IV. The Einstein Elevator

- The elevator is a macroscopic box , which is small compared to the curvature scale, and which is undergoing free fall.

- The “walls” of the elevator are not rigid bodies.

- Instead, they are made of comoving “dust” undergoing free fall.

- The gravitational field (Riemann curvature tensor) is revealed by the way the walls of the elevator are distorted by the gravitational tidal forces.

Motivation

- Study one liter of dark energy

  "Extract" a dark-energy dominated 10 cm x 10 cm x 10 cm cube.

  Terminate the surfaces with periodic boundary conditions.

  This liter of space will satisfy FRW equation.

  Therefore it grows exponentially.

  Problem to solve: understand the microscopic mechanism responsible for this growth.

- Then generalize the elevator construction:

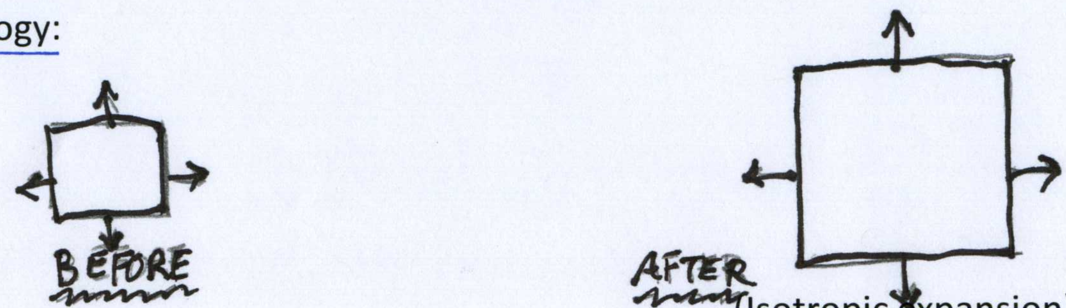
  General FRW cosmology

  Painleve-Gullstrand metrics: “go with the flow”

  Apply to the loop-gravity description

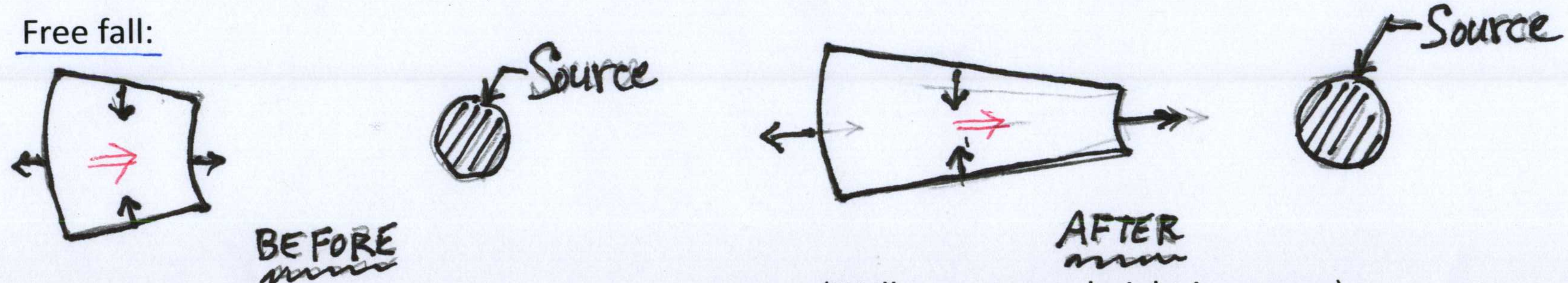
THREE EXAMPLES

- FRW cosmology:



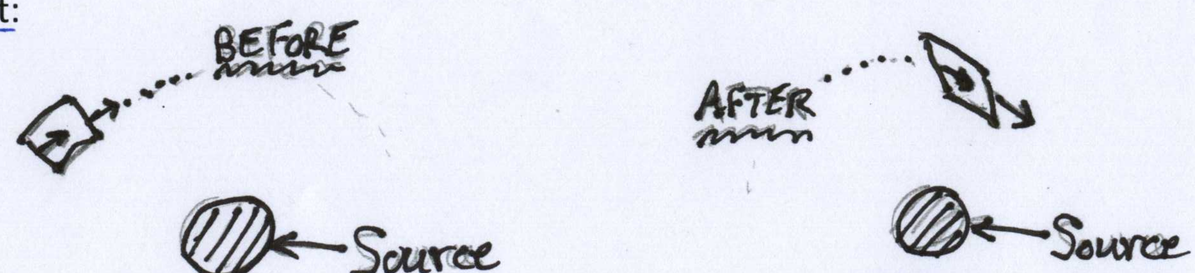
(Isotropic expansion)

