

**Chapter 4**  
**Polynomial Functions**

**Section 4-7**  
**Transformations of Polynomial Functions**

## Describing Transformations of Polynomial Functions

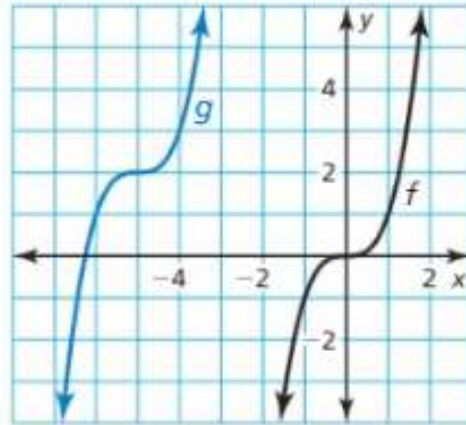
You can transform graphs of polynomial functions in the same way you transformed graphs of linear functions, absolute value functions, and quadratic functions. Examples of transformations of the graph of  $f(x) = x^4$  are shown below.

### Core Concept

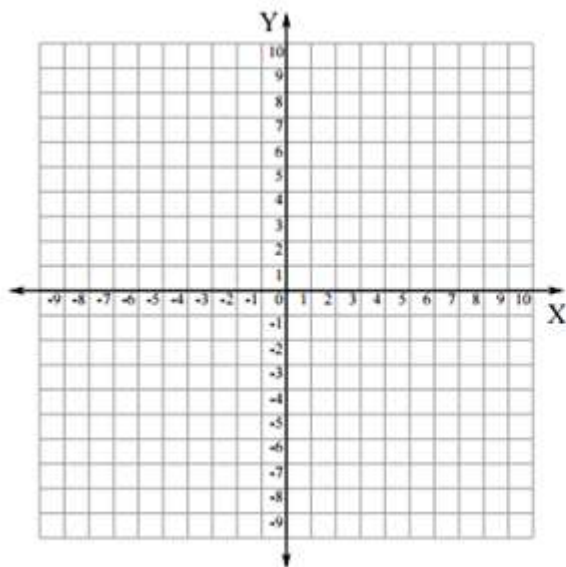
Transformation	$f(x)$ Notation	Examples
<b>Horizontal Translation</b> Graph shifts left or right.	$f(x - h)$	$g(x) = (x - 5)^4$ 5 units right $g(x) = (x + 2)^4$ 2 units left
<b>Vertical Translation</b> Graph shifts up or down.	$f(x) + k$	$g(x) = x^4 + 1$ 1 unit up $g(x) = x^4 - 4$ 4 units down
<b>Reflection</b> Graph flips over $x$ - or $y$ -axis.	$f(-x)$ $-f(x)$	$g(x) = (-x)^4 = x^4$ over $y$ -axis $g(x) = -x^4$ over $x$ -axis
<b>Horizontal Stretch or Shrink</b> Graph stretches away from or shrinks toward $y$ -axis.	$f(ax)$	$g(x) = (2x)^4$ shrink by a factor of $\frac{1}{2}$ $g(x) = \left(\frac{1}{2}x\right)^4$ stretch by a factor of 2
<b>Vertical Stretch or Shrink</b> Graph stretches away from or shrinks toward $x$ -axis.	$a \cdot f(x)$	$g(x) = 8x^4$ stretch by a factor of 8 $g(x) = \frac{1}{4}x^4$ shrink by a factor of $\frac{1}{4}$

**EXAMPLE 1****Translating a Polynomial Function**

Describe the transformation of  $f(x) = x^3$  represented by  $g(x) = (x + 5)^3 + 2$ .  
Then graph each function.



- ▶ 1. Describe the transformation of  $f(x) = x^4$  represented by  $g(x) = (x - 3)^4 - 1$ .  
Then graph each function.



X	Y

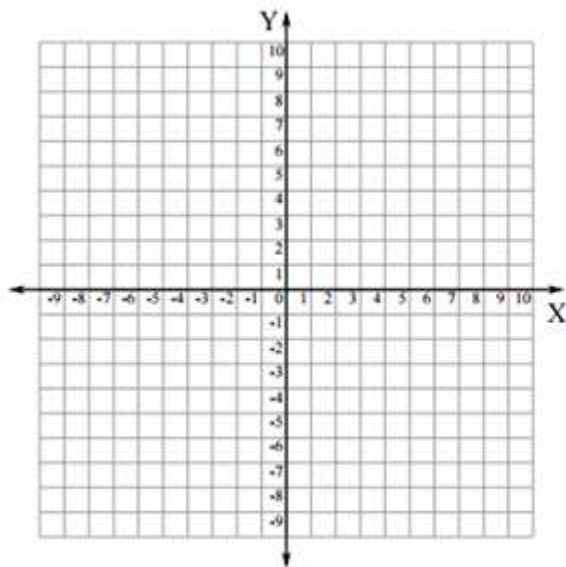
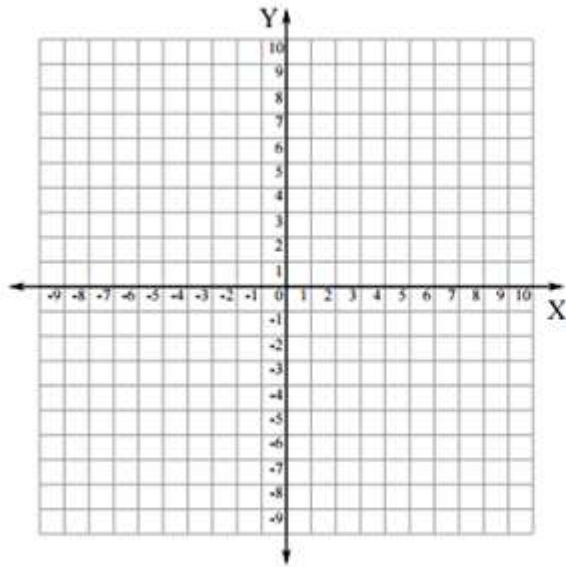
X	Y

