Effects of Phase Noise on Performance of PCC-OFDM

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Abstract

This paper investigates the effects of phase noise on the performance of Polynomial Cancellation Coded Orthogonal Frequency Division Multiplexing (PCC-OFDM). Phase noise will cause two effects on PCC-OFDM, one is a Common Phase Error (CPE) and the other is Interchannel Interference (ICI). CPE is constant for all subcarriers in a symbol so that it can be corrected using existing techniques while the ICI is untracked and contributes to the Bit Error Rate (BER) degradation. It is shown that the ICI cancellation properties of PCC-OFDM can effectively reduce the ICI caused by phase noise.

1. Introduction

In Orthogonal Frequency Division Multiplexing (OFDM) systems, the insertion of the up and down frequency conversion between the modulator and the demodulator will almost certainly introduce phase noise even if zero frequency offset can be achieved [1]. Phase noise is a random perturbation of the phase of the oscillator's output signal. It can also be interpreted as instability of an oscillator. In the literature [2], phase noise is normally modeled as an ideal phase modulation in the oscillator's signal with a constant amplitude and unique frequency. The effects of phase noise on the performance of OFDM systems have been investigated in the literature [3]. The effects of the phase noise in an OFDM system result from the combination of the phase noise in both oscillators of the transmitter and the receiver. Without loss of generality, in this paper, the local oscillator in the receiver will be considered as the phase noise source.

A frequently used model for phase noise in the literature is where a free-running oscillator is employed. In this case the phase noise is modeled as a Wiener process with zero mean and a Lorentzian power spectral density [4]. The variance of the phase noise increases linearly with the time. However, in a practical OFDM system, the frequency of the oscillator is stabilized by means of a phase locked loop (PLL). The PLL changes the statistics of the oscillator phase noise [5]. Therefore the overall phase noise spectrum depends on both the properties of the free-running oscillator and the components of the PLL. In this study we assume that a PLL is always employed. We will model the phase noise in PCC-OFDM systems as a stationary process with zero mean and a finite variance, the bandwidth of the phase noise is much smaller than the symbol rate $1/T$. This indicates a slowly varying phase error. In addition, the power density spectrum is assumed known in this study.

2. Background of PCC-OFDM

Polynomial Cancellation Coded OFDM (PCC-OFDM) is a coding scheme for OFDM. PCC-OFDM maps each data transmitted data onto weighted groups of subcarriers [6]. This technique was designed to cancel the interchannel interference (ICI) caused by frequency offset [7]. However, in this paper we will show that PCC-OFDM can also substantially reduce the ICI caused by phase noise.

Figure 1 shows a simplified block diagram of a PCC-OFDM communication system. The high speed QAM data stream enters a serial to parallel converter to be converted into n lower speed parallel substreams. The ith vector to be transmitted is represented by $d_{0,i},\cdots,d_{n-1,i}$. They are mapped onto the values $a_{0,i}\cdots a_{N-1,i}$ that modulate the N subcarriers in the ith symbol period. For conventional OFDM, $n=N$ and $a_{k,i}=d_{k,i}$, there is a simple one-to-one mapping of data values onto the subcarriers. In this paper, the case where each data value to be transmitted is mapped onto pairs of subcarriers is considered, so that $n=N/2$.

In the channel, the signal is filtered by the channel response $h(t)$, additive noise $n(t)$ is injected. At the receiver the phase noise $\theta(t)$ is introduced. The ith output vector of the Discrete Fourier Transform (DFT) is $z_{0,i}\cdots z_{N-1,i}$. The demodulated subcarriers are then weighted and added to generate the data estimates $v_{0,i}\cdots v_{n-1,i}$. For a pair of PCC-OFDM subcarriers $z_{2M,i}$, $z_{2M+1,i}$ an estimate is calculated using $v_{M,i}=\left(z_{2M,i}-z_{2M+1,i}\right)/2$.

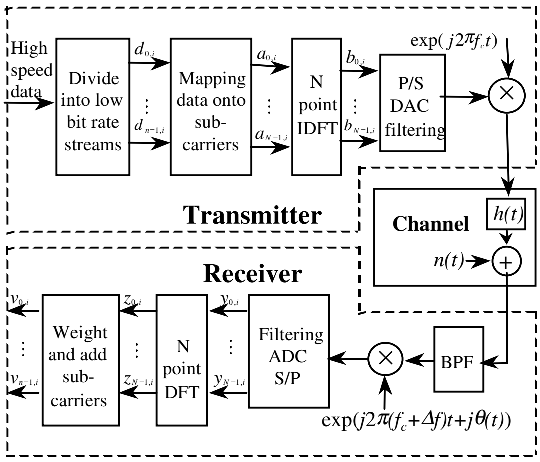

Figure 1 A PCC-OFDM system with phase noise and frequency offset

3. Phase noise in PCC-OFDM

The kth sample in the ith output vector of the Inverse Fast Fourier Transform (IFFT) is given by

