Math 1497 - Sample Test 3

1.   Sketch the following parametric curve and find the equation of the tangent at the point of self intersection

$$x = \frac{1 + t + t^2 - t^3}{1 + t^2}, \quad y = \frac{2}{1 + t^2}.$$

2.   Graph the following polar equations

$$r = 2 + 2\sin\theta, \quad r = 2\sin 2\theta.$$

3.   Find the area inside one leaf of the rose described by

$$r = 2\sin 3\theta.$$

4.   Find the area of the following:

   (i)   inside $r = 2 + 2\sin\theta$,
   (ii)   inside the outer loop and outside the inner loop of $r = 1 - 2\sin\theta$,
   (iii)   outside $r = \cos 2\theta$ and inside $r = \sin 2\theta$ on $\left[0, \frac{\pi}{2}\right]$.

5.   Find the projection of the vector $\overrightarrow{u}$ onto $\overrightarrow{v}$ where $\overrightarrow{u} = <2, 3>$,   and $\overrightarrow{v} = <4, 2>$. Sketch both vectors, the projected vector and the orthogonal complement.

6. (i)   Find the equation of the plane that contains the
          vector $<1, 2, 4>$ and the points $(1, 1, 1)$ and $(-2, 3, 7)$.
    (ii) Find the equation of the plane that contains the
          points $(1, 3, 5)$, $(2, -1, 2)$ and $(0, 4, 6)$.

7. (i)   Find the equation of the line that passes through the points $(1, 2, 4)$ and $(-2, 3, 7)$.
    (ii) Find the equation of the line perpendicular to the plane $x + 2y - 3z = 6$
          passing through the point $(1, -1, 3)$.

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