Performance Analysis of SC-FDMA and OFDMA Systems Based on PAPR by Making Comparisons between Raised Cosine (RC) and Root Raised Cosine (RRC) Shaping Filter

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ABSTRACT

High-speed and secure data transmission has become one of the most important requirements for mobile phone users at the moment. Orthogonal Frequency Division Multiple Access (OFDMA) is attractive techniques to achieve these requirements. High spectral efficiency and robustness to multipath phenomena made the OFDMA adopted in the downlink multiple transmissions by several technologies such as WiMAX and long term evolution (LTE). Despite of these features, OFDMA suffered from high peak to average power ratio (PAPR). Alternative technique to OFDMA is Single Carrier Frequency Division Multiple access (SC-FDMA). Low PAPR made SC-FDMA becomes an attractive transmission especially for the uplink multiple access of the (LTE) technology. In this paper we study the performance analysis of SC-FDMA and OFDMA based on peak to average power ratio PAPR by making comparisons between using raised cosine and root raised cosine pulse shaping filter with different types of modulation.

Keywords: SC-FDMA, OFDMA, peak to average power ratio (PAPR), Raised cosine RC, Root Raised Cosine RRC pulse shaping filter.

INTRODUCTION

Recently, higher data rate has become one of the important requirements in the communication systems, especially in the wireless communication fields. Orthogonal Frequency Division Multiple Access (OFDMA) is an extended form of OFDM system in order to implement the multiuser communication systems. For many features such as high data rate and robustness to multipath effect, OFDMA is adopted for both uplink and downlink transmission In WiMAX while it is adopted by 3GPP for the LTE [1][2]. The major drawback of OFDMA as in OFDM is sensitive to frequency offset and high peak to average power ratio [3][4]. High PAPR is considered the key that the 3GPP adopted SC-FDMA for the uplink transmission scheme for LTE technology [3][5][10]. SC-FDMA is attractive multiple access that considers an extended technique of SC-FDE. It is combined the low peak to average power ratio result from the use of single carrier modulation such as global system for mobile GSM and code division multiple access CDMA and the robustness to multipath effect and flexibility allocation of frequency such as OFDMA [25].

METHODOLOGY

In this paper we analyze the PAPR for both SC-FDMA and OFDMA systems and we obtained the results analysis. Two types of shaping filter raised cosine and root raised cosine were used to reduce the PAPR. Matlab program was used to achieve the performance of the mention two systems.

Overview of OFDMA system

Orthogonal Frequency Division Multiple Access (OFDMA) is an extended form of OFDM system in order to implement multiuser communication system. [1][2]. The major functionality of OFDMA is very similar to OFDM, in OFDM system the single user is receive data at any time on all sub-carriers while OFDMA distribute the sub-carriers at the same time to all users ; thereby OFDMA takes an advantage of the orthogonality between sub-carrier to mitigate the inter-symbol interference [3][4]. Figure (1) shows
the main blocks of OFDMA system Model. The overall system model is similar to that in OFDM[1][5][15]. First the high rate transmitting sequences of data bit stream is passed through serial to parallel convertor for modulation onto M parallel sub-carriers [1][15]. After serial to parallel convertor the sequence is mapped into multi-level complex numbers by using different modulation schemes such as (BPSK, QPSK, M-QAM) [16][17][23]. The Inverse Fast Fourier Transform (IFFT) in the transmitter and Fast Fourier Transform (FFT) in the receiver are considered the main and backbone parts of OFDMA system for signal processing. It used to convert the frequency domain complex sequence to time domain complex sequence and vice versa [5][10][12]. Acyclic prefix (CP) of length greater than the channel response acts like guard interval and copies a portion of the samples at the end of the time domain sample block to the beginning as well as it preserves orthogonality between the sub-carrier and duplicate of the signal’s last part and prevents inter-symbol interference (ISI) [16][17]. Another reason for adding the cyclic prefix is to transfer the time domain convolution to a circular domain convolution between the OFDM symbols and the channel response. This makes the frequency domain implementation becomes a point-wise multiplication between the complex symbols allocated in each orthogonal sub-carrier and the corresponding channel frequency response. Additive white Gaussian noise (AWGN) is added to the transmitted signal as channel modeling; this allows the multipath to be controlled [9][15].

