# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise A, Question 1**

**Question:**

Write out each of the following as a sum of terms, and hence calculate the sum of the series.

**a** $\sum_{r=1}^{10} r$

**b** $\sum_{p=3}^{8} p^2$

**c** $\sum_{r=1}^{10} r^3$

**d** $\sum_{p=1}^{10} (2p^2 + 3)$

**e** $\sum_{r=0}^{5} (7r + 1)^2$

**f** $\sum_{i=1}^{4} 2i(3 - 4i^2)$

**Solution:**

**a** $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

**b** $3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 = 199$

**c** $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3 = 3025$

{notice that this result is the square of the result for (a)}

**d** $5 + 11 + 21 + 35 + 53 + 75 + 101 + 131 + 165 + 203 = 800$

**e** $1 + 64 + 225 + 484 + 841 + 1296 = 2911$

**f** $-2 - 52 - 198 - 488 = -740$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise A, Question 2**

**Question:**

Write each of the following as a sum of terms, showing the first three terms and the last term.

**a** $\sum_{r=1}^{n}(7r-1)$

**b** $\sum_{r=1}^{n}(2r^3+1)$

**c** $\sum_{j=1}^{n}(j-4)(j+4)$

**d** $\sum_{p=3}^{k}p(p+3)$

**Solution:**

**a** $6+13+20+\ldots+(7n-1)$

**b** $3+17+55+\ldots+(2n^3+1)$

**c** $-15-12-7+\ldots+(n-4)(n+4)$

**d** $18+28+40+\ldots+k(k+3)$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise A, Question 3**

### Question:

In each part of this question write out, as a sum of terms, the two series defined by $\sum f(r)$; for example, in part **c**, write out the series $\sum_{r=1}^{10} r^2$ and $\sum_{r=1}^{10} r$. Hence, state whether the given statements relating their sums are true or not.

**a** $\sum_{r=1}^{n} (3r+1) = \sum_{r=2}^{n+1} (3r-2)$

**b** $\sum_{r=1}^{n} 2r = \sum_{r=0}^{n} 2r$

**c** $\sum_{r=1}^{10} r^2 = \left(\sum_{1}^{10} r\right)^2$

**d** $\sum_{r=1}^{4} r^3 = \left(\sum_{r=1}^{4} r\right)^2$

**e** $\sum_{r=1}^{n} \left(3r^2+4\right) = 3\sum_{r=1}^{n} r^2 + 4$

### Solution:

**a** The two series are exactly the same, $4+7+10+\ldots+(3n+1)$, and so their sums are the same.

**b** The two series are exactly the same, $2+4+6+\ldots+2n$, and so their sums are the same.

**c** The statement is not true.

$\sum_{r=1}^{r=10} r^2 = 1^2+2^2+3^2+\ldots+10^2 = 385$ (using your calculator)

$\left(\sum_{r=1}^{10} r\right)^2 = (1+2+3+\ldots 10)^2 = 55^2 = 3025.$

[This one example is enough to prove $\sum_{r=1}^{n} r^2 = \left(\sum_{r=1}^{n} r\right)^2$ for all $n$ is not true]

**d** This statement is true.

$\sum_{r=1}^{4} r^3 \quad = \quad 1^3+2^3+3^3+4^3 = 100$

$\left(\sum_{r=1}^{4} r\right)^2 \quad = \quad (1+2+3+4)^2 = 10^2 = 100$

[This does not prove that $\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} r\right)^2$ for *all n*; but it is true and this will be proved in Chapter 6]

**e** The statement is not true.

$$\sum_{r=1}^{n}\left(3r^2+4\right) = \left\{3\times 1^2+4\right\}+\left\{3\times 2^2+4\right\}+\left\{3\times 3^2+4\right\}+\ldots+\left\{3n^2+4\right\}$$
$$= 3\{1^2+2^2+3^2+\ldots+n^2\}+4n$$
$$3\sum_{r=1}^{n} r^2+4 = 3\{1^2+2^2+3^2+\ldots+n^2\}+4$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise A, Question 4**

### Question:

Express these series using $\sum$ notation.

**a** $3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

**b** $1 + 8 + 27 + 64 + 125 + 216 + 243 + 512$

**c** $11 + 21 + 35 + \ldots + (2n^2 + 3)$

**d** $11 + 21 + 35 + \ldots + (2n^2 - 4n + 5)$

**e** $3 \times 5 + 5 \times 7 + 7 \times 9 + \ldots + (2r - 1)(2r + 1) + \ldots$ to $k$ terms.

### Solution:

Answers are not unique (two examples are given, and any letter may be used for $r$)

**a** $\sum_{r=3}^{10} r, \quad \sum_{r=1}^{8} (r+2)$

**b** $\sum_{r=1}^{8} r^3, \quad \sum_{r=2}^{9} (r-1)^3$

**c** $\sum_{r=2}^{n} (2r^2+3), \quad \sum_{r=3}^{n+1} (2r^2-4r+5)$

**d** $\sum_{r=3}^{n} (2r^2-4r+5), \quad \sum_{r=2}^{n-1} (2r^2+3).$

**e** $\sum_{r=2}^{k+1} (2r-1)(2r+1), \quad \sum_{r=1}^{k} (2r+1)(2r+3)$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise B, Question 1**

**Question:**

Use the result for $\sum_{r=1}^{n} r$ to calculate

**a** $\sum_{r=1}^{36} r$

**b** $\sum_{r=1}^{99} r$

**c** $\sum_{p=10}^{55} p$

**d** $\sum_{r=100}^{200} r$

**e** $\sum_{r=1}^{k} r + \sum_{r=k+1}^{80} r$, where $k < 80$.

**Solution:**

**a** $\frac{36 \times 37}{2} = 666$

**b** $\frac{99 \times 100}{2} = 4950$

**c** $\sum_{p=1}^{55} p - \sum_{p=1}^{9} p = \frac{55 \times 56}{2} - \frac{9 \times 10}{2} = 1540 - 45 = 1495$

**d** $\sum_{r=1}^{200} r - \sum_{r=1}^{99} r = \frac{200 \times 201}{2} - \frac{99 \times 100}{2} = 20100 - 4950 = 15150$

**e** $\sum_{r=1}^{k} r + \sum_{r=k+1}^{80} r = \sum_{r=1}^{80} r = \frac{80 \times 81}{2} = 3240$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise B, Question 2**

