Three-Dimensional finite element forward modeling for Arbitrary anisotropy magnetotelluric

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SUMMARY

Magnetotelluric (MT) studies increasingly show indications for the existence of electrically anisotropic domains within the earth's crust and mantle. With the models in the MT method becoming more and more complicated,conventional modeling methods based on structured grids cannot satisfy the accuracy requirement. So, we adopt three dimensional finite element method with tetrahedral edge elements in the MT anisotropic modeling.First,we describe the numerical realization of the FEM algorithm briefly. Then we verify the FE algorithm using the DTM1 model.We further outline characteristics of the electric current density and induction vector for a number of geologically motivated standard anomalies with varying azimuthal anisotropy for 3D conductivity anomaly to support the assessment of field observations.

Keywords:MT,anisotropy, forward modeling,vector finite element, tetrahedron

INTRODUCTION

For modeling MT rsponses in 2D anisotropic structures, the finite difference algorithm and the finite element code were publisehed by Pek&Verner (1997) and Li (2008). Magnetotelluric (MT) studies increasingly show indications for the existence of electrically anisotropic domains within the earth's crust and mantle. With the models in the MT method becoming more and more complicated,conventional modeling methods based on structured grids cannot satisfy the accuracy requirement. The FE method has long been used in 3D EM geophysical problems (Nam et al., 2007; Schwarzbach,Börner, & Spitzer, 2011; Ren, Kalscheuer, Greenhalgh,& Maurer, 2013; Grayver & Bürg, 2014).To better simulate electrical anisotropy of the complex earth, we adopt three dimensional finite element method with tetrahedral edge elements. As the observation of time varying natural electromagnetic field components at the surface is usually expressed by impedance tensor and induction vector,this study demonstrates their behavior for various 3D settings of anisotropic conductivity anomalies.

3D forward modeling approach

Consider a 3D conductivity model with general anisotropy.The electric field satisfies the following secondary field(Newman, 2002) equation:

$$\nabla\times\nabla\times\mathbf{E}_s - i\omega\mu\sigma\mathbf{E}_s = i\omega\mu(\sigma-\sigma_0)\mathbf{E}_p \qquad (1)$$

$$\mathbf{n}\times\mathbf{E}_s = 0 \qquad (2)$$

Total field:

$$E = E_P + E_S \qquad (3)$$

With the electric conductivity tensor:

$$\delta = \begin{bmatrix} \delta_{xx} & \delta_{xy} & \delta_{xz} \\ \delta_{xy} & \delta_{yy} & \delta_{yz} \\ \delta_{xz} & \delta_{yz} & \delta_{zz} \end{bmatrix} \qquad (4)$$

The differential equation (1) is approximated by vector finite element method. The computation domain is discretized into tetrahedra using Tetgen (Si,2015) (shown in Fig. 1). We reformulate the resulting complex linear system to its real form and utilize a block-diagonal preconditioner. The CG method with Auxiliary Space Maxwell pre-conditioner was used to solve the two smaller real-valued equation required by application of the block-diagonal pre-conditioner. The resulting linear system was stored using parallel compressed row storage format and then solved by cooperation of all processes.

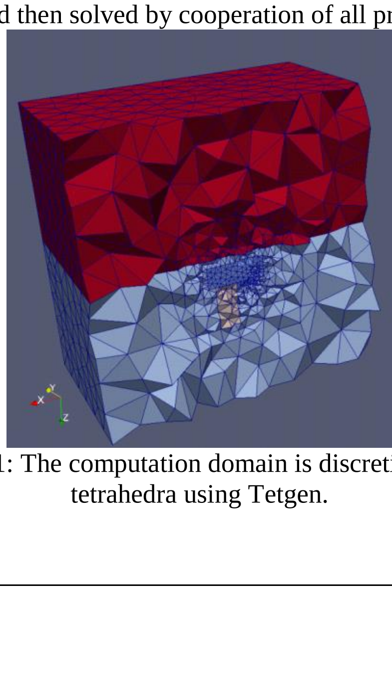

Figure 1: The computation domain is discretized into tetrahedra using Tetgen.

Numerical Examples

We present simulate MT resposes for DTM1 model (show in Fig.2).For comparision,our isotropic results agree very well with those obtained by using the FD (Mackie)and FE(Han`s).Then,The effectiveness of our anisotropic results consists of two models: one is MARE2DEM model, the other is 2D model computed by 3D anisotropic modeling (show in Fig.4).