- Free fall:



(Walls converge, height increases)

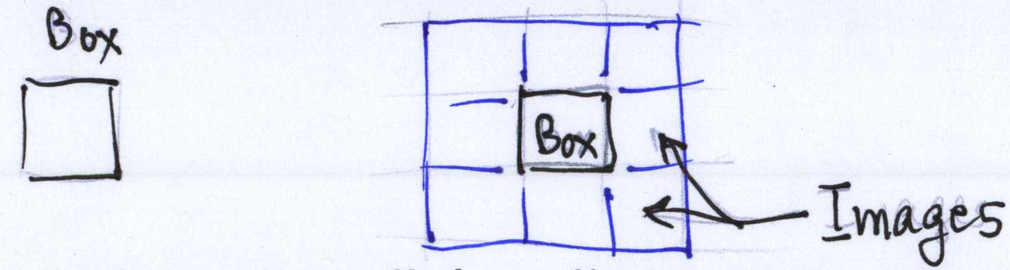
- Circular orbit:



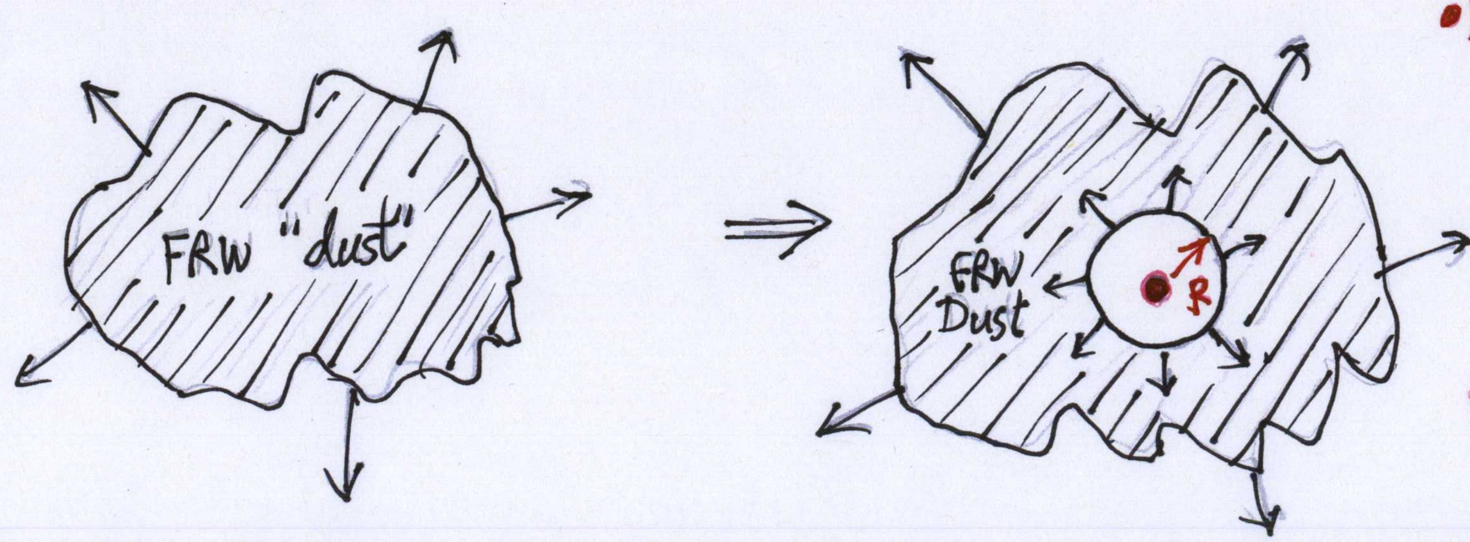
(Shear)