### Question:

Given that $\sum_{r=1}^{n} r = 528$,

**a** show that $n^2 + n - 1056 = 0$

**b** find the value of $n$.

### Solution:

**a** $\frac{n}{2}(n+1) = 528 \Rightarrow n(n+1) = 1056 \Rightarrow n^2 + n - 1056 = 0$

**b** Factorising: $(n-32)(n+33) = 0$ (or use "the formula") $\Rightarrow n = 32$, as $n$ cannot be negative.

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise B, Question 3**

### Question:

**a** Find $\sum_{k=1}^{2n-1} k$.

**b** Hence show that $\sum_{k=n+1}^{2n-1} k = \frac{3n}{2}(n-1),\ n \geq 2$.

### Solution:

**a** $\frac{(2n-1)\{(2n-1)+1\}}{2} = \frac{(2n-1)(2n)}{2} = n(2n-1)$

**b**

$$\sum_{k=1}^{2n-1} k - \sum_{k=1}^{n} k = n(2n-1) - \frac{n}{2}(n+1) = \frac{n}{2}\{2(2n-1)-(n+1)\} = \frac{n}{2}(3n-3)$$

$$= \frac{3n}{2}(n-1)$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise B, Question 4**

### Question:

Show that $\sum_{r=k-1}^{2k} r = \frac{(k+2)(3k-1)}{2}, k \geq 1$

### Solution:

$$\sum_{r=1}^{2k} r - \sum_{r=1}^{k-2} r = \frac{2k}{2}(2k+1) - \frac{(k-2)}{2}(k-1) = \frac{(4k^2+2k)-(k^2-3k+2)}{2}$$

$$= \frac{3k^2+5k-2}{2} = \frac{(3k-1)(k+2)}{2}$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise B, Question 5**

### Question:

**a** Show that $\sum_{r=1}^{n^2} r - \sum_{r=1}^{n} r = \frac{n(n^3-1)}{2}$.

**b** Hence evaluate $\sum_{r=10}^{81} r$.

### Solution:

**a** $\frac{n^2(n^2+1)}{2} - \frac{n(n+1)}{2} = \frac{n}{2}\left\{n(n^2+1)-(n+1)\right\} = \frac{n}{2}(n^3-1)$

**b** $\sum_{r=10}^{81} r = \sum_{r=1}^{9^2} r - \sum_{r=1}^{9} r = \frac{9}{2}(9^3-1)$ [using part (a)] = 3276.

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise C, Question 1**

**Question:**

(In this exercise use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} 1$.)

Calculate the sum of the series:

**a** $\sum_{r=1}^{55} (3r - 1)$

**b** $\sum_{r=1}^{90} (2 - 7r)$

**c** $\sum_{r=1}^{46} (9 + 2r)$

**Solution:**

**a** $3\sum_{r=1}^{55} r - \sum_{r=1}^{55} 1 = 3 \times \frac{55 \times 56}{2} - 55 = 4565$

**b** $2\sum_{r=1}^{90} 1 - 7\sum_{r=1}^{90} r = 2 \times 90 - 7 \times \frac{90 \times 91}{2} = -28485$

**c** $9\sum_{r=1}^{46} 1 + 2\sum_{r=1}^{46} r = 9 \times 46 + 2 \times \frac{46 \times 47}{2} = 2576$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise C, Question 2**

### Question:

Show that

**a** $\sum_{r=1}^{n}(3r+2)=\frac{n}{2}(3n+7)$

**b** $\sum_{i=1}^{2n}(5i-4)=n(10n-3)$

**c** $\sum_{r=1}^{n+2}(2r+3)=(n+2)(n+6)$

**d**

$\sum_{p=3}^{n}(4p+5)=(2n+11)(n-2)$

### Solution:

**a** $3\sum_{r=1}^{n}r+2\sum_{r=1}^{n}1=3\times\frac{n}{2}(n+1)+2n=\frac{n}{2}(3n+3+4)=\frac{n}{2}(3n+7)$

**b** $5\sum_{i=1}^{2n}i-4\sum_{i=1}^{2n}1=5\times\frac{2n}{2}(2n+1)-4(2n)=n(10n+5-8)=n(10n-3)$

**c** $2\sum_{r=1}^{n+2}r+3\sum_{r=1}^{n+2}1=2\times\frac{(n+2)}{2}(n+3)+3(n+2)=(n+2)(n+3+3)=(n+2)(n+6)$

**d**

$$\left\{4\sum_{p=1}^{n}p+5\sum_{p=1}^{n}1\right\}-\sum_{p=1}^{2}(4p+5)=\left\{4\times\frac{n}{2}(n+1)+5n\right\}-(9+13)$$

$$=2n^2+7n-22=(2n+11)(n-2)$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise C, Question 3**

### Question:

**a** Show that $\sum_{r=1}^{k}(4r-5)=2k^2-3k$.

**b** Find the smallest value of $k$ for which $\sum_{r=1}^{k}(4r-5)>4850$.

### Solution:

**a** $4\sum_{r=1}^{k}r-5\sum_{r=1}^{k}1=4\times\frac{k}{2}(k+1)-5k=2k^2-3k$

**b** $2k^2-3k>4850 \Rightarrow 2k^2-3k-4850>0 \Rightarrow (2k+97)(k-50)>0$,

so $k>50$ [$k$ is positive] $\Rightarrow k=51$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise C, Question 4**

**Question:**

Given that $u_r = ar + b$ and $\sum_{r=1}^{n} u_r = \frac{n}{2}(7n + 1)$, find the constants $a$ and $b$.

