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On the Influences of Starting/Prior Models in Three-Dimensional Magnetotelluric Inversion

Hao Dong[1] and Alan G. Jones[1,2]
[1] China University of Geosciences Beijing, China
[2] Formerly Dublin Institute for Advanced Studies, Ireland

SUMMARY

In electromagnetic inversions, most iterative optimization methods start with an initial guess (starting model) in the model space, which can strongly affect the global convergence of the method. However, most of inversions in practice just use a homogeneous half space for the starting/prior model, which limits the exploration of model space and may lead to convergence into a local minimum. In this study we test the influence of different types of starting/prior models on inversions. Two different classes of initial models, namely random model class and 1D prior model class, are tested against the homogeneous half space models. While inversions with different initial models can all archive a reasonable data fitting, an inappropriate prior/starting model may bring confusing features to the final result. An "averaged random inversions" model is also tested to provide a way to investigate the data without arbitrarily assuming a prior underground structure.

Keywords: magnetotellurics, inversion, starting model, prior model, random model

INTRODUCTION

Due to the (low) efficiency of three-dimensional (3D) electromagnetic (EM) forward modelling, most of the 3D EM inversion algorithms utilize iterative optimization methods that evaluate Hessians or Jacobians (or their approximations). Comparing with global optimization methods like Monte-Carlo, simulated annealing and genetic algorithms, which search the whole model domain, a typical iterative optimization method starts with an initial guess of the model and searches far smaller fraction of the model domain. Hence the global convergence of the iterative methods can strongly depend on the initial guess (starting and/or prior model) of the optimization.

However, in 3D EM inversion practices, especially magnetotelluric (MT) inversions, most practitioners merely utilize a (typically 100 Ωm) homogeneous half-space as starting/prior model. This could easily lead to convergences "trapped" in some local minimum rather than the global minimum solution. Moreover, for half-space starting models, model regions with little data constrains or resolutions could be left unchanged (= prior/starting value) in the inversion, resulting in confusing structure patterns (a.k.a. 'blobs' or 'measles') for interpretation.

To address the above problems, different initial search points (starting models) are investigated to test the influence of prior and starting models on inversions. In this study, a series of MT inversion tests are performed with different starting model using the Dublin Secret Model I (DSM1) synthetic dataset. A fully random (Monte-Carlo style) starting model class is tested against a prior (stitched 1D Bostick resistivity-depth model) starting model class and the homogeneous half space starting model class, plus other start models.

Inversion Dataset

The synthetic MT dataset of the inversion experiments is from the first 3D MT workshop [*Miensopust et al.*, 2013] held in 2008 by the Dublin Institute for Advanced Studies. The dataset is obtained from a secret model of Dublin Secret Model I, or DSM I, which was used for evaluating the efficiency of the inversion algorithms. The real model is a sprial shape 1 Ωm condutor which extents from ~5km to ~50km (Figure 1.). The shape and distribution of the anomaly in the true model is shown in the resistivity maps at different depth in Figure 2.

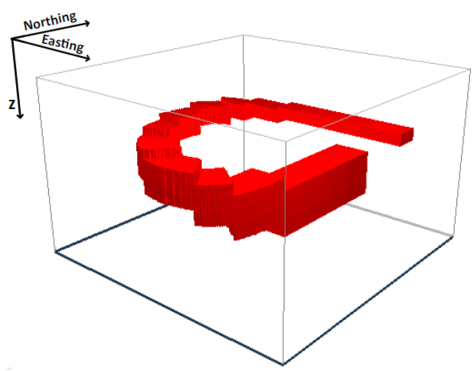

Figure 1. the 3D body of the 1 Ωm conductive anormaly from the true DSM1 model

The dataset has 100 stations on 10 profiles (10 by 10) covering a spacial range of 50 by 50 km, with 18 periods ranging from 0.56 s to 10 000 s. For the following inversion tests, all data stations and frequencies are used in the inversions.

