Raining Results

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The Relaxation Times

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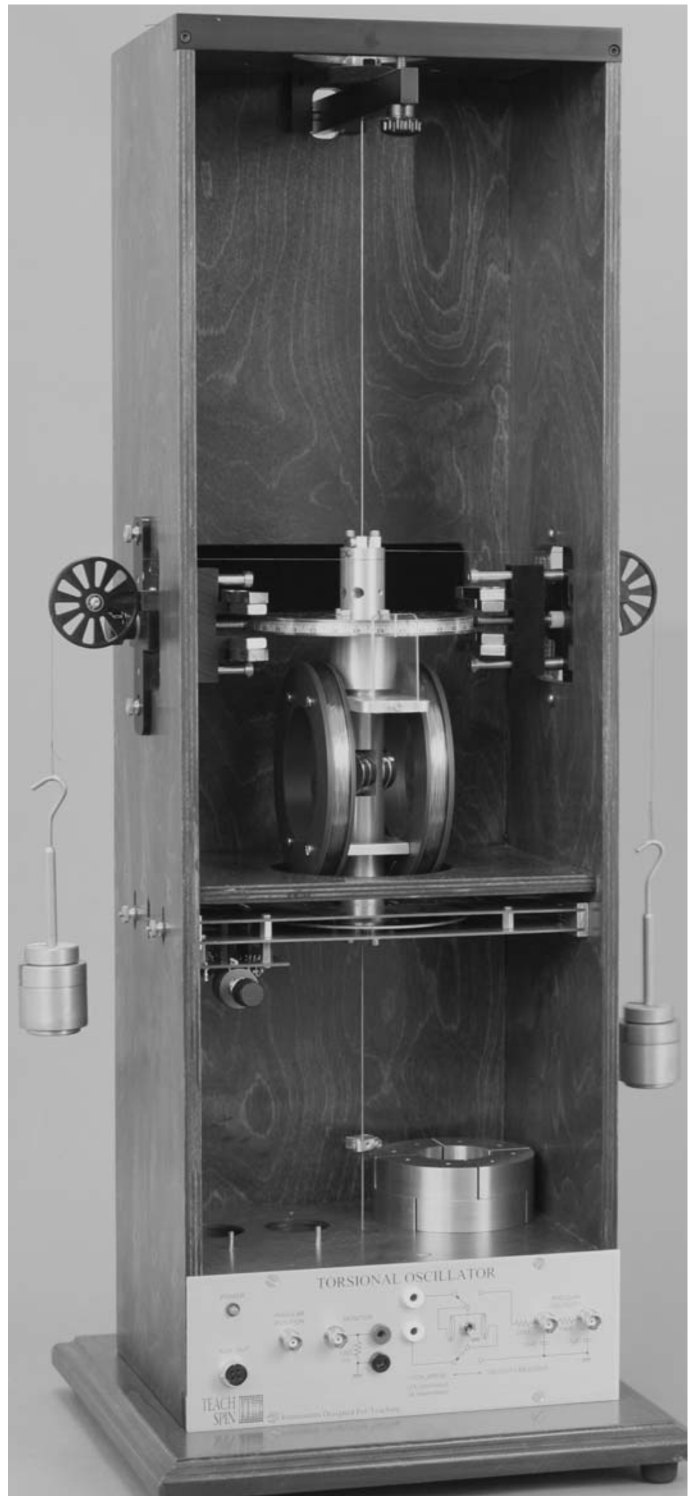
TORSIONAL OSCILLATOR

'An Instrument for All Seasons'

What's your nomination for the most generally-applicable model system in all of physics? We suggest that it's the simple harmonic oscillator. We've decided to rescue it from the theorists and offer our Torsional Oscillator to make all sorts of experiments visible, tactile, and quantitative. With this new apparatus, there are great learning opportunities accessible ***to all levels of physics instruction***, from first-year college through first-year graduate school. This Newsletter introduces those features appropriate for first-year students of mechanics. Future newsletters will discuss more advanced experiments.