**Solution:**

$$\sum_{r=1}^{n}(ar + b) = \frac{an}{2}(n + 1) + bn = \frac{an^2 + (a + 2b)n}{2}$$

Comparing with $\frac{7n^2 + n}{2} \Rightarrow a = 7$ and $a + 2b = 1$

So $a = 7$, $b = -3$

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## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise C, Question 5**

### Question:

**a** Show that $\sum_{r=1}^{4n-1}(1+3r) = 24n^2 - 2n - 1 \; n \geq 1$.

**b** Hence calculate $\sum_{r=1}^{99}(1+3r)$.

### Solution:

**a** $\sum_{r=1}^{4n-1} 1 + 3\sum_{r=1}^{4n-1} r = (4n-1) + 3 \times \frac{(4n-1)(4n)}{2} = (4n-1)(1+6n) = 24n^2 - 2n - 1$

**b** Substituting $n = 25$ into above result gives 14949

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## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise C, Question 6**

**Question:**

Show that $\sum_{r=1}^{2k+1}(4-5r)=-(2k+1)(5k+1), k\geq 0$

**Solution:**

$$4\sum_{r=1}^{2k+1}1-5\sum_{r=1}^{2k+1}r=4(2k+1)-5\frac{(2k+1)}{2}(2k+2)=(2k+1)\{4-5(k+1)\}$$

$$=(2k+1)(-1-5k)=-(2k+1)(5k+1)$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise D, Question 1**

### Question:

Verify that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$ is true for $n = 1, 2$ and 3.

### Solution:

For $n = 1$, $\sum_{r=1}^{n} r^2 = 1^2 = 1$, $\frac{n}{6}(n+1)(2n+1) = \frac{1}{6}(1+1)(2+1) = 1$

For $n = 2$, $\sum_{r=1}^{n} r^2 = 1^2 + 2^2 = 5$, $\frac{n}{6}(n+1)(2n+1) = \frac{2}{6}(2+1)(4+1) = 5$

For $n = 3$, $\sum_{r=1}^{n} r^2 = 1^2 + 2^2 + 3^2 = 14$, $\frac{n}{6}(n+1)(2n+1) = \frac{3}{6}(3+1)(6+1) = 14$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise D, Question 2**

### Question:

**a** By writing out each series, evaluate $\sum_{r=1}^{n} r$ for $n = 1, 2, 3$ and 4.

**b** By writing out each series, evaluate $\sum_{r=1}^{n} r^3$ for $n = 1, 2, 3$ and 4.

**c** What do you notice about the corresponding results for each value of $n$ ?

### Solution:

**a** $\sum_{r=1}^{1} r = 1$;  $\sum_{r=1}^{2} r = 1 + 2 = 3$;  $\sum_{r=1}^{3} r = 1 + 2 + 3 = 6$;  $\sum_{r=1}^{4} r = 1 + 2 + 3 + 4 = 10$

**b** $\sum_{r=1}^{1} r^3 = 1$;  $\sum_{r=1}^{2} r^3 = 1^3 + 2^3 = 9$;  $\sum_{r=1}^{3} r^3 = 1^3 + 2^3 + 3^3 = 36$;  $\sum_{r=1}^{4} r^3 = 1^3 + 2^3 + 3^3 + 4^3 = 100$

**c** The results for (b) are the square of the results for (a)

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise D, Question 3**

### Question:

Using the appropriate formula, evaluate

**a** $\sum_{r=1}^{100} r^2$

**b** $\sum_{r=20}^{40} r^2$

**c** $\sum_{r=1}^{30} r^3$

**d** $\sum_{r=25}^{45} r^3$

### Solution:

**a** $\frac{100}{6} \times 101 \times 201 = 338350$

**b** $\sum_{r=1}^{40} r^2 - \sum_{r=1}^{19} r^2 = \frac{40}{6} \times 41 \times 81 - \frac{19}{6} \times 20 \times 39 = 22140 - 2470 = 19670$

**c** $\frac{30^2 \times 31^2}{4} = 216225$

**d** $\sum_{r=1}^{45} r^3 - \sum_{r=1}^{24} r^3 = \frac{45^2 \times 46^2}{4} - \frac{24^2 \times 25^2}{4} = 1071225 - 90000 = 981225$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise D, Question 4**

### Question:

Use the formula for $\sum_{r=1}^{n} r^2$ or $\sum_{r=1}^{n} r^3$ to find the sum of

**a** $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 52^2$

**b** $2^3 + 3^3 + 4^3 + \ldots + 40^3$

**c** $26^2 + 27^2 + 28^2 + 29^2 + \ldots + 100^2$

**d** $1^2 + 2^2 + 3^2 + \ldots + (k+1)^2$

**e** $1^3 + 2^3 + 3^3 + \ldots + (2n-1)^3$

### Solution:

**a** $\sum_{r=1}^{52} r^2 = \frac{52}{6} \times 53 \times 105 = 48230$

**b** $\sum_{r=1}^{40} r^3 - 1 = \frac{40^2 \times 41^2}{4} - 1 = 672399$

**c** $\sum_{r=1}^{100} r^2 - \sum_{r=1}^{25} r^2 = \frac{100}{6} \times 101 \times 201 - \frac{25}{6} \times 26 \times 51 = 338350 - 5525 = 332825$

**d** $\sum_{r=1}^{k+1} r^2 = \frac{(k+1)}{6}(k+2)(2k+3)$

**e** $\sum_{r=1}^{2n-1} r^3 = \frac{(2n-1)^2(2n)^2}{4} = n^2(2n-1)^2$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise D, Question 5**

### Question:

For each of the following series write down, in terms of $n$, the sum, giving the result in its simplest form

**a** $\sum_{r=1}^{2n} r^2$

**b** $\sum_{r=1}^{n^2-1} r^2$

**c** $\sum_{i=1}^{2n-1} i^2$

**d** $\sum_{r=1}^{n+1} r^3$

**e** $\sum_{k=n+1}^{3n} k^3, n > 0.$

### Solution:

**a** $\frac{(2n)}{6}(2n+1)(4n+1) = \frac{n}{3}(2n+1)(4n+1)$

**b** $\frac{(n^2-1)n^2(2n^2-1)}{6}$

**c** $\frac{(2n-1)}{6}(2n)[2(2n-1)+1] = \frac{(2n-1)}{6}(2n)(4n-1) = \frac{n}{3}(2n-1)(4n-1)$

**d** $\frac{(n+1)^2(n+2)^2}{4}$

**e**

$$\sum_{r=1}^{3n} k^3 - \sum_{r=1}^{n} k^3 = \frac{(3n)^2(3n+1)^2}{4} - \frac{n^2(n+1)^2}{4} = \frac{n^2}{4}\{9(3n+1)^2 - (n+1)^2\}$$

$$= \frac{n^2}{4}\{3(3n+1)-(n+1)\}\{3(3n+1)+(n+1)\}[\text{using } a^2-b^2=(a-b)(a+b)]$$

$$= \frac{n^2}{4}\{(8n+2)(10n+4)\}$$

$$= n^2(4n+1)(5n+2)$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise D, Question 6**