Inversion Settings

The inversion tests are performed with a modular system for the inversion of geophysical electromagnetic data (ModEM) [*Kelbert et al.*, 2014], which utilize a staggered grid finite difference method for forward calculation and a non-linear conjugate algorithm for model line search.
Like most iterative optimization methods, ModEM also utilizes a Tikhonov regularized object function that penalizes the data residue and the "roughness" of the model resistivity. Any model that differs from a prior model will also be penalized, as shown in the following object function for the inversion:

$$\Phi = (d - F(m))^T C_d^{-1}(d - F(m)) + \lambda(m - m_{pri})^T C_m^{-1}(m - m_{pri}) \quad (1)$$

A damping factor "λ" is used to balance the weight of data residue and model roughness in the object function. ModEM uses a strategy of decreasing λ to stabilize the model convergence. In the following tests all inversions start with a λ of 10.

It should be noted that the inversion scheme of ModEM does not have an explicit initial model. The starting model is by default also the prior model. One must prepare a separate model perturbation model and apply for the model covariance to calculate an explicit starting model that is different from a prior model.

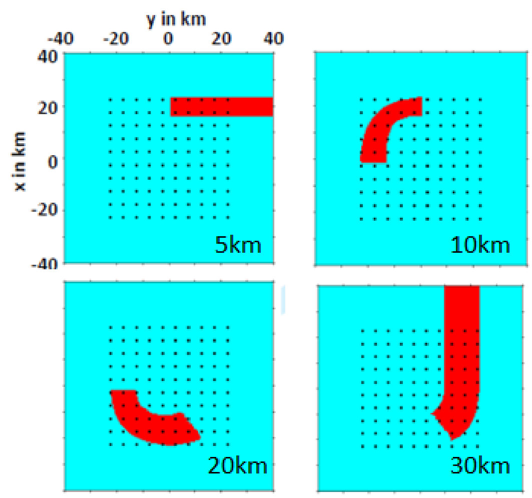

Figure 2. Horizontal maps of the true model at 5-30km

Same inversion and forward parameters are used in three different types of starting/prior models.

1. Homogeneous half space as starting/prior model
2. "Bostick" type as starting/prior model
3. "random" type as starting/prior model

The "Bostick" type is a 3D model generated with the interpolated 1D structure beneath each site location. The 1D resistivity is calculated with Bostick transformation from the geometric average of XY and YX apparent resistivity from each site [*Jones et al.*, 2005].

The "random structure" type is a series of randomly generated 3D models. In each model, every single resistivity cell is independently determined by a random number evenly distributed from 10 to 1000 Ωm in logarithm. Ten different random models are generated using the same approach.

Prior model tests

In this section, the ModEM default settings are used: the prior model is at the same time the starting model.

Starting from the 100Ωm half space prior model, which is the common approach for most inversion practices, the shape and location of the conductive sprial can be relatively well resolved (Figure 3) in the final result. This of course can be expected as the background of the true model is exactly 100 Ωm, which make it a perfect prior model. On the other hand, the resolution of the lower part of the spiral seems to decrease below the shallow conductor, which can be also expected, given the diffusion nature of MT signal.

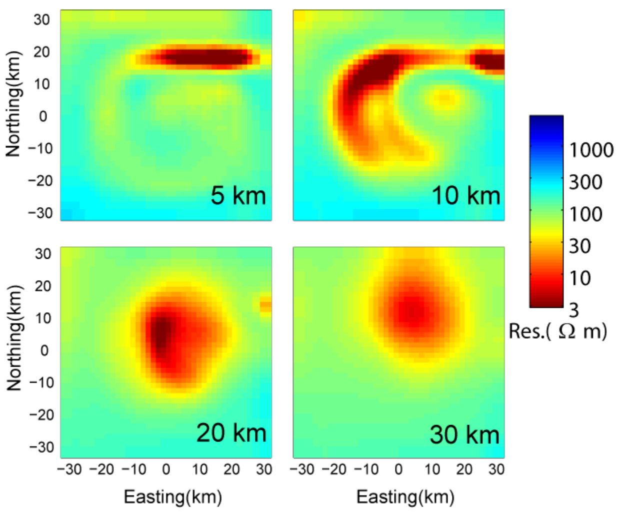

Figure 3. Inversion result from an 100Ωm half space starting/prior model

The inversion result from the "Bostick" type prior model proves to be more interesting. While the shallow structures seems to be better resolved than the half-space prior model, the deeper structures apparently have the "smearing down" 1D signature. As shown in Figure 2, the depth of the conductor seems to be better resolved. However, the inversion result keeps too much of the prior model information. As the objective function penalize changes from prior information, the search scheme will tend to stay around the prior model where MT data lose its resolution.

