OFDM Peak-to-Average Power Reduction Scheme with Spectral Masking

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***Abstract*— Clipping and filtering can be a simple and effective method of reducing the peak-to-average power ratio (PAPR) for an orthogonal frequency division multiplexed (OFDM) signal. To be effective the clipping must be performed on an oversampled version of the time domain signal, but this results in out-of-band (OOB) power. It has previously been shown that the OOB signals can be filtered out using a discrete Fourier transform (DFT) based filter. However filtering causes peak regrowth, and an increase in PAPR. In this paper it is shown that if the OOB power is limited rather than completely eliminated much less peak regrowth occurs. Simulation results are presented for HIPERLAN2.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) technology is used in many broadband communications systems due to its robustness against frequency selective fading, impulse noise, high bandwidth efficiency and relatively simple receiver implementation.

One of the main disadvantages of OFDM is its high peak-to-average power ratio (PAPR). OFDM transmitters therefore require very linear output amplifiers with wide dynamic range. These are inefficient, expensive and in applications such as wireless local area networks (WLAN) can cause excessive drain on the batteries of portable equipment. Any amplifier nonlinearity causes intermodulation products resulting in unwanted out-of-band (OOB) power. Although PAPR is very large for OFDM, high magnitude peaks occur relatively rarely and most of the transmitted power is concentrated in signals of low amplitude.

The simplest approach to reducing the PAPR in OFDM signals is to clip the high amplitude signals. A variety of clipping techniques have been described in the literature [1], [2]. Some clip the outputs of the inverse discrete Fourier transform (IDFT) before interpolation. However the signal must be interpolated before analogue to digital conversion, and this will cause peak regrowth [3].

To avoid the problem of peak regrowth, the signal can be clipped after interpolation. However this causes significant OOB power. Some papers have described clipping of the interpolated signal followed by filtering [1], [4], [5], [6]. Two different filtering techniques are described in these papers. In [1] a conventional time invariant linear filter is used, while [4] describes a DFT based filter. This filter passes all of the in-band components and removes the OOB components by nulling the associated discrete frequencies. This approach also has the advantage of causing no intersymbol interference (ISI).

Both forms of filtering result in some peak regrowth. If necessary, this peak regrowth can be reduced by repeating the clipping and filtering operation [4].

Clipping also results in some in-band distortion, but it has been shown that in practice the effect of this on overall bit error rate (BER) is negligible [7]. This is because the main effect of clipping is to shrink the overall signal constellation rather than to add clipping noise. Moreover, the clipping noise is added at the transmitter, rather than the receiver and is subject to frequency selective channel fading along with the wanted component signals.

In [6] the performance of these different filtering techniques are compared. These results suggest that the time-invariant linear filter results in less peak regrowth and lower PAPR than two other techniques including the DFT based filter. The simulation scheme in [6] makes use of a lattice wave-digital filter (LWDF). However this scheme results in greater OOB power than the DFT based filter. In this paper, we show that the improvement in the PAPR with the LWDF in [6] is mainly due to the difference in the amount of the OOB power that is allowed at the output of the filter. We present simulation results for the DFT based filter if some OOB power is allowed at the output of the filter. The amount of OOB power is set according to the spectral mask specified in the HIPERLAN2 standard. Simulation results show that for the same OOB power the DFT based filter causes less peak regrowth than the linear filter considered in [6].

This paper is organized as follows. In Section II we describe the OOB power in OFDM systems and PAPR reduction technique with spectral masking is introduced in Section III. We present simulation results on PAPR reduction and the BER performance of the system in Section IV and finally Section V contains some concluding remarks.

II. Out-of-band power in OFDM systems

The spectrum of the OFDM signal has some OOB power. The form of the OOB spectrum depends very strongly on the detailed design of the OFDM transmitter. However, the OOB

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power is mainly due to two factors: the sidelobes of the spectra of the in-band subcarriers and intermodulation products due to nonlinearities in the transmitter output. In the case of an OFDM system with no windowing of each output symbol, then each subcarrier has a $\sin(x)/x$ spectrum. This results in relatively high sidelobes. The nonlinearities are due to the non-ideal amplifier characteristics, or due to deliberate clipping of the signal to eliminate signal peaks. It has been shown that the limiting factor becomes the amplifier nonlinearity when clipping is used [8].