### Question:

Show that

**a** $\sum_{r=2}^{n} r^2 = \frac{1}{6}(n-1)(2n^2+5n+6)$

**b** $\sum_{r=n}^{2n} r^2 = \frac{n}{6}(n+1)(14n+1)$

### Solution:

**a** $\frac{n}{6}(n+1)(2n+1) - 1 = \frac{2n^3+3n^2+n-6}{6} = \frac{(n-1)(2n^2+5n+6)}{6}$ [use factor theorem]

**b**

$$\sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2 = \frac{2n}{6}(2n+1)(4n+1) - \frac{(n-1)}{6}n(2n-1)$$

$$= \frac{n}{6}\{2(2n+1)(4n+1) - (n-1)(2n-1)\}$$

$$\frac{n}{6}\{(16n^2+12n+2) - (2n^2-3n+1)\} = \frac{n}{6}(14n^2+15n+1)$$

$$= \frac{n}{6}(14n+1)(n+1)$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise D, Question 7**

### Question:

**a** Show that $\sum_{k=n}^{2n} k^3 = \frac{3n^2(n+1)(5n+1)}{4}$

**b** Find $\sum_{k=30}^{60} k^3$.

### Solution:

**a**

$$\sum_{k=1}^{2n} k^3 - \sum_{k=1}^{n-1} k^3 = \frac{(2n)^2(2n+1)^2}{4} - \frac{(n-1)^2 n^2}{4}$$

$$= \frac{n^2}{4}\{4(2n+1)^2 - (n-1)^2\}$$

$$= \frac{n^2}{4}[\{2(2n+1)+(n-1)\}\{2(2n+1)-(n-1)\}$$ "Difference of two squares"

$$= \frac{n^2}{4}(5n+1)(3n+3) = \frac{3n^2}{4}(5n+1)(n+1)$$

**b** Substituting $n = 30$ into (a) gives 3 159 675

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise D, Question 8**

### Question:

**a** Show that $\sum_{r=1}^{2n} r^3 = n^2(2n+1)^2$.

**b** By writing out the series for $\sum_{r=1}^{n}(2r)^3$, show that $\sum_{r=1}^{n}(2r)^3 = 8\sum_{r=1}^{n} r^3$.

**c** Show that $1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3$ can be written as $\sum_{r=1}^{2n} r^3 - \sum_{r=1}^{n}(2r)^3$.

**d** Hence show that the sum of the cubes of the first $n$ odd natural numbers, $1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3$, is $n^2(2n^2-1)$.

### Solution:

**a** $\sum_{r=1}^{2n} r^3 = \frac{(2n)^2(2n+1)^2}{4} = n^2(2n+1)^2$.

**b** $\sum_{r=1}^{n}(2r)^3 = 2^3 + 4^3 + 6^3 + \ldots + (2n)^3 = 2^3\{1^3 + 2^3 + 3^3 + \ldots + n^3\} = 8\sum_{r=1}^{n} r^3$.

**c**

$$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = \{1^3 + 2^3 + 3^3 + \ldots + (2n-1)^3 + (2n)^3\} - \{2^3 + 4^3 + 6^3 + \ldots + (2n)^3\}$$
$$= \sum_{r=1}^{2n} r^3 - \sum_{r=1}^{n}(2r)^3.$$

**d** Using the results in parts (b) and (c), $1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = \sum_{r=1}^{2n} r^3 - 8\sum_{r=1}^{n} r^3$

$$= n^2(2n+1)^2 - 8\sum_{r=1}^{n} r^3 \text{ (using(a))}$$
$$= n^2(2n+1)^2 - \frac{8n^2(n+1)^2}{4}$$
$$= n^2[(2n+1)^2 - 2(n+1)^2]$$
$$= n^2[(4n^2+4n+1) - 2(n^2+2n+1)]$$
$$= n^2(2n^2-1)$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise E, Question 1**

**Question:**

Use the formulae for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$, $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} 1$, where appropriate, to find

**a** $\sum_{m=1}^{30} (m^2 - 1)$

**b** $\sum_{r=1}^{40} r(r+4)$

**c** $\sum_{r=1}^{80} r(r^2+3)$

**d** $\sum_{r=11}^{35} (r^3 - 2)$.

**Solution:**

**a** $\sum_{m=1}^{30} m^2 - 30 = \frac{30 \times 31 \times 61}{6} - 30 = 9425$

**b** $\sum_{r=1}^{40} r^2 + 4 \sum_{r=1}^{40} r = \frac{40 \times 41 \times 81}{6} + 4 \times \frac{40 \times 41}{2} = 22140 + 3280 = 25420$

**c** $\sum_{r=1}^{80} r^3 + 3 \sum_{r=1}^{80} r = \frac{80^2 \times 81^2}{4} + 3 \times \frac{80 \times 81}{2} = 10497600 + 9720 = 10507\,320$

**d** $\sum_{r=1}^{35} (r^3 - 2) - \sum_{r=1}^{10} (r^3 - 2) = \sum_{r=1}^{35} r^3 - 2(35) - \left[ \sum_{r=1}^{10} r^3 - 2(10) \right]$

$\sum_{r=1}^{35} r^3 - \sum_{r=1}^{10} r^3 - 2(35-10) = \frac{35^2 \times 36^2}{4} - \frac{10^2 \times 11^2}{4} - 50 = 396900 - 3025 - 50 = 393825.$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise E, Question 2**

**Question:**

Use the formulae for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$, and $\sum_{r=1}^{n} r$, where appropriate, to find

**a** $\sum_{r=1}^{n} (r^2 + 4r)$

**b** $\sum_{r=1}^{n} r(2r^2 - 1)$

**c** $\sum_{r=1}^{2n} r^2(1 + r)$, giving your answer in its simplest form.

**Solution:**

**a** $\sum_{r=1}^{n} r^2 + 4\sum_{r=1}^{n} r = \frac{n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} = \frac{n(n+1)\{(2n+1)+12\}}{6} = \frac{n}{6}(n+1)(2n+13)$

**b** $2\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r = \frac{2n^2(n+1)^2}{4} - \frac{n(n+1)}{2} = \frac{n(n+1)\{n(n+1)-1\}}{2} = \frac{n}{2}(n+1)(n^2+n-1)$

**c**

$$\sum_{r=1}^{2n} r^2 + \sum_{r=1}^{2n} r^3 = \frac{2n(2n+1)(4n+1)}{6} + \frac{(2n)^2(2n+1)^2}{4} = \frac{n(2n+1)\{(4n+1)+3n(2n+1)\}}{3}$$

$$= \frac{n}{3}(2n+1)(6n^2+7n+1) = \frac{n}{3}(n+1)(2n+1)(6n+1)$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise E, Question 3**

### Question:

**a** Write out $\sum_{r=1}^{n} r(r+1)$ as a sum, showing at least the first three terms and the final term.

**b** Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to calculate

$1\times 2+2\times 3+3\times 4+4\times 5+5\times 6+\ldots+60\times 61$.