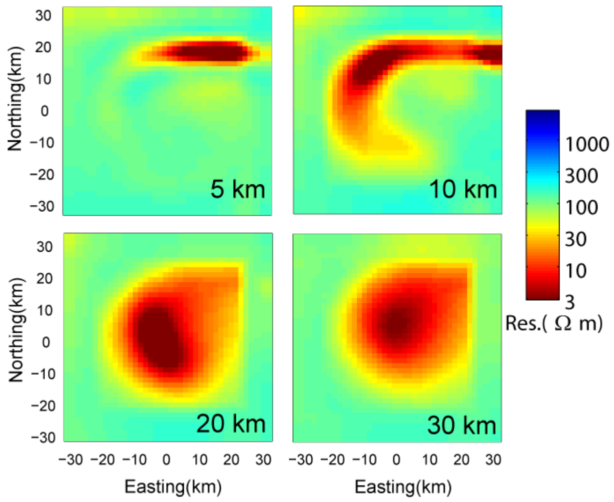

Figure 4. Inversion result from an 1D Bostick prior model

Despite the randomness of the starting models, all the ten random class prior model tests can converge to final models that fit the observed data to an impressive level. Interestingly, the major model feature (the spiral conductor) can always be retrieved and recognized with different random prior models, although the other part of the model remains relatively unchanged as the prior model (e.g. Figure 5).

As in the case of random prior models, the model covariance has little to do to help generating a smooth model, as the smoothness of a prior model is not penalized in (1).

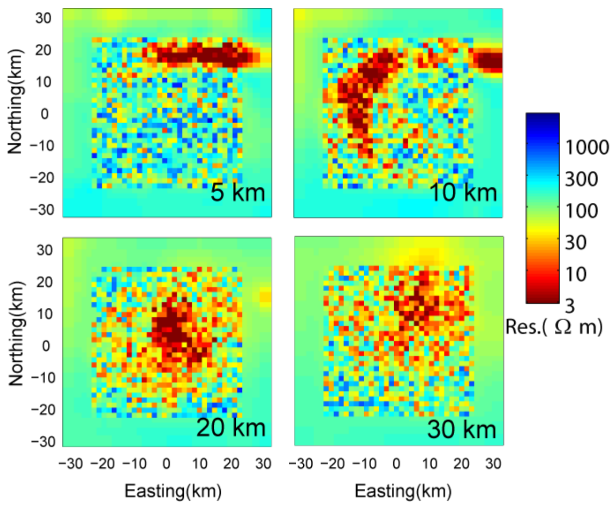

Figure 5. Inversion result from one of the "random" prior model tests (rand0)

In conclusion, with three quite different classes of prior models, the ModEM scheme actually did a good job fitting the data for each case. Apparently, all these different results are local minimums of a possible model space. And through the common However, as determined in the objective function, the prior information will always be present (although this is why it is called "prior") even where the data have good constrains on the model (see depth of 5km and 10km in Figure 5). As such a separate group of experiments are performed to test the influence of explicit starting models for inversions in the following session.

Starting model tests

This section shows inversion results from different starting models that differ from prior models. All of the following tests use a same prior model of 100 Ωm half space.

Figure 6 shows the inversion result from a "Bostick" type starting model and 100 Ωm half space model. Interestingly, although the model search started quite far away from the prior model, the inversion result seems almost identical to the result from half space prior/starting model in the last session (Figure 3). Few model features from the starting model cannot be found in the final result.

In theory, a starting model that is closer to the true solution should reduce the search effort in the model space. However, while the initial normalized RMS is considerably smaller than the half space starting model (3.51 vs. 7.47), the actual runtime of the inversion and the total number of iterations seems to be otherwise increased (68 vs. 62 iterations) when using a "Bostick" starting model.