$$b_{k,i} = \frac{1}{N}\sum_{l=0}^{N-1} a_{l,i} \exp\left(\frac{j2\pi kl}{N}\right),\ k = 0,1,\cdots,N-1 \quad (1)$$

where $a_{l,i}$ is the lth subcarrier in the ith input vector of the IFFT. Assuming the additive noise in the channel is $n(t)$ with zero mean and finite variance, and the signal is sampled at Nyquist rate, then the received signal in the presence of phase noise is given by

$$y_{k,i} = \exp(j\theta(k))b_{k,i} + n_{k,i},\ \ k = 0,1,\cdots,N-1 \quad (2)$$

where $\theta(k)$ is the discrete phase noise of $\theta(t)$. $n_{k,i}$ is the kth sample of the additive noise in the mth symbol. Thus the mth demodulated subcarrier is given by

$$z_{m,i} = \sum_{k=0}^{N-1} y_{k,i} \exp\left(\frac{-j2\pi km}{N}\right) + W_{m,i} \quad (3)$$

where $W_{m,i}$ is the Fast Fourier Transform (FFT) of the channel noise, which is also Additive White Gaussian Noise (AWGN) with zero mean and finite variance. Substituting equation (1) and (2) to (3) gives

$$z_{m,i} = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} a_{l,i} \exp\left(\frac{j2\pi k(l-m)}{N}\right)\exp(j\theta(k)) + W_{m,i} \quad (4)$$

Using $a_{2M,i} = -a_{2M+1,i} = d_{M,i}$ for PCC-OFDM, we can obtain the expression for the $2M$th PCC-OFDM subcarrier,

$$z_{2M,i} = \frac{1}{N}\sum_{L=0}^{N/2-1} d_{L,i} \sum_{k=0}^{N-1}\left\{\exp\left(\frac{j4\pi k(L-M)}{N}\right) - \exp\left(\frac{j2\pi k(2(L-M)+1)}{N}\right)\right\}\exp(j\theta(k)) + W_{2M,i} \quad (5)$$

Defining

$$\psi_n = \frac{1}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-j2\pi kn}{N}\right)\exp(j\theta(k)) \quad (6)$$

where $n = -(N-1),\cdots,0,\cdots,(N-1)$, the $2M$th subcarrier can be represented by

$$z_{2M,i} = d_{M,i}(\psi_0 - \psi_{-1}) + \sum_{\substack{L=0 \\ L\neq M}}^{N/2-1} d_{L,i}\left(\psi_{2(M-L)} - \psi_{2(M-L)-1}\right) + W_{2M,i} \quad (7)$$

The quantity ψ_n is the discrete Fourier Transform of $\exp(j\theta(k))$, evaluated at the frequency n/N. ψ_n depends on the frequency spectrum of the phase noise. The properties of each term in (7) will be analyzed later after we have derived the expression of the relevant output of the weighting and adding block for the subcarrier pair. If $\theta(k)$ is a constant, then $\psi_n = 0$, for $n \neq 0$, there is no ICI. Furthermore, for $\theta(k) = 0$, we obtain $\psi_0 = 1$, the demodulated subcarrier is equal to its original data to be transmitted.

Under the condition of a small phase noise $\theta(k)$, using the approximation $\exp(j\theta(k)) \approx 1 + j\theta(k)$, (7) can be written as

$$\psi_n \approx \frac{1}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-j2\pi kn}{N}\right)(1 + j\theta(k)) \quad (8)$$

For $n \neq 0$, (8) is given by

$$\psi_n \approx \frac{j}{N}\sum_{k=0}^{N-1}\exp\left(\frac{-j2\pi kn}{N}\right)\theta(k) \quad (9)$$

ψ_n is now the FFT of the phase noise, which is the spectrum of the phase noise $\theta(k)$.

For $n = 0$, we get

$$\psi_0 \approx 1 + \frac{j}{N}\sum_{k=0}^{N-1}\theta(k) \quad (10)$$

Similarly we can get the $(2M+1)$th demodulated subcarrier

$$z_{2M+1,i} = d_{M,i}(\psi_1 - \psi_0) + \sum_{\substack{L=0 \\ L\neq M}}^{N/2-1} d_{L,i}\left(\psi_{2(M-L)+1} - \psi_{2(M-L)}\right) + W_{2M+1,i} \quad (11)$$

The corresponding output of the subcarrier pair from the weighting and adding block is then given by

$$\begin{aligned} v_{M,i} &= \left(z_{2M,i} - z_{2M+1,i}\right)/2 \\ &= \frac{1}{2} d_{M,i}(-\psi_{-1} + 2\psi_0 - \psi_1) \\ &\quad + \frac{1}{2}\sum_{\substack{L=0 \\ L\neq M}}^{N/2-1} d_{L,i}\left(-\psi_{2(M-L)-1} + 2\psi_{2(M-L)} - \psi_{2(M-L)+1}\right) \end{aligned} \quad (12)$$