TeachSpin's oscillating structure consists of an aluminum hub and several permanently attached components including a copper disc with a diameter of 0.12 m. Mounted on a stretched, vertical, length of steel music wire, the structure can rotate ±90° (or more) about that vertical axis. While the taut wire provides support and a restoring torque, the copper disc provides most of the rotational inertia. On the periphery of the disc, a novel 'radian protractor' introduces students to the physicists' way of measuring angles.

Torque and the resulting angular acceleration are key variables in this example of rotational motion about a fixed axis. We've provided several ways to exert known torques on the rotor. In the simplest version, two strings are wrapped in opposite directions around the hub of the rotor. The strings are led over low-friction pulleys to hangers on which a variety of masses can be hung. The hanging masses can be seen in the photograph at the right. As weights are added to the hangers, the rotor system accelerates, oscillates and finally settles down at a new, displaced, equilibrium position where the net torque is zero. The twist in the torsion fiber supplies a counter-torque to balance the gravitational one.

The torsion constant of the steel fiber can be found from measurements of the gravitational torques and the resulting angular displacement. The hanging weights create a total gravitational torque of 2rmg, where r is the radius of the rotor's hub. Figure 1 shows a graph of angular displacement vs. gravitational torque. The result is manifestly linear, showing that the counter-torque generated by the elasticity in the fiber is accurately modeled by the equation $\tau = -\kappa\theta$, Hooke's Law for torsional elasticity. The numerical value of the torsion constant κ can be calculated from the slope of the graph. The 'embarrassingly good' fit of this data is characteristic of all of the data generated by this apparatus.

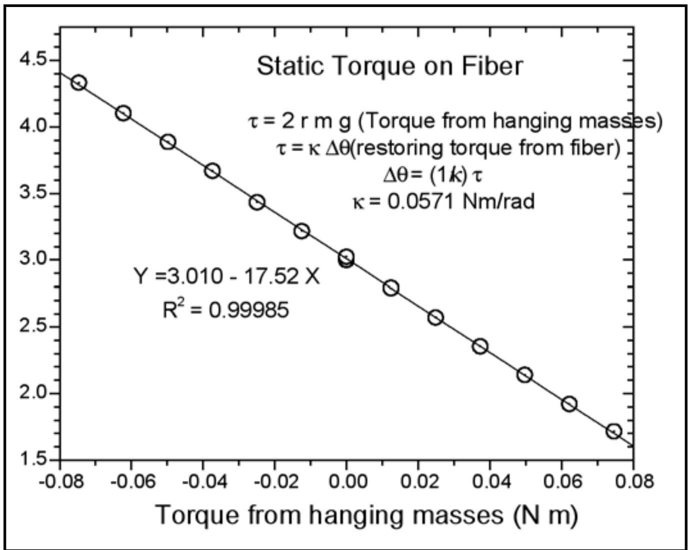


Figure 1 Angular Deflection vs. Applied Torque

There is much more to measure. We supply four fibers, of differing diameters, and students can measure κ for any of them. Students can check a claim of elasticity theory that κ should be proportional to the fourth power of the fiber diameter. They can also vary the tension in the torsion fiber, and can show (surprisingly) that this makes negligible difference in its torsion constant.

So far, the data shown make use of ‘eyeball reading’ of the angular displacement. The 'radian protractor' scale is easily read to 0.01 radians, or better, using the anti-parallax sighting plate mounted in front of the copper disc. But the rotor is also equipped with a non-contact, analog, capacitive position sensor, which provides a real-time output voltage proportional to the angular deflection of the rotor. The hanging masses providing the data in Figure 1 also provide a set of angle locations at which to measure the output of this sensor. The data is plotted in Figure 2.