### Solution:

**a** $1\times 2+2\times 3+3\times 4+\ldots+n(n+1)$

**b** Putting $n = 60$: $\sum_{r=1}^{60} r^2+\sum_{r=1}^{60} r=\frac{60\times 61\times 121}{6}+\frac{60\times 61}{2}=73810+1830=75640$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise E, Question 4**

### Question:

**a** Show that $\sum_{r=1}^{n}(r+2)(r+5)=\frac{n}{3}(n^2+12n+41)$.

**b** Hence calculate $\sum_{r=10}^{50}(r+2)(r+5)$.

### Solution:

**a**

$$\begin{aligned}\sum_{r=1}^{n}(r^2+7r+10) &= \sum_{r=1}^{n}r^2+7\sum_{r=1}^{n}r+10\sum_{r=1}^{n}1\\ &= \frac{n}{6}(n+1)(2n+1)+7\frac{n}{2}(n+1)+10n\\ &= \frac{n}{6}\{(2n^2+3n+1)+21(n+1)+60\}\\ &= \frac{n}{6}(2n^2+24n+82)=\frac{n}{3}(n^2+12n+41)\end{aligned}$$

**b** Substituting $n = 50$ and $n = 9$ in the formula in (a), and subtracting, gives 51 660.

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise E, Question 5**

### Question:

**a** Show that $\sum_{r=2}^{n}(r-1)r(r+1)=\frac{(n-1)n(n+1)(n+2)}{4}$.

**b** Hence find the sum of the series $13\times14\times15+14\times15\times16+15\times16\times17+\ldots+44\times45\times46$.

### Solution:

**a**

$$\sum_{r=2}^{n}(r^3-r)=\sum_{r=1}^{n}(r^3-r)=\sum_{r=1}^{n}r^3-\sum_{r=1}^{n}r=\frac{n^2(n+1)^2}{4}-\frac{n}{2}(n+1)$$
$$=\frac{n(n+1)}{4}(n^2+n-2)$$
$$=\frac{n}{4}(n+1)\{n^2+n-2\}$$
$$=\frac{n}{4}(n+1)(n+2)(n-1)=\frac{(n-1)n(n+1)(n+2)}{4}$$

**b** $\sum_{r=14}^{45}(r-1)r(r+1)=\sum_{r=2}^{45}(r-1)r(r+1)-\sum_{r=2}^{13}(r-1)r(r+1)=\frac{44\times45\times46\times47}{4}-\frac{12\times13\times14\times15}{4}$

= 1 062 000

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise E, Question 6**

### Question:

Find the following sums, and check your results for the cases $n = 1, 2$ and 3.

**a** $\sum_{r=1}^{n}(r^3 - 1)$

**b** $\sum_{r=1}^{n}(2r-1)^2$

**c** $\sum_{r=1}^{n} r(r+1)^2$

### Solution:

**a** $\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} 1 = \frac{n^2(n+1)^2}{4} - n = \frac{n}{4}\{n(n+1)^2 - 4\} = \frac{n}{4}(n^3 + 2n^2 + n - 4)$

When $n = 1$ : $\sum_{r=1}^{1}(r^3 - 1) = 0;$ $\frac{n}{4}(n^3 + 2n^2 + n - 4) = \frac{1 \times 0}{4} = 0$

When $n = 2$ : $\sum_{r=1}^{2}(r^3 - 1) = 0 + 7 = 7;$ $\frac{n}{4}(n^3 + 2n^2 + n - 4) = \frac{2 \times 14}{4} = 7$

When $n = 3$ : $\sum_{r=1}^{3}(r^3 - 1) = 0 + 7 + 26 = 33;$ $\frac{n}{4}(n^3 + 2n^2 + n - 4) = \frac{3 \times 44}{4} = 33$

**b**

$$\sum_{r=1}^{n}(4r^2 - 4r + 1) = 4\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1 = \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$$
$$= \frac{n}{3}\left\{2(2n^2 + 3n + 1) - 6(n+1) + 3\right\} = \frac{n}{3}(4n^2 - 1)$$

When $n = 1$ : $\sum_{r=1}^{1}(4r^2 - 4r + 1) = 1;$ $\frac{n}{3}(4n^2 - 1) = \frac{1 \times 3}{3} = 1$

When $n = 2$ : $\sum_{r=1}^{2}(4r^2 - 4r + 1) = 1 + 9 = 10;$ $\frac{n}{3}(4n^2 - 1) = \frac{2 \times 15}{3} = 10$

When $n = 3$ : $\sum_{r=1}^{3}(4r^2 - 4r + 1) = 1 + 9 + 25 = 35;$ $\frac{n}{3}(4n^2 - 1) = \frac{3 \times 35}{3} = 35$

**c**

$$\sum_{r=1}^{n}(r^3+2r^2+r) = \sum_{r=1}^{n} r^3+2\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} r = \frac{n^2(n+1)^2}{4}+\frac{2n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{12}\{3n(n+1)+4(2n+1)+6\} = \frac{n(n+1)}{12}\{3n^2+11n+10\}$$

$$= \frac{n}{12}(n+1)(n+2)(3n+5)$$

When $n = 1$ : $\sum_{r=1}^{1} r(r+1)^2 = 1\times 4 = 4;$ $\frac{n}{12}(n+1)(n+2)(3n+5) = \frac{1\times 2\times 3\times 8}{12} = 4$

