

Solutionbank 4

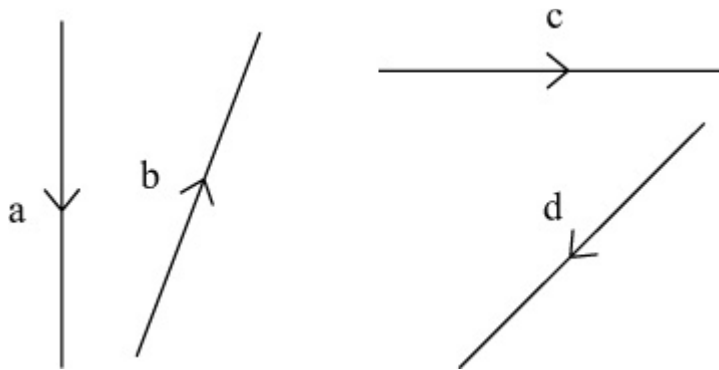
Edexcel AS and A Level Modular Mathematics

Vectors

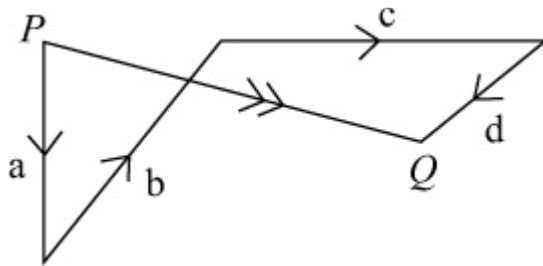
Exercise A, Question 1

Question:

The diagram shows the vectors **a**, **b**, **c** and **d**. Draw a diagram to illustrate the vector addition $a + b + c + d$.



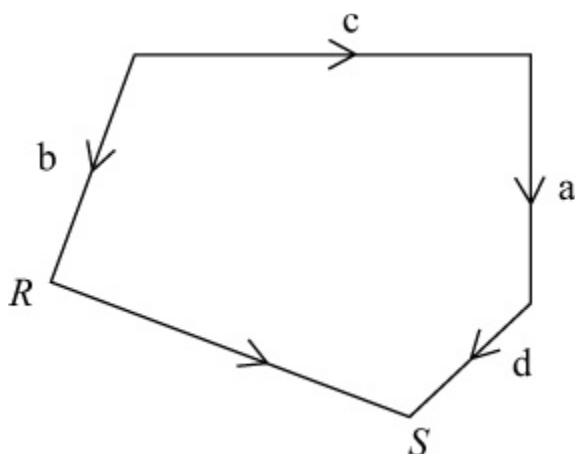
Solution:



$$a + b + c + d = PQ$$

(Vector goes from the start of **a** to the finish of **d**).

The vectors could be added in a different order, e.g. $b + c + a + d$:



$$\text{Here } b + c + a + d = RS$$

$$(RS = PQ)$$

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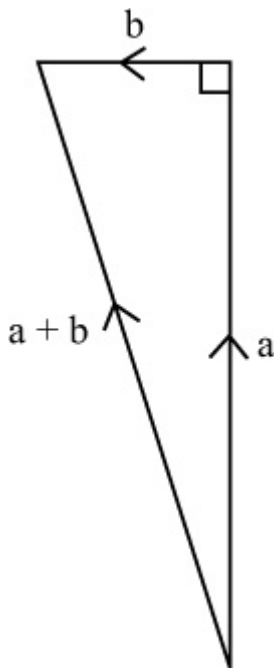
Vectors

Exercise A, Question 2

Question:

The vector \mathbf{a} is directed due north and $|\mathbf{a}| = 24$. The vector \mathbf{b} is directed due west and $|\mathbf{b}| = 7$. Find $|\mathbf{a} + \mathbf{b}|$.

Solution:



$$|\mathbf{a}| = 24$$

$$|\mathbf{b}| = 7$$

$$|\mathbf{a} + \mathbf{b}|^2 = 24^2 + 7^2 = 625$$

$$\therefore |\mathbf{a} + \mathbf{b}| = 25$$

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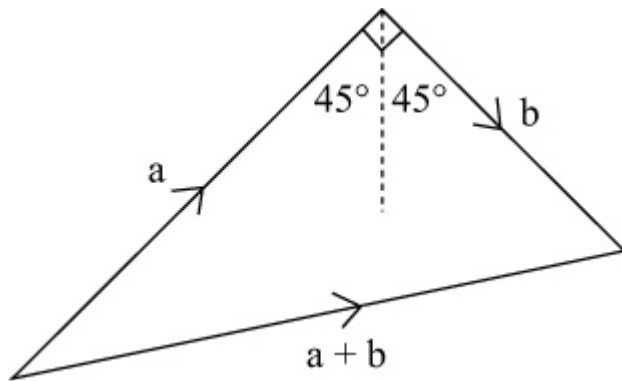
Vectors

Exercise A, Question 3

Question:

The vector **a** is directed north-east and $|a| = 20$. The vector **b** is directed south-east and $|b| = 13$. Find $|a + b|$.

Solution:



$$|a| = 20$$

$$|b| = 13$$

$$|a + b|^2 = 20^2 + 13^2 = 569$$

$$|a + b| = \sqrt{569} = 23.9 \text{ (3 s.f.)}$$

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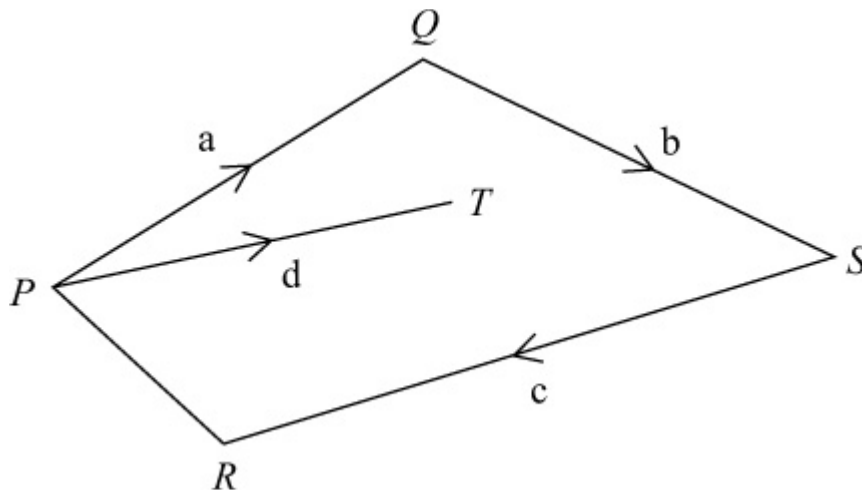
Vectors

Exercise A, Question 4

Question:

In the diagram, $PQ = a$, $QS = b$, $SR = c$ and $PT = d$. Find in terms of **a**, **b**, **c** and **d**:

- (a) QT
- (b) PR
- (c) TS
- (d) TR



Solution:

- (a) $QT = QP + PT = -a + d$
- (b) $PR = PQ + QS + SR = a + b + c$
- (c) $TS = TP + PQ + QS = -d + a + b = a + b - d$
- (d) $TR = TP + PR = -d + (a + b + c) = a + b + c - d$

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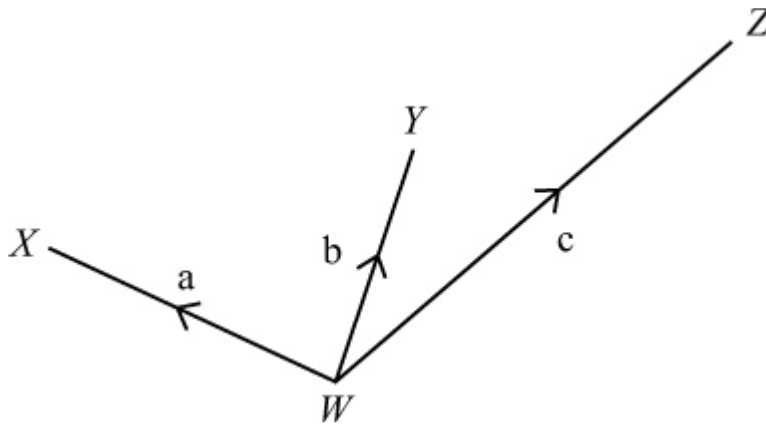
Vectors

Exercise A, Question 5

Question:

In the diagram, $WX = a$, $WY = b$ and $WZ = c$. It is given that $XY = YZ$. Prove that $a + c = 2b$.

($2b$ is equivalent to $b + b$).



Solution:

$$XY = XW + WY = -a + b$$

$$YZ = YW + WZ = -b + c$$

Since $XY = YZ$,

$$-a + b = -b + c$$

$$b + b = a + c$$

$$a + c = 2b$$

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Vectors

Exercise B, Question 1

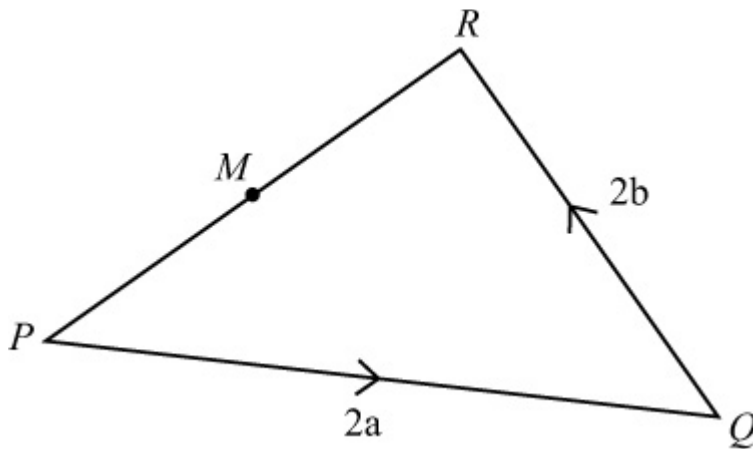
Question:

In the triangle PQR, $PQ = 2a$ and $QR = 2b$. The mid-point of PR is M .

Find, in terms of a and b :

- (a) PR
- (b) PM
- (c) QM .

Solution:



$$(a) \ PR = PQ + QR = 2a + 2b$$

$$(b) \ PM = \frac{1}{2}PR = \frac{1}{2} \left(2a + 2b \right) = a + b$$

$$(c) \ QM = QP + PM = -2a + a + b = -a + b$$

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Exercise B, Question 2

Question:

$ABCD$ is a trapezium with AB parallel to DC and $DC = 3AB$. M is the mid-point of DC , $AB = a$ and $BC = b$.

Find, in terms of a and b :

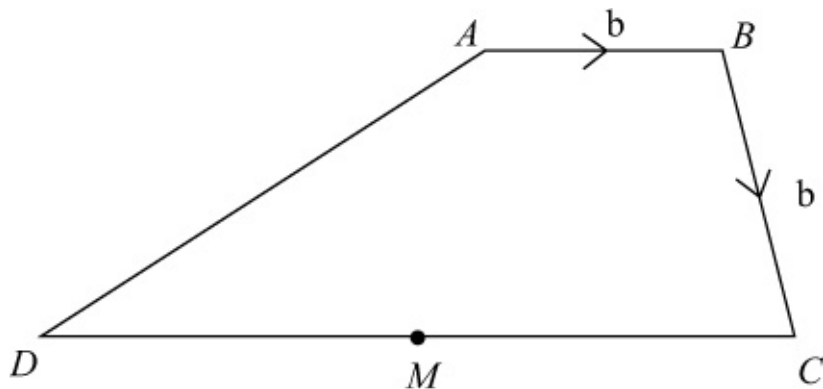
(a) AM

(b) BD

(c) MB

(d) DA .

Solution:



Since $DC = 3AB$, $DC = 3a$

Since M is the mid-point of DC , $DM = MC = \frac{3}{2}a$

$$(a) \ AM = AB + BC + CM = a + b - \frac{3}{2}a = -\frac{1}{2}a + b$$

$$(b) \ BD = BC + CD = b - 3a$$

$$(c) \ MB = MC + CB = \frac{3}{2}a - b$$

$$(d) \ DA = DC + CB + BA = 3a - b - a = 2a - b$$

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Exercise B, Question 3

Question:

In each part, find whether the given vector is parallel to $a - 3b$:

(a) $2a - 6b$

(b) $4a - 12b$

(c) $a + 3b$

(d) $3b - a$

(e) $9b - 3a$

(f) $\frac{1}{2}a - \frac{2}{3}b$

Solution:

(a) $2a - 6b = 2(a - 3b)$
Yes, parallel to $a - 3b$.

(b) $4a - 12b = 4(a - 3b)$
Yes, parallel to $a - 3b$.

(c) $a + 3b$ is not parallel to $a - 3b$

(d) $3b - a = -1(a - 3b)$
Yes, parallel to $a - 3b$.

(e) $9b - 3a = -3(a - 3b)$
Yes, parallel to $a - 3b$.

(f) $\frac{1}{2}a - \frac{2}{3}b = \frac{1}{2}\left(a - \frac{4}{3}b\right)$
No, not parallel to $a - 3b$.

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Exercise B, Question 4

Question:

The non-zero vectors **a** and **b** are not parallel. In each part, find the value of λ and the value of μ :

(a) $a + 3b = 2\lambda a - \mu b$

(b) $(\lambda + 2)a + (\mu - 1)b = 0$

(c) $4\lambda a - 5b - a + \mu b = 0$

(d) $(1 + \lambda)a + 2\lambda b = \mu a + 4\mu b$

(e) $(3\lambda + 5)a + b = 2\mu a + (\lambda - 3)b$

Solution:

(a) $a + 3b = 2\lambda a - \mu b$
 $1 = 2\lambda$ and $3 = -\mu$
 $\lambda = \frac{1}{2}$ and $\mu = -3$

(b) $(\lambda + 2)a + (\mu - 1)b = 0$
 $\lambda + 2 = 0$ and $\mu - 1 = 0$
 $\lambda = -2$ and $\mu = 1$

(c) $4\lambda a - 5b - a + \mu b = 0$
 $4\lambda - 1 = 0$ and $-5 + \mu = 0$
 $\lambda = \frac{1}{4}$ and $\mu = 5$

(d) $(1 + \lambda)a + 2\lambda b = \mu a + 4\mu b$
 $1 + \lambda = \mu$ and $2\lambda = 4\mu$
 Since $2\lambda = 4\mu$, $\lambda = 2\mu$
 $1 + 2\mu = \mu$
 $\mu = -1$ and $\lambda = -2$

(e) $(3\lambda + 5)a + b = 2\mu a + (\lambda - 3)b$
 $3\lambda + 5 = 2\mu$ and $1 = \lambda - 3$
 $\lambda = 4$ and $2\mu = 12 + 5$

$$\lambda = 4 \quad \text{and} \quad \mu = 8 \frac{1}{2}$$

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Exercise B, Question 5

Question:

In the diagram, $OA = \mathbf{a}$, $OB = \mathbf{b}$ and C divides AB in the ratio 5:1.

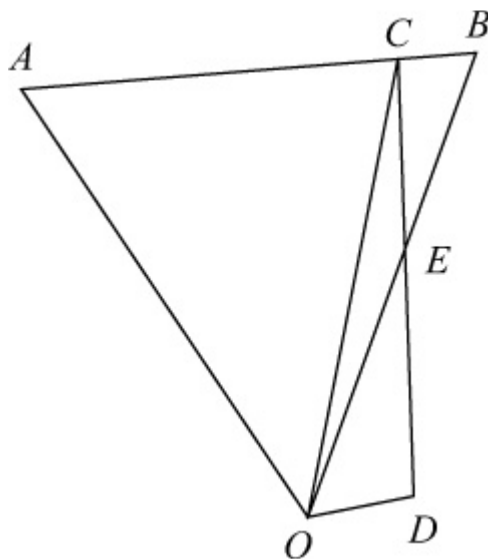
(a) Write down, in terms of \mathbf{a} and \mathbf{b} , expressions for AB , AC and OC .
Given that $OE = \lambda \mathbf{b}$, where λ is a scalar:

(b) Write down, in terms of \mathbf{a} , \mathbf{b} and λ , an expression for CE .
Given that $OD = \mu (\mathbf{b} - \mathbf{a})$, where μ is a scalar:

(c) Write down, in terms of \mathbf{a} , \mathbf{b} , λ and μ , an expression for ED .
Given also that E is the mid-point of CD :

(d) Deduce the values of λ and μ .