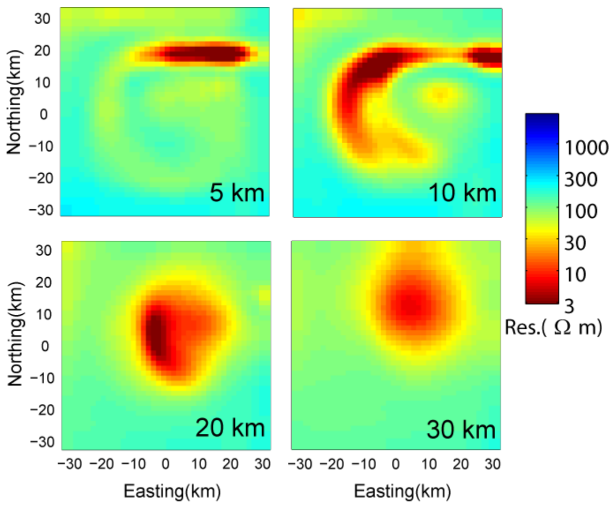

Figure 6. Inversion result with a "Bostick" starting model and an 100Ωm half space prior model

Similarly, despite the randomness of the starting models, all the ten "random" type tests result in a similar image (e.g. Figure 7) like the result from a half space starting model. The convergence of these models are almost always "bind" to a local minimum that is not far away from the prior model, regardless of the starting model. It turns out that the inversion search is still strongly influenced by the prior model, even with a separate starting model.

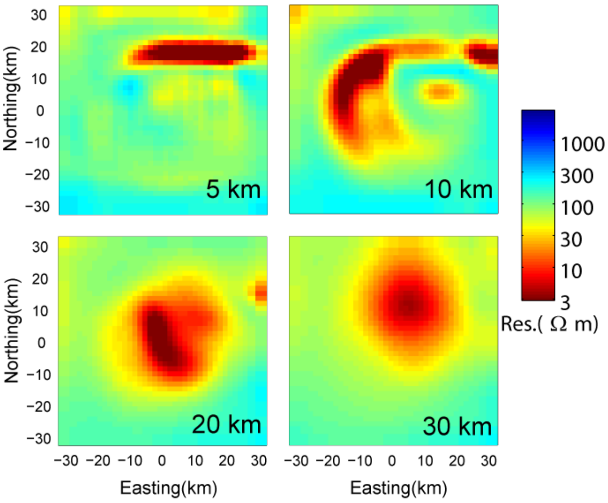

Figure 7. Inversion result from one of the "random" starting models with an 100Ωm half space prior model (rand0)

Discussions

As the tests suggests, the current model covariance mechanism (in ModEM) does a good job stabilizing the inversion with the prior settings. However, as the default object function penalizes any model that differs from the prior model, the model search cannot get too far away from the prior model. Hence users should be very careful when selecting their prior models. As shown in tests above, even for a very simple model of DSM 1, an inappropriate prior model will almost certainly bring confusing features to the final result.

Obviously, the influence of the prior model can be reduced by decreasing the damping factor λ of in (1). However, this will also otherwise result in a rougher model. Hence a more flexible model covariance mechanism is needed to separate the penalization from prior model and the model roughness. On the other hand, common feature in different "random" models could suggest the "robust" structures that should appear in the global optimal solution.

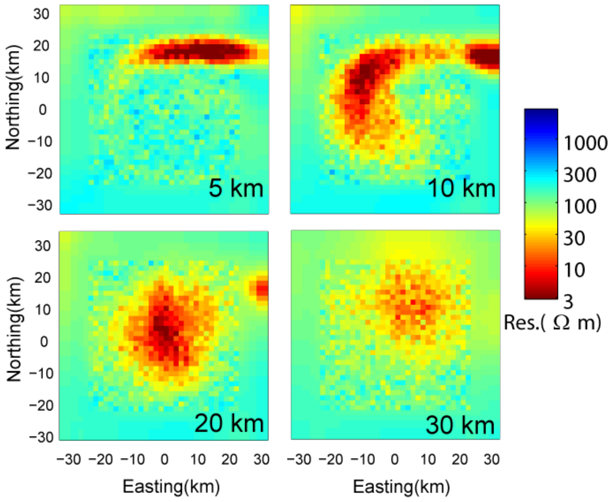

Figure 8. Inversion result from an "averaged" model of ten inversions with "random" starting and prior models.

Figure 8 shows an "average" model of ten random prior model test results. This may provide a way to interpret the data when a prior model cannot be easily determined. The randomness of the final model with respect to the initial model could also suggest the resolution of inversions.

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