International Journal of Modern Science and Technology
http://www.ijmst.co/
Vol. 1, No. 4, July 2016. Page 138-143.
ISSN: 2456-0235

Research Article

Numerical Solutions of Mathematical Model on Effects of Biological Control on Cereal Aphid Population Dynamics

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Abstract

Aphids can be of high risk in any crop if not contained. They can cause significant yield reduction by damage done to crops directly or indirectly either through direct feeding or diseases caused by virus transmission. It is also important to note that aphids have a complex lifecycle and dispersal patterns. Most researchers use already existing models and often extend them to incorporate extra parameters that represent specific features of the aphid that they would wish to analyze, without necessarily testing or determining the extent of effectiveness of the model. In this study, we have therefore gone an extra mile to construct two sets of mathematical models, by adjusting the function representing the prey-predator interaction then comparing them to determine which model gives a more accurate analysis of data.

Keywords: Mathematical model; Numerical solution; Population dynamics; Biological control; Aphids

Introduction

Cereals are the world's most important food crops. There are pest species that interfere with human activity or cause injury, loss or irritation to a crop, stored products, animals or people. Aphids have caused considerable damage to cereals [1-8]. This has stimulated research on the population dynamics of aphids and on the loss or damage they cause. Carter [9-10] discussed forecasting cereal aphid outbreaks. They modeled the effects of coccinellids, parasitoids and disease. In their model, they used steps of model initialization, data input, hourly temperatures, immigration, development and survival, reproduction and morph determination, predators, output, crop development model and input variable. They then used the models to study the interactions between state variables, which then quantified all the properties that describe the state of the system.

Sapoukhina [11] looked at a reaction-diffusion-advection model for the dynamics of populations under biological control. In their study, they assumed that the control agent was the predator species that had the ability to perceive the heterogeneity of pest distribution. The advection term represented the predator density movement according to a basic prey taxis assumption: acceleration of predators is proportional to the prey gradient. The prey population reproduced logistically, and the local population interactions followed the Holling Type II function [12]. Their spatially explicit model subdivided the predation process into random movement represented by diffusion, directed movement was described by prey taxis, local prey encounters, and consumption modeled by trophic function. The model enabled studying the effects of large-scale predator spatial activity on population dynamics.

Kindlmann [4] came up with a logistic model with variable carrying capacity and growth rate affected by cumulative density to study the population dynamics of aphids. Lopes [7] presented a flux-based model to describe an aphid-parasitoidal system in a closed structured environment. They applied this approach to the *Aphis gossypii* and to one of its parasitoids, *Lysiphlebu testaceipes* in a melon green house. They developed a model showing host-parasitoidal interactions. The model represented the level of plant infestation as a continuous

Received: 06.08.2016; Received after Revision: 10.08.2016; Accepted: 10.07.2016; Published: 13.08.2016

variable corresponding to the number of plants bearing a given density of pests at a given time. They used partial differential equations to describe the variation of this variable, which was coupled to an ordinary differential equation and a delay-differential equation that described the parasitized host population and the parasitoid population, respectively.

Bampfylde and Lewis in 2007 [12] presented a management alternative for the control of pest species through intraguild predation for the spatially homogeneous system. They extended the model to include movement of predator and prey in the spatial context. They considered a spatially homogeneous system and found the conditions for predator and prey to exclude each other, to coexist and for alternative stable states. Mohr [8] presented a general framework for age-structured predator-prey systems where individuals were divided into two classes, juveniles and adults, and several possible interactions considered. They used the Rosenzweig-MacArthur prey-predator model which they extended to include delay. They then reduced the initial system of partial differential equations to a system of (neutral) delay differential equations with one or two delays [13-14].

In this paper, we have extended the mathematical background given by Rosenzweig-MacArthur prey-predator model using the work done by Kindlmann [5-6]. We first formulate two sets of Rosenzweig-MacArthur prey-predator model with one predator and the prey, and then solve them analytically and numerically. The second set of the model seeks to modify and thus give a more accurate analysis of data compared to the first set of the model.

Assumptions

i. The economic threshold level for adopting control measures has not been attained.
ii. The carrying capacity of cereal aphid varies.
iii. The cumulative density is the regulatory term that slows down the instantaneous rate of increase.