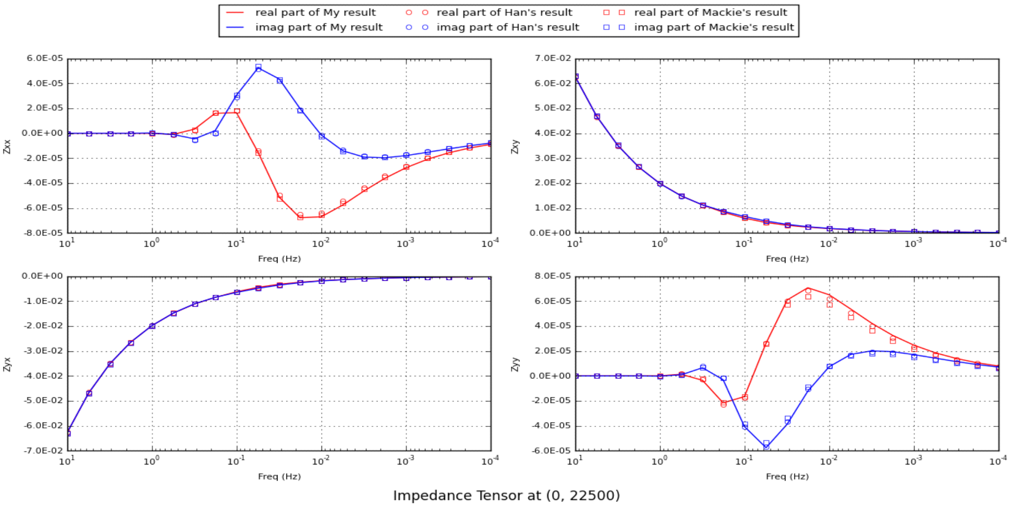

Fig. 2 Comparision of 3D isotropic model(DTM1)

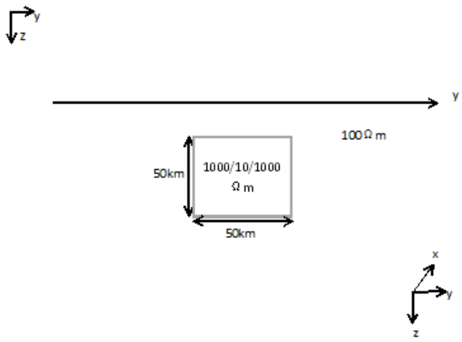

Fig. 3 2D anisotropic model

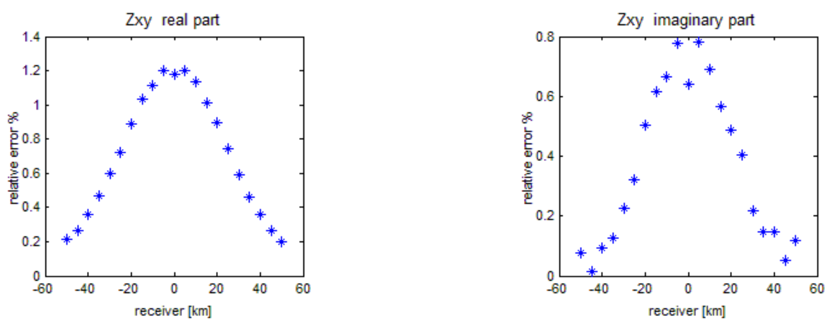

Fig. 4 Comparision of two anisotropic model

Effects of anisotropy

We simulated a simple anisotropic block of 50Km * 50Km*40Km,which is embedded into an isotropic homogeneous half-space.3D anisotroic anomaly, principal resistivities: $\rho_x/\rho_y/\rho_z$=1000/10/1000 Ω m, azimuth γ =0°, ρ_{bg}=100 Ω m(show in Fig.5),Parkinson Arrow: Tipper real,Colors: ρ_{xy}, Φ_{xy} (show in Fig.6). azimuth γ =45°, ρ_{bg}=100 Ω m(show in Fig.7),Parkinson Arrow: Tipper real,Colors: ρ_{xy}, Φ_{xy} (show in Fig.8). All calculations are made at a period of T=20s. The computation platform is a PC cluster, each node has two quad-core Intel Xeon E5530 2.4 GHz processors and 16 GB memory. The run time is about 122 seconds using 32

CPU cores. This model is used to demonstrate the effect of horizontal anisotropy upon the magnetotelluric responses. In general the magnetic field variations depend on the spatial pattern of the electric current distribution,but the electric field variations both on current and charge distribution. The induction vectors compared to reflect the features of the bottom of medium anisotropy(show in Fig.6,8). rhoa and phase values show a very different pattern above and in the vicinity of a conductivity anomaly.