Fig. 1 shows the spectrum of a HIPERLAN2 transmission without windowing and nonlinearities. The power has been averaged across a 1 MHz bandwidth as specified by the HIPERLAN2 standard. A simple way to reduce the level of these sidelobes is to apply some windowing to each symbol before transmission [8]. Filtering of the signal can also reduce the OOB power. If the signal is clipped (without filtering) the out-of-band power increases. Fig. 2 shows the spectrum when the signal is clipped at CR = 2dB and no filtering is applied. Clipping ratio is defined as the ratio of the clipping level to the root-mean-square power of the unclipped baseband signal,

$$\mathrm{CR} = 20\log_{10}\left(\frac{A}{\sigma}\right)\ \mathrm{dB}. \tag{1}$$

III. PAPR and DFT based filtering with out-of-band masking

A. PAPR

The PAPR of a continuous-time OFDM signal cannot be computed accurately by sampling the signal at Nyquist rate. Hence oversampling is essential to produce accurate PAPR estimates. The discrete time domain OFDM signal oversampled by a factor of I_1 can be expressed as,

$$x(n) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} a(k)\exp\left(\frac{j2\pi nk}{NI_1}\right) \tag{2}$$

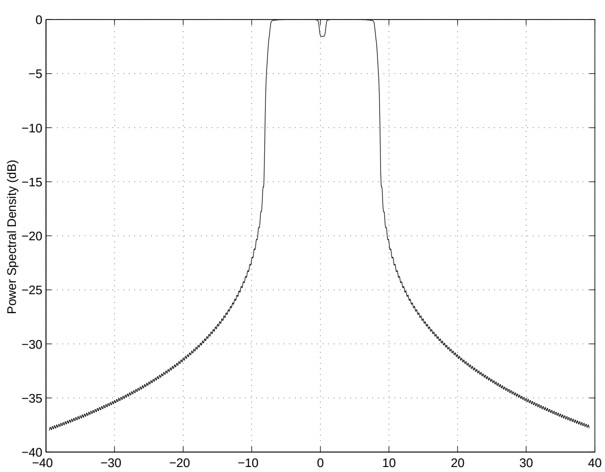

Fig. 1. Power spectral density of the HIPERLAN2 OFDM signal.

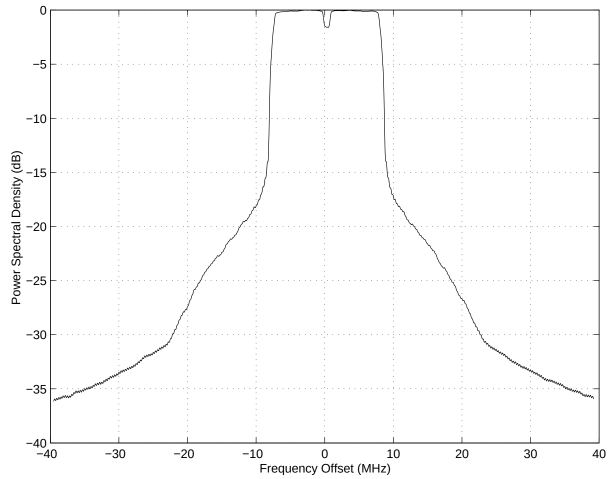

Fig. 2. Power spectra of the clipped HIPERLAN2 OFDM signal. CR = 2.0 dB.

for $n = 0, 1, \ldots, NI_1 - 1$. The constructive superposition of complex subcarriers can contribute large peaks for the time domain OFDM signal resulting in a high PAPR. When there are enough subcarriers for the central limit theorem to apply, the real and imaginary components of the OFDM signal have Gaussian distributions. Thus the amplitude has a Rayleigh distribution. Clipping and filtering changes the probability distribution of the amplitude. In this paper we measure the effectiveness of the PAPR reduction techniques in terms of the cumulative distribution of the amplitude, as this is directly related to the OOB power which will result from any amplifier nonlinearities.