Lattice Cosmology

Periodic boundary conditions; "hall of mirrors" (or "universal covering space)

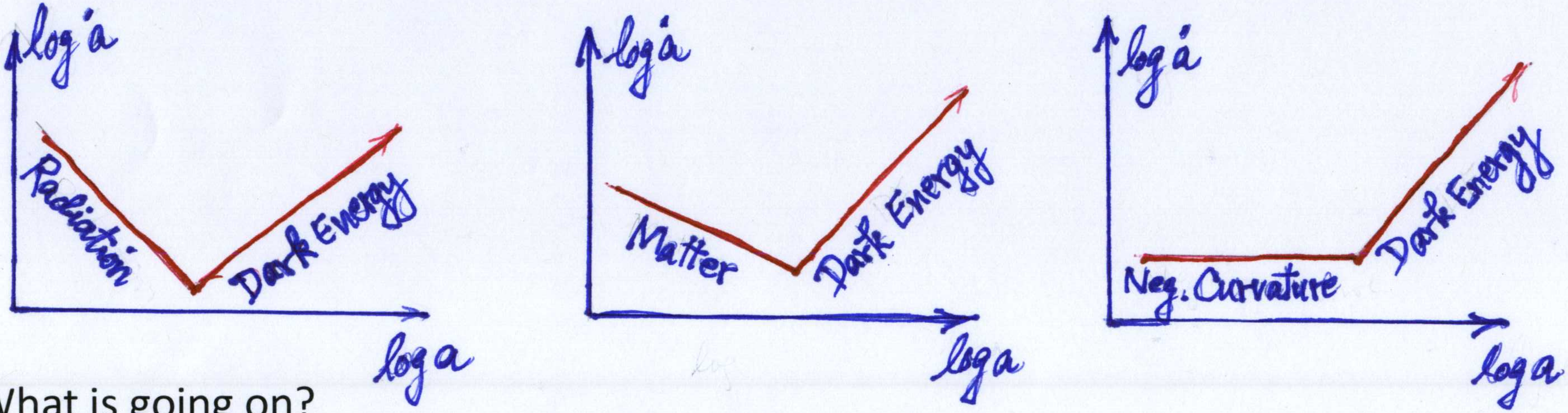


"Swiss cheese theorem" (or "Einstein-Strauss vacuole")



- Remove the dust
- Replace with $M = \frac{4}{3}\pi R^3 \rho$
- No effect on exterior spacetime

Simple Solutions (FRW Language)



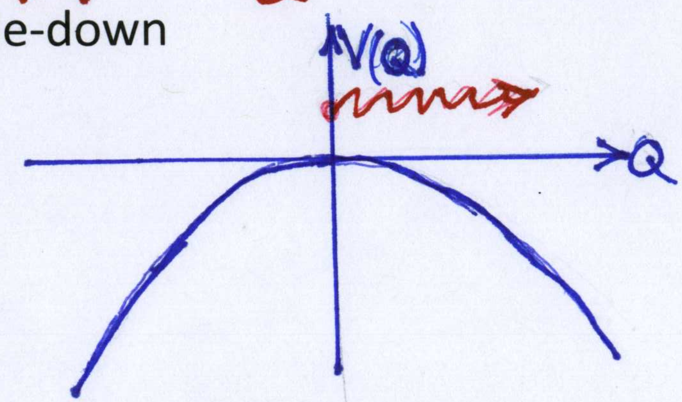
What is going on?

FRW: $\left(\frac{\dot a}{a}\right)^2 = H^2 + \frac{\text{const}}{a^n}$

Let

$a^n = \frac{c\,Q^2}{n^2 H^2}$ $\quad \frac{1}{n^2}\left(\frac{\dot Q}{Q}\right)^2 = H^2 + \frac{\text{const}}{Q^2}$ $\quad \dot Q^2 = n^2 H^2 Q^2 + \text{const.}$

Mechanical analogy: upside-down oscillator potential



Matter Plus Dark Energy (FRW)