When $n = 2$ : $\sum_{r=1}^{2} r(r+1)^2 = 4+2\times 9 = 22;$ $\frac{n}{12}(n+1)(n+2)(3n+5) = \frac{2\times 3\times 4\times 11}{12} = 22$

When $n = 3$ : $\sum_{r=1}^{3} r(r+1)^2 = 22+3\times 16 = 70;$ $\frac{n}{12}(n+1)(n+2)(3n+5) = \frac{3\times 4\times 5\times 14}{12} = 70$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise E, Question 7**

### Question:

**a** Show that $\sum_{r=1}^{n} r^2(r-1) = \frac{n}{12}(n^2-1)(3n+2)$.

**b** Deduce the sum of $1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + \ldots + 30 \times 31^2$.

### Solution:

**a**

$$\begin{aligned}\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r^2 &= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6} \\ &= \frac{n(n+1)}{12}\{3n(n+1) - 2(2n+1)\} \\ &= \frac{n(n+1)}{12}(3n^2 - n - 2) \\ &= \frac{n(n+1)(n-1)(3n+2)}{12} = \frac{n(n^2-1)(3n+2)}{12}\end{aligned}$$

**b** As $\sum_{r=2}^{31} r^2(r-1) = \sum_{r=1}^{31} r^2(r-1)$, substitute $n = 31$ in (a); sum = 235 600

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise E, Question 8**

### Question:

**a** Show that $\sum_{r=2}^{n}(r-1)(r+1)=\frac{n}{6}(2n+5)(n-1)$.

**b** Hence sum the series $1\times3+2\times4+3\times5+\ldots+35\times37$.

### Solution:

**a** [ $\sum_{r=2}^{n}(r^2-1)=\sum_{r=1}^{n}(r^2-1)$ as when $r=1$ the term is zero]

$$\sum_{r=1}^{n}(r^2-1)=\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}1=\frac{n}{6}(n+1)(2n+1)-n$$
$$=\frac{n}{6}\{(2n^2+3n+1)-6\}$$
$$=\frac{n}{6}(2n^2+3n-5)$$
$$=\frac{n}{6}(2n+5)(n-1)$$

**b** $1\times3+2\times4+3\times5+\ldots+35\times37=\sum_{r=1}^{36}(r-1)(r+1)$

Substituting $n = 36$ into result in (a) gives 16 170

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise E, Question 9**

### Question:

**a** Write out the series defined by $\sum_{r=7}^{12} r(2+3r)$, and hence find its sum.

**b** Show that $\sum_{r=n+1}^{2n} r(2+3r) = \frac{n}{2}(14n^2+15n+3)$.

**c** By substituting the appropriate value of $n$ into the formula in **b**, check that your answer for **a** is correct.

### Solution:

**a** $7\times 23+8\times 26+9\times 29+10\times 32+11\times 35+12\times 38 = 1791$.

**b** $\sum_{r=n+1}^{2n} (2r+3r^2) = \sum_{r=1}^{2n}(2r+3r^2) - \sum_{r=1}^{n}(2r+3r^2)$

$$\sum_{r=1}^{n}(2r+3r^2) = 2\sum_{r=1}^{n} r + 3\sum_{r=1}^{n} r^2 = n(n+1)+\frac{n}{2}(n+1)(2n+1)$$
$$= \frac{n}{2}(n+1)\{2+(2n+1)\}$$
$$= \frac{n}{2}(n+1)(2n+3)$$

$$\Rightarrow \quad \sum_{r=1}^{2n}(2r+3r^2) = n(2n+1)(4n+3)$$

$$\sum_{r=n+1}^{2n}(2r+3r^2) = n(2n+1)(4n+3)-\frac{n}{2}(n+1)(2n+3)$$
$$= \frac{n}{2}\{2(2n+1)(4n+3)-(n+1)(2n+3)\}$$
$$= \frac{n}{2}\{(16n^2+20n+6)-(2n^2+5n+3)\}$$
$$= \frac{n}{2}(14n^2+15n+3)$$

**c** Substituting $n = 6$ gives 1791

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise E, Question 10**

### Question:

Find the sum of the series $1 \times 1 + 2 \times 3 + 3 \times 5 + \ldots$ to $n$ terms.

### Solution:

Series can be written as $\sum_{r=1}^{n} r(2r-1)$

$$\sum_{r=1}^{n} r(2r-1) = 2\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r = 2 \times \frac{n}{6}(n+1)(2n+1) - \frac{n}{2}(n+1)$$

$$= \frac{n(n+1)\{2(2n+1)-3\}}{6}$$

$$= \frac{n(n+1)(4n-1)}{6}$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 1**

### Question:

**a** Write down the first three terms and the last term of the series given by $\sum_{r=1}^{n}(2r+3^r)$.

**b** Find the sum of this series.

**c** Verify that your result in **b** is correct for the cases $n = 1, 2$ and 3.

### Solution:

**a** $(2+3)+(4+3^2)+(6+3^3)+\ldots+(2n+3^n)$ $[=5+13+33+\ldots+(2n+3^n)]$

**b** $\sum_{r=1}^{n}(2r+3^r)=2\sum_{r=1}^{n}r+\sum_{r=1}^{n}3^r=n(n+1)+\frac{3}{2}(3^n-1)$ [AP + GP]

**c**

$n=1$: (b) gives $2+3=5$, agrees with (a)

$n=2$: (b) gives $6+12=18$, agrees with (a)

$n=3$: (b) gives $12+39=51$, agrees with (a)

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 2**

**Question:**

Find

**a** $\sum_{r=1}^{50}(7r+5)$

**b** $\sum_{r=1}^{40}(2r^2-1)$

**c** $\sum_{r=33}^{75} r^3$.

**Solution:**

**a** $7\sum_{r=1}^{50} r+5\sum_{r=1}^{50} 1=\frac{7\times 50\times 51}{2}+5(50)=9175$

**b** $2\sum_{r=1}^{40} r^2-\sum_{r=1}^{40} 1=\frac{40(41)(81)}{3}-40=44\ 240$

**c** $\sum_{r=1}^{75} r^3-\sum_{r=1}^{32} r^3=\frac{75^2\times 76^2}{4}-\frac{32^2\times 33^2}{4}=7\ 843\ 716$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 3**