Justification of the study

Pests pose a challenge to crop farming. They cause reduction in yields or even no yields at all when catastrophic, which then results in reduction in food production and economic loss. Some control strategies put in place to curb pest menace may also affect negatively human health and the ecosystem. The model(s) we have developed in this research is used to project stable systems of control type by seeking control methods with such characteristics that supply stability to the system to ensure maximum aphid reduction. This will enable the stakeholders in the agricultural sector to maintain the density of the aphid population at equilibrium below the economic injury level.

Limitations of the study

To forecast pest aphid abundance and for appropriate decision making, it may be necessary to have accurate estimates of aphid abundance and population growth rates. This is best done by undertaking a field experiment incorporating the particular aspect of the system and the characteristics of the interacting species that we are studying that has a direct bearing on the results. However, this is a challenge since this study will use data collected from previous field experiments, because of time constraints. Therefore estimation of some parameter values in the system may be a challenge.

Results and discussions

The model formed from the interaction between the prey and predator is as follows.

Case I:

$$\left.\begin{aligned} \frac{dN}{dt} &= N(r-b)\left(1-\frac{N}{K}\right) - \frac{aNP}{D+ahN} \\ \frac{dP}{dt} &= \mu P - e\frac{aNP}{D+ahN} \end{aligned}\right\} \quad (1)$$

Case II:

$$\left.\begin{aligned} \frac{dN}{dt} &= N(r-b)\left(1-\frac{N}{K}\right) - \frac{aNP}{D+ahN+P} \\ \frac{dP}{dt} &= \mu P - e\frac{aNP}{D+ahN+P} \end{aligned}\right\} \quad (2)$$

Without loss of generality we simplify the models by taking, $ah = 1$. Hence the above equations (1) and (2) respectively in Case I and Case II respectively can be written as indicated in equations (3) and (4) below,

Case I:

$$\left.\begin{aligned} \frac{dN}{dt} &= N(r-b)\left(1-\frac{N}{K}\right) - \frac{aNP}{D+N} \\ \frac{dP}{dt} &= \mu P - e\frac{aNP}{D+N} \end{aligned}\right\} \quad (3)$$

Case II:

$$\left.\begin{aligned}\frac{dN}{dt} &= N(r-b)\left(1-\frac{N}{K}\right) - \frac{aNP}{D+N+P}\\ \frac{dP}{dt} &= \mu P - e\frac{aNP}{D+N+P}\end{aligned}\right\} \quad (4)$$

where N>0 and P>0, respectively. This implies that all the parameters in the model are positive. We then perform non-dimensionalization to reduce the number of parameters in the model in equation (4) and (5) by reducing$\bar{t}$, $\bar{N}$ and $\bar{P}$ into non-dimensional form using,

$$t = \frac{t}{r}, N = \bar{N}K, P = \bar{P}eK.$$

Then, further by setting the parameters $\bar{u} - \frac{aek}{r}$, $\bar{\mu} - \frac{\mu}{r}$, $\bar{K} - \frac{K}{e}$ then dropping the sign, we find that the equations (3) and (4) take the form in equations (5) and (6) respectively.

Case I:

$$\frac{dN}{dt} = N\left(1-\frac{b}{r}\right)(1-N) - \frac{aNP}{D+N} \quad 5(a)$$

$$\frac{dP}{dt} = \mu P - \frac{aNP}{D+N} \quad 5(b)$$

Case II:

$$\frac{dN}{dt} - N\left(1-\frac{b}{r}\right)(1-N) - \frac{aNP}{D+N+P} \quad 6(a)$$

$$\frac{dP}{dt} = \mu P - \frac{aN\,P}{D+N+P} \quad 6(b)$$

N(0)>0 and P(0)>0, respectively.

The table 1 below gives all the parameter values used in the numerical simulations.

Table 1. Parameter values used in simulations [13]

Parameter	Value
r	2
b	0
K	200
μ	0.3
a	20
D	10

Numerical simulation results: Case I Model

The simulation of Case I Model has been done to find out the dynamics of aphids. Figures 1(a) and 1(b) show sustained oscillations in aphid and ladybird dynamics respectively. These oscillations result from the interaction between the aphid and the ladybird. The results show that the aphid/ ladybird populations increase/ decrease until they reach their respective equilibrium levels depending on the initial conditions.

