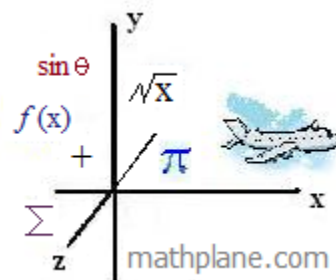


# Simplifying Negative Exponents and Variables

Brief notes, examples, and Worksheet (& solutions)



Exponent Rules: Notes and Examples

Exponent *definition*:

$$X^A = X_1 \cdot X_2 \cdot X_3 \cdot \dots \cdot X_{A-2} \cdot X_{A-1} \cdot X_A$$

Examples:  $4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$

$$(-2)^7 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = -128$$

$$(-2)^6 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 64$$

Rule #1 ('Addition Rule')

$$X^A \cdot X^B = X^{A+B}$$

Examples:

$$X^3 \cdot X^5 = X^8$$

$$5^3 \cdot 5^2 = 125 \cdot 25 = 3125 = 5^5$$

Note:

$$Y^2 \cdot Y^4 = Y^6$$

$$\underbrace{(Y \times Y)}_2 \cdot \underbrace{(Y \times Y \times Y \times Y)}_4 = \underbrace{Y \times Y \times Y \times Y \times Y \times Y}_{6 \text{ total Y's}}$$

Rule #2: ('Multiplication Rule')

$$(X^A)^B = X^{AB}$$

Examples:  $(X^4)^3 = X^{12}$

$$(4^2)^4 = 4^8 = 16^4 = 65536$$

Note:

$$(Y^5)^3 = \underbrace{(Y \times Y \times Y \times Y \times Y)}_5 \cdot \underbrace{(Y \times Y \times Y \times Y \times Y)}_5 \cdot \underbrace{(Y \times Y \times Y \times Y \times Y)}_5 = Y^{15}$$

3 groups of 5 Y's  
total: 3 x 5 = 15 Y's

Rule #3: ('zero exponent')

$$X^0 = 1$$

Examples:  $Y^0 = 1$

$$8^0 = 1$$

$$(3cd)^0 = 1$$

Note:  $Z^5 \cdot Z^{-5} = Z^0 = 1$

addition rule --- then, zero exponent rule

What is  $0^0$ ?  $0^A = 0$  because  $0 \cdot 0 \cdot 0 \cdot 0 \dots = 0$   
(if  $A \neq 0$ )

$$X^0 = 1 \quad (\text{zero exponent rule})$$

$$0^0 = 1$$

$$\frac{\overbrace{Z \times Z \times Z \times Z \times Z}^{Z^5}}{\underbrace{Z \times Z \times Z \times Z \times Z}_{Z^5}} = 1$$

Rule #4: ('negatives' or 'reciprocal rule')

$$X^{(-A)} = \frac{1}{X^A}$$

Examples:

$$X^{-3} = \frac{1}{X^3}$$

$$5^{-2} = \frac{1}{25} \quad \text{It is not equal to -25!!!}$$

$$\left(\frac{1}{3}\right)^{-4} = 81$$

Note:

$$Y^{(-A)} = \boxed{Y^{(-A)}} \cdot \frac{Y^A}{Y^A} = \frac{Y^{(-A)} \cdot Y^A}{Y^A} = \frac{Y^{(-A+A)}}{Y^A} = \frac{Y^0}{Y^A} = \boxed{\frac{1}{Y^A}}$$

multiply by one
exponent addition rule
zero exponent

Rule #5: ('base rule')

$$X^A \cdot Y^A = (XY)^A$$

Examples:  $5^3 \cdot 7^3 = 125 \times 343 = 42875 = 35^3$

$$= (5 \times 5 \times 5) \times (7 \times 7 \times 7) = (5 \times 7) \times (5 \times 7) \times (5 \times 7)$$

$$4^{\frac{1}{2}} \cdot 16^{\frac{1}{2}} = 64^{(1/2)} = 8$$

$$\sqrt{4} \times \sqrt{16} = \sqrt{4 \times 16} = \sqrt{64}$$

Rule #6: ('rational exponents')

$$X^{(1/2)} = \sqrt{X} \qquad X^{\left(\frac{A}{B}\right)} = \sqrt[B]{X^A}$$

Examples:  $25^{(1/2)} = \sqrt{25} = 5$

$$8^{(1/3)} = \sqrt[3]{8} = 2 \quad (\text{'cubed root of 8'})$$

$$121^{(.5)} = 11$$

Note:

