Global club goods and the fragmented global financial safety net

Supplementary Appendix

The following is a political economy club goods model of the global financial safety net. The motivation for this model is an inability of public goods-based theories to explain why the global financial safety net is fragmented and permits highly uneven access.

Club goods are defined as non-rival and excludable goods. In the article, I argue that the liquidity that constitutes the global financial safety net is a club good because it is non-rival and excludable. The model shows that the Founder of the global financial safety net regime finds it advantageous to exploit the excludability character of liquidity. While the Founder may disburse emergency liquidity through a multilateral regime, a series of bilateral arrangements, or through a dual regime that combines multilateral and bilateral segments, the model finds that the dual regime gives the global financial safety net provider the highest payoff. For borrower states, it is shown that accessing the dual regime through the multilateral arrangement is relatively expensive while accessing loans bilaterally is relatively inexpensive. Table 1 lists the variables used in the model.

Table 1: Summary of model variables

Variable	Definition
$\boldsymbol{\Omega_i}$	Fraction of state *i*'s global reserve currency deposits claimed by citizens of the GFSNP.
$\boldsymbol{\alpha}$	The fraction of deposits held in the global reserve currency.
$\boldsymbol{\pi}$	The probability of a liquidity crisis when a borrower state cannot access the global financial safety net.
$\boldsymbol{\tau}$	The cost of borrowing applied to all states within the multilateral regime.
$\boldsymbol{\rho_i}$	The cost of borrowing applied to state *i* in a bilateral regime.
$\boldsymbol{\delta}$	Ideal "price" (i.e., policy) for borrower in bilateral regime.
$\boldsymbol{\mu}$	Ideal "price" (i.e., policy) for GFSNP in bilateral regime.
$\boldsymbol{\gamma}$	Amount by which the probability of contagion is reduced due to lending by the GFSNP.

I Multilateral Regime

The following is the solution to the multilateral regime. Recall that $\tau^* = \alpha(1 - \Omega_m^*)$.

$$\int_0^{\Omega_m} \tau \, d\Omega \; - \; \pi \int_{\Omega_m}^1 \alpha\Omega \; d\Omega$$

$$= \alpha(1 - \Omega)\Omega - \frac{\alpha\pi}{2} + \frac{\Omega^2 \alpha\pi}{2}$$

FOC with respect to Ω:

$$\alpha - \; 2\,\Omega\alpha \; + \; \Omega\alpha\pi = \; 0$$

$$\Omega_m^* = \frac{1}{2 - \pi}$$

$$\Rightarrow \quad \tau^* = \frac{\alpha(1-\pi)}{2-\pi}$$

Total utility under a multilateral regime:

$$U_m = \frac{\alpha(1 - \pi)^2}{2(2 - \pi)}$$

II Bilateral Regime

The following is the solution to the bilateral regime. Recall that $\rho_i = \frac{2\delta - 2\pi\alpha\Omega_i + \pi\alpha}{2}$.

$$\int_0^{\Omega_b} \rho_b \; d\Omega$$

$$\int_0^{\Omega_b} \frac{2\delta - 2\pi\alpha\Omega_i + \pi\alpha}{2} \; d\Omega$$

$$= \frac{2\delta\Omega - \pi\alpha\Omega^2 + \pi\alpha\Omega}{2}$$

FOC with respect to Ω:

$$\frac{2\delta - 2\pi\alpha\Omega + \pi\alpha}{2} = 0$$

$$\Omega_b^* = \frac{2\delta + \pi\alpha}{2\pi\alpha}$$

Total utility under a bilateral regime:

$$U_b = \int_0^{\frac{2\delta+\pi\alpha}{2\pi\alpha}} \frac{2\delta - 2\pi\alpha\Omega_b^* + \pi\alpha}{2} \; d\Omega$$

$$U_b = \frac{(2\delta + \pi\alpha)^2}{8\pi\alpha}$$

III Dual Regime

The following is the solution to the dual regime.

$$\int_0^{\Omega_d} \tau_d \, d\Omega \quad + \quad \int_{\Omega_d}^{\frac{2\delta+\pi\alpha}{2\pi\alpha}} \rho_d \, d\Omega$$

$$= \alpha(1-\Omega)\Omega + \frac{2\delta\frac{2\delta+\pi\alpha}{2\pi\alpha} - \pi\alpha\frac{2\delta+\pi\alpha}{2\pi\alpha}^2 + \pi\alpha\frac{2\delta+\pi\alpha}{2\pi\alpha}}{2} - \frac{2\delta\Omega - \pi\alpha\Omega^2 + \pi\alpha\Omega}{2}$$