E



Solution:

$$(a) AB = AO + OB = -\mathbf{a} + \mathbf{b}$$

$$AC = \frac{5}{6}AB = \frac{5}{6} \left(-\mathbf{a} + \mathbf{b} \right)$$

$$OC = OA + AC = \mathbf{a} + \frac{5}{6} \left(-\mathbf{a} + \mathbf{b} \right) = \frac{1}{6}\mathbf{a} + \frac{5}{6}\mathbf{b}$$

$$(b) OE = \lambda \mathbf{b}$$

$$CE = CO + OE = - \left(\frac{1}{6}\mathbf{a} + \frac{5}{6}\mathbf{b} \right) + \lambda \mathbf{b} = - \frac{1}{6}\mathbf{a} + \left(\lambda - \frac{5}{6} \right) \mathbf{b}$$

(c) $OD = \mu (\mathbf{b} - \mathbf{a}) :$

$$ED = EO + OD = - \lambda \mathbf{b} + \mu (\mathbf{b} - \mathbf{a}) = - \mu \mathbf{a} + (\mu - \lambda) \mathbf{b}$$

(d) If E is the mid-point of CD , $CE = ED$:

$$- \frac{1}{6}\mathbf{a} + \left(\lambda - \frac{5}{6} \right) \mathbf{b} = - \mu \mathbf{a} + (\mu - \lambda) \mathbf{b}$$

Since \mathbf{a} and \mathbf{b} are not parallel

$$- \frac{1}{6} = - \mu \quad \Rightarrow \quad \mu = \frac{1}{6}$$

and

$$\left(\lambda - \frac{5}{6} \right) = (\mu - \lambda)$$

$$\Rightarrow 2\lambda = \mu + \frac{5}{6}$$

$$\Rightarrow 2\lambda = 1$$

$$\Rightarrow \lambda = \frac{1}{2}$$

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Exercise B, Question 6

Question:

In the diagram $OA = \mathbf{a}$, $OB = \mathbf{b}$, $3OC = 2OA$ and $4OD = 7OB$.
The line DC meets the line AB at E .

(a) Write down, in terms of \mathbf{a} and \mathbf{b} , expressions for

(i) AB

(ii) DC

Given that $DE = \lambda DC$ and $EB = \mu AB$ where λ and μ are constants:

(b) Use $\triangle EBD$ to form an equation relating to \mathbf{a} , \mathbf{b} , λ and μ .

Hence:

(c) Show that $\lambda = \frac{9}{13}$.

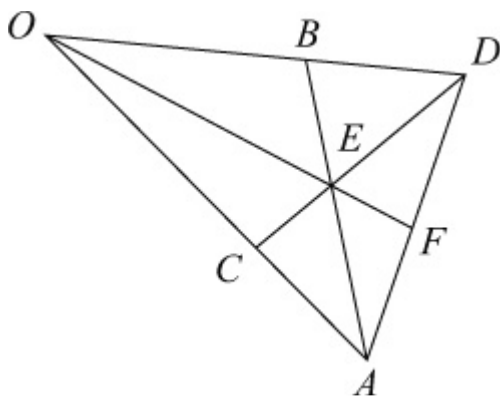
(d) Find the exact value of μ .

(e) Express OE in terms of \mathbf{a} and \mathbf{b} .

The line OE produced meets the line AD at F .

Given that $OF = kOE$ where k is a constant and that $AF = \frac{1}{10} (7\mathbf{b} - 4\mathbf{a})$:

(f) Find the value of k . **(E)**



Solution:

(a) $OC = \frac{2}{3}OA = \frac{2}{3}\mathbf{a}$, $OD = \frac{7}{4}OB = \frac{7}{4}\mathbf{b}$

(i) $AB = AO + OB = -\mathbf{a} + \mathbf{b}$

$$(ii) DC = DO + OC = \frac{2}{3}a - \frac{7}{4}b$$

$$(b) DE = \lambda DC \text{ and } EB = \mu AB.$$

From $\triangle EBD$, $DE = DB + BE$

$$\text{Since } OD = \frac{7}{4}b, BD = OD - OB = \frac{7}{4}b - b = \frac{3}{4}b$$

$$\therefore DB = -\frac{3}{4}b$$

$$\text{So } \lambda DC = DB - \mu AB$$

$$\lambda \left(\frac{2}{3}a - \frac{7}{4}b \right) = -\frac{3}{4}b - \mu \left(-a + b \right)$$

$$\left(\frac{2}{3}\lambda - \mu \right) a + \left(\frac{3}{4} + \mu - \frac{7}{4}\lambda \right) b = 0$$

$$(c) \text{ So } \frac{2}{3}\lambda - \mu = 0 \Rightarrow \mu = \frac{2}{3}\lambda$$

$$\text{and } \frac{3}{4} + \mu - \frac{7}{4}\lambda = 0$$

$$\Rightarrow \frac{3}{4} + \frac{2}{3}\lambda - \frac{7}{4}\lambda = 0$$

$$\Rightarrow \frac{13}{12}\lambda = \frac{3}{4}$$

$$\Rightarrow \lambda = \frac{3}{4} \times \frac{12}{13} = \frac{9}{13}$$

$$(d) \mu = \frac{2}{3}\lambda = \frac{2}{3} \times \frac{9}{13} = \frac{6}{13}$$

$$(e) OE = OB + BE = OB - \mu AB = b - \frac{6}{13} \left(-a + b \right) = \frac{6}{13}a + \frac{7}{13}b$$

$$(f) OF = kOE \text{ and } AF = \frac{7}{10}b - \frac{4}{10}a.$$

$$OF = \frac{6k}{13}a + \frac{7k}{13}b$$

From $\triangle OFA$, $OF = OA + AF$

$$\frac{6k}{13}a + \frac{7k}{13}b = a + \left(\frac{7}{10}b - \frac{4}{10}a \right) = \frac{6}{10}a + \frac{7}{10}b$$

$$\text{So } \frac{6k}{13} = \frac{6}{10} \quad \left(\text{and } \frac{7k}{13} = \frac{7}{10} \right)$$

$$\Rightarrow k = \frac{13}{10}$$

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Exercise B, Question 7

Question:

In $\triangle OAB$, P is the mid-point of AB and Q is the point on OP such that $OQ = \frac{3}{4}OP$. Given that $OA = \mathbf{a}$ and $OB = \mathbf{b}$, find, in terms of \mathbf{a} and \mathbf{b} :

(a) AB

(b) OP

(c) OQ

(d) AQ

The point R on OB is such that $OR = kOB$, where $0 < k < 1$.

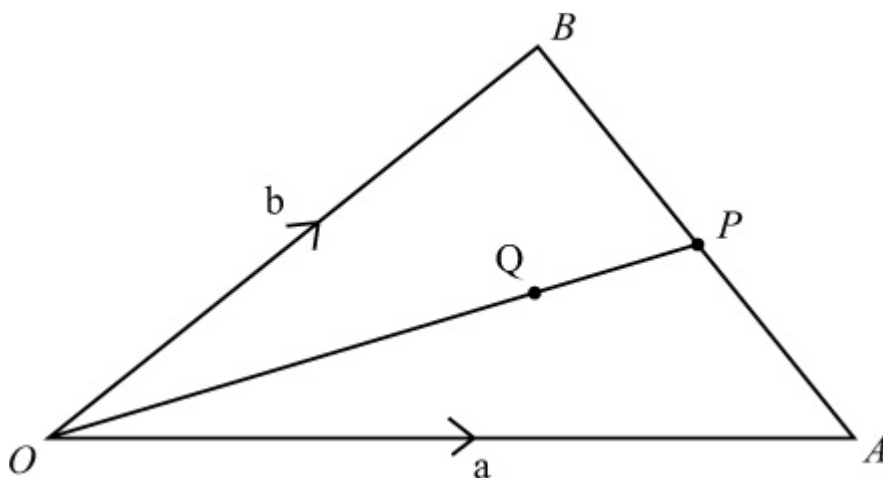
(e) Find, in terms of \mathbf{a} , \mathbf{b} and k , the vector AR .

Given that AQR is a straight line:

(f) Find the ratio in which Q divides AR and the value of k .

E

Solution:



$$BP = PA \text{ and } OQ = \frac{3}{4}OP$$

(a) $AB = AO + OB = -\mathbf{a} + \mathbf{b}$

$$(b) \quad OP = OA + AP = OA + \frac{1}{2}AB = a + \frac{1}{2} \left(-a + b \right) = \frac{1}{2}a + \frac{1}{2}b$$

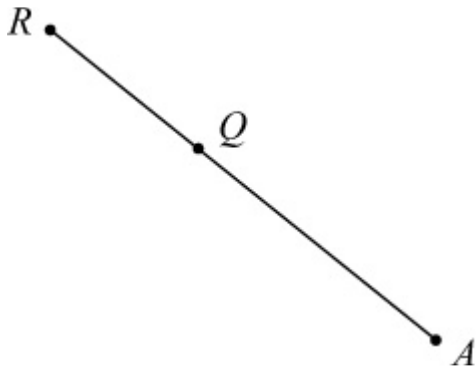
$$(c) \quad OQ = \frac{3}{4}OP = \frac{3}{4} \left(\frac{1}{2}a + \frac{1}{2}b \right) = \frac{3}{8}a + \frac{3}{8}b$$

$$(d) \quad AQ = AO + OQ = -a + \left(\frac{3}{8}a + \frac{3}{8}b \right) = -\frac{5}{8}a + \frac{3}{8}b$$

(e) Given $OR = kOB$ ($0 < k < 1$)

$$\text{In } \triangle OAR, \quad AR = AO + OR = -a + kb$$

(f) Since AQR is a straight line, AR and AQ are parallel vectors.



Suppose $AQ = \lambda AR$

$$-\frac{5}{8}a + \frac{3}{8}b = \lambda \left(-a + kb \right)$$

$$\text{So } -\frac{5}{8} = -\lambda \quad \Rightarrow \quad \lambda = \frac{5}{8}$$

$$\text{and } \frac{3}{8} = \lambda k$$

$$\Rightarrow k = \frac{3}{8\lambda}$$

$$\Rightarrow k = \frac{3}{5}$$

Since $AQ = \frac{5}{8}AR$, $RQ = \frac{3}{8}AR$

So Q divides AR in the ratio 5:3.

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Exercise B, Question 8

Question:

In the figure $OE : EA = 1 : 2$, $AF : FB = 3 : 1$ and $OG : OB = 3 : 1$. The vector $OA = \mathbf{a}$ and the vector $OB = \mathbf{b}$.

Find, in terms of \mathbf{a} , \mathbf{b} or \mathbf{a} and \mathbf{b} , expressions for:

(a) OE

(b) OF

(c) EF

(d) BG

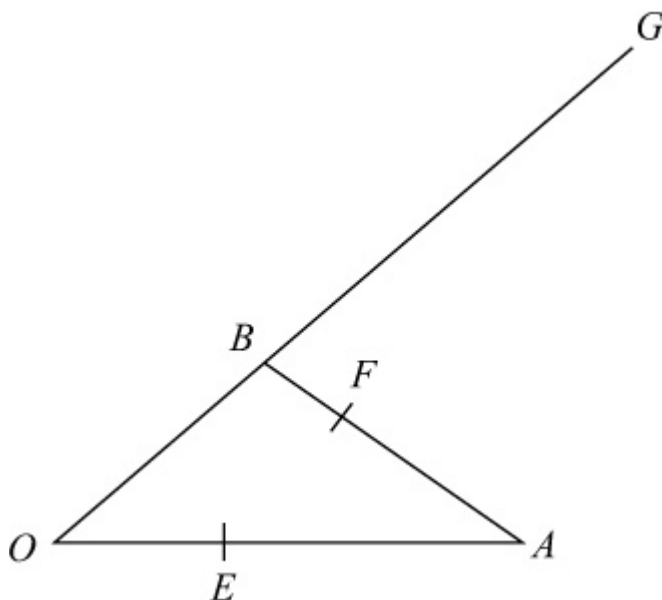
(e) FB

(f) FG

(g) Use your results in (c) and (f) to show that the points E , F and G are collinear and find the ratio $EF : FG$.

(h) Find EB and AG and hence prove that EB is parallel to AG .

E



Solution:

$$(a) \text{OE} = \frac{1}{3}\text{OA} = \frac{1}{3}\mathbf{a}$$

$$\begin{aligned} (b) \text{OF} &= \text{OA} + \text{AF} = \text{OA} + \frac{3}{4}\text{AB} \\ &= \mathbf{a} + \frac{3}{4} \left(\mathbf{b} - \mathbf{a} \right) \\ &= \mathbf{a} + \frac{3}{4}\mathbf{b} - \frac{3}{4}\mathbf{a} \\ &= \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b} \end{aligned}$$

$$\begin{aligned} (c) \text{EF} &= \text{EA} + \text{AF} = \frac{2}{3}\text{OA} + \frac{3}{4}\text{AB} \\ &= \frac{2}{3}\mathbf{a} + \frac{3}{4} \left(\mathbf{b} - \mathbf{a} \right) \\ &= \frac{2}{3}\mathbf{a} + \frac{3}{4}\mathbf{b} - \frac{3}{4}\mathbf{a} \\ &= -\frac{1}{12}\mathbf{a} + \frac{3}{4}\mathbf{b} \end{aligned}$$

$$(d) \text{BG} = 2\text{OB} = 2\mathbf{b}$$

$$(e) \text{FB} = \frac{1}{4}\text{AB} = \frac{1}{4} \left(\mathbf{b} - \mathbf{a} \right) = -\frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$$

$$(f) \text{FG} = \text{FB} + \text{BG} = -\frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b} + 2\mathbf{b} = -\frac{1}{4}\mathbf{a} + \frac{9}{4}\mathbf{b}$$

$$(g) \text{FG} = -\frac{1}{4}\mathbf{a} + \frac{9}{4}\mathbf{b} = 3 \left(-\frac{1}{12}\mathbf{a} + \frac{3}{4}\mathbf{b} \right) = 3\text{EF}$$

So EF and FG are parallel vectors.

So E, F and G are collinear.

$$\text{EF} : \text{FG} = 1 : 3$$

$$(h) \text{EB} = \text{EO} + \text{OB} = -\frac{1}{3}\mathbf{a} + \mathbf{b}$$

$$\text{AG} = \text{AO} + \text{OG} = -\mathbf{a} + 3\mathbf{b} = 3 \left(-\frac{1}{3}\mathbf{a} + \mathbf{b} \right) = 3\text{EB}$$

So EB is parallel to AG.

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Exercise C, Question 1

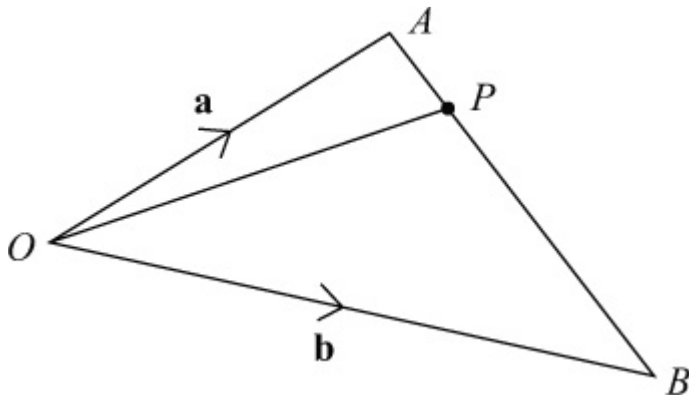
Question:

The points A and B have position vectors \mathbf{a} and \mathbf{b} respectively (referred to the origin O).

The point P divides AB in the ratio 1:5.

Find, in terms of \mathbf{a} and \mathbf{b} , the position vector of P .

Solution:



$$AP : PB = 1 : 5$$

$$\text{So } AP = \frac{1}{6}AB = \frac{1}{6}(\mathbf{b} - \mathbf{a})$$

$$OP = OA + AP = \mathbf{a} + \frac{1}{6}(\mathbf{b} - \mathbf{a})$$

$$= \mathbf{a} + \frac{1}{6}\mathbf{b} - \frac{1}{6}\mathbf{a}$$

$$= \frac{5}{6}\mathbf{a} + \frac{1}{6}\mathbf{b}$$

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Vectors

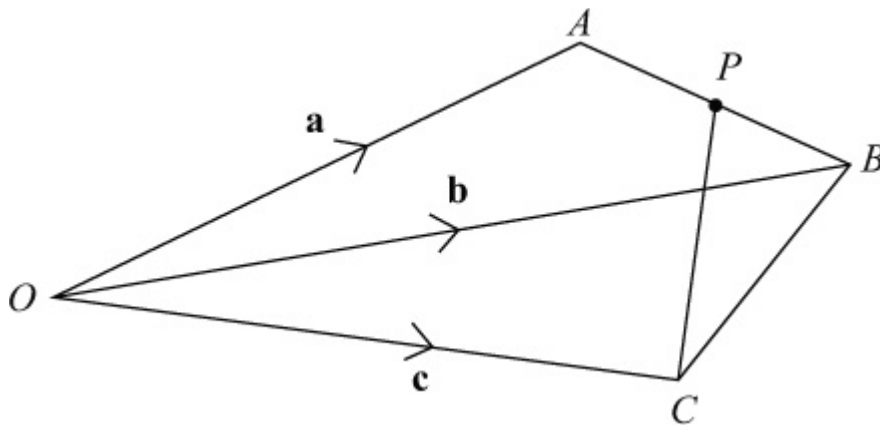
Exercise C, Question 2

Question:

The points A , B and C have position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} respectively (referred to the origin O). The point P is the mid-point of AB .

Find, in terms of \mathbf{a} , \mathbf{b} and \mathbf{c} , the vector PC .

