

**Elmwood Press**  
**Core Mathematics C4**  
**Paper I**  
**(Mark Scheme)**

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## Worked Solutions

### Edexcel C4 Paper I

1. (a)  $\frac{dy}{d\theta} = \frac{1}{(1 + \cos \theta)}(-\sin \theta), \frac{dx}{d\theta} = 2 \cos 2\theta$

$$\frac{dy}{dx} = \frac{-\sin \theta}{(1 + \cos \theta)2 \cos 2\theta}$$

where  $\theta = \frac{\pi}{6}$ , gradient =  $\frac{-\frac{1}{2}}{\left(1 + \frac{\sqrt{3}}{2}\right) \cdot 2 \cdot \frac{1}{2}} = -\frac{1}{2\left(1 + \frac{\sqrt{3}}{2}\right)}$

$$= -\frac{1}{2 + \sqrt{3}} = -\frac{(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} = \sqrt{3} - 2 \quad (5)$$

(b) gradient = 0 where  $\sin \theta = 0$

i.e. where  $\theta = 0$

at  $\theta = 0, x = 0, y = \ln 2$

gradient is zero at  $(0, \ln 2)$  (3)

2.  $\int A \, dA = \int e^{\frac{1}{10}t} dt$

$$\frac{A^2}{2} = 10e^{\frac{1}{10}t} + c$$

$A = 20, t = 0: \frac{400}{2} = 10 + c, c = 190$

$$\therefore \frac{A^2}{2} = 10e^{\frac{1}{10}t} + 190$$

when  $t = 20, \frac{A^2}{2} = 10e^2 + 190$

$$A = 23 \text{ (2 sig. fig.)} \quad (7)$$

3. (a)  $\frac{1}{y} \frac{dy}{dx} + 3x^2 - 2 = 0$

$$\frac{dy}{dx} = y(2 - 3x^2) \quad (3)$$

(b) (i)  $e^x \frac{dy}{dx} + ye^x + 2y \frac{dy}{dx} = 0$

$$\frac{dy}{dx} (e^x + 2y) = -ye^x$$

$$\frac{dy}{dx} = \frac{-ye^x}{e^x + 2y}$$

at  $(0, 3) \frac{dy}{dx} = \frac{-3}{1+6} = -\frac{3}{7}$  (3)

(ii) equation of tangent at  $(0, 3)$  is  $y - 3 = -\frac{3}{7}x$

$$3x + 7y = 21 \quad (2)$$

4. (a) (i)  $\cos 2x = 1 - 2 \sin^2 x$  (1)

hence  $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

(ii)  $\int \sin^2 x = \frac{1}{2} \int (1 - \cos 2x) dx = \frac{1}{2}x - \frac{1}{4} \sin 2x + c$  (2)

(b) Integrating by parts,

$$\int_0^{\frac{\pi}{8}} x \frac{d}{dx} \left( -\frac{1}{2} \cos 2x \right) dx = \left[ -\frac{x}{2} \cos 2x \right]_0^{\frac{\pi}{8}} + \int_0^{\frac{\pi}{8}} \frac{1}{2} \cos 2x \, dx$$

$$= \left[ -\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x \right]_0^{\frac{\pi}{8}}$$

$$= -\frac{\pi}{16} \cdot \frac{1}{\sqrt{2}} + \frac{1}{4} \cdot \frac{1}{\sqrt{2}} - (0 + 0) = \frac{4 - \pi}{16\sqrt{2}} \quad (5)$$

5. (a)  $\vec{OM} = \mathbf{i} + 2\mathbf{j}$ ,  $\vec{ON} = \mathbf{i} + 2\mathbf{k}$  (2)

(b) line  $OM$ :  $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

line  $AB$ :  $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

at intersection of  $OM$  and  $AB$ ,  $0 + \lambda = 2 + 0 \Rightarrow \lambda = 2$

$$0 + 2\lambda = 0 + \mu \Rightarrow \mu = 4$$

point of intersection is  $\begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$ .