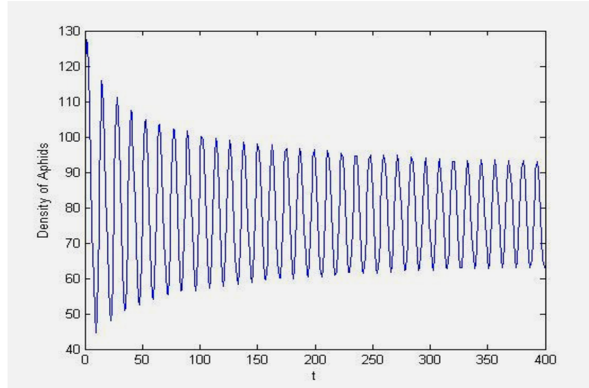

Figure1(a). Graph showing the density of prey N against time t

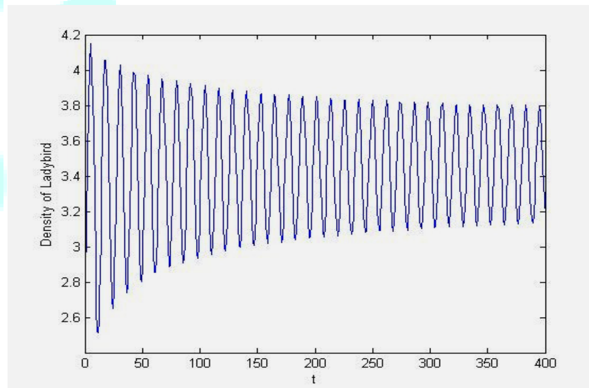

Figure 1(b). Graph showingthe density of predator P against time

Figures 2(a) and 2(b) shows interaction of predator with prey with time, which yields sustained oscillations converging to a stable equilibrium. From the two diagrams, we can clearly observe that the prey population increases when the number of predators is low, and prey population decreases when the predator's population increases. Also, the predator population decreases when there are no prey. The oscillations arise from the predation effect on the prey, whose aim is to suppress the prey. It can also be deduced that a higher prey population size than the predator will give a realistic ecological dynamics, whereas higher predator population size will give a rapid extinction of predator population.

Numerical simulation results: Case II Model

The simulation of Case II Model has been done to find out the dynamics of aphids. Figures 3(a) and 3(b) show sustained oscillations in aphid and ladybird dynamics respectively. These oscillations result from the interaction between the aphid and the ladybird. The results show that the aphid/ ladybird populations increase/ decrease until they reach their respective equilibrium levels depending on the initial conditions.