$$Y^{(1/2)} \cdot Y^{(1/2)} = Y^1 \qquad \sqrt{Y} \cdot \sqrt{Y} = Y$$

(addition exponent rule)

$$8^{(1/3)} \cdot 8^{(1/3)} \cdot 8^{(1/3)} = 8^1 = 8$$

Example: Simplify  $\frac{x^5}{x^8}$

Method 1: Use Exponent Rules

$$\frac{x^5}{x^8} \Rightarrow x^{5-8} = x^{-3} = \frac{1}{x^3}$$

"Subtraction Rule"      "Negatives/Reciprocal Rule"

Method 2: Count the variables

$$\frac{x^5}{x^8} \Rightarrow \frac{\overbrace{xxxxx}^5}{\underbrace{xxxxxxxx}_8} = \frac{1}{xxx} = \frac{1}{x^3}$$

5 cancel out

Example: Simplify  $\frac{a^2 b^3 c^4}{a^5 b^3 c^8}$

This is a 3-part problem: 1) simplify the a's 2) simplify the b's 3) simplify the c's

$$\frac{a^2 b^3 c^4}{a^5 b^3 c^8} = \frac{a^2 b^3 c^4}{a^3 b^3 c^8} = \frac{(1) c^4}{a^3 c^8} = \frac{(1)}{a^3 c^4} = \frac{1}{a^3 c^4}$$

2 a's      3 b's      4 c's  
5 a's      3 b's      8 c's

so, two a's cancel out      so, three b's cancelled      so, four c's cancel out

Exponents and Variables Examples:

$x = -2$        $y = 4$        $z = \frac{1}{3}$

a)  $x^{-2}$

b)  $xy^0z^3$

c)  $\frac{y^{-3}zx^5}{x^2z^2}$

d)  $\frac{5x^3y^3}{25xy^{-2}z^{-1}}$

e)  $\frac{(x^2y^{-3}z^4)^2}{x^{-1}y^{-3}z^6}$

a)  $x^{-2} = \frac{1}{x^2}$       simplify the variable  
 $\frac{1}{(-2)^2}$       substitute  
 $\frac{1}{4}$

b)  $xy^0z^3 = (-2)(4)^0(\frac{1}{3})^3$       substitute  
 $-2 \cdot 1 \cdot \frac{1}{27}$       simplify  
 $\frac{-2}{27}$

c)  $\frac{y^{-3}zx^5}{x^2z^2} = \frac{x^5z}{x^2y^3z^2}$       rearrange variables (reciprocal rule)  
 $\frac{x^3}{y^3z}$       simplify  
 $\frac{(-2)^3}{(4)^3(\frac{1}{3})}$       substitute

d)  $\frac{5x^3y^3}{25xy^{-2}z^{-1}} = \frac{1x^2y^5z^1}{5}$   
 simplify each variable and the numbers  
 then, substitute  $\frac{1 \cdot (-2)^2 \cdot (4)^5 \cdot (\frac{1}{3})}{5}$   
 $\frac{1024}{15}$

e)  $\frac{(x^2y^{-3}z^4)^2}{x^{-1}y^{-3}z^6} = \frac{x^4y^{-6}z^8}{x^{-1}y^{-3}z^6}$  (exponent rule)       $\frac{-8}{\frac{64}{3}} = \frac{-3}{8}$   
 $\frac{x^5z^2}{y^3}$  (addition/subtraction rule)  
 then, substitute  $\frac{(-2)^5 \cdot (\frac{1}{3})^2}{(4)^3} = \frac{-1}{18}$

"To check the power,  
just plug it in!"



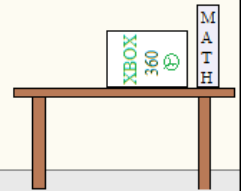
Exponents

$$3^x = 27$$
$$x = 3$$

→

$$3^{(3)} = 27$$

Mr. Volt  
Algebra



Alex gets the connection!