On the receiver side the same operations are applied inversely, first the guard band was removed; the N-point DFT was used to convert the time domain signal to frequency domain signal. After that the signal was demapped. Parallel to serial convertor was used to convert the signal back to serial form. One of the most important processing in the receiver side is the channel estimation and equalization for estimating the channel frequency response and compensating the distortion signal. Modulation and detection block are the last operation for implementing the OFDM system to recover the transmitted signal information [5][15].

**Overview of SC-FDMA system**

The block diagram for SC-FDMA and overall complexity is similar to OFDMA. The difference between the two technologies is the use of discrete Fourier transform DFT and the IDFT in the transmitter and the receiver side respectively [5][17][18]. Naturally SC-FDMA system technology offers frequency diversity gain over OFDMA system technology due to the use of DFT process at the beginning of the transmitter system by spreading all input information data over the multiple sub-carriers. DFT in the transmitter is inserted before the IFFT to transfer the time data stream into frequency domain implementation and IDFT in the receiver is inserted after the frequency domain equalizer to convert the frequency domain signal back to time domain signal. For this reason the SC-FDMA sometimes is called DFT spread OFDM [2][9][19]. The block diagram for transmitter and receiver of SC-FDMA is shown in figure (2). A brief description of the SC-FDMA operating system is: the modulation complex symbol in time domain is converted to frequency domain data stream using M-point Discrete Fourier Transform DFT, after that the sub-carrier mapper in frequency domain is mapping the data to the desired location on the total channel bandwidth. The output frequency domain signal from the sub-carrier mapper is converted again to the time domain signal by using N-point IFFT. Typically the size of IFFT is greater than the size of the DFT (N>M) where the unused input to the IFFT is set to zero and the output of IFFT will be a signal with single carrier properties that mean a signal with low power variations [9][13][17][18].
The benefit of using DFT is to mitigate the variation of the transmission power; thereby increase the possibility of the power amplifier efficiency. The Parallel to serial converter block was added in the SC-FDMA transmitter to convert the slow data rate signal to high data rate signal to perform sequential transmission rather than parallel as in OFDMA. [9][11]. In reality the transmitter performs two other processing tasks where a cyclic prefix of length greater than or roughly equal to the channel spreading delay is added to the total transmission block as guard interval and a pulse shaping linear filter is added to reduce the out of band signal energy. In spite of the cyclic prefix, the transmission of data still suffer from inter-symbol interference and this problem require a high equalization processing at the receiver side to compensate channel distortion. This problem is not a major issue as equalization is performed at the base station [9].

Both SC-FDMA and OFDMA perform equalization in the frequency domain after the discrete Fourier transform operation by dividing the DFT of the received signal by the DFT of the channel frequency response. The implementation in the frequency domain takes an advantage over the time domain equalization as it requires less computational power. After compensation for channel distortion, the output from the equalization process is Inverse Discrete Fourier Transformed (IDFT) to convert the signal to time domain. The final stage is converting the signal to serial form and detecting the origin data bit stream by using a detection operation such as hard decision detector [5][7][9].

Sub-carrier mapping schemes

In Localized mapper the output of DFT is mapped to a subset of the adjacent sub-carrier; therefore it take only fraction of the total system bandwidth while in the distributed mapper the output complex symbols are mapped equally over the entire total system bandwidth and this lead to zero amplitude for the remaining sub-carriers [1][5][9]. Interleaved SC-FDMA (IFDMA) is considered a special case of the distributed SC-FDMA where the occupied sub-carriers are equally spaced over the total system bandwidth also it is considered very efficient in the transmitter because it has the ability to modulate the input data signal in the time domain without using the DFT and IDFT [2][5][9]. The distributed FDMA (DFDMA) is considered more robust to frequency selective fading and offers additional frequency diversity gain than localized FDMA due to the fact that information is spread across the entire channel bandwidth. Combination of Localized SC-FDMA with channel-dependent scheduling can probably offer multi-user diversity in frequency selective channel conditions[1][5][9][10]. In case of IFDMA, the frequency domain modulated signal at the output of the mapper is:

$$y_l = \begin{cases} \frac{y_l}{Q} & l = qk, 0 \leq k \leq M-1 \\ 0 & \text{otherwise} \end{cases}$$

Q is the bandwidth spreading factor of the IFDMA.