FOC with respect to Ω:

$$\alpha - 2\alpha\Omega - \delta + \pi\alpha\Omega - \frac{\pi\alpha}{2} = 0$$

$$\Omega_d^* = \frac{\alpha(2-\pi) - 2\delta}{2\alpha(2-\pi)}$$

Total utility under a dual regime:

$$U_d = \alpha(1-\Omega_d^*)\Omega_d^* + \int_{\frac{\alpha(2-\pi)-2\delta}{2\alpha(2-\pi)}}^{\frac{2\delta+\pi\alpha}{2\pi\alpha}} \frac{2\delta - 2\pi\alpha\Omega_d^* + \pi\alpha}{2} \, d\Omega$$

$$U_d = \frac{4\delta^2 + \alpha^2\pi(2-\pi)}{4\pi\alpha(2-\pi)}$$

IV Alternative Dual Regime

The following is the solution to an alternative dual regime where bilateral arrangements are reserved for low Ω states and a multilateral arrangement is reserved for high Ω states.

$$\int_0^{\Omega_d} \frac{2\delta - 2\pi\alpha\Omega + \pi\alpha}{2}\, d\Omega \quad + \quad \int_{\Omega_d}^{X} \tau_d \; dX - \pi \int_X^1 \alpha\Omega \; dX$$

$$= \frac{2\delta\Omega - \pi\alpha\Omega^2 + \pi\alpha\Omega}{2} + \alpha(1-X)X - \alpha(1-X)\Omega - \pi\alpha\Omega + \pi\alpha\Omega X$$

FOC with respect to Ω:

$$\frac{2\delta - 2\pi\alpha\Omega + \pi\alpha}{2} - \pi\alpha + \pi\alpha X = 0$$

$$\Omega_d^* = \frac{2\delta - \pi\alpha + 2\pi\alpha X}{2\pi\alpha}$$

Optimal X:

$$\tau(X - \Omega)$$

$$= \alpha(1-X)(X-\Omega)$$

$$X^* = \frac{1 + \Omega_d^*}{2}$$

$$\Rightarrow \quad \Omega_d^* = \frac{2\delta}{2-\pi\alpha}$$

$$\Rightarrow \quad X^* = \frac{1+\frac{2\delta}{2-\pi\alpha}}{2} = \frac{2-\pi\alpha+2\delta}{2(2-\pi\alpha)}$$

Total utility under alternative dual regime:

$$U_{ad} = \int_0^{\frac{2\delta}{2-\pi\alpha}} \frac{2\delta - 2\pi\alpha\Omega + \pi\alpha}{2}\, d\Omega \quad + \quad \int_{\frac{2\delta}{2-\pi\alpha}}^{\frac{2-\pi\alpha+2\delta}{2(2-\pi\alpha)}} \tau_d\, d\Omega - \pi \int_{\frac{2-\pi\alpha+2\delta}{2(2-\pi\alpha)}}^{1} \alpha\Omega\, dX$$

$$U_{ad} = \frac{\alpha}{4} - \frac{(4-\alpha)\delta^2}{(2-\pi\alpha)^2} + \frac{(4\delta-\alpha)\delta}{2-\pi\alpha}$$

Proof that $U_{ad} < U_d$

$$\frac{\alpha}{4} - \frac{(4-\alpha)\delta^2}{(2-\pi\alpha)^2} + \frac{(4\delta-\alpha)\delta}{2-\pi\alpha} \leq \frac{4\delta^2 + \alpha^2\pi(2-\pi)}{4\pi\alpha(2-\pi)}$$

This inequality has been verified using Mathematica and holds $\forall\, \alpha \in [0,1], \pi \in [0,1]$, and $\delta \in \left[0, \frac{2-\pi\alpha}{2}\right)$. Note that the restriction $\delta \in \left[0, \frac{2-\pi\alpha}{2}\right)$ merely implies that $\Omega \in [0,1]$.

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V Proposition 1: Dual Regime Dominant Strategy

Recall $U_m = \frac{\alpha(1-\pi)^2}{2(2-\pi)}$, $U_d = \frac{4\delta^2+\alpha^2\pi(2-\pi)}{4\pi\alpha(2-\pi)}$, and $U_b = \frac{(2\delta+\pi\alpha)^2}{8\pi\alpha}$.

Dual vs. Multilateral Regime
$U_d \geq U_m$

$$\frac{4\delta^2 + \alpha^2\pi(2-\pi)}{4\pi\alpha(2-\pi)} \geq \frac{\alpha(1-\pi)^2}{2(2-\pi)}$$

$$4\delta^2 + \alpha^2\pi(2-\pi) \geq 2\pi\alpha^2(1-\pi)^2$$

$$4\delta^2 \geq \alpha^2\pi[2(1-\pi)^2 - 2 + \pi]$$

$$4\delta^2 \geq -\alpha^2\pi^2(3-2\pi)$$

Which is true $\forall\ \alpha \in (0,1],\ \pi \in (0,1],\ \delta \in [0,\infty)$.

