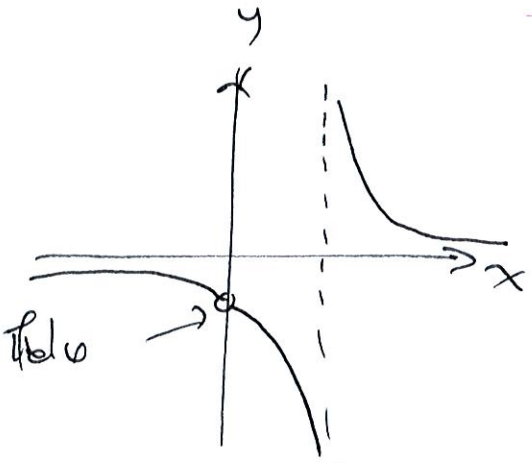
Math 1496 - Calc I

consider $f(x) = \dfrac{x}{x^2 - x}$

at $x = 0$ $\quad f(0) = \dfrac{0}{0}$ "$\frac{0}{0}$" so we need to do something else

$$\lim_{x \to 0} \frac{x}{x(x-1)} = \frac{1}{0-1} = -1$$ Analytically

Graphically



2 things going on here

(1) $\lim\limits_{x \to 1} \dfrac{x}{x^2 - x} = ?$

(2) $\lim\limits_{x \to \infty} \dfrac{x}{x^2 - x} = ?$

from graph

$\lim\limits_{x \to 1^-} \dfrac{x}{x^2 - x} = -\infty$

$\lim\limits_{x \to 1^+} \dfrac{x}{x^2 - x} = \infty$

infinite limits

$\lim\limits_{x \to -\infty} \dfrac{x}{x^2 - x} = 0$ $\quad \lim\limits_{x \to \infty} \dfrac{x}{x^2 - x} = 0$

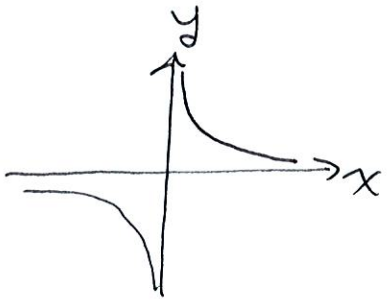
limits at infinity

Infinite Limits

if $\lim_{x \to a} f(x) = \pm\infty$

then $f(x)$ becomes unbounded as x approaches a and we say that $x = a$ is a vertical asymptote.

ex $f(x) = \frac{1}{x}$



$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

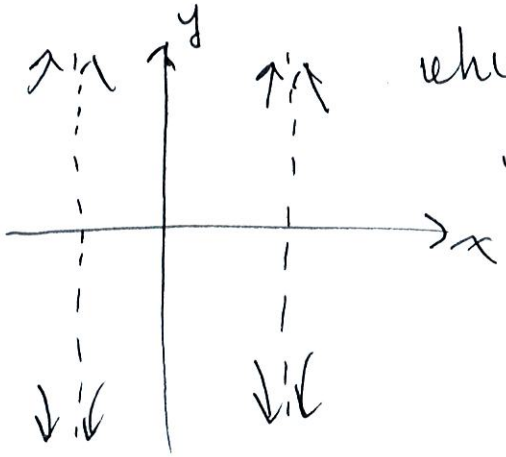
$\lim_{x \to 0^+} \frac{1}{x} = +\infty$

Note: if $x < 0$ $\frac{1}{x} < 0$

so as $x \to 0^-$ $\frac{1}{x} \to \infty$ from the negative side

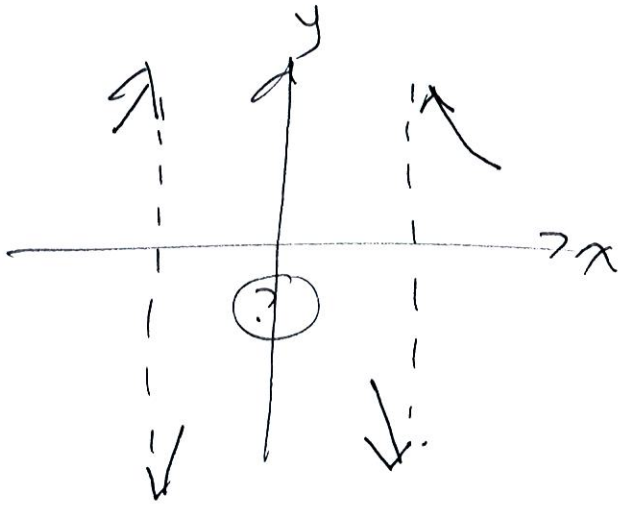
ex $f(x) = \frac{1}{x^2 - 1} = \frac{1}{(x-1)(x+1)}$

2 VA $x = \pm 1$



which - an easy way is using a sign chart

x		-1		1	
$x-1$	$-$	$-$	$-$	0	$+$
$x+1$	$-$	0	$+$	$+$	$+$
$(x-1)(x+1)$	$+$	0	$-$	0	$+$
$\frac{1}{(x-1)(x+1)}$	$+$	∞	$-$	∞	$+$



$$f(x) = \frac{-x}{x^2-4} = \frac{-x}{(x-2)(x+2)}$$

VA $x = \pm 2$

$y = 0$ when $x = 0$

x		-2		0		2	
$-x$	$+$	$+$	$+$	0	$-$	$-$	$-$
$x-2$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x+2$	$-$	0	$+$	$+$	$+$	$+$	$+$
$\frac{-x}{(x-2)(x+2)}$	$+$	∞	$-$	0	$+$	∞	$-$