**Question:**

Given that $\sum_{r=1}^{n} U_r = n^2 + 4n$,

**a** find $\sum_{r=1}^{n-1} U_r$.

**b** Deduce an expression for $U_n$.

**c** Find $\sum_{r=n}^{2n} U_r$.

**Solution:**

**a** Replacing $n$ with $(n-1)$ gives $(n-1)^2 + 4(n-1) = n^2 + 2n - 3$

**b** $U_n = \sum_{r=1}^{n} U_r - \sum_{r=1}^{n-1} U_r = n^2 + 4n - (n^2 + 2n - 3) = 2n + 3$

**c** $\sum_{r=1}^{2n} U_r - \sum_{r=1}^{n-1} U_r = (4n^2 + 8n) - (n^2 + 2n - 3) = 3n^2 + 6n + 3 = 3(n+1)^2$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 4**

**Question:**

Evaluate $\sum_{r=1}^{30} r(3r-1)$

**Solution:**

$$3\sum_{r=1}^{30} r^2 - \sum_{r=1}^{30} r = \frac{3\times 30\times 31\times 61}{6} - \frac{30\times 31}{2} = 28365 - 465 = 27900$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 5**

### Question:

Find $\sum_{r=1}^{n} r^2(r-3)$.

### Solution:

$$\sum_{r=1}^{n} r^3 - 3\sum_{r=1}^{n} r^2 = \frac{n^2}{4}(n+1)^2 - \frac{n}{2}(n+1)(2n+1)$$
$$= \frac{n}{4}(n+1)\{n(n+1) - 2(2n+1)\}$$
$$= \frac{n}{4}(n+1)(n^2 - 3n - 2)$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 6**

### Question:

Show that $\sum_{r=1}^{2n}(2r-1)^2 = \frac{2n}{3}(16n^2-1)$.

### Solution:

$$\begin{aligned}
4\sum_{r=1}^{2n} r^2 - 4\sum_{r=1}^{2n} r + \sum_{r=1}^{2n} 1 &= \frac{4}{3}n(2n+1)(4n+1) - 4n(2n+1) + 2n \\
&= \frac{n}{3}\{4(2n+1)(4n+1) - 12(2n+1) + 6\} \\
&= \frac{n}{3}\{32n^2 + 24n + 4 - 12(2n+1) + 6\} \\
&= \frac{n}{3}(32n^2 - 2) \\
&= \frac{2n}{3}(16n^2 - 1)
\end{aligned}$$

# Solutionbank FP1

## Edexcel AS and A Level Modular Mathematics

### Series

**Exercise F, Question 7**

### Question:

**a** Show that $\sum_{r=1}^{n} r(r+2) = \frac{n}{6}(n+1)(2n+7)$.

**b** Using this result, or otherwise, find in terms of *n*, the sum of $3\log 2 + 4\log 2^2 + 5\log 2^3 + \ldots + (n+2)\log 2^n$.

### Solution:

**a**

$$\sum_{r=1}^{n} r^2 + 2\sum_{r=1}^{n} r = \frac{n}{6}(n+1)(2n+1) + 2\frac{n}{2}(n+1)$$
$$= \frac{n}{6}(n+1)\{(2n+1)+6\}$$
$$= \frac{n}{6}(n+1)(2n+7)$$

**b**

The series is : $\sum_{r=1}^{n}(r+2)\log 2^r = \sum_{r=1}^{n} r(r+2)\log 2$ as $\log 2^r = r \log 2$

$= \log 2 \sum_{r=1}^{n} r(r+2)$ as $\log 2$ is a constant

$= \frac{n}{6}(n+1)(2n+7)\log 2$

# Solutionbank FP1

## Edexcel AS and A Level Modular Mathematics

### Series

**Exercise F, Question 8**

### Question:

Show that $\sum_{r=n}^{2n} r^2 = \frac{n}{6}(n+1)(an+b)$, where $a$ and $b$ are constants to be found.

### Solution:

$$\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2 = \frac{(2n)(2n+1)(4n+1)}{6} - \frac{(n-1)n(2n-1)}{6}$$

$$= \frac{n}{6}\{2(8n^2+6n+1)-(2n^2-3n+1)\}$$

$$= \frac{n}{6}(14n^2+15n+1)$$

$$= \frac{n}{6}(n+1)(14n+1) \qquad a = 14, \quad b = 1$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 9**

### Question:

**a** Show that $\sum_{r=1}^{n}(r^2 - r - 1) = \frac{n}{3}(n-2)(n+2)$.

**b** Hence calculate $\sum_{r=10}^{40}(r^2 - r - 1)$.

### Solution:

**a**

$$\begin{aligned}\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1 &= \frac{n}{6}(n+1)(2n+1) - \frac{n}{2}(n+1) - n \\ &= \frac{n}{6}\{(n+1)(2n+1) - 3(n+1) - 6\} \\ &= \frac{n}{6}(2n^2 - 8) \\ &= \frac{n}{3}(n^2 - 4) \\ &= \frac{n}{3}(n-2)(n+2)\end{aligned}$$

**b** $\sum_{r=10}^{40}(r^2 - r - 1) = \sum_{r=1}^{40}(r^2 - r - 1) - \sum_{r=1}^{9}(r^2 - r - 1)$

Substitute $n = 40$ and $n = 9$ into the result for part (a), and subtract.

The result is 21280 – 230 = 21049

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 10**

**Question:**

**a** Show that $\sum_{r=1}^{n} r(2r^2+1) = \frac{n}{2}(n+1)(n^2+n+1)$.

**b** Hence calculate $\sum_{r=26}^{58} r(2r^2+1)$.

**Solution:**

**a**

$$2\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r = \frac{n^2(n+1)^2}{2} + \frac{n}{2}(n+1)$$
$$= \frac{n}{2}(n+1)\{n(n+1)+1\}$$
$$= \frac{n}{2}(n+1)(n^2+n+1)$$

**b** Substitute $n = 58$ and $n = 25$ into the result for (a), and subtract. The result = 5654178.

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 11**

### Question:

Find

**a** $\sum_{r=1}^{n} r(3r-1)$

**b** $\sum_{r=1}^{n} (r+2)(3r+5)$

**c** $\sum_{r=1}^{n} (2r^3-2r+1)$.

### Solution:

**a** $3\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r = \frac{n(n+1)(2n+1)}{2} - \frac{n(n+1)}{2} = \frac{n(n+1)}{2}(2n+1-1) = n^2(n+1)$