(c)  $\vec{MN} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$ ,  $|\vec{MN}| = \sqrt{8}$ ,  $\vec{MO} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$ ,  $|\vec{MO}| = \sqrt{5}$

$$\vec{MN} \cdot \vec{MO} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} = 4$$

$\therefore 4 = \sqrt{8}\sqrt{5} \cos \theta$ , where  $\theta =$  angle required

$$\theta = 50.8^\circ$$

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6. (a)  $\int_1^2 \left(2x + \frac{1}{x}\right) dx = \left[x^2 + \ln x\right]_1^2 = 4 + \ln 2 - (1 + \ln 1) = 3 + \ln 2$  (3)

(b) (i)  $\int x e^x dx = \int x \frac{d}{dx}(e^x) dx = x e^x - \int e^x dx = x e^x - e^x + c$  (3)

(ii) volume =  $\pi \int_0^1 x e^x dx = \pi \left[ e^x(x-1) \right]_0^1 = \pi [e \times 0 - 1(-1)] = \pi$  (4)

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7. (a)  $f(x) = \frac{1}{x+3} + \frac{3}{x-1}$  (using 'cover up' rule) (3)

(b)  $\frac{1}{3+x} = \frac{1}{3\left(1 + \frac{x}{3}\right)}$   
 $= \frac{1}{3} \left(1 + \frac{x}{3}\right)^{-1}$

$$f(x) = \frac{1}{3} \left(1 + \frac{x}{3}\right)^{-1} - 3(1-x)^{-1} \quad \text{[note change of sign]}$$

$$= \frac{1}{3} \left[ 1 + (-1)\frac{x}{3} + \frac{(-1)(-2)}{2} \frac{x^2}{9} + \dots \right]$$

$$- 3 \left[ 1 + (-1)(-x) + \frac{(-1)(-2)}{2} (-x)^2 + \dots \right]$$

$$= \frac{1}{3} - \frac{x}{9} + \frac{x^2}{27} - 3 - 3x - 3x^2$$

$$= -\frac{8}{3} - \frac{28}{9}x - \frac{80}{27}x^2 \quad (4)$$

(c) valid for  $|x| < 1$  (i.e.  $-1 < x < 1$ ) (1)

(d)  $f(x) = (x+3)^{-1} + 3(x-1)^{-1}$

$$f'(x) = -(x+3)^{-2} - 3(x-1)^{-2}$$

$$= -\frac{1}{(x+3)^2} - \frac{3}{(x-1)^2}$$

$f'(x) < 0$  as both  $(x+3)^2$  and  $(x-1)^2$  are always positive. (3)

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$$8. (a) \frac{dy}{dx} = \frac{(1+x^2) \cdot 1 - x \cdot 2x}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2}$$

$$\frac{dy}{dx} = 0 \text{ at } 1-x^2 = 0$$

$$\text{i.e. } x = 1, -1$$

$$\text{when } x = 1, y = \frac{1}{2}$$

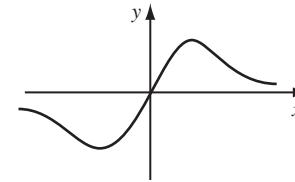
$$x = -1, y = -\frac{1}{2}$$

$$(b) \frac{d^2y}{dx^2} = \frac{(1+x^2)^2(-2x) - (1-x^2)2(1+x^2) \cdot 2x}{(1+x^2)^4}$$

$$\text{when } x = 1, \frac{d^2y}{dx^2} = \frac{-8-0}{2^4}, \text{ which is } < 0 \quad \therefore \text{max. value}$$

$$x = -1, \frac{d^2y}{dx^2} = \frac{8-0}{2^4}, \text{ which is } > 0 \quad \therefore \text{min. value} \quad (3)$$

(c)



(3)

$$(d) \text{ area} = \int_0^2 \frac{x}{1+x^2} dx = \left[ \frac{1}{2} \ln(1+x^2) \right]_0^2 = \frac{1}{2} \ln 5 \quad (3)$$