Math 2471-Sample Test 1

1. Find the unit tangent and unit normal vector for the following vector functions

$$(i) \quad \vec{r}(t) = < 2t, t^2 >$$
$$(ii) \quad \vec{r}(t) = < e^t \cos t, e^t \sin t >$$

2. Prove the limits either exist or do not exist. In the former case use the squeeze theorem.

$$(i) \quad \lim_{(x,y)->(0,0)} \frac{x^2 + xy + y^2}{x^2 + y^2} \qquad (ii) \quad \lim_{(x,y)->(0,0)} \frac{y - x^3}{y + x^3}$$
$$(iii) \quad \lim_{(x,y)->(0,0)} \frac{x^3 + y^3}{x^2 + y^2} \qquad (iv) \quad \lim_{(x,y)->(0,0)} \frac{x^4 + 2y^4}{x^2 + y^2}$$

3. Find the equation of the tangent plane to the given surface at the specified point

$$x^2y + xz + yz^2 = 3, \quad P(1, 2, -1)$$

4. If $z = x^2 - y^2$, calculate the following chain rules:

$$(i) \ \frac{dz}{dt} \ \text{ if } \ x = \cos t, \ \text{ and } \ y = \sin t$$
$$(ii) \ \frac{\partial z}{\partial r} \ \text{ and } \ \frac{\partial z}{\partial s} \ \text{ if } \ x = \frac{\cos s}{r} \ \text{ and } \ y = \frac{\sin s}{r}$$

5. Find the directional derivative of $z = x^2 + 3xy + y^2$ at $(1, 1)$ in the direction of $< -3, 4 >$. In what direction should you move for maximum increase?

6. Sketch and name the following surfaces

$$(i) \quad -x^2 + y^2 + z^2 = 1 \qquad (ii) \quad x^2 - y^2 - z^2 = 1$$
$$(iii) \quad x^2 - y + z^2 = 0 \qquad (iv) \quad -y^2 + z = 0$$