Dual vs. Bilateral Regime
$U_d \geq U_b$

$$\frac{4\delta^2 + \pi\alpha^2(2-\pi)}{4\pi\alpha(2-\pi)} \geq \frac{(2\delta+\pi\alpha)^2}{8\pi\alpha}$$

$$\frac{8\delta^2 + 4\pi\alpha^2 - 2\pi^2\alpha^2}{2\alpha(2-\pi)} \geq \frac{4\delta^2(2-\pi) + 4\delta\pi\alpha(2-\pi) + \pi^2\alpha^2(2-\pi)}{2\alpha(2-\pi)}$$

$$4\delta^2 - 4\delta\alpha(2-\pi) + 4\alpha^2 - 4\pi\alpha^2 + \pi^2\alpha^2 \geq 0$$

$$[2\delta - \alpha(2-\pi)]^2 \geq 0$$

Which is true $\forall\ \alpha \in (0,1],\ \pi \in (0,1],\ \delta \in [0,\infty)$.

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VI Proposition 2: Bilateral Lower Cost for Borrower

Recall: $\Omega_d^* = \frac{\alpha(2-\pi)-2\delta}{2\alpha(2-\pi)}$, $\tau_d^* = \alpha(1-\Omega_d^*)$, and $\rho_d^* = \frac{2\delta-2\pi\alpha\Omega_i+\pi\alpha}{2}$.

Multilateral cost at Ω_d^* ≥ Bilateral cost at Ω_d^*

$$\alpha(1-\Omega_d^*) \geq \delta - \frac{2\delta - 2\pi\alpha\Omega_d^* + \pi\alpha}{2}$$

$$\alpha - \alpha\Omega_d^* \geq \delta - \delta + \pi\alpha\Omega_d^* - \frac{\pi\alpha}{2}$$

$$1 + \frac{\pi}{2} \geq -\Omega_d^*(1-\pi)$$

Which is true $\forall\ \Omega \in [0,1],\ \pi \in [0,1]$. Given that ρ_d^* is highest at Ω_d^*, $\tau_d^* \geq \delta - \rho_d^*\ \forall\ \Omega > \Omega_d$.

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VII Multilateral Regime with Joint Products

The following is the solution to the multilateral regime. The additional subscript "p" denotes the public goods regime. Recall that $\tau^*_{m,p} = \alpha\left(1 - \Omega^*_{m,p}\right)$.

Recall that $\gamma \in [0,1]$ is the amount by which contagion to state *j* are reduced by the GFSNP's lending to state *i* and that the total amount of the public good produced by the GFSNP equals $\Omega_i \int_0^1 \gamma$, where Ω_i equals $\Omega_{m,p}$, $\Omega_{b,p}$, or $\Omega_{d,p}$ if the regime is multilateral, bilateral, or dual.

$$\int_0^{\Omega_{m,p}} \left(\tau_{m,p} + \alpha\Omega_{m,p}\right) d\Omega \;-\; \pi \int_{\Omega_{m,p}}^{1} \alpha\Omega_{m,p}\, d\Omega \;+\; \Omega_{m,p} \int_0^1 \gamma\alpha\, d\Omega$$

$$= \alpha\left(1 - \Omega_{m,p}\right)\Omega_{m,p} + \frac{\alpha\Omega_{m,p}^2}{2} - \frac{\pi\alpha}{2} + \frac{\pi\alpha\Omega_{m,p}^2}{2} + \gamma\alpha\Omega_{m,p}$$

FOC with respect to Ω:

$$\alpha - 2\alpha\Omega_{m,p} + \alpha\Omega_{m,p} + \pi\alpha\Omega_{m,p} + \gamma\alpha = 0$$

$$\Omega^*_{m,p} = \frac{1+\gamma}{1-\pi}$$

However, because $\Omega_m \in [0,1] \Rightarrow \Omega^*_m = 1$.

$\Rightarrow \tau^* = 0$

Total utility under a multilateral regime:

$$\int_0^1 \left(\tau_{m,p} + \alpha\Omega_{m,p}\right) d\Omega \;-\; \pi \int_1^1 \alpha\Omega_{m,p}\, d\Omega \;+\; \Omega_{m,p} \int_0^1 \gamma\alpha\, d\Omega$$

$$U_{m,p} = \frac{\alpha}{2} + \gamma$$

VIII Bilateral Regime with Joint Products

The following is the solution to the bilateral regime.