Solution:



$$PC = PO + OC = -OP + OC$$

$$\text{But } OP = OA + AP = OA + \frac{1}{2}AB = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$$

$$\text{So } PC = -\left(\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) + \mathbf{c} = -\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} + \mathbf{c}$$

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Vectors

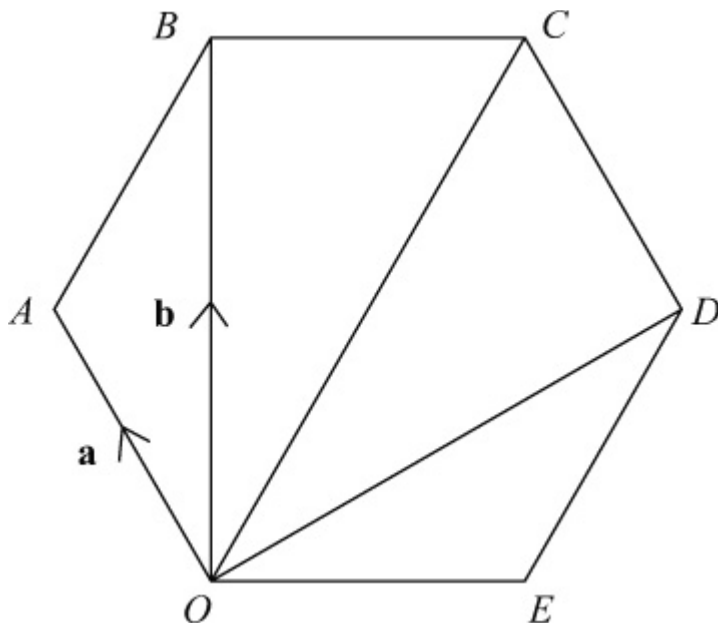
Exercise C, Question 3

Question:

$OABCDE$ is a regular hexagon. The points A and B have position vectors \mathbf{a} and \mathbf{b} respectively, referred to the origin O .

Find, in terms of \mathbf{a} and \mathbf{b} , the position vectors of C , D and E .

Solution:



$$OC = 2AB = 2(\mathbf{b} - \mathbf{a}) = -2\mathbf{a} + 2\mathbf{b}$$

$$OD = OC + CD = OC + AO = (-2\mathbf{a} + 2\mathbf{b}) - \mathbf{a} = -3\mathbf{a} + 2\mathbf{b}$$

$$OE = OD + DE = OD + BA = (-3\mathbf{a} + 2\mathbf{b}) + (\mathbf{a} - \mathbf{b}) = -2\mathbf{a} + \mathbf{b}$$

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Exercise D, Question 1

Question:

Given that $a = 9i + 7j$, $b = 11i - 3j$ and $c = -8i - j$, find:

(a) $a + b + c$

(b) $2a - b + c$

(c) $2b + 2c - 3a$

(Use column matrix notation in your working.)

Solution:

$$(a) \ a + b + c = \begin{pmatrix} 9 \\ 7 \end{pmatrix} + \begin{pmatrix} 11 \\ -3 \end{pmatrix} + \begin{pmatrix} -8 \\ -1 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \end{pmatrix}$$

$$(b) \ 2a - b + c = 2 \begin{pmatrix} 9 \\ 7 \end{pmatrix} + \begin{pmatrix} -11 \\ 3 \end{pmatrix} + \begin{pmatrix} -8 \\ -1 \end{pmatrix} \\ = \begin{pmatrix} 18 \\ 14 \end{pmatrix} + \begin{pmatrix} -11 \\ 3 \end{pmatrix} + \begin{pmatrix} -8 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 16 \end{pmatrix}$$

$$(c) \ 2b + 2c - 3a = 2 \begin{pmatrix} 11 \\ -3 \end{pmatrix} + 2 \begin{pmatrix} -8 \\ -1 \end{pmatrix} - 3 \begin{pmatrix} 9 \\ 7 \end{pmatrix} \\ = \begin{pmatrix} 22 \\ -6 \end{pmatrix} + \begin{pmatrix} -16 \\ -2 \end{pmatrix} + \begin{pmatrix} -27 \\ -21 \end{pmatrix} = \begin{pmatrix} -21 \\ -29 \end{pmatrix}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise D, Question 2

Question:

The points A , B and C have coordinates $(3, -1)$, $(4, 5)$ and $(-2, 6)$ respectively, and O is the origin.

Find, in terms of \mathbf{i} and \mathbf{j} :

(a) the position vectors of A , B and C

(b) AB

(c) AC

Find, in surd form:

(d) $|OC|$

(e) $|AB|$

(f) $|AC|$

Solution:

$$(a) \mathbf{a} = 3\mathbf{i} - \mathbf{j}, \quad \mathbf{b} = 4\mathbf{i} + 5\mathbf{j}, \quad \mathbf{c} = -2\mathbf{i} + 6\mathbf{j}$$

$$\begin{aligned} (b) AB &= \mathbf{b} - \mathbf{a} = (4\mathbf{i} + 5\mathbf{j}) - (3\mathbf{i} - \mathbf{j}) \\ &= 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{i} + \mathbf{j} \\ &= \mathbf{i} + 6\mathbf{j} \end{aligned}$$

$$\begin{aligned} (c) AC &= \mathbf{c} - \mathbf{a} = (-2\mathbf{i} + 6\mathbf{j}) - (3\mathbf{i} - \mathbf{j}) \\ &= -2\mathbf{i} + 6\mathbf{j} - 3\mathbf{i} + \mathbf{j} \\ &= -5\mathbf{i} + 7\mathbf{j} \end{aligned}$$

$$(d) |OC| = |-2\mathbf{i} + 6\mathbf{j}| = \sqrt{(-2)^2 + 6^2} = \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$$

$$(e) |AB| = |\mathbf{i} + 6\mathbf{j}| = \sqrt{1^2 + 6^2} = \sqrt{37}$$

$$(f) |AC| = |-5\mathbf{i} + 7\mathbf{j}| = \sqrt{(-5)^2 + 7^2} = \sqrt{74}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise D, Question 3

Question:

Given that $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = 5\mathbf{i} - 12\mathbf{j}$, $\mathbf{c} = -7\mathbf{i} + 24\mathbf{j}$ and $\mathbf{d} = \mathbf{i} - 3\mathbf{j}$, find a unit vector in the direction of **a**, **b**, **c** and **d**.

Solution:

$$|\mathbf{a}| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\text{Unit vector} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{5} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$|\mathbf{b}| = \sqrt{5^2 + (-12)^2} = \sqrt{169} = 13$$

$$\text{Unit vector} = \frac{\mathbf{b}}{|\mathbf{b}|} = \frac{1}{13} \begin{pmatrix} 5 \\ -12 \end{pmatrix}$$

$$|\mathbf{c}| = \sqrt{(-7)^2 + 24^2} = \sqrt{625} = 25$$

$$\text{Unit vector} = \frac{\mathbf{c}}{|\mathbf{c}|} = \frac{1}{25} \begin{pmatrix} -7 \\ 24 \end{pmatrix}$$

$$|\mathbf{d}| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$$

$$\text{Unit vector} = \frac{\mathbf{d}}{|\mathbf{d}|} = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise D, Question 4

Question:

Given that $\mathbf{a} = 5\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \lambda\mathbf{i} + 3\mathbf{j}$, and that $|\mathbf{3a} + \mathbf{b}| = 10$, find the possible values of λ .

Solution:

$$3\mathbf{a} + \mathbf{b} = 3 \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} \lambda \\ 3 \end{pmatrix} = \begin{pmatrix} 15 \\ 3 \end{pmatrix} + \begin{pmatrix} \lambda \\ 3 \end{pmatrix} = \begin{pmatrix} 15 + \lambda \\ 6 \end{pmatrix}$$

$$|\mathbf{3a} + \mathbf{b}| = 10, \text{ so}$$

$$\sqrt{(15 + \lambda)^2 + 6^2} = 10$$

$$(15 + \lambda)^2 + 6^2 = 100$$

$$225 + 30\lambda + \lambda^2 + 36 = 100$$

$$\lambda^2 + 30\lambda + 161 = 0$$

$$(\lambda + 7)(\lambda + 23) = 0$$

$$\lambda = -7, \lambda = -23$$

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Edexcel AS and A Level Modular Mathematics

Vectors

Exercise E, Question 1

Question:

Find the distance from the origin to the point $P (2 , 8 , - 4)$.

Solution:

$$\text{Distance} = \sqrt{2^2 + 8^2 + (-4)^2} = \sqrt{4 + 64 + 16} = \sqrt{84} \approx 9.17 \text{ (3 s.f.)}$$

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Edexcel AS and A Level Modular Mathematics

Vectors

Exercise E, Question 2

Question:

Find the distance from the origin to the point $P (7 , 7 , 7)$.

Solution:

$$\text{Distance} = \sqrt{7^2 + 7^2 + 7^2} = \sqrt{49 + 49 + 49} = \sqrt{147} = 7\sqrt{3} \approx 12.1 \text{ (3 s.f.)}$$

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Vectors

Exercise E, Question 3

Question:

Find the distance between A and B when they have the following coordinates:

- (a) $A (3 , 0 , 5)$ and $B (1 , - 1 , 8)$
- (b) $A (8 , 11 , 8)$ and $B (- 3 , 1 , 6)$
- (c) $A (3 , 5 , - 2)$ and $B (3 , 10 , 3)$
- (d) $A (- 1 , - 2 , 5)$ and $B (4 , - 1 , 3)$

Solution:

$$\begin{aligned} \text{(a) } AB &= \sqrt{(3-1)^2 + [0 - (-1)]^2 + (5-8)^2} \\ &= \sqrt{2^2 + 1^2 + (-3)^2} \\ &= \sqrt{14} \approx 3.74 \end{aligned}$$

$$\begin{aligned} \text{(b) } AB &= \sqrt{[8 - (-3)]^2 + (11-1)^2 + (8-6)^2} \\ &= \sqrt{11^2 + 10^2 + 2^2} \\ &= \sqrt{225} = 15 \end{aligned}$$

$$\begin{aligned} \text{(c) } AB &= \sqrt{(3-3)^2 + (5-10)^2 + [(-2) - 3]^2} \\ &= \sqrt{0^2 + (-5)^2 + (-5)^2} \\ &= \sqrt{50} = 5\sqrt{2} \approx 7.07 \end{aligned}$$

$$\begin{aligned} \text{(d) } AB &= \sqrt{[(-1) - 4]^2 + [(-2) - (-1)]^2 + (5-3)^2} \\ &= \sqrt{(-5)^2 + (-1)^2 + 2^2} \\ &= \sqrt{30} \approx 5.48 \end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Vectors

Exercise E, Question 4

Question:

The coordinates of A and B are $(7, -1, 2)$ and $(k, 0, 4)$ respectively. Given that the distance from A to B is 3 units, find the possible values of k .

Solution:

$$AB = \sqrt{(7 - k)^2 + (-1 - 0)^2 + (2 - 4)^2} = 3$$

$$\sqrt{(49 - 14k + k^2) + 1 + 4} = 3$$

$$49 - 14k + k^2 + 1 + 4 = 9$$

$$k^2 - 14k + 45 = 0$$

$$(k - 5)(k - 9) = 0$$

$$k = 5 \text{ or } k = 9$$

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Edexcel AS and A Level Modular Mathematics

Vectors

Exercise E, Question 5

Question:

The coordinates of A and B are $(5, 3, -8)$ and $(1, k, -3)$ respectively.

Given that the distance from A to B is $3\sqrt{10}$ units, find the possible values of k .

Solution:

$$AB = \sqrt{(5-1)^2 + (3-k)^2 + [-8 - (-3)]^2} = 3\sqrt{10}$$

$$\sqrt{16 + (9 - 6k + k^2) + 25} = 3\sqrt{10}$$

$$16 + 9 - 6k + k^2 + 25 = 9 \times 10$$

$$k^2 - 6k - 40 = 0$$

$$(k + 4)(k - 10) = 0$$

$$k = -4 \text{ or } k = 10$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise F, Question 1

Question:

Find the modulus of:

(a) $3\mathbf{i} + 5\mathbf{j} + \mathbf{k}$

(b) $4\mathbf{i} - 2\mathbf{k}$

(c) $\mathbf{i} + \mathbf{j} - \mathbf{k}$

(d) $5\mathbf{i} - 9\mathbf{j} - 8\mathbf{k}$

(e) $\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$

Solution:

$$\begin{aligned} \text{(a) } |3\mathbf{i} + 5\mathbf{j} + \mathbf{k}| &= \sqrt{3^2 + 5^2 + 1^2} \\ &= \sqrt{9 + 25 + 1} = \sqrt{35} \end{aligned}$$

$$\begin{aligned} \text{(b) } |4\mathbf{i} - 2\mathbf{k}| &= \sqrt{4^2 + 0^2 + (-2)^2} \\ &= \sqrt{16 + 4} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{(c) } |\mathbf{i} + \mathbf{j} - \mathbf{k}| &= \sqrt{1^2 + 1^2 + (-1)^2} \\ &= \sqrt{1 + 1 + 1} = \sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{(d) } |5\mathbf{i} - 9\mathbf{j} - 8\mathbf{k}| &= \sqrt{5^2 + (-9)^2 + (-8)^2} \\ &= \sqrt{25 + 81 + 64} = \sqrt{170} \end{aligned}$$

$$\begin{aligned} \text{(e) } |\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}| &= \sqrt{1^2 + 5^2 + (-7)^2} \\ &= \sqrt{1 + 25 + 49} = \sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3} \end{aligned}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise F, Question 2

Question:

Given that $\mathbf{a} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix}$, find in column

matrix form:

(a) $\mathbf{a} + \mathbf{b}$

(b) $\mathbf{b} - \mathbf{c}$

(c) $\mathbf{a} + \mathbf{b} + \mathbf{c}$

(d) $3\mathbf{a} - \mathbf{c}$

(e) $\mathbf{a} - 2\mathbf{b} + \mathbf{c}$

(f) $|\mathbf{a} - 2\mathbf{b} + \mathbf{c}|$

Solution:

$$(a) \mathbf{a} + \mathbf{b} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ -1 \end{pmatrix}$$

$$(b) \mathbf{b} - \mathbf{c} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \\ -5 \end{pmatrix}$$

$$(c) \mathbf{a} + \mathbf{b} + \mathbf{c} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 14 \\ -3 \\ 1 \end{pmatrix}$$

$$(d) 3\mathbf{a} - \mathbf{c} = 3 \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 15 \\ 0 \\ 6 \end{pmatrix} - \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \\ 4 \end{pmatrix}$$

$$\begin{aligned} \text{(e) } \mathbf{a} - 2\mathbf{b} + \mathbf{c} &= \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} - 2 \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} + \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \\ 10 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{(f) } |\mathbf{a} - 2\mathbf{b} + \mathbf{c}| &= \sqrt{8^2 + (-6)^2 + 10^2} \\ &= \sqrt{64 + 36 + 100} \\ &= \sqrt{200} = \sqrt{100\sqrt{2}} = 10\sqrt{2} \end{aligned}$$

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Vectors

Exercise F, Question 3

Question:

The position vector of the point A is $2\mathbf{i} - 7\mathbf{j} + 3\mathbf{k}$ and $\mathbf{AB} = 5\mathbf{i} + 4\mathbf{j} - \mathbf{k}$. Find the position of the point B .

Solution:

$\mathbf{AB} = \mathbf{b} - \mathbf{a}$, so $\mathbf{b} = \mathbf{AB} + \mathbf{a}$

$$\mathbf{b} = \begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -7 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \\ 2 \end{pmatrix}$$

Position vector of B is $7\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

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Edexcel AS and A Level Modular Mathematics

Vectors

Exercise F, Question 4

Question:

Given that $\mathbf{a} = t\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, and that $|\mathbf{a}| = 7$, find the possible values of t .

Solution:

$$|\mathbf{a}| = \sqrt{t^2 + 2^2 + 3^2} = 7$$
$$\sqrt{t^2 + 4 + 9} = 7$$
$$t^2 + 4 + 9 = 49$$
$$t^2 = 36$$
$$t = 6 \text{ or } t = -6$$

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Vectors

Exercise F, Question 5

Question:

Given that $\mathbf{a} = 5t\mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$, and that $|\mathbf{a}| = 3\sqrt{10}$, find the possible values of t .

Solution:

$$|\mathbf{a}| = \sqrt{(5t)^2 + (2t)^2 + t^2} = 3\sqrt{10}$$

$$\sqrt{25t^2 + 4t^2 + t^2} = 3\sqrt{10}$$

$$\sqrt{30t^2} = 3\sqrt{10}$$

$$30t^2 = 9 \times 10$$

$$t^2 = 3$$

$$t = \sqrt{3} \text{ or } t = -\sqrt{3}$$

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Edexcel AS and A Level Modular Mathematics

Vectors

Exercise F, Question 6

Question:

The points A and B have position vectors $\begin{pmatrix} 2 \\ 9 \\ t \end{pmatrix}$ and $\begin{pmatrix} 2t \\ 5 \\ 3t \end{pmatrix}$ respectively.