Worksheet and Solutions →

Positive/Negative Exponents & Variables

Simplify the following:

1)  $(3a^2)^3$

2)  $(2b^3)^2$

3)  $(-2c^3)^{-2}$

4)  $(-4d^2)(-5^{-3})$

5)  $(3e^2)(-2e^0)^2$

6)  $(-8f^2)(fg^{-2})$

7)  $h^{-2}(h^2j^3)^4(-jk)^5$

8)  $(3k^7lm)^{-2}(l^2m)^0$

9)  $\frac{6p^3q^2}{4pq^{-2}}$

10)  $\frac{a^3b^2c^{-1}}{b^2cd^{-3}}$

11)  $\frac{-2tu^5v^{10}}{(4t^2v)^2}$

12)  $\frac{5r(s^0t)^{-3}}{(5st^2)^2}$

12)  $\frac{(x^2y^{-3})(3xy)}{(xy^2)^2(6x^2y)}$

13)  $\frac{(7x^9y^8)^0(3xy^2)^{-2}}{-2(6x^2)(xy^{-2})}$

14)  $\frac{(a^2b^5)^3}{(ab)^{-2}} \div \frac{(ab^{-1})}{(a^0b)^2}$

15)  $\frac{def}{(d^2e^3f)^{-1}} \div fed$

Simplify the following:

1)  $(3a^2)^3$

$$27a^6$$

2)  $(2b^3)^2$

$$4b^6$$

3)  $(-2c^3)^{-2}$

$$\frac{1}{(-2c^3)^2} = \frac{1}{(-2c^3)(-2c^3)}$$

$$\frac{1}{4c^6}$$

4)  $(-4d^2)(-5^{-3})$

$$(-4d^2) \frac{-1}{125}$$

$$\frac{4d^2}{125}$$

5)  $(3e^2)(-2e^0)^2$

$$(3e^2)(-2)^2$$

$$12e^2$$

6)  $(-8f^2)(fg^{-2})$

$$\frac{-8f^2}{1} \cdot \frac{f}{g^2}$$

$$\frac{-8f^3}{g^2}$$

7)  $h^{-2}(h^2j^3)^4(-jk)^5$

$$\frac{h^8 j^{12}}{h^2} \cdot (-1)^5 j^5 k^5$$

$$-h^6 j^{17} k^5$$

8)  $(3k^7lm)^{-2}(l^2m)^0$

$$\frac{1}{(3k^7lm)^2}$$

$$\frac{1}{9k^{14}l^2m^2}$$

9)  $\frac{6p^3q^2}{4pq^{-2}}$

$$\frac{3p^2q^4}{2}$$

10)  $\frac{a^3b^2c^{-1}}{b^2cd^{-3}}$

$$\frac{a^3d^3}{c^2}$$

11)  $\frac{-2tu^5v^{10}}{(4t^2v)^2}$

$$\frac{-2tu^5v^{10}}{16t^4v^2}$$

$$\frac{-u^5v^8}{8t^3}$$

12)  $\frac{5r(s^0t)^{-3}}{(5st^2)^2}$

$$\frac{5r(t)^{-3}}{25s^2t^4}$$

$$\frac{r}{5s^2t^7}$$

12)  $\frac{(x^2y^{-3})(3xy)}{(xy^2)^2(6x^2y)}$

$$\frac{\frac{x^2}{y^3} \cdot 3xy}{x^2y^4 \cdot 6x^2y}$$

$$\frac{\frac{3x^3}{y^2}}{6x^4y^5}$$

$$\frac{1}{2xy^7}$$

13)  $\frac{(7x^9y^8)^0(3xy^2)^{-2}}{-2(6x^2)(xy^{-2})}$

$$\frac{1 \cdot 3^{-2}x^{-2}y^{-4}}{-12x^3y^{-2}}$$

$$\frac{\frac{1}{9}}{-12x^5 \cdot y^2}$$

$$\frac{-1}{108x^5y^2}$$

14)  $\frac{(a^2b^5)^3}{(ab^{-2})^2} \div \frac{(ab^{-1})}{(a^0b)^2}$

$$\frac{a^6b^{15}}{a^{-2}b^{-2}} \div \frac{ab^{-1}}{b^2}$$

$$a^8b^{17} \div \frac{a}{b^3}$$

$$a^8b^{17} \cdot \frac{b^3}{a}$$

$$a^7b^{20}$$

15)  $\frac{def}{(d^2e^3f)^{-1}} \div fed$

$$\frac{\cancel{def}}{(d^2e^3f)^{-1}} \cdot \frac{1}{\cancel{fed}}$$

$$d^2e^3f$$