**EXAMPLE 2****Transforming Polynomial Functions**

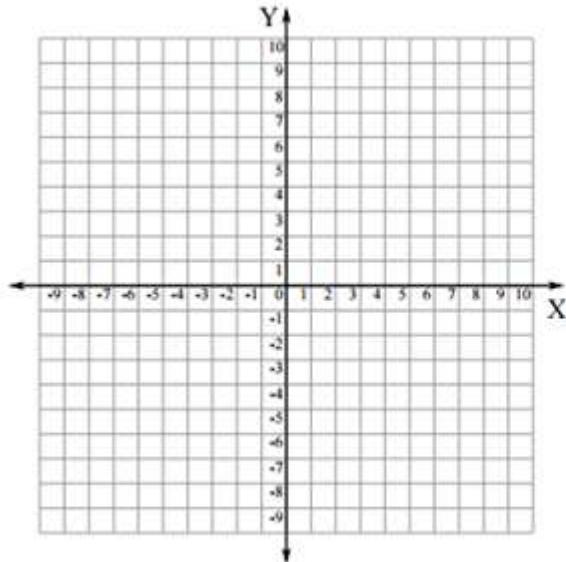
Describe the transformation of  $f$  represented by  $g$ . Then graph each function.

a.  $f(x) = x^4$ ,  $g(x) = -\frac{1}{4}x^4$

b.  $f(x) = x^5$ ,  $g(x) = (2x)^5 - 3$



2. Describe the transformation of  $f(x) = x^3$  represented by  $g(x) = 4(x + 2)^3$ . Then graph each function.



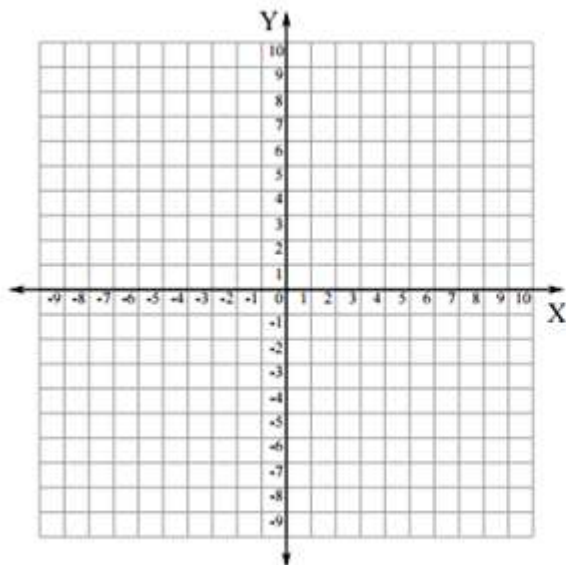
## Writing Transformations of Polynomial Functions

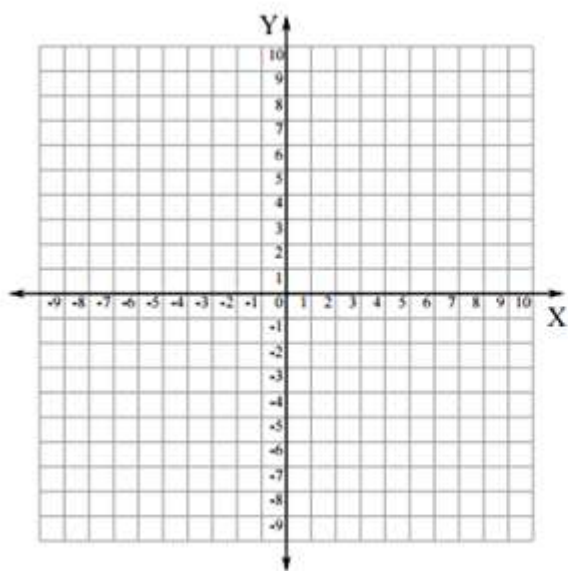
### EXAMPLE 3 Writing Transformed Polynomial Functions

Let  $f(x) = x^3 + x^2 + 1$ . Write a rule for  $g$  and then graph each function. Describe the graph of  $g$  as a transformation of the graph of  $f$ .

a.  $g(x) = f(-x)$

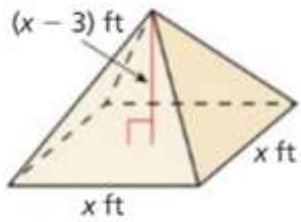
b.  $g(x) = 3f(x)$





**EXAMPLE 4** Writing a Transformed Polynomial Function

Let the graph of  $g$  be a vertical stretch by a factor of 2, followed by a translation 3 units up of the graph of  $f(x) = x^4 - 2x^2$ . Write a rule for  $g$ .

**EXAMPLE 5** Modeling with Mathematics

The function  $V(x) = \frac{1}{3}x^3 - x^2$  represents the volume (in cubic feet) of the square pyramid shown. The function  $W(x) = V(3x)$  represents the volume (in cubic feet) when  $x$  is measured in yards. Write a rule for  $W$ . Find and interpret  $W(10)$ .