**b**

$$3\sum_{r=1}^{n} r^2 + 11\sum_{r=1}^{n} r + 10\sum_{r=1}^{n} 1 = \frac{n(n+1)(2n+1)}{2} + \frac{11n(n+1)}{2} + 10n$$
$$= \frac{n}{2}\{(2n^2+3n+1)+11(n+1)+20\}$$
$$= \frac{n}{2}(2n^2+14n+32) = n(n^2+7n+16)$$

**c**

$$2\sum_{r=1}^{n} r^3 - 2\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1 = \frac{n^2(n+1)^2}{2} - n(n+1) + n$$
$$= \frac{n}{2}\{n(n+1)^2 - 2(n+1) + 2]$$
$$= \frac{n}{2}\{n(n+1)^2 - 2n\} = \frac{n^2}{2}(n^2+2n-1)$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 12**

### Question:

**a** Show that $\sum_{r=1}^{n} r(r+1) = \frac{n}{3}(n+1)(n+2)$.

**b** Hence calculate $\sum_{r=31}^{60} r(r+1)$.

### Solution:

**a**

$$\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r = \frac{n}{6}(n+1)(2n+1) + \frac{n}{2}(n+1) \quad = \frac{n}{6}(n+1)\{2n+1+3\}$$
$$= \frac{n}{3}(n+1)(n+2)$$

**b** Substitute $n = 60$ and $n = 30$ into the result for part (a), and subtract. The result = 65720.

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 13**

### Question:

**a** Show that $\sum_{r=1}^{n} r(r+1)(r+2) = \frac{n}{4}(n+1)(n+2)(n+3)$.

**b** Hence evaluate $3 \times 4 \times 5 + 4 \times 5 \times 6 + 5 \times 6 \times 7 + \ldots + 40 \times 41 \times 42$.

### Solution:

**a**

$$\sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r^2 + 2\sum_{r=1}^{n} r = \frac{n^2}{4}(n+1)^2 + \frac{n}{2}(n+1)(2n+1) + n(n+1)$$
$$= \frac{n}{4}(n+1)\{(n(n+1) + 2(2n+1) + 4\}$$
$$= \frac{n}{4}(n+1)(n+2)(n+3)$$

**b** $3 \times 4 \times 5 + 4 \times 5 \times 6 + 5 \times 6 \times 7 + \ldots + 40 \times 41 \times 42 = \sum_{r=3}^{40} r(r+1)(r+2)$

$$\sum_{r=3}^{40} r(r+1)(r+2) = \sum_{r=1}^{40} r(r+1)(r+2) - \sum_{r=1}^{2} r(r+1)(r+2)$$
$$= \frac{40 \times 41 \times 42 \times 43}{4} - \frac{2 \times 3 \times 4 \times 5}{4} = 740430$$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 14**

### Question:

**a** Show that $\sum_{r=1}^{n} r\{2(n-r)+1\} = \frac{n}{6}(n+1)(2n+1)$.

**b** Hence sum the series $(2n-1)+2(2n-3)+3(2n-5)+\ldots+n$

### Solution:

**a** Series can be written as $(2n+1)\sum_{r=1}^{n} r - 2\sum_{r=1}^{n} r^2$ as $n$ is a constant.

$$= (2n+1)\frac{n}{2}(n+1) - \frac{n}{3}(n+1)(2n+1)$$
$$= \frac{n}{6}(n+1)(2n+1)$$

**b** $\sum_{r=1}^{n} r[2(n-r)+1] = (2n-1)+2[(2n-4)+1]+3[(2n-6)+1]+\ldots+n[2(n-n)+1]$

$= (2n-1)+2(2n+3)+3(2n+5)+\ldots+n$, the series in part (b).

The sum, therefore, is $\frac{n}{6}(n+1)(2n+1)$

# Solutionbank FP1
## Edexcel AS and A Level Modular Mathematics

### Series
**Exercise F, Question 15**

### Question:

**a** Show that when $n$ is even,

$$1^3 - 2^3 + 3^3 - \ldots - n^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 - 16\left(1^3 + 2^3 + 3^3 + \ldots + \left(\frac{n}{2}\right)^3\right)$$

$$= \sum_{r=1}^{n} r^3 - 16\sum_{r=1}^{\frac{n}{2}} r^3.$$

**b** Hence show that, for $n$ even, $1^3 - 2^3 + 3^3 - \ldots - n^3 = -\frac{n^2}{4}(2n+3)$

**c** Deduce the sum of $1^3 - 2^3 + 3^3 - \ldots - 40^3$.

### Solution:

**a**

$$\begin{aligned}
1^3 - 2^3 + 3^3 - \ldots - n^3 &= (1^3 + 2^3 + 3^3 + \ldots + n^3) - 2(2^3 + 4^3 + 6^3 + \ldots + n^3) \\
&= (1^3 + 2^3 + 3^3 + \ldots + n^3) - 2\left\{2^3\left(1^3 + 2^3 + 3^3 + \ldots + \left(\frac{n}{2}\right)^3\right)\right\} \text{ as } n \text{ is even} \\
&= (1^3 + 2^3 + 3^3 + \ldots + n^3) - 16\left\{1^3 + 2^3 + 3^3 + \ldots + \left(\frac{n}{2}\right)^3\right\} \\
&= \sum_{r=1}^{n} r^3 - 16\sum_{r=1}^{\frac{n}{2}} r^3 \text{ [As } n \text{ is even, } \frac{n}{2} \text{ is an integer]}
\end{aligned}$$

**b**

$$\begin{aligned}
\sum_{r=1}^{n} r^3 - 16\sum_{r=1}^{\frac{n}{2}} r^3 &= \frac{n^2}{4}(n+1)^2 - 16\frac{\left(\frac{n}{2}\right)^2\left(\frac{n}{2}+1\right)^2}{4} \\
&= \frac{n^2}{4}(n+1)^2 - 4\frac{n^2}{4}\frac{(n+2)^2}{4} \\
&= \frac{n^2}{4}(n+1)^2 - \frac{n^2}{4}(n+2)^2 \\
&= \frac{n^2}{4}\{(n+1)^2 - (n+2)^2\} \\
&= \frac{n^2}{4}(-2n-3) = -\frac{n^2}{4}(2n+3)
\end{aligned}$$

**c** Substituting $n = 40$, gives $-33200$.