In case of LFDMA, the frequency domain modulated signal at the output of the mapper is:

$$y_l = \begin{cases} y_l & 0 \leq l \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

where $N$ is the order of IFFT

Pulse shaping filter

Pulse shaping filter is used in the transmitter side in case of SC-FDMA system to mitigate the out of band signal energy at the output of IFFT and avoids the overlap of the symbols in time domain; thereby avoiding inter-symbol interference. The complex pass band transmitted signal is represented as in [5]:

Figure (2) SC-FDMA system
\[ X(t) = e^{j\omega_c t} \sum_{n=0}^{N-1} x(n)p(t - nT) \quad (3) \]

Where \( \omega_c \) is the carrier frequency of the system, and \( p(t) \) is the baseband pulse, \( T \) is the symbol duration of the transmitted signal.

**Raised Cosine (RC) filter**

Raised Cosine is considered the most common type of pulse shaping filter and it is widely used for wireless communication system [9][21]. The time domain baseband pulse representation is:

\[
pRC(t) = \frac{\sin\left(\frac{\pi t}{T}\right) \ast \cos\left(\frac{\pi a t}{T}\right)}{1 - \left(\frac{4a t}{T}\right)^2} \quad (4)\]

Where \( a \) is the roll of factor, \( 0 \leq a \leq 1 \), and it is defined as a portion of the total symbol period and it measures the sharpness of the frequency response characteristics of the pulse shaping filter [8].

**Root Raised Cosine (RRC) filter**

Root Raised Cosine filter is another type of pulse shaping filter. The time domain baseband pulse representation as in [9][14][20][22] is given by:

\[
pRRC(t) = \frac{4a t}{\pi \sqrt{\pi}} \cdot \sin\left(\frac{\pi t}{T}(1 - a)\right) + \cos\left(\frac{\pi t}{T}(1 + a)\right) \div \left(1 - \left(\frac{4a t}{T}\right) z\right) \quad (5)\]

**Peak to average power ratio (PAPR)**

PAPR is considered an important parameter for studying the main difference of SC-FDMA and OFDMA and it indicates the power efficiency for the transmitted signal. Low PAPR makes the SC-FDMA more preferred than OFDMA in the uplink multiple access for Long Term Evolution (LTE) due to it is inherently single carrier modulation structure [10]. There is theoretical expression between the PAPR and the power efficiency of the transmitted signal and it is given by [5]

\[
\eta = \eta_{\text{max}} \cdot 10^{-\text{PAPR}/20} \quad (6)
\]

Where \( \eta \) is the power efficiency and \( \eta_{\text{max}} \) is the maximum power efficiency. It is clear from the relationship, high PAPR indicates the low power efficiency and low PAPR indicate high power efficiency. PAPR is defined as the ratio of peak power amplitude of the signal to the average power amplitude of the transmitted signal [3][9]. The relationship to explain PAPR as in [6][5][24] is:

\[
PAPR = \frac{\text{peak power of } x(t)}{\text{average power of } x(t)} \quad (7)
\]

The peak to average power ratio (PAPR) for the complex pass-band transmitted signal is represented as in [5][24]:

\[
PAPR = \frac{1}{N^T} \int_0^{N^T} |x(t)|^2 dt \quad (8)
\]

Where \( N \) represented the number of sub-carrier.

In case there is no pulse shaping applied to the signal, the symbol rate sampling gives the same PAPR as in the continuous case because SC-FDMA uses single carrier modulation. The relationship that explains the PAPR without pulse shaping is implemented as in [5][24]:

\[
PAPR = \frac{\text{Max}_{n=0,1...,N-1} |x(n)|^2}{\sum_{n=0}^{N-1} |x(n)|^2} \quad (9)
\]

The Complementary Cumulative Distribution Function (CCDF) is defined as the probability that PAPR is greater than a certain PAPR value \( \text{PAPR}_0 \) (\( \text{Pr}\{\text{PAPR} > \text{PAPR}_0\} \)) [5][9][24].

