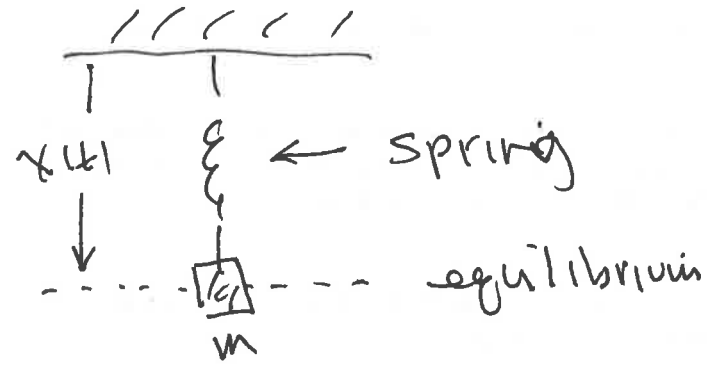
Math 3331 - ODEs

lect 21

Mass - Spring Prob



Restoring force $F = -kx$ Hooke's law

Newton's 2nd law

$$F = ma = m\frac{d^2x}{dt^2} = -kx$$

or $$m\frac{d^2x}{dt^2} + kx = 0$$

Dividing by m and letting $\omega^2 = \frac{k}{m}$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

M

Ms

3

W

MS

5

F

Meeting

7

Soccer

10

System

12

open

14

T3

17

19

21

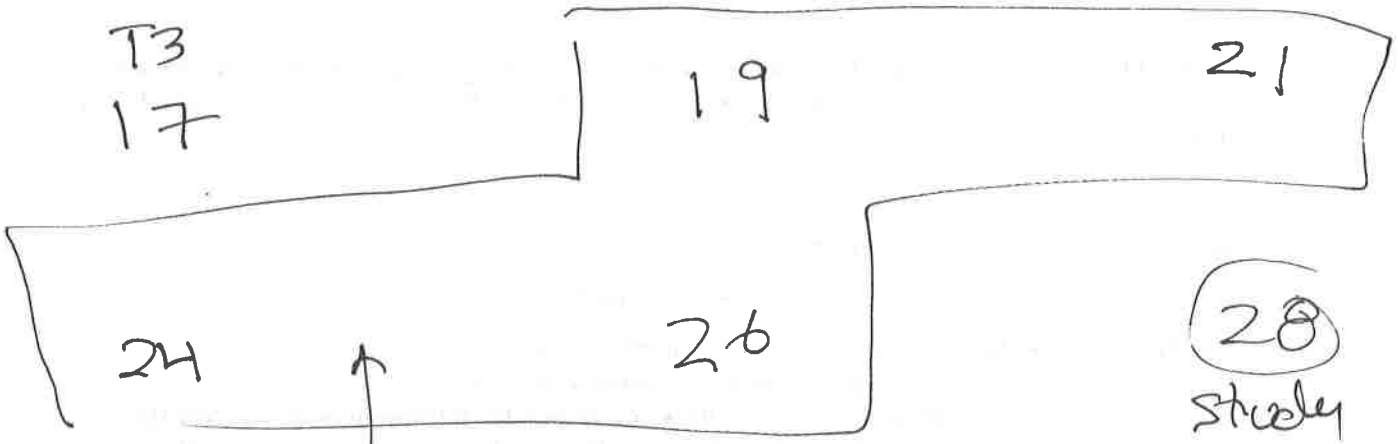
24

26

28

Study

Seminars



Soln $x = e^{rt}$ $\dot{x} = re^{rt}$ $\ddot{x} = r^2 e^{rt}$

sub $(r^2 + \omega^2) e^{rt} = 0$

$r^2 + \omega^2 = 0$ CE.

$r = \pm i\omega$ complex

$x(t) = c_1 \cos \omega t + c_2 \sin \omega t$

we usually give IC.

$x(0) = ?$ $\dot{x}(0) = ?$ $\dot{x} = \frac{dx}{dt}$

Ex A force of 25.6 N is required to stretch a spring 0.2 m. If a mass of 2 kg is attached and is stretched 0.2 m and released (no velocity) find the motion of the mass.

1st - Find k

$F = kx$ so $25.6 = k(.2)$

so $k = \frac{25.6}{.2} = 128$

so $m\ddot{x} + kx = 0$

becomes $2\ddot{x} + 128x = 0$

or $\ddot{x} + 64x = 0$

$x = c_1 \cos 8t + c_2 \sin 8t$

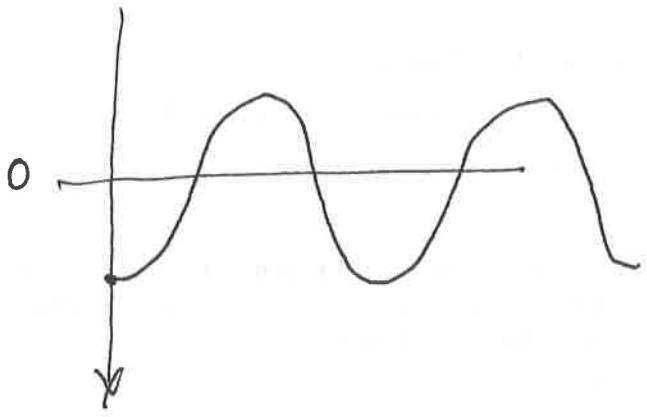
$x(0) = .2 \quad \dot{x}(0) = 0$

so $.2 = c_1$ so $x = .2 \cos 8t + c_2 \sin 8t$

$\frac{dx}{dt} = -8(.2)\sin 8t + 8c_2 \cos 8t$

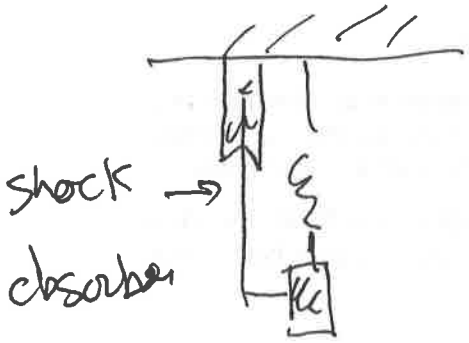
$\dot{x}(0) = 0 \Rightarrow c_2 = 0$

and $x(t) = .2 \cos 8t$



time

Now suppose we damp the system?



How does the ODE change?