- (a) Find AB .
- (b) Find, in terms of t , $|AB|$.
- (c) Find the value of t that makes $|AB|$ a minimum.
- (d) Find the minimum value of $|AB|$.

Solution:

$$(a) AB = b - a = \begin{pmatrix} 2t \\ 5 \\ 3t \end{pmatrix} - \begin{pmatrix} 2 \\ 9 \\ t \end{pmatrix} = \begin{pmatrix} 2t - 2 \\ -4 \\ 2t \end{pmatrix}$$

$$(b) |AB| = \sqrt{(2t - 2)^2 + (-4)^2 + (2t)^2}$$

$$= \sqrt{4t^2 - 8t + 4 + 16 + 4t^2}$$

$$= \sqrt{8t^2 - 8t + 20}$$

$$(c) \text{ Let } |AB|^2 = p, \text{ then } p = 8t^2 - 8t + 20$$

$$\frac{dp}{dt} = 16t - 8$$

$$\text{For a minimum, } \frac{dp}{dt} = 0, \text{ so } 16t - 8 = 0, \text{ i.e. } t = \frac{1}{2}$$

$$\frac{d^2p}{dt^2} = 16, \text{ positive, } \therefore \text{ minimum}$$

$$(d) \text{ When } t = \frac{1}{2},$$

$$|AB| = \sqrt{8t^2 - 8t + 20} = \sqrt{2 - 4 + 20} = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

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Edexcel AS and A Level Modular Mathematics

Vectors

Exercise F, Question 7

Question:

The points A and B have position vectors $\begin{pmatrix} 2t+1 \\ t+1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} t+1 \\ 5 \\ 2 \end{pmatrix}$ respectively.

- (a) Find AB .
- (b) Find, in terms of t , $|AB|$.
- (c) Find the value of t that makes $|AB|$ a minimum.
- (d) Find the minimum value of $|AB|$.

Solution:

$$(a) \quad AB = b - a = \begin{pmatrix} t+1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 2t+1 \\ t+1 \\ 3 \end{pmatrix} = \begin{pmatrix} -t \\ 4-t \\ -1 \end{pmatrix}$$

$$(b) \quad |AB| = \sqrt{(-t)^2 + (4-t)^2 + (-1)^2}$$

$$= \sqrt{t^2 + 16 - 8t + t^2 + 1}$$

$$= \sqrt{2t^2 - 8t + 17}$$

$$(c) \quad \text{Let } |AB|^2 = P, \text{ then } P = 2t^2 - 8t + 17$$

$$\frac{dP}{dt} = 4t - 8$$

For a minimum, $\frac{dP}{dt} = 0$, so $4t - 8 = 0$, i.e. $t = 2$

$$\frac{d^2P}{dt^2} = 4, \text{ positive, } \therefore \text{ minimum}$$

$$(d) \quad \text{When } t = 2, \quad |AB| = \sqrt{2t^2 - 8t + 17} = \sqrt{8 - 16 + 17} = \sqrt{9} = 3$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 1

Question:

The vectors **a** and **b** each have magnitude 3 units, and the angle between **a** and **b** is 60° . Find **a.b**.

Solution:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta = 3 \times 3 \times \cos 60^\circ = 3 \times 3 \times \frac{1}{2} = \frac{9}{2}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 2

Question:

In each part, find $\mathbf{a} \cdot \mathbf{b}$:

(a) $\mathbf{a} = 5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$

(b) $\mathbf{a} = 10\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - 5\mathbf{j} - 12\mathbf{k}$

(c) $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = -\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

(d) $\mathbf{a} = 2\mathbf{i} - \mathbf{k}$, $\mathbf{b} = 6\mathbf{i} - 5\mathbf{j} - 8\mathbf{k}$

(e) $\mathbf{a} = 3\mathbf{j} + 9\mathbf{k}$, $\mathbf{b} = \mathbf{i} + 12\mathbf{j} - 4\mathbf{k}$

Solution:

$$(a) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} = 10 - 2 - 6 = 2$$

$$(b) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 10 \\ -7 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -5 \\ -12 \end{pmatrix} = 30 + 35 - 48 = 17$$

$$(c) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix} = -1 - 1 - 4 = -6$$

$$(d) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -5 \\ -8 \end{pmatrix} = 12 + 0 + 8 = 20$$

$$(e) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 0 \\ 3 \\ 9 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 12 \\ -4 \end{pmatrix} = 0 + 36 - 36 = 0$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 3

Question:

In each part, find the angle between **a** and **b**, giving your answer in degrees to 1 decimal place:

(a) $a = 3i + 7j$, $b = 5i + j$

(b) $a = 2i - 5j$, $b = 6i + 3j$

(c) $a = i - 7j + 8k$, $b = 12i + 2j + k$

(d) $a = -i - j + 5k$, $b = 11i - 3j + 4k$

(e) $a = 6i - 7j + 12k$, $b = -2i + j + k$

(f) $a = 4i + 5k$, $b = 6i - 2j$

(g) $a = -5i + 2j - 3k$, $b = 2i - 2j + 11k$

(h) $a = i + j + k$, $b = i - j + k$

Solution:

$$(a) a \cdot b = \begin{pmatrix} 3 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 1 \end{pmatrix} = 15 + 7 = 22$$

$$|a| = \sqrt{3^2 + 7^2} = \sqrt{58}$$

$$|b| = \sqrt{5^2 + 1^2} = \sqrt{26}$$

$$\sqrt{58}\sqrt{26}\cos\theta = 22$$

$$\cos\theta = \frac{22}{\sqrt{58}\sqrt{26}}$$

$$\theta = 55.5^\circ \text{ (1 d.p.)}$$

$$(b) a \cdot b = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 3 \end{pmatrix} = 12 - 15 = -3$$

$$|a| = \sqrt{2^2 + (-5)^2} = \sqrt{29}$$

$$|b| = \sqrt{6^2 + 3^2} = \sqrt{45}$$

$$\sqrt{29}\sqrt{45}\cos\theta = -3$$

$$\cos\theta = \frac{-3}{\sqrt{29}\sqrt{45}}$$

$$\theta = 94.8^\circ \text{ (1 d.p.)}$$

$$(c) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 1 \\ -7 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 2 \\ 1 \end{pmatrix} = 12 - 14 + 8 = 6$$

$$|\mathbf{a}| = \sqrt{1^2 + (-7)^2 + 8^2} = \sqrt{114}$$

$$|\mathbf{b}| = \sqrt{12^2 + 2^2 + 1^2} = \sqrt{149}$$

$$\sqrt{114}\sqrt{149}\cos\theta = 6$$

$$\cos\theta = \frac{6}{\sqrt{114}\sqrt{149}}$$

$$\theta = 87.4^\circ \text{ (1 d.p.)}$$

$$(d) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 11 \\ -3 \\ 4 \end{pmatrix} = -11 + 3 + 20 = 12$$

$$|\mathbf{a}| = \sqrt{(-1)^2 + (-1)^2 + 5^2} = \sqrt{27}$$

$$|\mathbf{b}| = \sqrt{11^2 + (-3)^2 + 4^2} = \sqrt{146}$$

$$\sqrt{27}\sqrt{146}\cos\theta = 12$$

$$\cos\theta = \frac{12}{\sqrt{27}\sqrt{146}}$$

$$\theta = 79.0^\circ \text{ (1 d.p.)}$$

$$(e) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 6 \\ -7 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -12 - 7 + 12 = -7$$

$$|\mathbf{a}| = \sqrt{6^2 + (-7)^2 + 12^2} = \sqrt{229}$$

$$|\mathbf{b}| = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6}$$

$$\sqrt{229}\sqrt{6}\cos\theta = -7$$

$$\cos\theta = \frac{-7}{\sqrt{229}\sqrt{6}}$$

$$\theta = 100.9^\circ \text{ (1 d.p.)}$$

$$(f) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix} = 24 + 0 + 0 = 24$$

$$|\mathbf{a}| = \sqrt{4^2 + 5^2} = \sqrt{41}$$

$$|\mathbf{b}| = \sqrt{6^2 + (-2)^2} = \sqrt{40}$$

$$\sqrt{41}\sqrt{40}\cos\theta = 24$$

$$\cos\theta = \frac{24}{\sqrt{41}\sqrt{40}}$$

$$\theta = 53.7^\circ \text{ (1 d.p.)}$$

$$(g) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} -5 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 11 \end{pmatrix} = -10 - 4 - 33 = -47$$

$$|\mathbf{a}| = \sqrt{(-5)^2 + 2^2 + (-3)^2} = \sqrt{38}$$

$$|\mathbf{b}| = \sqrt{2^2 + (-2)^2 + 11^2} = \sqrt{129}$$

$$\sqrt{38}\sqrt{129}\cos\theta = -47$$

$$\cos\theta = \frac{-47}{\sqrt{38}\sqrt{129}}$$

$$\theta = 132.2^\circ \text{ (1 d.p.)}$$

$$(h) \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 1 - 1 + 1 = 1$$

$$|\mathbf{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|\mathbf{b}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

$$\sqrt{3}\sqrt{3}\cos\theta = 1$$

$$\cos\theta = \frac{1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$$

$$\theta = 70.5^\circ \text{ (1 d.p.)}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 4

Question:

Find the value, or values, of λ for which the given vectors are perpendicular:

(a) $3\mathbf{i} + 5\mathbf{j}$ and $\lambda\mathbf{i} + 6\mathbf{j}$

(b) $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $\lambda\mathbf{i} - 4\mathbf{j} - 14\mathbf{k}$

(c) $3\mathbf{i} + \lambda\mathbf{j} - 8\mathbf{k}$ and $7\mathbf{i} - 5\mathbf{j} + \mathbf{k}$

(d) $9\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and $\lambda\mathbf{i} + \lambda\mathbf{j} + 3\mathbf{k}$

(e) $\lambda\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\lambda\mathbf{i} + \lambda\mathbf{j} + 5\mathbf{k}$

Solution:

$$\begin{aligned} \text{(a)} \quad \begin{pmatrix} 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} \lambda \\ 6 \end{pmatrix} &= 3\lambda + 30 = 0 \\ \Rightarrow 3\lambda &= -30 \\ \Rightarrow \lambda &= -10 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} \lambda \\ -4 \\ -14 \end{pmatrix} &= 2\lambda - 24 + 14 = 0 \\ \Rightarrow 2\lambda &= 10 \\ \Rightarrow \lambda &= 5 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \begin{pmatrix} 3 \\ \lambda \\ -8 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ -5 \\ 1 \end{pmatrix} &= 21 - 5\lambda - 8 = 0 \\ \Rightarrow 5\lambda &= 13 \\ \Rightarrow \lambda &= 2\frac{3}{5} \end{aligned}$$

$$\text{(d)} \quad \begin{pmatrix} 9 \\ -3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} \lambda \\ \lambda \\ 3 \end{pmatrix} = 9\lambda - 3\lambda + 15 = 0$$

$$\Rightarrow 6\lambda = -15$$

$$\Rightarrow \lambda = -2\frac{1}{2}$$

$$(e) \begin{pmatrix} \lambda \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} \lambda \\ \lambda \\ 5 \end{pmatrix} = \lambda^2 + 3\lambda - 10 = 0$$

$$\Rightarrow (\lambda + 5)(\lambda - 2) = 0$$

$$\Rightarrow \lambda = -5 \text{ or } \lambda = 2$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 5

Question:

Find, to the nearest tenth of a degree, the angle that the vector $9\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$ makes with:

- (a) the positive x -axis
- (b) the positive y -axis

Solution:

(a) Using $\mathbf{a} = 9\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i}$,

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 9 \\ -5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 9$$

$$|\mathbf{a}| = \sqrt{9^2 + (-5)^2 + 3^2} = \sqrt{115}$$

$$|\mathbf{b}| = 1$$

$$\sqrt{115} \cos \theta = 9$$

$$\cos \theta = \frac{9}{\sqrt{115}}$$

$$\theta = 32.9^\circ$$

(b) Using $\mathbf{a} = 9\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = \mathbf{j}$,

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 9 \\ -5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -5$$

$$|\mathbf{a}| = \sqrt{115}, \quad |\mathbf{b}| = 1$$

$$\sqrt{115} \cos \theta = -5$$

$$\cos \theta = \frac{-5}{\sqrt{115}}$$

$$\theta = 117.8^\circ$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 6

Question:

Find, to the nearest tenth of a degree, the angle that the vector $i + 11j - 4k$ makes with:

- (a) the positive y -axis
- (b) the positive z -axis

Solution:

(a) Using $a = i + 11j - 4k$ and $b = j$,

$$a \cdot b = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 11$$

$$|a| = \sqrt{1^2 + 11^2 + (-4)^2} = \sqrt{138}$$

$$|b| = 1$$

$$\sqrt{138} \cos \theta = 11$$

$$\cos \theta = \frac{11}{\sqrt{138}}$$

$$\theta = 20.5^\circ$$

(b) Using $a = i + 11j - 4k$ and $b = k$,

$$a \cdot b = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = -4$$

$$|a| = \sqrt{138}, \quad |b| = 1$$

$$\sqrt{138} \cos \theta = -4$$

$$\cos \theta = \frac{-4}{\sqrt{138}}$$

$$\theta = 109.9^\circ$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 7

Question:

The angle between the vectors $i + j + k$ and $2i + j + k$ is θ . Calculate the exact value of $\cos \theta$.

Solution:

Using $a = i + j + k$ and $b = 2i + j + k$,

$$a \cdot b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 2 + 1 + 1 = 4$$

$$|a| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|b| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$$

$$\sqrt{3}\sqrt{6}\cos\theta = 4$$

$$\cos\theta = \frac{4}{\sqrt{3}\sqrt{6}} = \frac{4}{\sqrt{3}\sqrt{3}\sqrt{2}} = \frac{4}{3\sqrt{2}}$$

$$= \frac{4}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 8

Question:

The angle between the vectors $\mathbf{i} + 3\mathbf{j}$ and $\mathbf{j} + \lambda \mathbf{k}$ is 60° .

Show that $\lambda = \pm \sqrt{\frac{13}{5}}$.

Solution:

Using $\mathbf{a} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = \mathbf{j} + \lambda \mathbf{k}$,

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ \lambda \end{pmatrix} = 0 + 3 + 0 = 3$$

$$|\mathbf{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$|\mathbf{b}| = \sqrt{1^2 + \lambda^2} = \sqrt{1 + \lambda^2}$$

$$\sqrt{10}\sqrt{1 + \lambda^2} \cos 60^\circ = 3$$

$$\sqrt{1 + \lambda^2} = \frac{3}{\sqrt{10} \cos 60^\circ} = \frac{6}{\sqrt{10}}$$

Squaring both sides:

$$1 + \lambda^2 = \frac{36}{10}$$

$$\lambda^2 = \frac{26}{10} = \frac{13}{5}$$

$$\lambda = \pm \sqrt{\frac{13}{5}}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 9

Question:

Simplify as far as possible:

(a) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) + \mathbf{b} \cdot (\mathbf{a} - \mathbf{c})$, given that \mathbf{b} is perpendicular to \mathbf{c} .

(b) $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b})$, given that $|\mathbf{a}| = 2$ and $|\mathbf{b}| = 3$.

(c) $(\mathbf{a} + \mathbf{b}) \cdot (2\mathbf{a} - \mathbf{b})$, given that \mathbf{a} is perpendicular to \mathbf{b} .

Solution:

$$\begin{aligned} \text{(a)} \quad & \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) + \mathbf{b} \cdot (\mathbf{a} - \mathbf{c}) \\ &= \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} + \mathbf{b} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{c} \\ &= 2\mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} \quad (\text{because } \mathbf{b} \cdot \mathbf{c} = 0) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) \\ &= \mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) + \mathbf{b} \cdot (\mathbf{a} + \mathbf{b}) \\ &= \mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{a} + \mathbf{b} \cdot \mathbf{b} \\ &= |\mathbf{a}|^2 + 2\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 \\ &= 4 + 2\mathbf{a} \cdot \mathbf{b} + 9 \\ &= 13 + 2\mathbf{a} \cdot \mathbf{b} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & (\mathbf{a} + \mathbf{b}) \cdot (2\mathbf{a} - \mathbf{b}) \\ &= \mathbf{a} \cdot (2\mathbf{a} - \mathbf{b}) + \mathbf{b} \cdot (2\mathbf{a} - \mathbf{b}) \\ &= 2\mathbf{a} \cdot \mathbf{a} - \mathbf{a} \cdot \mathbf{b} + 2\mathbf{b} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{b} \\ &= 2|\mathbf{a}|^2 - |\mathbf{b}|^2 \quad (\text{because } \mathbf{a} \cdot \mathbf{b} = 0) \end{aligned}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 10

Question:

Find a vector which is perpendicular to both **a** and **b**, where:

(a) $a = i + j - 3k$, $b = 5i - 2j - k$

(b) $a = 2i + 3j - 4k$, $b = i - 6j + 3k$

(c) $a = 4i - 4j - k$, $b = -2i - 9j + 6k$

Solution:

(a) Let the required vector be $xi + yj + zk$. Then

$$\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad \text{and} \quad \begin{pmatrix} 5 \\ -2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$x + y - 3z = 0$$

$$5x - 2y - z = 0$$

Let $z = 1$:

$$x + y = 3 \quad (\times 2)$$

$$5x - 2y = 1$$

$$2x + 2y = 6$$

$$5x - 2y = 1$$

Adding, $7x = 7 \Rightarrow x = 1$

$$1 + y = 3, \text{ so } y = 2$$

So $x = 1$, $y = 2$ and $z = 1$

A possible vector is $i + 2j + k$.