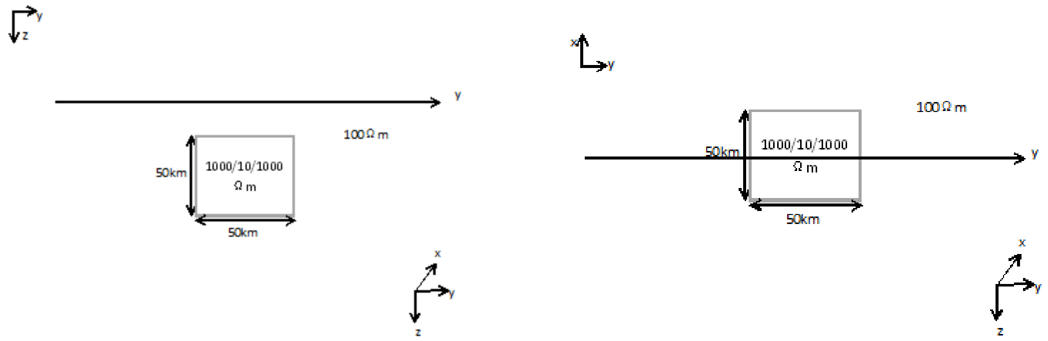

Fig. 5 The 3D triaxial anisotropic model

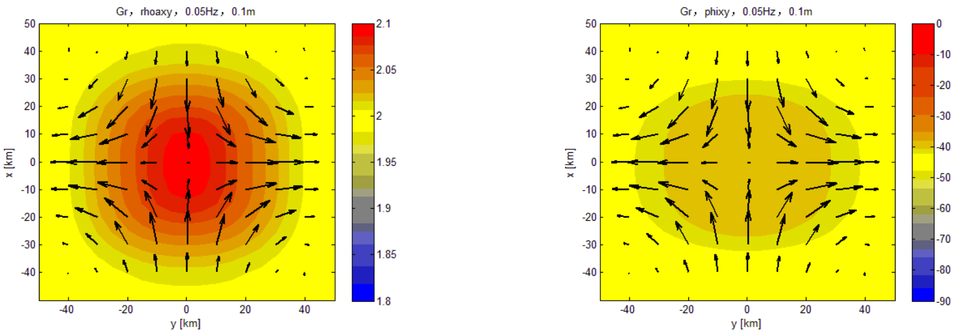

Fig. 6 The response of the triaxial anisotropy

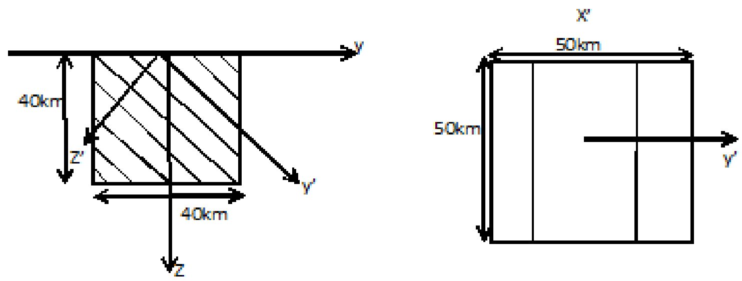

Fig. 7 The 3D varying azimuthal anisotropic model

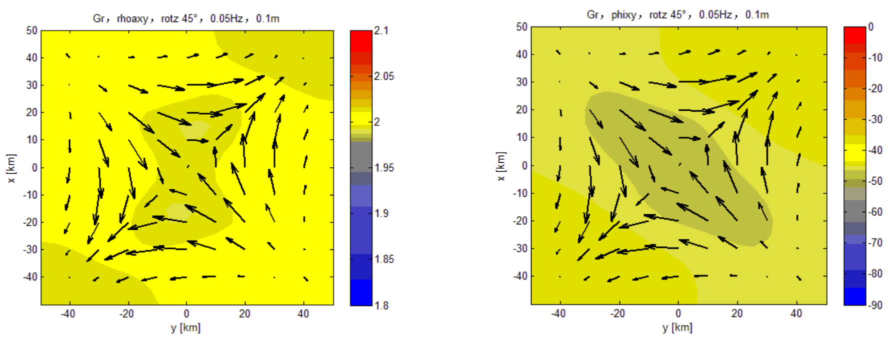

Fig. 8 The response of The 3D varying azimuthal anisotropy

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