Let

$$a(t) = \left(\sinh \tfrac{3}{2} Ht\right)^{2/3}$$

Then

$$\left(\frac{\dot a}{a}\right)^2 = \frac{H^2 \cosh^2 \frac{3}{2}Ht}{\sinh^2 \frac{3}{2}Ht} = H^2\left(1 + \frac{1}{\sinh^2 3/2\,Ht}\right)$$

The normalization time t_0 is

$$\sinh 3/2\,Ht_0 \equiv 1 \propto 3/2\,Ht_0$$

Matter and dark energy contribute equally at t = t_0:

$$\Omega_m = \Omega_{DE} = \frac{1}{2}$$

The time t_0 occurs a bit earlier than today.

Mass inside $r = r_0$ is same as dark energy inside $r = r_0$.

Matter Plus Dark Energy (PG)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r u(r)\right) = \frac{8\pi}{M_{pl}^2}\mu^4 = 3H^2$$

Integrate:

Integration constant consistent with BH/swiss-cheese interpretation

$$r u(r) = \frac{2M}{M_{pl}^2} + H^2 r^3 = r\left(\frac{dr}{dt}\right)^2$$

Rewrite:

$$\frac{1}{r^2}\left(\frac{dr}{dt}\right)^2 = H^2 + \frac{2M}{M_{pl}^2 r^3}$$

Solution is the same:

$$r = r_0 \left(\sinh \frac{3}{2} H t\right)^{2/3}$$

V. Lagrangians and Hamiltonians

- Purposes:

A first look at the situation.

Complications: constraint equations.

Phase-space structure of the simple solutions.

Loop gravity is to large extent Hamiltonian-based (the appropriate pathway to quantization??)

Einstein-Hilbert Action

Curvature scalar

$$S_{EH} = \frac{M_{pl}^2}{16\pi}\int d^4x \sqrt{-g}\, R \equiv \int dt\, L_{EH}$$

determinant of $g_{\mu\nu}$ (Jacobian)

FRW example:

$$L = L_{EH} - V_{DE} - V_{Dust}$$

$$L_{EH} = \frac{M_{pl}^2}{16\pi}\int_{Box} d^3\xi \; a^3(t)\, R = \frac{M_{pl}^2}{16\pi}(V_0 a^3) R$$

$$V_{DE} = \int_{Box} d^3\xi \; a^3 \mu^4 = V_0 a^3 \mu^4 = \frac{8\pi}{3} H^2 M_{pl}^2 (V_0 a^3)$$

$$V_{Dust} = \int_{Box} d^3\xi \, a^3 \left(\frac{M}{V_0 a^3}\right) = M$$

Normalize appropriately for the upside-down oscillator:

$$L = -\left[\frac{\dot{Q}^2}{2} + \frac{\tilde{H}^2 Q^2}{2} + M\right] \qquad \tilde{H} \equiv \frac{3}{2}H \qquad \boxed{a^3 = \frac{3\pi Q^2}{M_{pl}^2 V_0}}$$

Note that the kinetic energy term has the "wrong sign".

A parts integration was ~~is~~ required for the EH term:

$$L_{EH} = \frac{3}{4}\frac{d}{dt}(Q\dot{Q}) - \frac{1}{2}\dot{Q}^2$$

The time derivative term is often canceled out with a surface term added to the action.

The Hamiltonian

$$P = -\dot{Q} \qquad H = P\dot{Q} - L = -\frac{P^2}{2} + \frac{\tilde{H}^2 Q^2}{2} + M$$

The "equation of motion": H = 0

$$\dot{Q}^2 = \tilde{H}^2 Q^2 + 2M \qquad \boxed{Q = \frac{\sqrt{2M}}{\tilde{H}} \sinh \tilde{H}t}$$

Deep issues here: "The Problem of ~~t~~ T time"

Other canonical variables:

We used $a^{3/2} \sim (\text{Volume})^{1/2}$

FRW uses $a \sim$ (Scale factor) = (length)

LQG uses $a^2 \sim$ (Area)

Evaluation of the action for the contents of the box:

$$S = \int_0^t dt\, L = \int dt\,[P\dot{Q} - H] = \int dt\, P\dot{Q} = \int^t P\,dQ$$

It is the "semiclassical phase" associated with the wave function of the box.