(b) Let the required vector be $xi + yj + zk$. Then

$$\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad \text{and} \quad \begin{pmatrix} 1 \\ -6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$2x + 3y - 4z = 0$$

$$x - 6y + 3z = 0$$

Let $z = 1$:

$$2x + 3y = 4$$

$$x - 6y = -3 \quad (\times 2)$$

$$2x + 3y = 4$$

$$2x - 12y = -6$$

$$\text{Subtracting, } 15y = 10 \Rightarrow y = \frac{2}{3}$$

$$2x + 2 = 4, \text{ so } x = 1$$

$$\text{So } x = 1, y = \frac{2}{3} \text{ and } z = 1$$

A possible vector is $i + \frac{2}{3}j + k$.

$$\text{Another possible vector is } 3 \left(i + \frac{2}{3}j + k \right) = 3i + 2j + 3k .$$

(c) Let the required vector be $xi + yj + zk$. Then

$$\begin{pmatrix} 4 \\ -4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad \text{and} \quad \begin{pmatrix} -2 \\ -9 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$4x - 4y - z = 0$$

$$-2x - 9y + 6z = 0$$

Let $z = 1$:

$$4x - 4y = 1$$

$$-2x - 9y = -6 \quad (\times 2)$$

$$4x - 4y = 1$$

$$-4x - 18y = -12$$

$$\text{Adding, } -22y = -11 \Rightarrow y = \frac{1}{2}$$

$$4x - 2 = 1, \text{ so } x = \frac{3}{4}$$

$$\text{So } x = \frac{3}{4}, y = \frac{1}{2} \text{ and } z = 1$$

A possible vector is $\frac{3}{4}i + \frac{1}{2}j + k$

$$\text{Another possible vector is } 4 \left(\frac{3}{4}i + \frac{1}{2}j + k \right) = 3i + 2j + 4k .$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

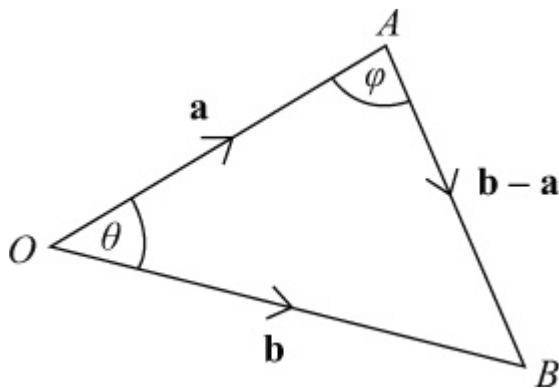
Exercise G, Question 11

Question:

The points A and B have position vectors $2\mathbf{i} + 5\mathbf{j} + \mathbf{k}$ and $6\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ respectively, and O is the origin.

Calculate each of the angles in $\triangle OAB$, giving your answers in degrees to 1 decimal place.

Solution:



Using \mathbf{a} and \mathbf{b} to find θ :

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix} = 12 + 5 - 2 = 15$$

$$|\mathbf{a}| = \sqrt{2^2 + 5^2 + 1^2} = \sqrt{30}$$

$$|\mathbf{b}| = \sqrt{6^2 + 1^2 + (-2)^2} = \sqrt{41}$$

$$\sqrt{30}\sqrt{41}\cos\theta = 15$$

$$\cos\theta = \frac{15}{\sqrt{30}\sqrt{41}}$$

$$\theta = 64.7^\circ$$

Using AO and AB to find ϕ :

$$\mathbf{AO} = -\mathbf{a} = \begin{pmatrix} -2 \\ -5 \\ -1 \end{pmatrix}$$

$$\mathbf{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -a \end{pmatrix} \cdot \begin{pmatrix} b-a \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -4 \\ -3 \end{pmatrix} = -8 + 20 + 3 = 15$$

$$|-a| = \sqrt{(-2)^2 + (-5)^2 + (-1)^2} = \sqrt{30}$$

$$|b-a| = \sqrt{4^2 + (-4)^2 + (-3)^2} = \sqrt{41}$$

$$\sqrt{30}\sqrt{41} \cos \phi = 15$$

$$\cos \phi = \frac{15}{\sqrt{30}\sqrt{41}}$$

$$\phi = 64.7^\circ \text{ (1 d.p.)}$$

(Since $|b-a| = |b|$, $AB = OB$, so the triangle is isosceles).

$$\angle OBA = 180^\circ - 64.7^\circ - 64.7^\circ = 50.6^\circ \text{ (1 d.p.)}$$

Angles are 64.7° , 64.7° and 50.6° (all 1 d.p.)

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise G, Question 12

Question:

The points A , B and C have position vectors $i + 3j + k$, $2i + 7j - 3k$ and $4i - 5j + 2k$ respectively.

(a) Find, as surds, the lengths of AB and BC .

(b) Calculate, in degrees to 1 decimal place, the size of $\angle ABC$.

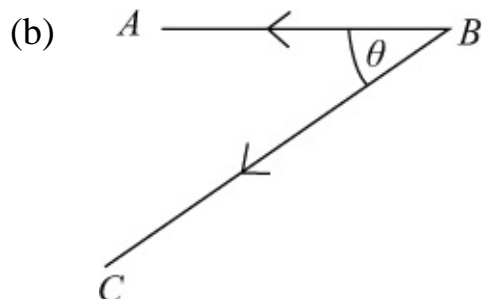
Solution:

$$(a) \ AB = b - a = \begin{pmatrix} 2 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -4 \end{pmatrix}$$

$$\text{Length of } AB = |AB| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{33}$$

$$BC = c - b = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 7 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ -12 \\ 5 \end{pmatrix}$$

$$\text{Length of } BC = |BC| = \sqrt{2^2 + (-12)^2 + 5^2} = \sqrt{173}$$



θ is the angle between BA and BC .

$$BA \cdot BC = \begin{pmatrix} -1 \\ -4 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -12 \\ 5 \end{pmatrix} = -2 + 48 + 20 = 66$$

$$\sqrt{33}\sqrt{173} \cos \theta = 66$$

$$\cos \theta = \frac{66}{\sqrt{33}\sqrt{173}}$$

$$\theta = 29.1^\circ \text{ (1 d.p.)}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

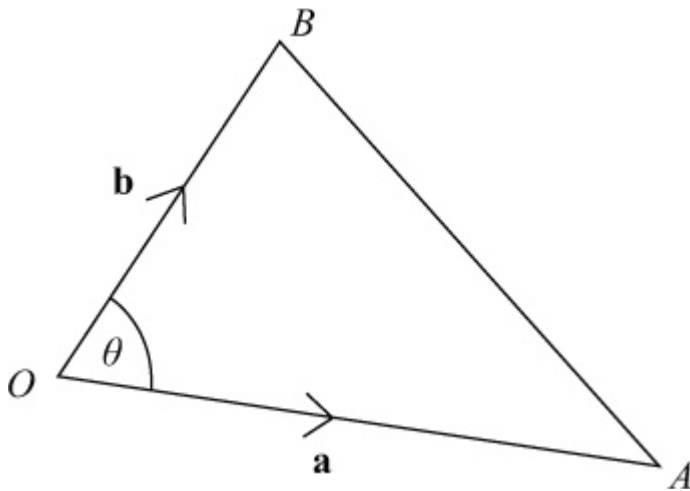
Exercise G, Question 13

Question:

Given that the points A and B have coordinates $(7, 4, 4)$ and $(2, -2, -1)$ respectively, use a vector method to find the value of $\cos \angle AOB$, where O is the origin.

Prove that the area of $\triangle AOB$ is $\frac{5\sqrt{29}}{2}$.

Solution:



The position vectors of A and B are

$$\mathbf{a} = \begin{pmatrix} 7 \\ 4 \\ 4 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 7 \\ 4 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = 14 - 8 - 4 = 2$$

$$|\mathbf{a}| = \sqrt{7^2 + 4^2 + 4^2} = \sqrt{81} = 9$$

$$|\mathbf{b}| = \sqrt{2^2 + (-2)^2 + (-1)^2} = \sqrt{9} = 3$$

$$9 \times 3 \times \cos \theta = 2$$

$$\cos \theta = \frac{2}{27}$$

$$\cos \angle AOB = \frac{2}{27}$$

$$\text{Area of } \angle AOB = \frac{1}{2} |\mathbf{a}| |\mathbf{b}| \sin \angle AOB$$

Using $\sin^2 \theta + \cos^2 \theta = 1$:

$$\sin^2 \angle AOB = 1 - \left(\frac{2}{27} \right)^2 = \frac{725}{27^2}$$

$$\sin \angle AOB = \sqrt{\frac{725}{27^2}} = \frac{\sqrt{25 \times 29}}{27} = \frac{5\sqrt{29}}{27}$$

$$\text{Area of } \triangle AOB = \frac{1}{2} \times 9 \times 3 \times \frac{5\sqrt{29}}{27} = \frac{5\sqrt{29}}{2}$$

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Edexcel AS and A Level Modular Mathematics

Vectors

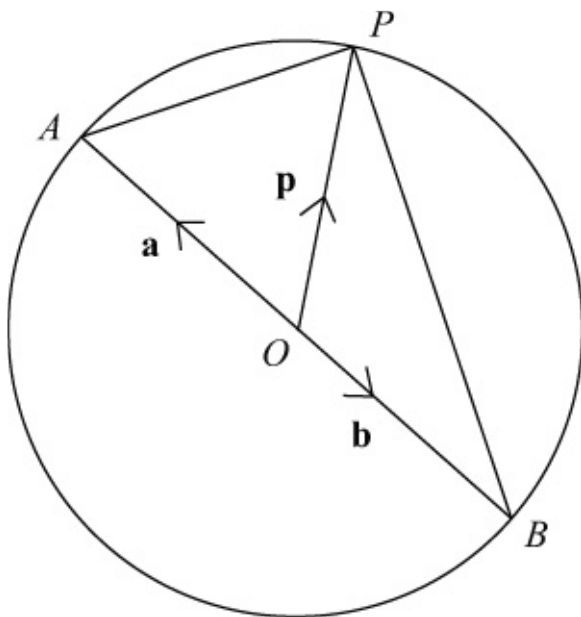
Exercise G, Question 14

Question:

AB is a diameter of a circle centred at the origin O , and P is any point on the circumference of the circle.

Using the position vectors of A , B and P , prove (using a scalar product) that AP is perpendicular to BP (i.e. the angle in the semicircle is a right angle).

Solution:



Let the position vectors, referred to origin O , of A , B and P be \mathbf{a} , \mathbf{b} and \mathbf{p} respectively.

Since $|OA| = |OB|$ and AB is a straight line, $\mathbf{b} = -\mathbf{a}$

$$AP = \mathbf{p} - \mathbf{a}$$

$$BP = \mathbf{p} - \mathbf{b} = \mathbf{p} - (-\mathbf{a}) = \mathbf{p} + \mathbf{a}$$

$$\begin{aligned} AP \cdot BP &= (\mathbf{p} - \mathbf{a}) \cdot (\mathbf{p} + \mathbf{a}) = \mathbf{p} \cdot (\mathbf{p} + \mathbf{a}) - \mathbf{a} \cdot (\mathbf{p} + \mathbf{a}) \\ &= \mathbf{p} \cdot \mathbf{p} + \mathbf{p} \cdot \mathbf{a} - \mathbf{a} \cdot \mathbf{p} - \mathbf{a} \cdot \mathbf{a} \\ &= \mathbf{p} \cdot \mathbf{p} - \mathbf{a} \cdot \mathbf{a} \end{aligned}$$

$$\mathbf{p} \cdot \mathbf{p} = |\mathbf{p}|^2 \text{ and } \mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$$

Also $|\mathbf{p}| = |\mathbf{a}|$, since the magnitude of each vector equals the radius of the circle.

$$\text{So } AP \cdot BP = |\mathbf{p}|^2 - |\mathbf{a}|^2 = 0$$

Since the scalar product is zero, AP is perpendicular to BP .

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

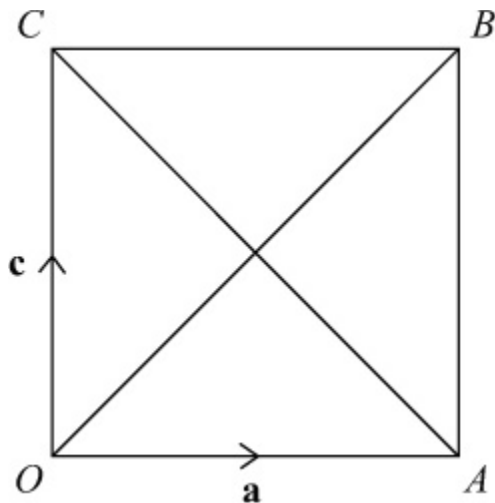
Vectors

Exercise G, Question 15

Question:

Use a vector method to prove that the diagonals of the square $OABC$ cross at right angles.

Solution:



Let the position vectors, referred to origin O , of A and C be \mathbf{a} and \mathbf{c} respectively.

$$AB = OC = \mathbf{c}$$

$$AC = \mathbf{c} - \mathbf{a}$$

$$OB = OA + AB = \mathbf{a} + \mathbf{c}$$

$$AC \cdot OB = (\mathbf{c} - \mathbf{a}) \cdot (\mathbf{a} + \mathbf{c}) = \mathbf{c} \cdot (\mathbf{a} + \mathbf{c}) - \mathbf{a} \cdot (\mathbf{a} + \mathbf{c})$$

$$= \mathbf{c} \cdot \mathbf{a} + \mathbf{c} \cdot \mathbf{c} - \mathbf{a} \cdot \mathbf{a} - \mathbf{a} \cdot \mathbf{c}$$

$$= \mathbf{c} \cdot \mathbf{c} - \mathbf{a} \cdot \mathbf{a}$$

$$= |\mathbf{c}|^2 - |\mathbf{a}|^2$$

But $|\mathbf{c}| = |\mathbf{a}|$, since the magnitude of each vector equals the length of the side of the square.

$$\text{So } AC \cdot OB = |\mathbf{c}|^2 - |\mathbf{a}|^2 = 0$$

Since the scalar product is zero; the diagonals cross at right angles.

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise H, Question 1

Question:

Find a vector equation of the straight line which passes through the point A, with position vector \mathbf{a} , and is parallel to the vector \mathbf{b} :

(a) $\mathbf{a} = 6\mathbf{i} + 5\mathbf{j} - \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$

(b) $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

(c) $\mathbf{a} = -7\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

(d) $\mathbf{a} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$

(e) $\mathbf{a} = \begin{pmatrix} 6 \\ -11 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

Solution:

(a) $\mathbf{r} = \begin{pmatrix} 6 \\ 5 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$

(b) $\mathbf{r} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(c) $\mathbf{r} = \begin{pmatrix} -7 \\ 6 \\ 2 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

(d) $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} + t \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$

$$(e) \mathbf{r} = \begin{pmatrix} 6 \\ -11 \\ 2 \end{pmatrix} + t \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$$

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Vectors

Exercise H, Question 2

Question:

Calculate, to 1 decimal place, the distance between the point P , where $t = 1$, and the point Q , where $t = 5$, on the line with equation:

$$(a) \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + t(3\mathbf{i} - 8\mathbf{j} - \mathbf{k})$$

$$(b) \mathbf{r} = (\mathbf{i} + 4\mathbf{j} + \mathbf{k}) + t(6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$$

$$(c) \mathbf{r} = (2\mathbf{i} + 5\mathbf{k}) + t(-3\mathbf{i} + 4\mathbf{j} - \mathbf{k})$$

Solution:

$$(a) t = 1: \quad \mathbf{p} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 3 \\ -8 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ -9 \\ 0 \end{pmatrix}$$

$$t = 5: \quad \mathbf{q} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + 5 \begin{pmatrix} 3 \\ -8 \\ -1 \end{pmatrix} = \begin{pmatrix} 17 \\ -41 \\ -4 \end{pmatrix}$$

$$\mathbf{PQ} = \mathbf{q} - \mathbf{p} = \begin{pmatrix} 17 \\ -41 \\ -4 \end{pmatrix} - \begin{pmatrix} 5 \\ -9 \\ 0 \end{pmatrix} = \begin{pmatrix} 12 \\ -32 \\ -4 \end{pmatrix}$$

$$\text{Distance} = |\mathbf{PQ}| = \sqrt{12^2 + (-32)^2 + (-4)^2} \\ = \sqrt{1184} = 34.4 \text{ (1 d.p.)}$$

$$(b) t = 1: \quad \mathbf{p} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}$$

$$t = 5: \quad \mathbf{q} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + 5 \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 31 \\ -6 \\ 16 \end{pmatrix}$$

$$\mathbf{PQ} = \mathbf{q} - \mathbf{p} = \begin{pmatrix} 31 \\ -6 \\ 16 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 24 \\ -8 \\ 12 \end{pmatrix}$$

$$\text{Distance} = |\mathbf{PQ}| = \sqrt{24^2 + (-8)^2 + 12^2} \\ = \sqrt{784} = 28 \text{ (exact)}$$

$$(c) t = 1: \quad \mathbf{p} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + 1 \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 4 \end{pmatrix}$$

$$t = 5: \quad \mathbf{q} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + 5 \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -13 \\ 20 \\ 0 \end{pmatrix}$$

$$\mathbf{PQ} = \mathbf{q} - \mathbf{p} = \begin{pmatrix} -13 \\ 20 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -12 \\ 16 \\ -4 \end{pmatrix}$$

$$\begin{aligned} \text{Distance} &= |\mathbf{PQ}| = \sqrt{(-12)^2 + 16^2 + (-4)^2} \\ &= \sqrt{416} = 20.4 \text{ (1 d.p.)} \end{aligned}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise H, Question 3

Question:

Find a vector equation for the line which is parallel to the z -axis and passes through the point $(4, -3, 8)$.