**Simulation & Results**

In this section we analyze the PAPR of the SC-FDMA and OFDMA systems. The total number of sub-carriers, \( M= 256 \), the number of sub-carrier per block, \( N= 64 \) and the maximum number of users, \( Q=4 \) due to \( M=Q \). We assumed raised cosine and root raised cosine pulse shaping filters in case of the SC-FDMA system and assume no pulse shaping in case of OFDMA system to analyse the influences on PAPR and also employed three types of constellation mapping (QPSK, 16-QAM, 64-QAM) for comparison. The results are observed for the value of \( \text{PAPR}_0 \) that exceed with probability less than 0.1% (\( \text{Pr}\{\text{PAPR} > \text{PAPR}_0\} = 10^{-3}\)). Figures (3), (4), (5) illustrate the comparison of CCDF of PAPR for IFDMA, LFDMA and OFDMA using QPSK, 16QAM and 64QAM modulation, for two cases ; with raised cosine and with root raised cosine pulse shaping filter and roll of
factor ($\alpha = 0.5$). It is clear from figure (3) that root raised cosine (RRC) gives better PAPR performance as compared with raised cosine (RC) filter with all types of modulation schemes. The impact is clear in case of IFDMA. With QPSK, root raised cosine gives PAPR equal to 3.5 dB whereas raised cosine gives 4.5 dB. This shows that root raised cosine reduced the PAPR by 1 dB as compared with raised cosine for IFDMA. The same thing occurs with 16QAM and With 64QAM in figures (4), (5), root raised cosine gives better PAPR by 0.5 dB as compared with raised cosine pulse shaping filter for IFDMA.

In case of LFDMA, we can notice that the performance with the two types of pulse shaping is approximately the same. In case of OFDMA, the PAPR is decreasing with the increase of the modulation order. In figures (6), (7), (8), (9), (10), (11) the performance is studied with different values of roll off factor. It is clear that, both raised cosine and root raised cosine filter are more harmful in term of PAPR for IFDMA compared to LFDMA. In case of raised cosine, the PAPR is decreased significantly for IFDMA as the roll of factor increases from 0 to 1 whereas PAPR of LFDMA hardly decreases. In case of root raised cosine, it is clear that PAPR decreases with increase in the roll of factor from 0 to 0.5 whereas PAPR increases hardly after roll of factor exceed ($\alpha = 0.5$) for both IFDMA and LFDMA. In all results there is a noticeable fact that SC-FDMA signals with both types (localized and interleaved) have lower PAPR as compared with OFDMA signals. The PAPR of LFDMA is higher than the PAPR of IFDMA, but it is lower than the PAPR of OFDMA. Furthermore raised cosine and root raised cosine pulse shaping filters noticeably increases the PAPR of IFDMA; therefore they should be designed strictly in order to restrict the PAPR without impact on system performance.
Figure (6) comparison of CCDF of PAPR for SC-FDMA (IFDMA, LFDMA) and OFDMA using QPSK with raised cosine filter

Figure (7) comparison of CCDF of PAPR for SC-FDMA (IFDMA, LFDMA) and OFDMA using QPSK with root raised cosine filter

Figure (8) comparison of CCDF of PAPR for SC-FDMA (IFDMA, LFDMA) and OFDMA using 16QAM with raised cosine filter

Figure (9) comparison of CCDF of PAPR for SC-FDMA (IFDMA, LFDMA) and OFDMA using 16QAM with root raised cosine filter

Figure (10) comparison of CCDF of PAPR for SC-FDMA (IFDMA, LFDMA) and OFDMA using 64QAM with raised cosine filter

Figure (11) comparison of CCDF of PAPR for SC-FDMA (IFDMA, LFDMA) and OFDMA using 64QAM with root raised cosine filter
Conclusion
In this paper we have focused on the performance analysis of SC-FDMA and OFDMA systems in terms of PAPR, different pulse shaping filters have also been analysed. Furthermore different types of modulation techniques (QPSK, 16QAM, and 64QAM) are considered. In case of adding raised cosine filter, the PAPR performance in IFDMA is shown to be better than LFDMA and OFDMA. Furthermore the LFDMA shows better performance as compared with OFDMA. PAPR increases in both types of SC-FDMA sub-carrier mapper with respect to the type of modulation used. QPSK has lower PAPR as compared with 16QAM and 64QAM; therefore it increases significantly in IFDMA for all modulation schemes while it increases hardly in LFDMA. The PAPR is observed to decrease significantly for IFDMA as the roll of factor increases from 0 to 1 whereas PAPR of LFDMA hardly decreases. Root raised cosine gives better PAPR performance as compared with raised cosine shaping filter. The important thing noticed in adding root raised cosine filter is that when the roll of factor increases from 0 to 0.5 for both types of SC-FDMA mapper, PAPR decreases then hardly increases after 0.5; therefore (α=0.5) is the optimal value that gives a better PAPR performance for both types of the mapper as shown in the simulation results.

References


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