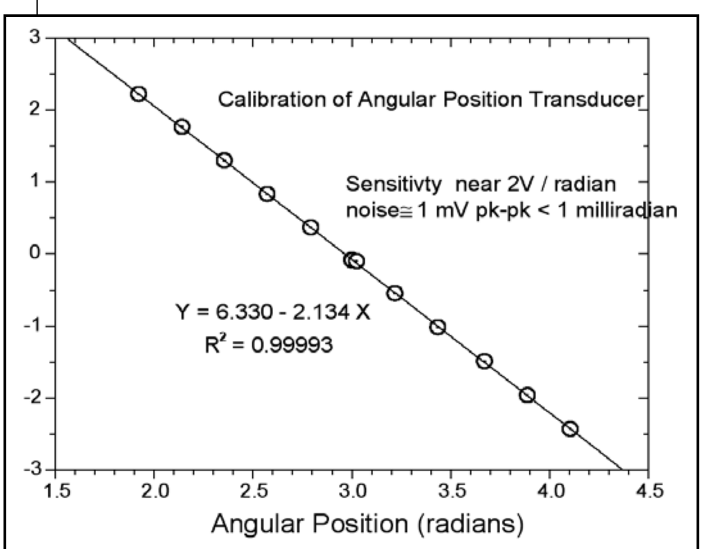


Figure 2 Position-Signal vs. Angular Deflection

A graph like this represents a very general experimental task: the calibration of a transducer. In this case, students get to deal with many real-life issues such as zero offset, zero stability, linearity of response, speed of response, stability of response, and noise level. And, in this case, they won't be disappointed by their results – our angle-to-voltage transducer is nearly ideal, and covers a range of ±1.5 radians with superb linearity and amazing signal-to-noise ratio.

Storing the voltage signal in an oscilloscope or computer, students can make detailed quantitative observations of the time dependence of the angular position of the rotor in a wide variety of experiments. For a simple release from displacement, a sinusoidal pattern appears which immediately tells them that this is an angular example of simple harmonic motion. And the precision of these measurements is quite amazing.

Figure 3 shows the position-transducer output for an oscillation amplitude of just 0.002 radians. This corresponds to a peak-to-peak motion of just 0.25 mm at the edge of the rotor. While nearly invisible to the eye, this motion is easily detected by the transducer.

Figure 3 also illustrates that we've chosen the parameters of our Torsional Oscillator so that the periods come out in a range of 1 to 2 seconds, a human time scale. This means that students can time oscillations by counting against a stopwatch. Using more complicated data-acquisition equipment and data-fitting methods, even higher precision is easily attained. And for these ‘slow-motion’ experiments, moments of maximum displacement to either side, or maximum velocity at the zero-crossing, can be readily seen, distinguished, and understood.

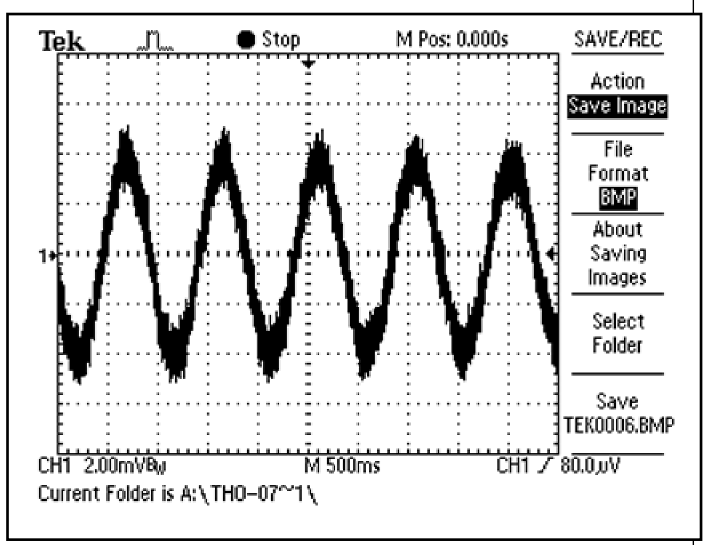


Figure 3 A Sinusoid for Oscillations of 2 mrad Amplitude

If the period can be measured so well, what then does it depend on? Not the amplitude of the oscillation, as students can verify to high precision. Rather, the period T depends on the restoring torque and the rotational of inertia, I, of the system. Most students have all but memorized the case of the one-dimensional mass-on-a-spring where $T = 2\pi (m/k)^{1/2}$. For this rotational system, theory gives the dependence as $T = 2\pi (I/\kappa)^{1/2}$.