Solution:

Vector $\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is in the direction of the z -axis.

The point $(4, -3, 8)$ has position vector $\begin{pmatrix} 4 \\ -3 \\ 8 \end{pmatrix}$.

The equation of the line is

$$\mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 8 \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise H, Question 4

Question:

Find a vector equation for the line which passes through the points:

- (a) $(2, 1, 9)$ and $(4, -1, 8)$
- (b) $(-3, 5, 0)$ and $(7, 2, 2)$
- (c) $(1, 11, -4)$ and $(5, 9, 2)$
- (d) $(-2, -3, -7)$ and $(12, 4, -3)$

Solution:

$$(a) \mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$$

$$\mathbf{b} - \mathbf{a} = \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

Equation is

$$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

$$(b) \mathbf{a} = \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 7 \\ 2 \\ 2 \end{pmatrix}$$

$$\mathbf{b} - \mathbf{a} = \begin{pmatrix} 7 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 10 \\ -3 \\ 2 \end{pmatrix}$$

Equation is

$$\mathbf{r} = \begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix} + t \begin{pmatrix} 10 \\ -3 \\ 2 \end{pmatrix}$$

$$(c) \mathbf{a} = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 5 \\ 9 \\ 2 \end{pmatrix}$$

$$\mathbf{b} - \mathbf{a} = \begin{pmatrix} 5 \\ 9 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix}$$

Equation is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 11 \\ -4 \end{pmatrix} + t \begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix}$$

$$(d) \mathbf{a} = \begin{pmatrix} -2 \\ -3 \\ -7 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 12 \\ 4 \\ -3 \end{pmatrix}$$

$$\mathbf{b} - \mathbf{a} = \begin{pmatrix} 12 \\ 4 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ -7 \end{pmatrix} = \begin{pmatrix} 14 \\ 7 \\ 4 \end{pmatrix}$$

Equation is

$$\mathbf{r} = \begin{pmatrix} -2 \\ -3 \\ -7 \end{pmatrix} + t \begin{pmatrix} 14 \\ 7 \\ 4 \end{pmatrix}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise H, Question 5

Question:

The point $(1, p, q)$ lies on the line l . Find the values of p and q , given that the equation is l is:

$$(a) \mathbf{r} = (2\mathbf{i} - 3\mathbf{j} + \mathbf{k}) + t(\mathbf{i} - 4\mathbf{j} - 9\mathbf{k})$$

$$(b) \mathbf{r} = (-4\mathbf{i} + 6\mathbf{j} - \mathbf{k}) + t(2\mathbf{i} - 5\mathbf{j} - 8\mathbf{k})$$

$$(c) \mathbf{r} = (16\mathbf{i} - 9\mathbf{j} - 10\mathbf{k}) + t(3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

Solution:

$$(a) \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -4 \\ -9 \end{pmatrix}$$

$$x = 1: \quad 2 + t = 1 \quad \Rightarrow \quad t = -1$$

$$\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ -4 \\ -9 \end{pmatrix} = \begin{pmatrix} 1 \\ -7 \\ -8 \end{pmatrix}$$

So $p = -7$ and $q = -8$.

$$(b) \mathbf{r} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -5 \\ -8 \end{pmatrix}$$

$$x = 1: \quad -4 + 2t = 1 \quad \Rightarrow \quad 2t = 5 \quad \Rightarrow \quad t = \frac{5}{2}$$

$$\mathbf{r} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix} + \frac{5}{2} \begin{pmatrix} 2 \\ -5 \\ -8 \end{pmatrix} = \begin{pmatrix} 1 \\ -6\frac{1}{2} \\ -21 \end{pmatrix}$$

So $p = -6\frac{1}{2}$ and $q = -21$.

$$(c) \mathbf{r} = \begin{pmatrix} 16 \\ -9 \\ -10 \end{pmatrix} + t \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$x = 1: \quad 16 + 3t = 1 \quad \Rightarrow \quad 3t = -15 \quad \Rightarrow \quad t = -5$$

$$\mathbf{r} = \begin{pmatrix} 16 \\ -9 \\ -10 \end{pmatrix} + \begin{pmatrix} -5 \\ -5 \\ -5 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -19 \\ -15 \end{pmatrix}$$

So $p = -19$ and $q = -15$.

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise I, Question 1

Question:

Determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

$$\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ -7 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 1 \\ 14 \\ 16 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} 2 + 2t \\ 4 + t \\ -7 + 3t \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 1 + s \\ 14 - s \\ 16 - 2s \end{pmatrix}.$$

$$\text{At an intersection point: } \begin{pmatrix} 2 + 2t \\ 4 + t \\ -7 + 3t \end{pmatrix} = \begin{pmatrix} 1 + s \\ 14 - s \\ 16 - 2s \end{pmatrix}$$

$$2 + 2t = 1 + s$$

$$4 + t = 14 - s$$

$$\text{Adding: } 6 + 3t = 15$$

$$\Rightarrow 3t = 9$$

$$\Rightarrow t = 3$$

$$2 + 6 = 1 + s$$

$$\Rightarrow s = 7$$

If the lines intersect, $-7 + 3t = 16 - 2s$ must be true.

$$-7 + 3t = -7 + 9 = 2$$

$$16 - 2s = 16 - 14 = 2$$

The z components are equal, so the lines do intersect.

Intersection point:

$$\begin{pmatrix} 2 + 2t \\ 4 + t \\ -7 + 3t \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \\ 2 \end{pmatrix}$$

Coordinates (8, 7, 2)

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise I, Question 2

Question:

Determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

$$\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} + t \begin{pmatrix} 9 \\ -2 \\ -1 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} 2 + 9t \\ 2 - 2t \\ -3 - t \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 3 + 2s \\ -1 - s \\ 2 + 3s \end{pmatrix}$$

$$\text{At an intersection point: } \begin{pmatrix} 2 + 9t \\ 2 - 2t \\ -3 - t \end{pmatrix} = \begin{pmatrix} 3 + 2s \\ -1 - s \\ 2 + 3s \end{pmatrix}$$

$$2 + 9t = 3 + 2s$$

$$2 - 2t = -1 - s \quad (\times 2)$$

$$2 + 9t = 3 + 2s$$

$$4 - 4t = -2 - 2s$$

$$\text{Adding: } 6 + 5t = 1$$

$$\Rightarrow 5t = -5$$

$$\Rightarrow t = -1$$

$$2 - 9 = 3 + 2s$$

$$\Rightarrow 2s = -10$$

$$\Rightarrow s = -5$$

If the lines intersect, $-3 - t = 2 + 3s$ must be true.

$$-3 - t = -3 + 1 = -2$$

$$2 + 3s = 2 - 15 = -13$$

The z components are not equal, so the lines do not intersect.

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise I, Question 3

Question:

Determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

$$\mathbf{r} = \begin{pmatrix} 12 \\ 4 \\ -6 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 8 \\ -2 \\ 6 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ -5 \end{pmatrix}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} 12 - 2t \\ 4 + t \\ -6 + 4t \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 8 + 2s \\ -2 + s \\ 6 - 5s \end{pmatrix}$$

$$\text{At an intersection point: } \begin{pmatrix} 12 - 2t \\ 4 + t \\ -6 + 4t \end{pmatrix} = \begin{pmatrix} 8 + 2s \\ -2 + s \\ 6 - 5s \end{pmatrix}$$

$$12 - 2t = 8 + 2s$$

$$4 + t = -2 + s \quad (\times 2)$$

$$12 - 2t = 8 + 2s$$

$$8 + 2t = -4 + 2s$$

$$\text{Adding: } 20 = 4 + 4s$$

$$\Rightarrow 4s = 16$$

$$\Rightarrow s = 4$$

$$12 - 2t = 8 + 8$$

$$\Rightarrow 2t = -4$$

$$\Rightarrow t = -2$$

If the lines intersect, $-6 + 4t = 6 - 5s$ must be true.

$$-6 + 4t = -6 - 8 = -14$$

$$6 - 5s = 6 - 20 = -14$$

The z components are equal, so the lines do intersect. Intersection point:

$$\begin{pmatrix} 12 - 2t \\ 4 + t \\ -6 + 4t \end{pmatrix} = \begin{pmatrix} 16 \\ 2 \\ -14 \end{pmatrix}.$$

Coordinates (16 , 2 , -14)

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise I, Question 4

Question:

Determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} + t \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} -2 \\ -9 \\ 12 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} 1 + 4t \\ 2t \\ 4 + 6t \end{pmatrix}, \mathbf{r} = \begin{pmatrix} -2 + s \\ -9 + 2s \\ 12 - s \end{pmatrix}$$

$$\text{At an intersection point: } \begin{pmatrix} 1 + 4t \\ 2t \\ 4 + 6t \end{pmatrix} = \begin{pmatrix} -2 + s \\ -9 + 2s \\ 12 - s \end{pmatrix}$$

$$1 + 4t = -2 + s$$

$$2t = -9 + 2s \quad (\times 2)$$

$$1 + 4t = -2 + s$$

$$4t = -18 + 4s$$

$$\text{Subtracting: } 1 = 16 - 3s$$

$$\Rightarrow 3s = 15$$

$$\Rightarrow s = 5$$

$$1 + 4t = -2 + 5$$

$$\Rightarrow 4t = 2$$

$$\Rightarrow t = \frac{1}{2}$$

If the lines intersect, $4 + 6t = 12 - s$ must be true.

$$4 + 6t = 4 + 3 = 7$$

$$12 - s = 12 - 5 = 7$$

The z components are equal, so the lines do intersect. Intersection point:

$$\begin{pmatrix} 1 + 4t \\ 2t \\ 4 + 6t \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}.$$

Coordinates (3, 1, 7)

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise I, Question 5

Question:

Determine whether the lines with the given equations intersect. If they do intersect, find the coordinates of their point of intersection.

$$\mathbf{r} = \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} 6 \\ -4 \\ 1 \end{pmatrix}$$

Solution:

$$\mathbf{r} = \begin{pmatrix} 3 + 2t \\ -3 + t \\ 1 - 4t \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 3 + 6s \\ 4 - 4s \\ 2 + s \end{pmatrix}$$

$$\text{At an intersection point: } \begin{pmatrix} 3 + 2t \\ -3 + t \\ 1 - 4t \end{pmatrix} = \begin{pmatrix} 3 + 6s \\ 4 - 4s \\ 2 + s \end{pmatrix}$$

$$3 + 2t = 3 + 6s$$

$$-3 + t = 4 - 4s \quad (\times 2)$$

$$3 + 2t = 3 + 6s$$

$$-6 + 2t = 8 - 8s$$

$$\text{Subtracting: } 9 = -5 + 14s$$

$$\Rightarrow 14s = 14$$

$$\Rightarrow s = 1$$

$$3 + 2t = 3 + 6$$

$$\Rightarrow 2t = 6$$

$$\Rightarrow t = 3$$

If the lines intersect, $1 - 4t = 2 + s$ must be true.

$$1 - 4t = 1 - 12 = -11$$

$$2 + s = 2 + 1 = 3$$

The z components are not equal, so the lines do not intersect.

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise J, Question 1

Question:

Find, to 1 decimal place, the acute angle between the lines with the given vector equations:

$$r = (2i + j + k) + t(3i - 5j - k)$$

$$\text{and } r = (7i + 4j + k) + s(2i + j - 9k)$$

Solution:

$$\text{Direction vectors are } a = \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} \text{ and } b = \begin{pmatrix} 2 \\ 1 \\ -9 \end{pmatrix}$$

$$\cos \theta = \frac{a \cdot b}{|a| |b|}$$

$$a \cdot b = \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -9 \end{pmatrix} = 6 - 5 + 9 = 10$$

$$|a| = \sqrt{3^2 + (-5)^2 + (-1)^2} = \sqrt{35}$$

$$|b| = \sqrt{2^2 + 1^2 + (-9)^2} = \sqrt{86}$$

$$\cos \theta = \frac{10}{\sqrt{35}\sqrt{86}}$$

$$\theta = 79.5^\circ \text{ (1 d.p.)}$$

The acute angle between the lines is 79.5° (1 d.p.)

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise J, Question 2

Question:

Find, to 1 decimal place, the acute angle between the lines with the given vector equations:

$$r = (i - j + 7k) + t(-2i - j + 3k)$$

$$\text{and } r = (8i + 5j - k) + s(-4i - 2j + k)$$

Solution:

$$\text{Direction vectors are } a = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix} \text{ and } b = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix}$$

$$\cos \theta = \frac{a \cdot b}{|a| |b|}$$

$$a \cdot b = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix} = 8 + 2 + 3 = 13$$

$$|a| = \sqrt{(-2)^2 + (-1)^2 + 3^2} = \sqrt{14}$$

$$|b| = \sqrt{(-4)^2 + (-2)^2 + 1^2} = \sqrt{21}$$

$$\cos \theta = \frac{13}{\sqrt{14}\sqrt{21}}$$

$$\theta = 40.7^\circ \text{ (1 d.p.)}$$

The acute angle between the lines is 40.7° (1 d.p.)

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise J, Question 3

Question:

Find, to 1 decimal place, the acute angle between the lines with the given vector equations:

$$r = (3i + 5j - k) + t(i + j + k)$$

$$\text{and } r = (-i + 11j + 5k) + s(2i - 7j + 3k)$$

Solution:

$$\text{Direction vectors are } a = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } b = \begin{pmatrix} 2 \\ -7 \\ 3 \end{pmatrix}$$

$$\cos \theta = \frac{a \cdot b}{|a| |b|}$$

$$a \cdot b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -7 \\ 3 \end{pmatrix} = 2 - 7 + 3 = -2$$

$$|a| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|b| = \sqrt{2^2 + (-7)^2 + 3^2} = \sqrt{62}$$

$$\cos \theta = \frac{-2}{\sqrt{3}\sqrt{62}}$$

$$\theta = 98.4^\circ \text{ (1 d.p.)}$$

This is the angle between the two vectors.

The acute angle between the lines is $180^\circ - 98.4^\circ = 81.6^\circ$ (1 d.p.).

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise J, Question 4

Question:

Find, to 1 decimal place, the acute angle between the lines with the given vector equations:

$$r = (i + 6j - k) + t(8i - j - 2k)$$

$$\text{and } r = (6i + 9j) + s(i + 3j - 7k)$$

Solution:

$$\text{Direction vectors are } a = \begin{pmatrix} 8 \\ -1 \\ -2 \end{pmatrix} \text{ and } b = \begin{pmatrix} 1 \\ 3 \\ -7 \end{pmatrix}$$

$$\cos \theta = \frac{a \cdot b}{|a| |b|}$$

$$a \cdot b = \begin{pmatrix} 8 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ -7 \end{pmatrix} = 8 - 3 + 14 = 19$$

$$|a| = \sqrt{8^2 + (-1)^2 + (-2)^2} = \sqrt{69}$$

$$|b| = \sqrt{1^2 + 3^2 + (-7)^2} = \sqrt{59}$$

$$\cos \theta = \frac{19}{\sqrt{69}\sqrt{59}}$$

$$\theta = 72.7^\circ \text{ (1 d.p.)}$$

The acute angle between the lines is 72.7° (1 d.p.)