B. New Technique with Spectral Masking

Fig. 3 shows the block diagram of the new DFT based PAPR reduction scheme with spectral masking. Vector $\mathbf{A}_i = a_0, \ldots, a_{N-1}$, which represents the data in each symbol i is converted from frequency to time domain using an oversize IDFT. N is the number of subcarriers in each OFDM symbol. For an oversampling factor of I_1, the input vector is extended by adding $N(I_1 - 1)$ zeros in the middle of the vector. That is,

$$\tilde{\mathbf{A}}_i = [a_{0,i}, a_{1,i}, \underbrace{0, \ldots, 0}_{N(I_1-1)\text{ zeros}}, \ldots, a_{N-1,i}] \tag{3}$$

This results in trigonometric interpolation of the OFDM time domain signal [9]. The interpolated signal is then clipped. In this paper we use "amplitude clipping" and the nonlinearity acts on the envelope of the complex baseband signal.

Amplitude clipping :

$$f(r) = \begin{cases} x_n & |x_n| \le A \\ Ae^{j\arg\{x_n\}} & |x_n| \ge A \end{cases} \tag{4}$$

Clipping is followed by frequency domain filtering to limit the OOB power. The filter consists of a DFT followed by an IDFT operation. The forward DFT transforms

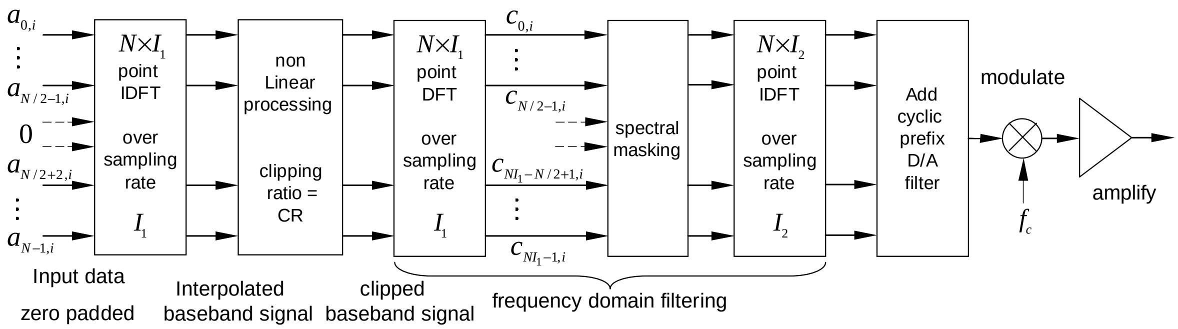

Fig. 3. Block diagram of the spectral masking PAPR reduced OFDM system.

the clipped signal back to the frequency domain. The in-band discrete frequency components of the clipped signal $c_0, \ldots, c_{N/2-1}, c_{NI_1-N/2+1}, \ldots, c_{NI_1-1}$ are passed unchanged while OOB spectral components are modified according to the spectral mask specifications given by the HIPERLAN2 standard [10]. This is different from the DFT filter described in [4] where all of the OOB components are nulled rather than limited.

The HIPERLAN2 standard [10] specifies the spectral mask which OFDM transmissions must meet. Fig. 4 shows this mask. Each OFDM symbol is composed of 64 subcarriers with a nominal bandwidth of 20 MHz. Of the 64 subcarriers, 47 are data carriers, 4 are pilots and 13 (the 0-th and some band-edge subcarriers) are not used and are set to zero. The band-edge subcarriers are not used as this reduces the analogue filtering requirements. The information bearing subcarriers have an effective bandwidth of 18 MHz while OOB power decays below 40 dBc at a frequency offset of 30 MHz away from the carrier frequency.