So now we have some theory just begging for experimental validation! A simple one-point check of the period seems appealing. The torsion constant, κ, has been measured, but just what is I for this particular torsional oscillator? Looking at the apparatus, students see that there is a large copper disk whose moment of inertia they are confident they can calculate, given its mass and dimensions. The disk, however, is rigidly attached to a rotor whose geometry is quite complicated. Interestingly, a moment of inertia calculated from the experimentally determined period and torsion constant, κ, turns out to be rather close to the moment of inertia of the copper disk alone. This will make sense to students when they realize that the mass of the rest of the rotor system is concentrated close to the axis. But a much more interesting set of measurements can give a precise measurement of I_0, the moment of inertia of the rotor system.

The key to this investigation is a set of precisely-crafted brass quadrants. Given the dimensions, masses and location of these quadrants, students can compute the ΔI that each quadrant contributes. By attaching them in pairs to the copper disk, students systematically change the rotational inertia. With n such quadrants in place, the moment of inertia becomes $I = I_0 + n\Delta I$. A little algebra leads to $(T/2\pi)^2 = (I_0/\kappa) + n\,(\Delta I/\kappa)$. Now the validity of the original theoretical equation can be checked by plotting $(T/2\pi)^2$ vs. n (the number of added quadrants). The graph is shown in Figure 4.

Rather than making and testing a "one-off" prediction, students are proposing and verifying a trend, and using a theoretically-motivated model to do so. As the equation predicts, the data give a linear variation. And important information can be gotten from *both* the slope and the intercept. For this particular plot, the slope gives $\Delta I/\kappa$. Using the computed value of ΔI, a dynamic value for the torsion constant κ can be determined. Since the intercept gives I_0/κ, this new value of κ can be used to find I_0, the rotational inertia of the rotor system.