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise J, Question 5

Question:

Find, to 1 decimal place, the acute angle between the lines with the given vector equations:

$$r = (2i + k) + t(11i + 5j - 3k)$$

$$\text{and } r = (i + j) + s(-3i + 5j + 4k)$$

Solution:

$$\text{Direction vectors are } a = \begin{pmatrix} 11 \\ 5 \\ -3 \end{pmatrix} \text{ and } b = \begin{pmatrix} -3 \\ 5 \\ 4 \end{pmatrix}$$

$$\cos \theta = \frac{a \cdot b}{|a| |b|}$$

$$a \cdot b = \begin{pmatrix} 11 \\ 5 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 5 \\ 4 \end{pmatrix} = -33 + 25 - 12 = -20$$

$$|a| = \sqrt{11^2 + 5^2 + (-3)^2} = \sqrt{155}$$

$$|b| = \sqrt{(-3)^2 + 5^2 + 4^2} = \sqrt{50}$$

$$\cos \theta = \frac{-20}{\sqrt{155}\sqrt{50}}$$

$$\theta = 103.1^\circ \text{ (1 d.p.)}$$

This is the angle between the two vectors.

The acute angle between the lines is $180^\circ - 103.1^\circ = 76.9^\circ$ (1 d.p.).

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise J, Question 6

Question:

The straight lines l_1 and l_2 have vector equations

$\mathbf{r} = (\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) + t(8\mathbf{i} + 5\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = (\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) + s(3\mathbf{i} + \mathbf{j})$ respectively, and P is the point with coordinates $(1, 4, 2)$.

- (a) Show that the point $Q(9, 9, 3)$ lies on l_1 .
- (b) Find the cosine of the acute angle between l_1 and l_2 .
- (c) Find the possible coordinates of the point R , such that R lies on l_2 and $PQ = PR$.

Solution:

$$(a) \text{ Line } l_1: \quad \mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix}$$

$$\text{When } t = 1, \mathbf{r} = \begin{pmatrix} 9 \\ 9 \\ 3 \end{pmatrix}$$

So the point $(9, 9, 3)$ lies on l_1 .

$$(b) \text{ Direction vectors are } \mathbf{a} = \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = 24 + 5 + 0 = 29$$

$$|\mathbf{a}| = \sqrt{8^2 + 5^2 + 1^2} = \sqrt{90}$$

$$|\mathbf{b}| = \sqrt{3^2 + 1^2 + 0^2} = \sqrt{10}$$

$$\cos \theta = \frac{29}{\sqrt{90}\sqrt{10}} = \frac{29}{\sqrt{900}} = \frac{29}{30}$$

$$(c) PQ = \sqrt{(9-1)^2 + (9-4)^2 + (3-2)^2} \\ = \sqrt{8^2 + 5^2 + 1^2} = \sqrt{90}$$

$$\text{Line } l_2: \quad \mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 + 3s \\ 4 + s \\ 2 \end{pmatrix}$$

Let the coordinates of R be $(1 + 3s, 4 + s, 2)$

$$PR = \sqrt{(1 + 3s - 1)^2 + (4 + s - 4)^2 + (2 - 2)^2}$$

$$= \sqrt{9s^2 + s^2} = \sqrt{10s^2}$$

$$PQ^2 = PR^2: \quad 90 = 10s^2$$

$$\Rightarrow s^2 = 9$$

$$\Rightarrow s = \pm 3$$

$$\text{When } s = 3, \mathbf{r} = \begin{pmatrix} 10 \\ 7 \\ 2 \end{pmatrix} \quad \mathbf{R}: \begin{pmatrix} 10, 7, 2 \end{pmatrix}$$

$$\text{When } s = -3, \mathbf{r} = \begin{pmatrix} -8 \\ 1 \\ 2 \end{pmatrix} \quad \mathbf{R}: \begin{pmatrix} -8, 1, 2 \end{pmatrix}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 1

Question:

With respect to an origin O , the position vectors of the points L , M and N are $(4i + 7j + 7k)$, $(i + 3j + 2k)$ and $(2i + 4j + 6k)$ respectively.

(a) Find the vectors ML and MN .

(b) Prove that $\cos \angle LMN = \frac{9}{10}$.

E

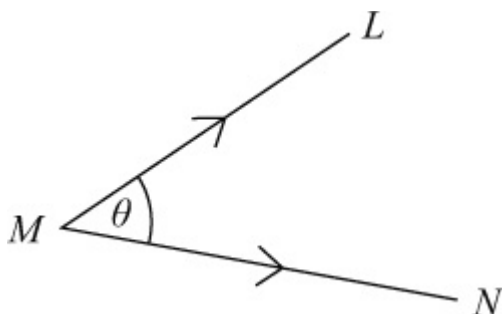
Solution:

$$l = \begin{pmatrix} 4 \\ 7 \\ 7 \end{pmatrix}, m = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, n = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$$

$$(a) ML = l - m = \begin{pmatrix} 4 \\ 7 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$$

$$MN = n - m = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

(b)



$$\cos \theta = \frac{ML \cdot MN}{|ML| |MN|}$$

$$ML \cdot MN = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} = 3 + 4 + 20 = 27$$

$$\begin{aligned} |ML| &= \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50} \\ |MN| &= \sqrt{1^2 + 1^2 + 4^2} = \sqrt{18} \\ \cos \theta &= \frac{27}{\sqrt{50}\sqrt{18}} = \frac{27}{\sqrt{25}\sqrt{2}\sqrt{9}\sqrt{2}} = \frac{27}{5 \times 3 \times 2} = \frac{9}{10}. \end{aligned}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

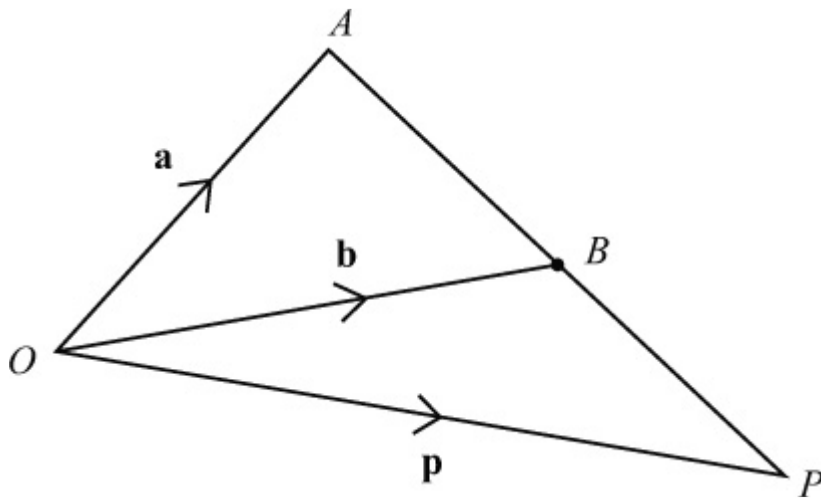
Exercise K, Question 2

Question:

The position vectors of the points A and B relative to an origin O are $5\mathbf{i} + 4\mathbf{j} + \mathbf{k}$, $-\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ respectively. Find the position vector of the point P which lies on AB produced such that $AP = 2BP$.

E

Solution:



$$\mathbf{a} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

$$OP = OA + AP = OA + 2AB$$

$$\mathbf{p} = \mathbf{a} + 2(\mathbf{b} - \mathbf{a}) = 2\mathbf{b} - \mathbf{a}$$

$$\mathbf{p} = 2 \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -7 \\ -2 \\ -5 \end{pmatrix}$$

The position vector of P is $-7\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$.

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 3

Question:

Points A, B, C, D in a plane have position vectors $\mathbf{a} = 6\mathbf{i} + 8\mathbf{j}$, $\mathbf{b} = \frac{3}{2}\mathbf{a}$, $\mathbf{c} = 6\mathbf{i} + 3\mathbf{j}$, $\mathbf{d} = \frac{5}{3}\mathbf{c}$ respectively. Write down vector equations of the lines AD and BC and find the position vector of their point of intersection.

E

Solution:

$$\mathbf{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}, \mathbf{b} = \frac{3}{2}\mathbf{a} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$$

$$\mathbf{c} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}, \mathbf{d} = \frac{5}{3}\mathbf{c} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

$$\text{Line } AD: \quad \mathbf{AD} = \mathbf{d} - \mathbf{a} = \begin{pmatrix} 10 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + t \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$\text{Line } BC: \quad \mathbf{BC} = \mathbf{c} - \mathbf{b} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 9 \\ 12 \end{pmatrix} = \begin{pmatrix} -3 \\ -9 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} 9 \\ 12 \end{pmatrix} + s \begin{pmatrix} -3 \\ -9 \end{pmatrix}$$

or

$$\mathbf{r} = \begin{pmatrix} 9 \\ 12 \end{pmatrix} + s \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\text{Where } AD \text{ and } BC \text{ intersect, } \begin{pmatrix} 6 + 4t \\ 8 - 3t \end{pmatrix} = \begin{pmatrix} 9 + s \\ 12 + 3s \end{pmatrix} \quad (\text{Using the last}$$

version of BC)

$$6 + 4t = 9 + s \quad (\times 3)$$

$$8 - 3t = 12 + 3s$$

$$18 + 12t = 27 + 3s$$

$$8 - 3t = 12 + 3s$$

$$\text{Subtracting: } 10 + 15t = 15$$

$$\Rightarrow 15t = 5$$

$$\Rightarrow t = \frac{1}{3}$$

$$\text{Intersection: } \mathbf{r} = \begin{pmatrix} 6 + 4t \\ 8 - 3t \end{pmatrix} = \begin{pmatrix} \frac{22}{3} \\ 7 \end{pmatrix}$$

$$\mathbf{r} = \frac{22}{3}\mathbf{i} + 7\mathbf{j}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 4

Question:

Find the point of intersection of the line through the points $(2, 0, 1)$ and $(-1, 3, 4)$ and the line through the points $(-1, 3, 0)$ and $(4, -2, 5)$.

Calculate the acute angle between the two lines.

E

Solution:

Line through $(2, 0, 1)$ and $(-1, 3, 4)$.

$$\text{Let } \mathbf{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}$$

$$\mathbf{b} - \mathbf{a} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$$

$$\text{Equation: } \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$$

$$\text{or } \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

Line through $(-1, 3, 0)$ and $(4, -2, 5)$.

$$\text{Let } \mathbf{c} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} \text{ and } \mathbf{d} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$$

$$\mathbf{d} - \mathbf{c} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \\ 5 \end{pmatrix}$$

$$\text{Equation: } \mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + s \begin{pmatrix} 5 \\ -5 \\ 5 \end{pmatrix}$$

$$\text{or } \mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

At the intersection point:
$$\begin{pmatrix} 2-t \\ t \\ 1+t \end{pmatrix} = \begin{pmatrix} -1+s \\ 3-s \\ s \end{pmatrix}$$

$$2 - t = -1 + s$$

$$t = 3 - s$$

$$1 + t = s$$

Adding the second and third equations:

$$1 + 2t = 3$$

$$2t = 2$$

$$t = 1$$

$$s = 2$$

Intersection point:

$$\mathbf{r} = \begin{pmatrix} 2-t \\ t \\ 1+t \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{Coordinates (1, 1, 2)}$$

Direction vectors of the lines are $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

Calling these **m** and **n**:

$$\cos \theta = \frac{\mathbf{m} \cdot \mathbf{n}}{|\mathbf{m}| |\mathbf{n}|}$$

$$\mathbf{m} \cdot \mathbf{n} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = -1 - 1 + 1 = -1$$

$$|\mathbf{m}| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|\mathbf{n}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

$$\cos \theta = \frac{-1}{\sqrt{3}\sqrt{3}} = \frac{-1}{3}$$

$$\theta = 109.5^\circ \text{ (1 d.p.)}$$

This is the angle between the two vectors.

The acute angle between the lines is $180^\circ - 109.5^\circ = 70.5^\circ$ (1 d.p.).

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 5

Question:

Show that the lines

$$r = (-2i + 5j - 11k) + \lambda(3i + j + 3k)$$

$$r = 8i + 9j + \mu(4i + 2j + 5k)$$

intersect. Find the position vector of their common point.

E

Solution:

$$r = \begin{pmatrix} -2 + 3\lambda \\ 5 + \lambda \\ -11 + 3\lambda \end{pmatrix}, r = \begin{pmatrix} 8 + 4\mu \\ 9 + 2\mu \\ 5\mu \end{pmatrix}$$

$$\text{At an intersection point: } \begin{pmatrix} -2 + 3\lambda \\ 5 + \lambda \\ -11 + 3\lambda \end{pmatrix} = \begin{pmatrix} 8 + 4\mu \\ 9 + 2\mu \\ 5\mu \end{pmatrix}$$

$$-2 + 3\lambda = 8 + 4\mu$$

$$5 + \lambda = 9 + 2\mu \quad (\times 2)$$

$$-2 + 3\lambda = 8 + 4\mu$$

$$10 + 2\lambda = 18 + 4\mu$$

$$\text{Subtracting: } -12 + \lambda = -10$$

$$\Rightarrow \lambda = 12 - 10$$

$$\Rightarrow \lambda = 2$$

$$-2 + 6 = 8 + 4\mu$$

$$\Rightarrow 4\mu = -4$$

$$\Rightarrow \mu = -1$$

If the lines intersect, $-11 + 3\lambda = 5\mu$:

$$-11 + 3\lambda = -11 + 6 = -5$$

$$5\mu = -5$$

The z components are equal, so the lines do intersect. Intersection point:

$$r = \begin{pmatrix} -2 + 3\lambda \\ 5 + \lambda \\ -11 + 3\lambda \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix} = 4i + 7j - 5k.$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 6

Question:

Find a vector that is perpendicular to both $2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

E.

Solution:

Let the required vector be $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

$$\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad \text{and} \quad \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$2x + y - z = 0$$

$$x + y - 2z = 0$$

Let $z = 1$:

$$2x + y = 1$$

$$x + y = 2$$

Subtracting: $x = -1, y = 3$

So $x = -1, y = 3$ and $z = 1$

A possible vector is $-\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 7

Question:

State a vector equation of the line passing through the points A and B whose position vectors are $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ respectively. Determine the position vector of the point C which divides the line segment AB internally such that $AC = 2CB$.

E

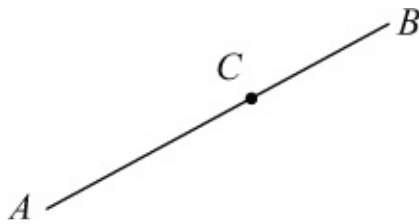
Solution:

$$\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

Equation of line:

$$\mathbf{r} = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$$

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}$$



but $AC = 2CB$

Position vector of C :

$$\begin{aligned} \mathbf{c} &= \mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a}) \\ &= \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + \frac{2}{3} \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \frac{7}{3} \end{pmatrix} \\ &= \mathbf{i} + \mathbf{j} + \frac{7}{3}\mathbf{k} \end{aligned}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 8

Question:

Vectors \mathbf{r} and \mathbf{s} are given by

$$\mathbf{r} = \lambda \mathbf{i} + (2\lambda - 1)\mathbf{j} - \mathbf{k}$$

$$\mathbf{s} = (1 - \lambda)\mathbf{i} + 3\lambda\mathbf{j} + (4\lambda - 1)\mathbf{k}$$

where λ is a scalar.

(a) Find the values of λ for which \mathbf{r} and \mathbf{s} are perpendicular.

When $\lambda = 2$, \mathbf{r} and \mathbf{s} are the position vectors of the points A and B respectively, referred to an origin O .

(b) Find AB .

(c) Use a scalar product to find the size of $\angle BAO$, giving your answer to the nearest degree.