IV. Performance of PAPR Reduction scheme with Spectral Masking

In all simulations 4-QAM modulation was used. Figs. 5 and 6 show the complementary cumulative distribution function (CCDF) of PAPR for CR values of 4.5 and 2.5 dB respectively. Clearly if OOB masking is applied, it can be observed that negligible peak regrowth occurs. Fig. 7 shows the CCDF of PAPR by CCDF of PAPR by applying a different spectral

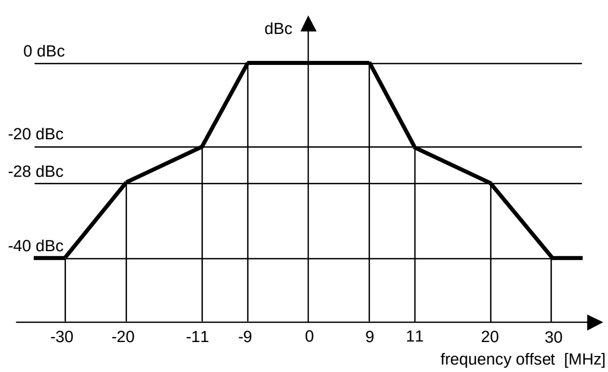

Fig. 4. Spectral power mask of OFDM based HIPERLAN2.

mask for CR = 4.5 dB. The applied mask is 3 dB below than that the mask of HIPERLAN2. In this case some peak growth is observed due to the distortion of the OOB components. However it is negligible compared to that of DFT based filter. Although the peak regrowth with spectral masking is negligible for the case of HIPERLAN2, this will not be true for applications such as digital video broadcasting where the OOB power requirements are much more stringent.

A. BER Performance

The BER performance of a clipped OFDM system depends on the in-band distortion. Hence the signal-to-clipping noise ratio (SCNR) which is defined as,

$$\text{SCNR}(k,i) = \frac{\alpha^2 E\left[|a(k,i)|^2\right]}{E\left[|N_d(k,i)|^2\right]} = \frac{\alpha^2\sigma^2}{E\left[|N_d(k,i)|^2\right]} \quad (5)$$

and depends on the subcarrier index k is a good measure of the BER performance. In (5) $N_d(k,i)$ is the distortion noise component due to clipping for the kth subcarrier and α is a constant. It can be calculated by extending the Bussgang's theorem to the complex case [4] and is given by,

$$\alpha = (1 - e^{-\frac{A^2}{\sigma^2}}) + \sqrt{\frac{\pi A^2}{4\sigma^2}}\,\text{erfc}\left(\frac{A}{\sigma}\right) \quad (6)$$

where $\text{erfc}(x) = 2/\sqrt{\pi}\int_x^\infty e^{-y^2}dy$.

With the DFT based filter and nulling there is a trade off between the minimum PAPR and the in-band distortion. Using masking instead of nulling results in a lower PAPR with no increase in the in-band distortion, and so allows a given PAPR target to be achieved with less in-band distortion. Fig. 8 shows the BER performance results in a two-path fading channel where signal power of the echo path is half of that of the direct path. The delay of the echo signal equals 10 OFDM sample periods and perfect channel state information was assumed at the receiver. Cyclic prefix length equals 16. $I_1 = I_2 = 4$. The BER performance of the two OFDM systems, with no clipping and clipped at CR = 6 dB are almost identical having error rates of well beyond 10^{-6} at $E_b/N_0 = 18$ dB. The overall effect of channel fading is to minimize the effect of transmitter-added noise on the error performance of the OFDM system [7].

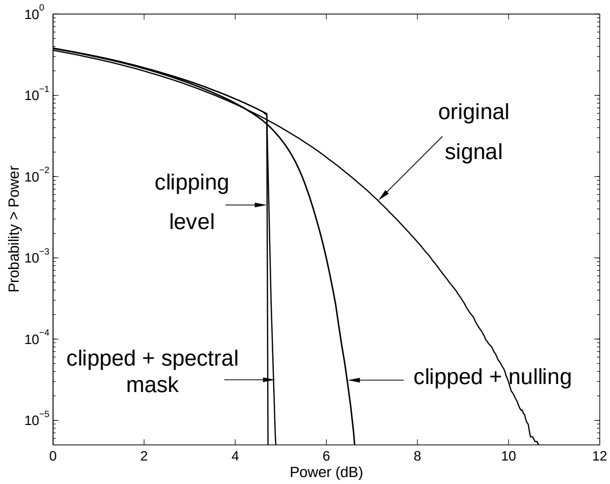

Fig. 5. CCDF for OFDM signals after clipping and spectral masking. CR = 4.5 dB.