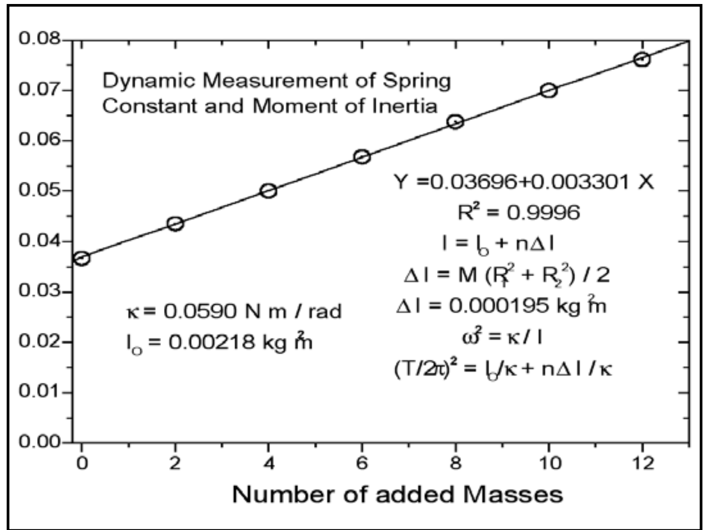
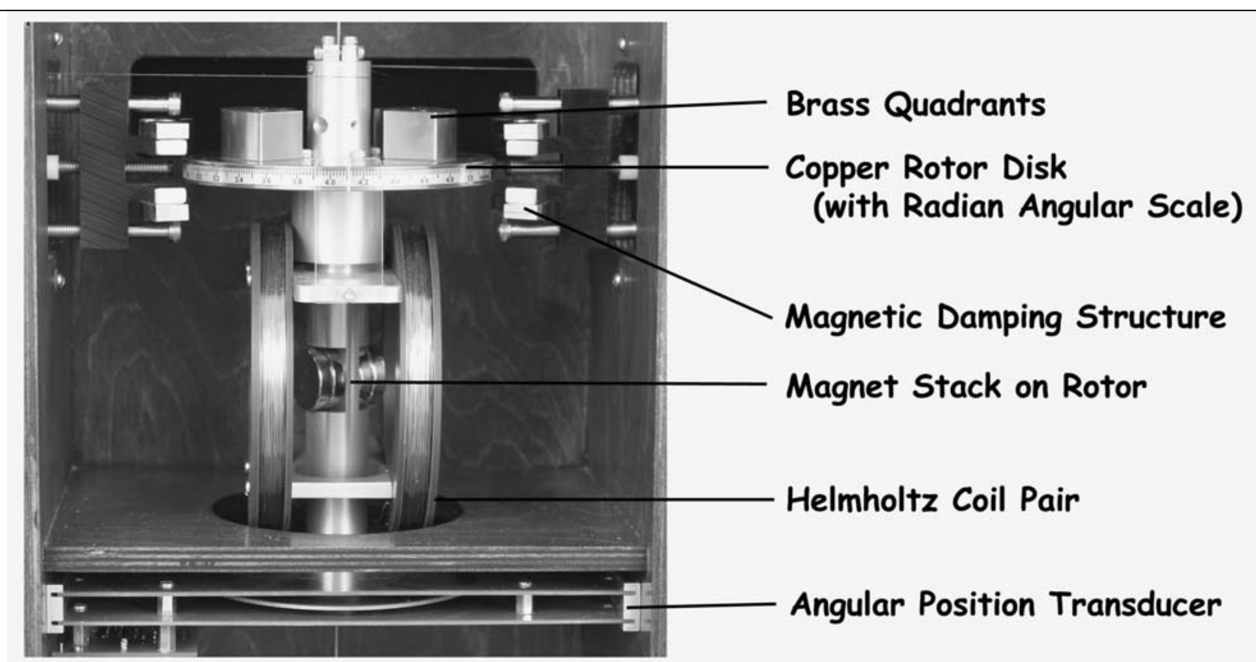


Figure 4 $(T/2\pi)^2$ vs. number of added quadrants

The students now have a fully-characterized mechanical system, and they have some experience using the angular displacement sensor. But what about angular velocity? As can be seen from the annotated photograph, when the system oscillates, a permanent magnet stack mounted on the rotor shaft moves within the Helmholtz coils. Using this apparatus, students get a preview of Faraday's Law as they monitor the emf induced in the coil system whenever the rotor structure is moving.

Figure 5 shows the signals simultaneously acquired from the angular position sensor and from this coil output. Certainly, the new signal does not give position information, but how can students convince themselves that it is angular velocity? If students model the angular-position signal with a sinusoid, they can predict, in quantitative units of radians/second, what the angular-velocity signal ought to look like. They can then calibrate the velocity sensor in Volts out per (radian/second) of angular velocity. In the process, they will have learned what differentiation means outside the mathematics classroom.



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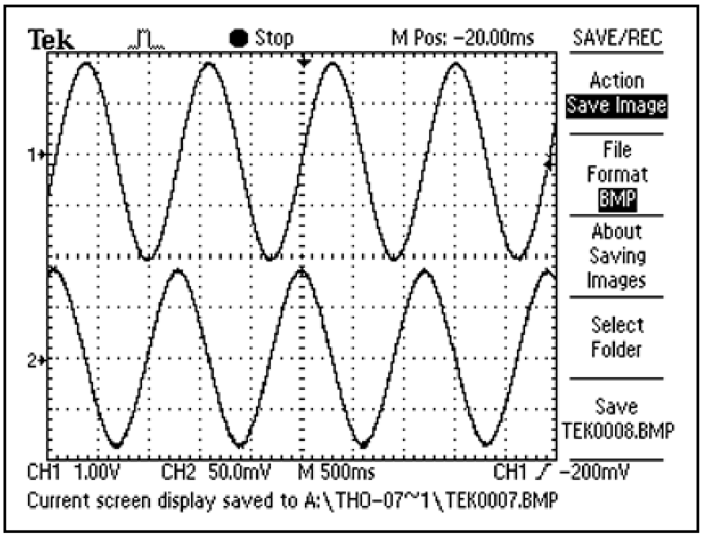