Substituting equation (10) into (12) gives

$$\begin{aligned} v_{M,i} &= d_{M,i} \\ &\quad + \frac{1}{2} d_{M,i}[-\psi_{-1} + 2(\psi_0 - 1) - \psi_1] \\ &\quad + \frac{1}{2}\sum_{\substack{L=0 \\ L\neq M}}^{N/2-1} d_{L,i}\left(-\psi_{2(M-L)+1} + 2\psi_{2(M-L)} - \psi_{2(M-L)-1}\right) \\ &\quad + \left(W_{2M,i} - W_{2M+1,i}\right)/2 \end{aligned} \quad (13)$$

where the first term at the right hand is the wanted signal, the second term is the Common Phase Error (CPE), the third term is the ICI and the last one is the weighted AWGN noise. In OFDM, the CPE and ICI depend on the individual frequency spectrum of the phase noise [5]. In PCC-OFDM, the CPE and ICI depend on the combinations of phase noise spectra rather than individual spectra. This makes it possible to reduce the effects of phase noise by using a PLL that can fit the overall phase noise spectrum to a particular pattern. For the special case where the individual spectrum ψ_n has a linear relationship with frequency, then the ICI caused by the phase noise can be completely cancelled.

Substituting ψ_{-1}, ψ_0 and ψ_1 from (9) and (10) to the CPE term in (13), the CPE term can be obtained by

$$\begin{aligned} Y_{CM,i} &= \frac{1}{2} d_{M,i}[-\psi_{-1} + 2(\psi_0 - 1) - \psi_1] \\ &= \frac{j}{N} d_{M,i}\sum_{k=0}^{N-1}\left(1 - \cos\left(\frac{2\pi k}{N}\right)\right)\theta(k) \\ &= \frac{2j}{N} d_{M,i}\sum_{k=0}^{N-1}\sin^2\left(\frac{\pi k}{N}\right)\theta(k) \\ &= j d_{M,i}\Theta_0 \end{aligned} \quad (14)$$

where Θ_0 is a constant, $\Theta_0 = \frac{2}{N}\sum_{k=0}^{N-1}\sin^2\left(\frac{\pi k}{N}\right)\theta(k)$. This result indicates that all PCC-OFDM subcarriers experience a CPE. This rotation can be detected and therefore compensated using techniques provided in the literature [8]. One simple way to do this is to insert pilot tones in a symbol and estimate the rotation angle. Once the CPE is corrected in the pilot tones, the remaining phase noise in other subcarriers can be corrected.

Similarly, the ICI term in equation (13) can be presented by

$$\begin{aligned} Y_{IM,i} &= \frac{1}{2}\sum_{\substack{L=0 \\ L\neq M}}^{N/2-1} d_{L,i}\left(-\psi_{2(M-L)+1} + 2\psi_{2(M-L)} - \psi_{2(M-L)-1}\right) \\ &= \frac{2j}{N}\sum_{\substack{L=0 \\ L\neq M}}^{N/2-1} d_{L,i}\sum_{k=0}^{N-1}\sin^2\left(\frac{\pi k}{N}\right)\exp\left(\frac{j4\pi k(L-M)}{N}\right)\theta(k) \end{aligned} \quad (15)$$

This term is the contribution of the phase noise to all other subcarriers in a symbol. This result shows that the phase noise also causes loss of orthogonality and introduces ICI. Defining

$$\Theta_{L-M} = \frac{2}{N}\sum_{k=0}^{N-1}\sin^2\left(\frac{\pi k}{N}\right)\exp\left(\frac{j4\pi k(L-M)}{N}\right)\theta(k) \quad (16)$$

Equation (15) can be written as

$$Y_{IM,i} = j\sum_{\substack{L=0 \\ L\neq M}}^{N/2-1} d_{L,i}\Theta_{L-M} \quad (17)$$

We will now investigate the case where phase noise is white Gaussian. In this case, no correlation between the samples of phase noise is assumed, the possible difference between two consecutive samples is highest. The variance of the CPE can be calculated by

$$\begin{aligned} \mathrm{var}[\Theta_0] &= \frac{4}{N^2}\sum_{k=0}^{N-1}\sin^4\left(\frac{\pi k}{N}\right)E\left[\left|\theta(k)\right|^2\right] \\ &= \frac{3\sigma_\theta^2}{2N} \end{aligned} \quad (18)$$

where σ_θ^2 is the variance of the phase noise, $\sigma_\theta^2 = E\left[\left|\theta(k)\right|^2\right]$. Note the summation in (18) is a constant, $\sum_{k=0}^{N-1}\sin^4(\pi k/N) = 3N/8$. The variance of the CPE is proportional to the variance of the phase noise. Thus the variance of the CPE is then given by

$$\mathrm{var}[Y_{CM,i}] = \frac{3\sigma_\theta^2\sigma_s^2}{2N} = \frac{3\sigma_\theta^2}{2N} \tag{19}$$