E

Solution:

$$\mathbf{r} = \begin{pmatrix} \lambda \\ 2\lambda - 1 \\ -1 \end{pmatrix}, \text{ and } \mathbf{s} = \begin{pmatrix} 1 - \lambda \\ 3\lambda \\ 4\lambda - 1 \end{pmatrix}$$

(a) If \mathbf{r} and \mathbf{s} are perpendicular, $\mathbf{r} \cdot \mathbf{s} = 0$

$$\begin{aligned} \mathbf{r} \cdot \mathbf{s} &= \begin{pmatrix} \lambda \\ 2\lambda - 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 - \lambda \\ 3\lambda \\ 4\lambda - 1 \end{pmatrix} \\ &= \lambda(1 - \lambda) + 3\lambda(2\lambda - 1) - 1(4\lambda - 1) \\ &= \lambda - \lambda^2 + 6\lambda^2 - 3\lambda - 4\lambda + 1 \\ &= 5\lambda^2 - 6\lambda + 1 \\ \therefore 5\lambda^2 - 6\lambda + 1 &= 0 \\ (5\lambda - 1)(\lambda - 1) &= 0 \\ \lambda &= \frac{1}{5} \text{ or } \lambda = 1 \end{aligned}$$

$$(b) \lambda = 2: \quad \mathbf{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 6 \\ 7 \end{pmatrix}$$

$$\begin{aligned} \mathbf{AB} = \mathbf{b} - \mathbf{a} &= \begin{pmatrix} -1 \\ 6 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 8 \end{pmatrix} \\ &= -3\mathbf{i} + 3\mathbf{j} + 8\mathbf{k} \end{aligned}$$

(c) Using vectors \mathbf{AB} and \mathbf{AO} :

$$\mathbf{AB} = \begin{pmatrix} -3 \\ 3 \\ 8 \end{pmatrix}, \quad \mathbf{AO} = -\mathbf{a} = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$$

$$\cos \angle \mathbf{BAO} = \frac{\mathbf{AB} \cdot \mathbf{AO}}{|\mathbf{AB}| |\mathbf{AO}|}$$

$$\mathbf{AB} \cdot \mathbf{AO} = \begin{pmatrix} -3 \\ 3 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = 6 - 9 + 8 = 5$$

$$|\mathbf{AB}| = \sqrt{(-3)^2 + 3^2 + 8^2} = \sqrt{82}$$

$$|\mathbf{AO}| = \sqrt{(-2)^2 + (-3)^2 + 1^2} = \sqrt{14}$$

$$\cos \angle \mathbf{BAO} = \frac{5}{\sqrt{82}\sqrt{14}}$$

$$\angle \mathbf{BAO} = 82^\circ \text{ (nearest degree)}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 9

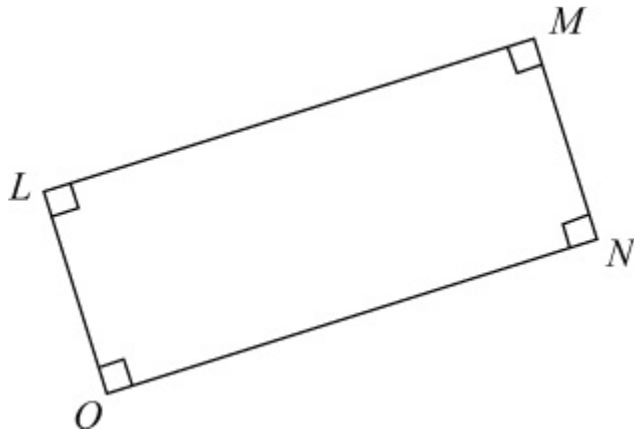
Question:

With respect to an origin O , the position vectors of the points L and M are $2\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$ and $5\mathbf{i} + \mathbf{j} + c\mathbf{k}$ respectively, where c is a constant. The point N is such that $OLMN$ is a rectangle.

- (a) Find the value of c .
- (b) Write down the position vector of N .
- (c) Find, in the form $\mathbf{r} = \mathbf{p} + t\mathbf{q}$, an equation of the line MN .

E

Solution:



$$(a) \mathbf{l} = \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{m} = \begin{pmatrix} 5 \\ 1 \\ c \end{pmatrix}$$

$$\mathbf{LM} = \mathbf{m} - \mathbf{l} = \begin{pmatrix} 5 \\ 1 \\ c \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ c-3 \end{pmatrix}$$

Since OL and LM are perpendicular, $OL \cdot LM = 0$

$$\begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ c-3 \end{pmatrix} = 0$$

$$6 - 12 + 3(c - 3) = 0$$

$$6 - 12 + 3c - 9 = 0$$

$$3c = 15$$

$$c = 5$$

$$(b) \mathbf{n} = \mathbf{ON} = \mathbf{LM} = \begin{pmatrix} 3 \\ 4 \\ c-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$$

$$\mathbf{n} = 3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

(c) The line MN is parallel to OL .

Using the point M and the direction vector \mathbf{l} :

$$\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 10

Question:

The point A has coordinates $(7, -1, 3)$ and the point B has coordinates $(10, -2, 2)$. The line l has vector equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} + \mathbf{k})$, where λ is a real parameter.

- (a) Show that the point A lies on the line l .
- (b) Find the length of AB .
- (c) Find the size of the acute angle between the line l and the line segment AB , giving your answer to the nearest degree.
- (d) Hence, or otherwise, calculate the perpendicular distance from B to the line l , giving your answer to two significant figures.

E

Solution:

$$(a) \text{ Line } l: \quad \mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$$

Point A is $(7, -1, 3)$

$$\text{Using } \lambda = 2, \mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ 3 \end{pmatrix}$$

So A lies on the line l .

$$(b) \quad AB = \sqrt{(10 - 7)^2 + [-2 - (-1)]^2 + (2 - 3)^2} \\ = \sqrt{3^2 + (-1)^2 + (-1)^2} = \sqrt{11}$$

$$(c) \quad AB = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 10 \\ -2 \\ 2 \end{pmatrix} - \begin{pmatrix} 7 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$$

Angle between the vectors $\begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$:

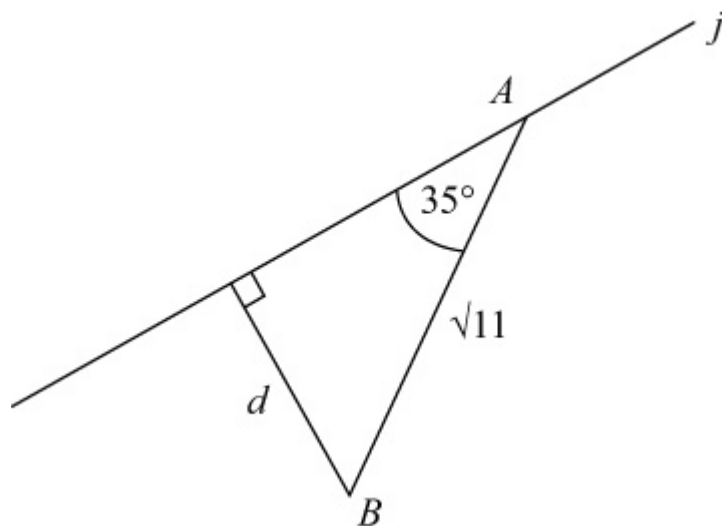
$$\begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = 9 + 1 - 1 = 9$$

The magnitude of each of the vectors is $\sqrt{11}$

$$\text{So } \cos \theta = \frac{9}{\sqrt{11}\sqrt{11}} = \frac{9}{11}$$

$$\Rightarrow \theta = 35^\circ \text{ (nearest degree)}$$

(d)



$$\sin 35^\circ = \frac{d}{\sqrt{11}}$$

$$d = \sqrt{11} \sin 35^\circ = 1.9 \text{ (2 s.f.)}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 11

Question:

Referred to a fixed origin O , the points A and B have position vectors $(5\mathbf{i} - \mathbf{j} - \mathbf{k})$ and $(\mathbf{i} - 5\mathbf{j} + 7\mathbf{k})$ respectively.

- (a) Find an equation of the line AB .
- (b) Show that the point C with position vector $4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ lies on AB .
- (c) Show that OC is perpendicular to AB .
- (d) Find the position vector of the point D , where $D \neq A$, on AB such that $|\mathbf{OD}| = |\mathbf{OA}|$.

E

Solution:

$$(a) \mathbf{a} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix}$$

$$\mathbf{b} - \mathbf{a} = \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix}$$

Equation of AB :

$$\mathbf{r} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} + t \begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix}$$

or

$$\mathbf{r} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

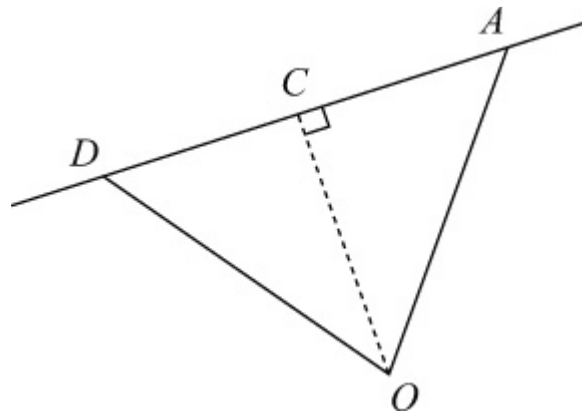
$$(b) \text{ Using } t = 1: \quad \mathbf{r} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} + 1 \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

So the point with position vector $4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ lies on AB .

$$(c) \mathbf{OC} \cdot \mathbf{AB} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix} = -16 + 8 + 8 = 0$$

Since the scalar product is zero, OC is perpendicular to AB .

(d)



Since $OD = OA$, $DC = CA$, so $DC = CA$.

$$\mathbf{CA} = \mathbf{a} - \mathbf{c} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$\mathbf{DC} = \mathbf{c} - \mathbf{d} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$\text{So } \mathbf{d} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$$

$$\mathbf{d} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 12

Question:

Referred to a fixed origin O , the points A , B and C have position vectors $(9\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, $(6\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$ and $(3\mathbf{i} + p\mathbf{j} + q\mathbf{k})$ respectively, where p and q are constants.

- (a) Find, in vector form, an equation of the line l which passes through A and B . Given that C lies on l :
- (b) Find the value of p and the value of q .
- (c) Calculate, in degrees, the acute angle between OC and AB . The point D lies on AB and is such that OD is perpendicular to AB .
- (d) Find the position vector of D .

E

Solution:

$$\mathbf{a} = \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 6 \\ 2 \\ 6 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix}$$

$$(a) \mathbf{b} - \mathbf{a} = \begin{pmatrix} 6 \\ 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$$

Equation of l :

$$\mathbf{r} = \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$$

(b) Since C lies on l ,

$$\begin{pmatrix} 3 \\ p \\ q \end{pmatrix} = \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$$

$$3 = 9 - 3t$$

$$3t = 6$$

$$t = 2$$

$$\text{So } p = -2 + 4t = 6$$

$$\text{and } q = 1 + 5t = 11$$

$$(c) \cos \theta = \frac{\text{OC} \cdot \text{AB}}{|\text{OC}| |\text{AB}|}$$

$$\text{OC} \cdot \text{AB} = \begin{pmatrix} 3 \\ 6 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix} = -9 + 24 + 55 = 70$$

$$|\text{OC}| = \sqrt{3^2 + 6^2 + 11^2} = \sqrt{166}$$

$$|\text{AB}| = \sqrt{(-3)^2 + 4^2 + 5^2} = \sqrt{50}$$

$$\cos \theta = \frac{70}{\sqrt{166}\sqrt{50}}$$

$$\theta = 39.8^\circ \text{ (1 d.p.)}$$

(d) If OD and AB are perpendicular, $\mathbf{d} \cdot (\mathbf{b} - \mathbf{a}) = 0$

$$\text{Since } \mathbf{d} \text{ lies on } AB, \text{ use } \mathbf{d} = \begin{pmatrix} 9 - 3t \\ -2 + 4t \\ 1 + 5t \end{pmatrix}$$

$$\begin{pmatrix} 9 - 3t \\ -2 + 4t \\ 1 + 5t \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix} = 0$$

$$-3(9 - 3t) + 4(-2 + 4t) + 5(1 + 5t) = 0$$

$$-27 + 9t - 8 + 16t + 5 + 25t = 0$$

$$50t = 30$$

$$t = \frac{3}{5}$$

$$\mathbf{d} = \begin{pmatrix} 9 - \frac{9}{5} \\ -2 + \frac{12}{5} \\ 1 + 3 \end{pmatrix} = \frac{36}{5}\mathbf{i} + \frac{2}{5}\mathbf{j} + 4\mathbf{k}$$

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 13

Question:

Referred to a fixed origin O , the points A and B have position vectors $(i + 2j - 3k)$ and $(5i - 3j)$ respectively.

(a) Find, in vector form, an equation of the line l_1 which passes through A and B . The line l_2 has equation $r = (4i - 4j + 3k) + \lambda(i - 2j + 2k)$, where λ is a scalar parameter.

(b) Show that A lies on l_2 .

(c) Find, in degrees, the acute angle between the lines l_1 and l_2 . The point C with position vector $(2i - k)$ lies on l_2 .

(d) Find the shortest distance from C to the line l_1 .

E

Solution:

$$(a) \mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix}$$

$$\mathbf{b} - \mathbf{a} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$$

Equation of l_1 :

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + t \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$$

(b) Equation of l_2 :

$$\mathbf{r} = \begin{pmatrix} 4 \\ -4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

$$\text{Using } \lambda = -3, \mathbf{r} = \begin{pmatrix} 4 \\ -4 \\ 3 \end{pmatrix} - 3 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$$

So A lies on the line l_2 .

$$(c) \text{ Direction vectors of } l_1 \text{ and } l_2 \text{ are } \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}.$$

Calling these \mathbf{m} and \mathbf{n} :

$$\cos \theta = \frac{\mathbf{m} \cdot \mathbf{n}}{|\mathbf{m}| |\mathbf{n}|}$$

$$\mathbf{m} \cdot \mathbf{n} = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = 4 + 10 + 6 = 20$$

$$|\mathbf{m}| = \sqrt{4^2 + (-5)^2 + 3^2} = \sqrt{50}$$

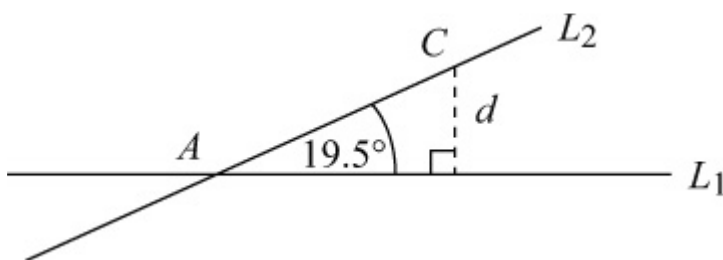
$$|\mathbf{n}| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{9} = 3$$

$$\cos \theta = \frac{20}{3\sqrt{50}}$$

$$\theta = 19.5^\circ \text{ (1 d.p.)}$$

The angle between l_1 and l_2 is 19.5° (1 d.p.).

$$(d) \mathbf{c} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$



$$|\mathbf{AC}| = \sqrt{(2-1)^2 + (0-2)^2 + [-1-(-3)]^2}$$

$$= \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{9} = 3$$

$$\sin \theta = \frac{d}{\mathbf{AC}}$$

$$d = \mathbf{AC} \sin \theta = 3 \times \frac{1}{3} = 1$$

The shortest distance from C to l_1 is 1 unit.

Solutionbank 4

Edexcel AS and A Level Modular Mathematics

Vectors

Exercise K, Question 14

Question:

Two submarines are travelling in straight lines through the ocean. Relative to a fixed origin, the vector equations of the two lines, l_1 and l_2 , along which they

travel are

$$r = 3i + 4j - 5k + \lambda (i - 2j + 2k)$$

$$\text{and } r = 9i + j - 2k + \mu (4i + j - k)$$

where λ and μ are scalars.

- (a) Show that the submarines are moving in perpendicular directions.
- (b) Given that l_1 and l_2 intersect at the point A , find the position vector of A .
The point B has position vector $10j - 11k$.
- (c) Show that only one of the submarines passes through the point B .
- (d) Given that 1 unit on each coordinate axis represents 100 m, find, in km, the distance AB .

E

Solution:

$$\text{(a) Line } l_1: \quad r = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

$$\text{Line } l_2: \quad r = \begin{pmatrix} 9 \\ 1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$$

Using the direction vectors:

$$\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = 4 - 2 - 2 = 0$$

Since the scalar product is zero, the directions are perpendicular.

$$\text{(b) At an intersection point: } \begin{pmatrix} 3 + \lambda \\ 4 - 2\lambda \\ -5 + 2\lambda \end{pmatrix} = \begin{pmatrix} 9 + 4\mu \\ 1 + \mu \\ -2 - \mu \end{pmatrix}$$

$$3 + \lambda = 9 + 4\mu \quad (\times 2)$$

$$4 - 2\lambda = 1 + \mu$$

$$6 + 2\lambda = 18 + 8\mu$$

$$4 - 2\lambda = 1 + \mu$$

$$\text{Adding: } 10 = 19 + 9\mu$$

$$\Rightarrow 9\mu = -9$$

$$\Rightarrow \mu = -1$$

$$3 + \lambda = 9 - 4$$

$$\Rightarrow \lambda = 2$$

$$\text{Intersection point: } \begin{pmatrix} 3 + \lambda \\ 4 - 2\lambda \\ -5 + 2\lambda \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$$

Position vector of A is $\mathbf{a} = 5\mathbf{i} - \mathbf{k}$.

$$(c) \text{ Position vector of } B: \quad \mathbf{b} = 10\mathbf{j} - 11\mathbf{k} = \begin{pmatrix} 0 \\ 10 \\ -11 \end{pmatrix}$$

For l_1 , to give zero as the x component, $\lambda = -3$.

$$\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} - 3 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -11 \end{pmatrix}$$

So B lies on l_1 .

For l_2 , to give -11 as the z component, $\mu = 9$.

$$\mathbf{r} = \begin{pmatrix} 9 \\ 1 \\ -2 \end{pmatrix} + 9 \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 45 \\ 10 \\ -11 \end{pmatrix}$$

So B does not lie on l_2 .

So only one of the submarines passes through B .

$$(d) \quad |AB| = \sqrt{(0 - 5)^2 + (10 - 0)^2 + [-11 - (-1)]^2}$$

$$= \sqrt{(-5)^2 + 10^2 + (-10)^2}$$

$$= \sqrt{225} = 15$$

Since 1 unit represents 100 m, the distance AB is $15 \times 100 = 1500 \text{ m} = 1.5 \text{ km}$.