For LambdaCDM, one gets $\left(\text{with } Q = \frac{\sqrt{2M}}{\tilde{H}} \sinh \tilde{H}t \right)$

$$S = \begin{cases} 2Mt & (t \text{ small}) \\ \left(\frac{2M}{\tilde{H}}\right) e^{2\tilde{H}t} \approx \frac{\tilde{H}Q^2}{2} & (t \text{ large}) \end{cases}$$

Phase accumulation is proportional to volume at large times.

A Curiosity

Consider pure deSitter space (dark energy only):

Question: For what volume does the phase accumulation become of order unity?

Answer: When the volume is of order QCD scale.

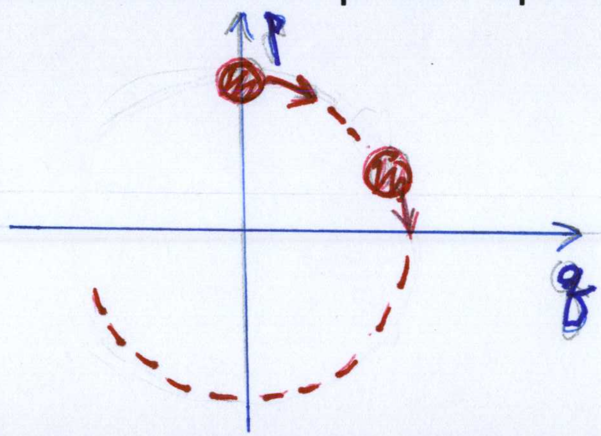
$$S'_{DE} = \frac{HM_{pl}^2}{16\pi} V(t) \equiv \frac{V(t)}{V_{QCD}} \qquad V_{QCD} \sim \frac{16\pi}{HM_{pl}^2} \sim \frac{10^{-60}}{M_{pl}^3} = \left(\frac{10^{-20}}{M_{pl}}\right)^3 \sim \Lambda_{QCD}^{-3}$$

- This is the "Zeldovich relation"—discovered in 1967.

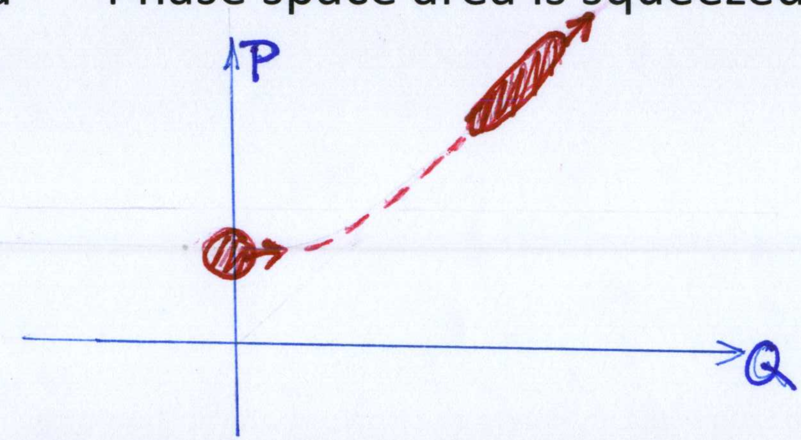
An open question: Is this numerology, or is there some physics here???

Phase Space Considerations

Classical oscillator
No deformation of phase space area



Upside down oscillator
Phase space area is squeezed



Phase space area is conserved (Liouville theorem)

Implications of phase-space squeezing:

- Thickness of phase space filament decreases exponentially during inflation and re-inflation.
- Quantum state of the "mature" universe is indistinguishable from classical ensemble.
- A wave-packet, NOT energy-eigenstate, description is indicated.
- This means a slight loosening of the Hamiltonian constraint (and, for cognoscenti, the Wheeler-deWitt equation)

Summary:

- FRW description is simple and robust—just the dynamics of an upside-down oscillator—one coordinate and one conjugate momentum.

- Phase space evolution involves filamentaion (squeezing), leading to a robust classical description under almost all circumstances.

- PG and comoving box descriptions lead to L being a total derivative (space or time), and the role of action principles becomes more obscure.