Note we have used $\sigma_s^2 = 1$ for 4QAM. The variance of the angle in ICI is given by

$$\begin{aligned}\mathrm{var}[\Theta_{L-M}] &= E[\Theta_{L-M}\Theta_{L-M}^*] \\ &= \frac{4}{N^2}\sum_{k=0}^{N-1}\sin^4\left(\frac{\pi k}{N}\right)E\left[|\theta(k)|^2\right] \\ &= \frac{3\sigma_\theta^2}{2N}\end{aligned} \tag{20}$$

Similarly, we can calculate the variance of the ICI from (17)

$$\begin{aligned}\mathrm{var}[Y_{IM,i}] &= \left(\frac{N}{2}+1\right)\frac{3}{2N}\sigma_s^2\sigma_\theta^2 \\ &= \left(\frac{3}{4}+\frac{3}{2N}\right)\sigma_\theta^2\end{aligned} \tag{21}$$

4. Simulation results

To demonstrate the effects of phase noise on the performance of PCC-OFDM systems, computer simulations were performed, where 4QAM was used to modulate 128 subcarriers. The number of symbols simulated was 10,000.

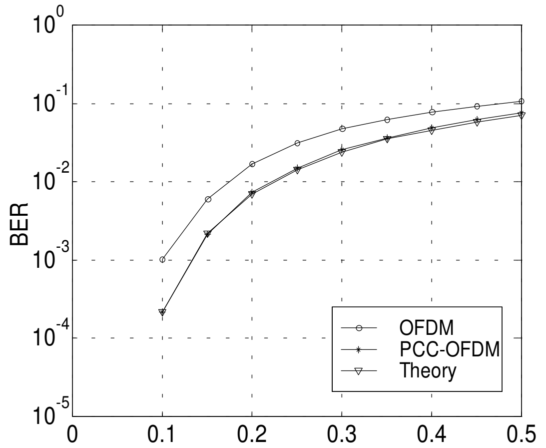

Figure 2 BER performance as a function of phase jitter for PCC-OFDM and OFDM, $N = 128$, no channel noise

Figure 2 shows the Bit Error Rate (BER) performance as a function of variance of phase noise for PCC-OFDM and OFDM systems where there is no channel noise. For a given BER of 10^{-3}, the phase noise variance for PCC-OFDM is about 0.03rad^2 lower than that for OFDM. The theoretical curve was calculated by using the phase noise variance given in (21) as the equivalent variance for OFDM. It matches the simulation results for PCC-OFDM very well.

Figure 3 shows the BER performance as a function of E_b/N_0 for PCC-OFDM and OFDM in the presence of phase noise. The variance of the phase noise is $0.04\,\mathrm{rad}^2$. For a given BER of 10^{-3}, E_b/N_0 for PCC-OFDM is about 3dB lower than that for OFDM. For comparison, the BER performance in a phase noise free AWGN channel is also presented. In an ideal ICI-free channel, OFDM and PCC-OFDM will have the same BER performance. The improvement on the performance for PCC-OFDM presented in the figure is due to the ICI cancellation property of PCC-OFDM.

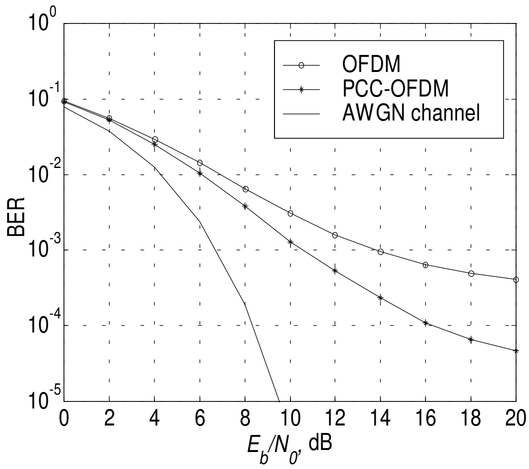

Figure 3 BER performance as a function of E_b/N_0 for PCC-OFDM and OFDM for a phase jitter of 0.04rad^2 and a phase noise free AWGN channel, $N = 128$.

5. Summary

The effects of phase noise on the performance of PCC-OFDM have been theoretically analyzed and simulated. Phase noise will cause two effects on PCC-OFDM, one is a CPE and the other is ICI. The expressions for the CPE and ICI term caused by phase noise in PCC-OFDM have been derived. Simulations show that the ICI cancellation properties of PCC-OFDM can effectively reduce the ICI caused by phase noise.

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