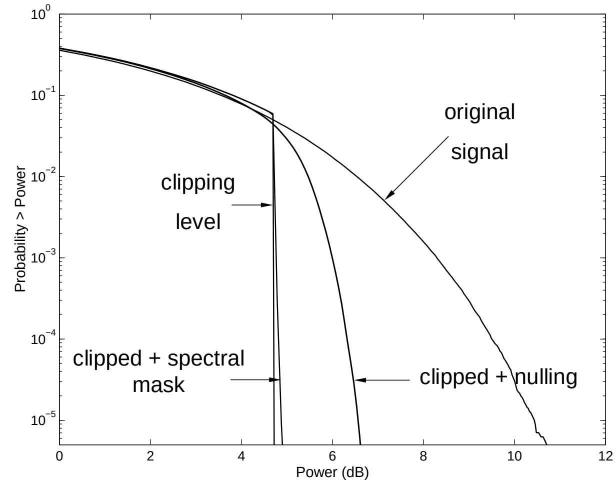

Fig. 7. CCDF for OFDM signals after clipping and spectral masking (3 dB below the HIPERLAN2 mask). CR = 4.5 dB.

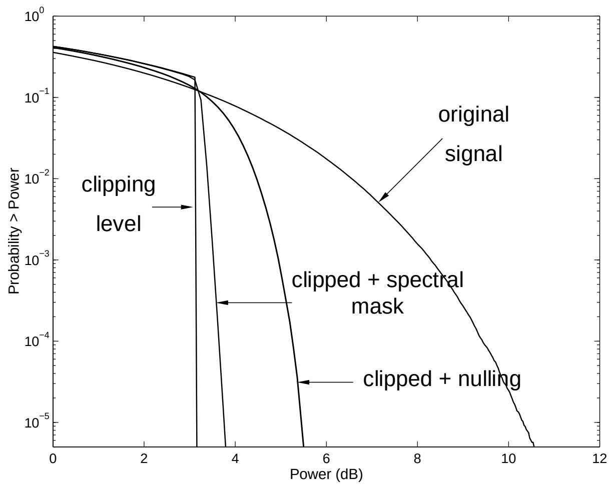

Fig. 6. CCDF for OFDM signals after clipping and spectral masking. CR = 2.5 dB.

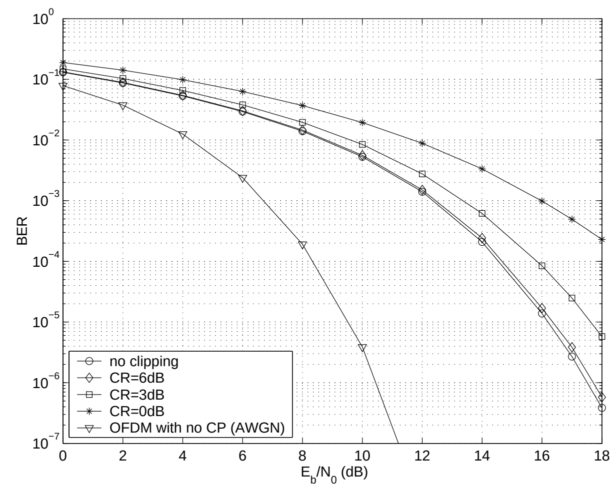

Fig. 8. BER versus E_b/N_0 in a multipath channel for varying CR values.

V. Conclusions

We have presented a detailed analysis of a DFT based filter with spectral masking to reduce the peak-to-average power ratio (PAPR) in OFDM signals. Spectral masking is used to limit the out-of-band (OOB) power and it is shown that the resulting peak-regrowth will be minimum. System degradation has been discussed in terms of in-band distortion and a bit error rate graph is presented in a frequency selective multipath channel. DFT based spectral masking filter introduces no in-band distortion and the effect of the clipping at the transmitter on the error performance of the OFDM system subject to multipath fading is minimal.

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