Nash product

$$\left[\rho_i - \delta - \pi(-\alpha\Omega_{b,p}) - \alpha\Omega_{b,p}\gamma\right]\left[\delta - \rho_i - \pi\left(-\alpha(1 - \Omega_{b,p})\right) - \alpha\Omega_{b,p}\gamma\right]$$

Optimal ρ is

$$\rho_{b,p}^{*} = \frac{2\delta - 2\alpha\Omega_{b,p}\pi + \alpha\pi}{2}$$

$$\int_0^{\Omega_{b,p}} \frac{2\delta - 2\alpha\Omega_{b,p}\pi + \alpha\pi}{2}\ d\Omega \ + \ \Omega_{b,p}\int_0^1 \gamma\alpha\ d\Omega$$

$$= \frac{2\delta\Omega_{b,p} - \alpha\Omega_{b,p}^2\pi + \alpha\Omega_{b,p}\pi}{2} \ + \ \gamma\alpha\Omega_{b,p}$$

FOC with respect to Ω:

$$\frac{2\delta - 2\pi\alpha\Omega_{b,p} + \pi\alpha}{2} + \gamma\alpha = 0$$

$$\Omega_{b,p}^{*} = \frac{2\delta + 2\alpha\gamma + \pi\alpha}{2\pi\alpha}$$

Total utility under a bilateral regime:

$$U_b = \int_0^{\frac{2\delta+2\alpha\gamma+\pi\alpha}{2\pi\alpha}} \frac{2\delta - 2\alpha\Omega_{b,p}\pi + \alpha\pi}{2}\ d\Omega \ + \ \frac{2\delta + 2\alpha\gamma + \pi\alpha}{2\pi\alpha}\int_0^1 \gamma\alpha\ d\Omega$$

$$U_b = \frac{[2\delta + \alpha(2\gamma + \pi)]^2}{8\alpha\pi}$$

IX Dual Regime with Joint Products

The following is the solution to the dual regime.

$$\int_0^{\Omega_{d,p}} (\tau_{d,p} + \alpha\Omega_{d,p})\, d\Omega \quad + \quad \int_{\Omega_{d,p}}^{\frac{2\delta+2\alpha\gamma+\pi\alpha}{2\pi\alpha}} \rho_{d,p}\, d\Omega \quad + \quad \frac{2\delta + 2\alpha\gamma + \pi\alpha}{2\pi\alpha} \int_0^1 \gamma\alpha\, d\Omega$$

$$= \alpha(1 - \Omega_{d,p})\Omega_{d,p} + \frac{\alpha\Omega_{d,p}^2}{2} + \frac{2\delta\left(\frac{2\delta + 2\alpha\gamma + \pi\alpha}{2\pi\alpha}\right) - \alpha\pi\left(\frac{2\delta + 2\alpha\gamma + \pi\alpha}{2\pi\alpha}\right)^2 + \alpha\pi\left(\frac{2\delta + 2\alpha\gamma + \pi\alpha}{2\pi\alpha}\right)}{2}$$
$$- \frac{2\delta\Omega_{d,p} - \alpha\pi\Omega_{d,p}^2 + \alpha\pi\Omega_{d,p}}{2} + \left(\frac{2\delta + 2\alpha\gamma + \pi\alpha}{2\pi\alpha}\right)\gamma$$

FOC with respect to Ω:

$$\alpha - 2\alpha\Omega_{d,p} + \alpha\Omega_{d,p} - \frac{2\delta - 2\pi\alpha\Omega_{d,p} + \pi\alpha}{2} = 0$$

$$\Omega_{d,p}^* = \frac{\alpha(2 - \pi) - 2\delta}{2\alpha(1 - \pi)}$$

Total utility under a dual regime:

$$U_{d,p} = \int_0^{\frac{\alpha(2-\pi)-2\delta}{2\alpha(1-\pi)}} (\tau_{d,p} + \alpha\Omega_{d,p})\, d\Omega \quad + \quad \int_{\frac{\alpha(2-\pi)-2\delta}{2\alpha(1-\pi)}}^{\frac{2\delta+2\alpha\gamma+\pi\alpha}{2\pi\alpha}} \rho_{d,p}\, d\Omega \quad + \quad \frac{2\delta + 2\alpha\gamma + \pi\alpha}{2\pi\alpha} \int_0^1 \gamma\alpha\, d\Omega$$

$$U_{d,p} = \frac{4\delta^2 + 4\gamma\alpha^2[(\gamma + \pi)(1 - \pi)] + \alpha^2\pi(4 - 3\pi) + 4\alpha\delta[2\gamma(1 - \pi) - \pi]}{8\alpha\pi(1 - \pi)}$$