Figure 5 – Upper Trace: Angular-Position Signal vs. Time
Lower Trace: Coil-emf vs. Time

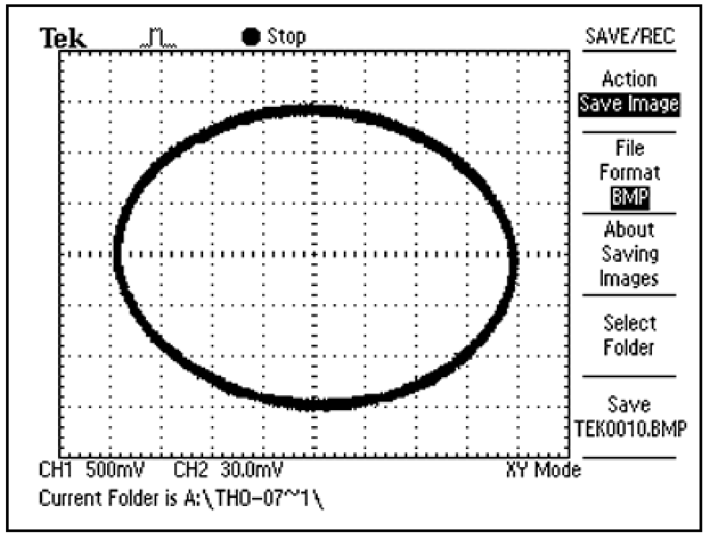

Figure 6 – Angular Velocity vs. Angular Position Signals:
A 'Phase Plane' Presentation

Once students have access to these two independent, real-time, analog signals, proportional to angular position and angular velocity respectively, they can appreciate the idea of a 'phase plane', a place to depict their values directly. An XY-display with the 'angular position' voltage on the x-axis, and 'angular velocity' voltage on the y-axis, yields beautiful elliptical paths repeated once per cycle of the periodic motion. We captured one for Figure 6.

Even more interesting is seeing directly how this 'moving dot' in the phase plane responds to hands-on intervention, or to artificial damping. Students can also think of this phenomenon in terms of potential-energy and kinetic-energy concepts. This might lead to creating graphs of $(1/2)\ \kappa\ \theta^2$ and $(1/2)\ I\ (d\theta/dt)^2$, to see how these quantities vary with time. Plotting their sum as a function of time provides a direct test of the conservation of mechanical energy in this system.

We are sure that you and your students can easily think of more experiments suitable for first-year mechanics students. And this apparatus will not be left behind when you move on to study electromagnetism. The same magnet-in-coil system you've seen as the velocity sensor can also be used in reverse as a way to supply magnetic torque to this system, proportional to the current you send into the coils. Our next Newsletter will lay out a whole list of second-semester electromagnetic physics that you can investigate with this 'torque drive' of the system. Measuring the magnetic moment, μ, of the permanent magnet, in real-world units of A m^2, is just the beginning of what you can do. How would you like your students to discover the three factors that arise from $\boldsymbol{\mu} \times \mathbf{B}$? How would you like to be able to drive this Torsional Oscillator with an arbitrary time-dependent torque waveform? How would you like to investigate three independent forms of damping in this Oscillator? Future newsletters will cover even more applications and investigations you can pursue with this apparatus.

For more information, see our website **(www.teachspin.com)** for work-in-progress coverage of this brand-new instrument, which made its debut at the 2008 March APS meeting and is now in production for first deliveries in the fall of 2008. Don't overlook our introductory US/Canada price is $1,995 per unit. And, if you want multiple units for your introductory lab, we will sell them to you at a discount. You are sure to think of lots more uses, in the advanced as well as the introductory labs. So here is your chance to spring the harmonic oscillator out of the hands of theorists. We think you will want to get one (or more) for your students.