

1. **Three-Player Chess (20 pts)** Consider a three-player game (similar to chess), where players alternate moves down a tree (player 1 (Red) moves first, then player 2 (Blue), then player 3 (Green), then Red, etc.), where every move sequence ultimately leads to a terminal state in a finite number of steps. Suppose every terminal state is one of the following: Win for Red, Win for Blue, Win for Green. A winning strategy for Red is a strategy that will guarantee a win for Red regardless of the strategies used by the other players, and analogously for Blue and Green. Prove that exactly one of the following is true:
  - (a) Red has a Winning Strategy
  - (b) Blue has a Winning Strategy
  - (c) Green has a Winning Strategy
  
2. **“Big Win” Chess (40 pts)** Consider a game very similar to chess, but with the following modification. If White successfully attacks Black King, and White still has his Queen remaining while Black does not have his Queen, then the outcome is a “Big Win” for White.
  - (a) Prove that exactly one of the following is true (20 pts):
    - i. White has a strategy that guarantees exactly a Big Win.
    - ii. White has a strategy that guarantees exactly a Win.
    - iii. Black has a strategy that guarantees exactly a Big Win.
    - iv. Black has a strategy that guarantees exactly a Win.
    - v. Both players have strategies that guarantee at least ties.
  - (b) Prove that exactly one of the following is true (20 pts):
    - i. White has a strategy that guarantees AT LEAST a Win (i.e., either a Win or a Big Win)
    - ii. Black has a strategy that guarantees AT LEAST a Win (i.e., either a Win or a Big Win)
    - iii. Both players have strategies that guarantee at least ties.
  
3. **Not von Neuman (40 pts)** Consider a finite game that has three possible outcomes (Win for White, Win for Black, Tie).
  - (a) Consider a hypothetical “theorem” that exactly one of the following is true:
    - i. White has a strategy that guarantees a Win.
    - ii. Black has a strategy that guarantees a Win.
    - iii. Both players have strategies that guarantee exactly ties.Show that this hypothetical theorem is false (20 pts).
  - (b) Consider a hypothetical “theorem” that exactly one of the following is true:
    - i. White has a strategy that guarantees at least a tie.
    - ii. Black has a strategy that guarantees at least a tie.Show that this hypothetical theorem is false (20 pts).