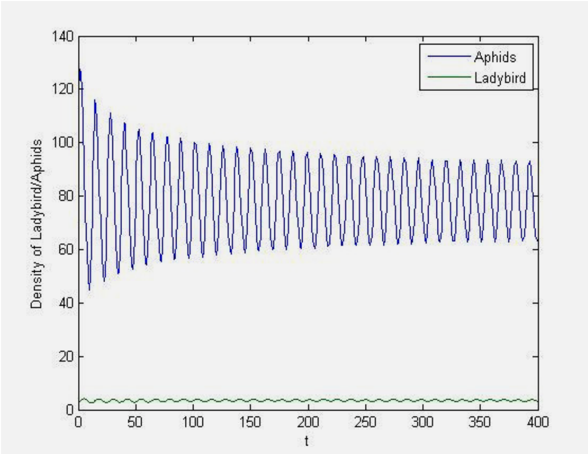

Figure 2(a). Graph showing prey N and predator P against time t

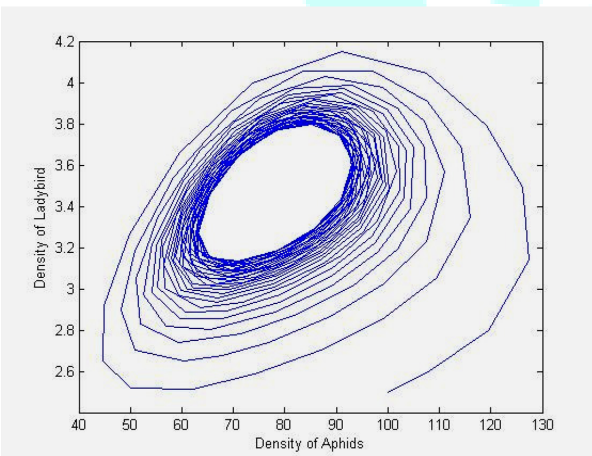

Figure 2(b). Graph showing phase portrait of prey N and predator P

Figures 4(a) and 4(b) shows interaction of predator with prey with time, which yields damped oscillations converging to a stable equilibrium. From the two diagrams, we can clearly observe that the prey population increases when the number of predators is low, and prey population decreases when the predator's population increases. Also, the predator population decreases when there are no prey. The oscillations arise from the predation effect on the prey, whose aim is to suppress the prey. It can also be deduced that a higher prey population size than the predator will give a realistic ecological dynamics, whereas higher predator population size will give a rapid extinction of predator population. From the simulation results given by both Cases, we observe that Figures 3(a) and 3(b) clearly depicts the oscillations in aphid's and ladybird' populations until they reach their respective equilibrium as compared to what we get from Figures 1(a) and 1(b).

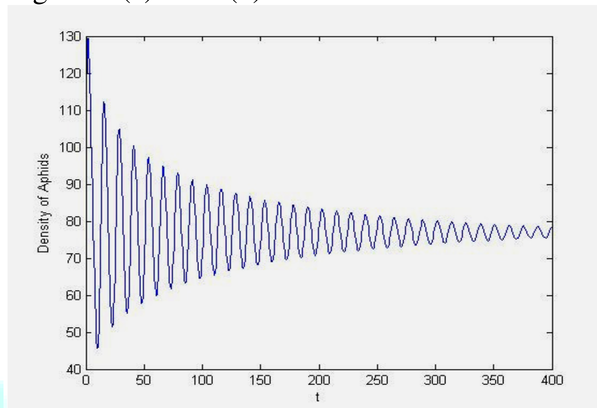

Figure 3(a): Graph showing the density of prey N against time t

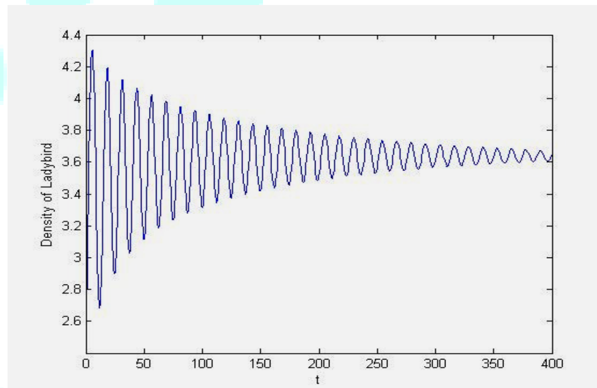

Figure 3(b). Graph showing the density of the predator P against time t

The equilibrium points for Figures 3(a) and 3(b) is clearly visible and can be located easily as compared to those of Figures 1(a) and 1(b). We also observe that the frequency of oscillations is higher in Figures 3(a) and 3(b) as it rapidly converges to their respective equilibrium points while it is lower in Figures 1(a) and 1(b). When we compare Figures 1 and 3, it is important to note that whereas we have used the same time limit and prey/ predator densities, the results from 1(a) and 1(b) shows that they need more time, than the 400 used, to get to the equilibrium point while those of 3 get to the equilibrium point within the 400 time duration. When we compare Figure 4(a) with 2(a), we arrive at the same differences already pointed out above. Figure 4(b) clearly depicts the phase portrait for the interaction between the aphid and the ladybird. We note that the equilibrium densities for the interacting species can easily be determined in 4(b) than in Figure 2(b). This is because the centre formed by the limit cycle is very small in 4(b) whereas it is very big in 2(b). From the small centre, it is easy to determine the equilibrium point than in a big centre where the degree of error may be higher.

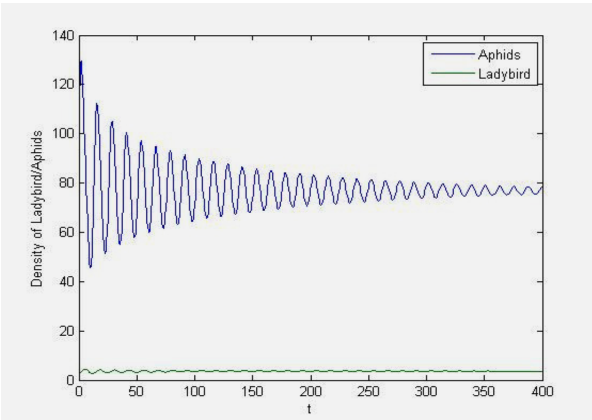

Figure 4(a). Graph showing a) prey N and predator P against time t

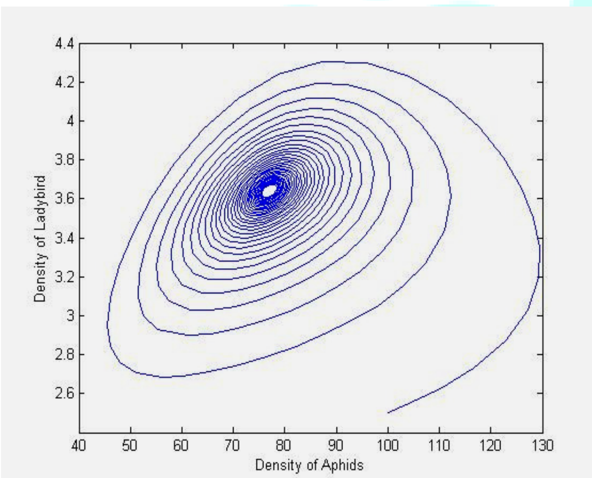

Figure 4(b). Graph showing phase portrait of prey N and predator P

In summary, numerical simulations of Case II Model presents relatively better and more accurate approximations, compared to those of Case I Model. This implies that Case II Model can give better predictions for the interaction between the prey and the predator. This is because we have introduced (added) an extra parameter, *P*, in the denominator of the functional response term $\left(\frac{aNP}{D+N+P}\right)$ for Case II as compared to $\left(\frac{aNP}{D+N}\right)$ for Case I. The denominator of the functional response term in both cases, form the carrying capacity, which is the limiting factor that regulates the prey-predator interactions. The introduction of an extra parameter P in the carrying capacity of Case II serves to increase the scope of our analysis because it enables us to look at more factors that affect the prey-predator interaction. This model also exhibits more complicated dynamics of the prey-predator dynamics and can be used to analyze a variety of parameters relevant to the system, hence can be used to give more accurate predictions of the system.

Conclusions

Aphids are important pests which cannot be ignored in cereal crop production, in the agricultural sector [15,16]. The damage they cause to these crops as well as loss of yields can be extensive if not contained. However, to contain these pests, it is important to understand its dynamics in relation to its interaction with its natural enemies like the ladybird. In mathematics, the best tool that can be used to understand this prey-predator dynamics is the models which have different variables and parameters that represent the various aspects of the dynamics of the prey-predator system that we are interested in. One major observation in this research is that when we modified the first model, we came up with more accurate results that improved our predictions of the prey-predator interaction, hence help in better decision making. This is because the denominator of the respective functional response terms $\left(\frac{aNP}{D+N}\right)and\left(\frac{aNP}{D+N+P}\right)$, forms the carrying capacity *K* per leaf [17-19]. The carrying capacity *K*, acts as the limiting factor that regulates the prey-predator interactions. In Case I, *K* = *D*+ *N*, which implies that the carrying capacity *K* is a sum of half saturation constant *D* and the prey population density *N*. In Case II, *K* = *D*+ *N*+*P*, which implies that the carrying capacity *K* is the sum total of half saturation constant *D*, the prey population density *N* and the predator population density *P*. The *P* in the carrying capacity *K* of Case II increases the number of parameters that influence the prey-predator interactions thus widens the scope of our analysis as compared to Case I. Therefore it is clearly demonstrated that increasing the number of parameters in the carrying capacity increases the number of factors to be considered and in turn improves the accuracy and the prediction ability of the model as indicated by Case II model. In reference to the objectives of our study, we have been able to achieve the following: development of a mathematical and give a modified version of the initial model and determine the extent of effectiveness of the modified model by comparing it to the initial one.

Conflict of Interest

Authors declare there are no conflicts of interest.

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