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Math 1496 Calc I

Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Rules

(i) $\frac{dc}{dx} = 0$

(ii) $\frac{d x^n}{dx} = n x^{n-1}$

(iii) $(f \pm g)' = f' \pm g'$

(iv) $(cf)' = cf'$

so if $f(x) = -x^2 + 4x + 3$

$f'(x) = -2x + 4$

Find the eq^n of the tangent to

$f(x) = -x^2 + 4x + 3$ at $x = 1$

we have the slope $f'(1) = -2(1) + 4 = 2$

& pt $f(1) = -1 + 4 + 3 = 6$

so tangent $y - 6 = 2(x - 1)$

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Product Rule

$(fg)' = f'g + fg'$

so $\frac{d}{dx}(x^2 \sin x) = 2x \sin x + x^2 \cos x$

$f = x^2 \quad g = \cancel{\cos}\, \sin x$

$f' = 2x \quad g' = \cos x$

Now about $\frac{d}{dx}\left(\frac{\sin x}{x^2}\right)$? $\quad \frac{d}{dx}(x^{-2} \sin x)$

$f = x^{-2} \quad g = \sin x$

$f' = -2x^{-3} \quad g' = \cos x$

$= -2x^{-3} \sin x + x^{-2} \cos x$

$= \frac{-2 \sin x}{x^3} + \frac{\cos x}{x^2} = \frac{-2 \sin x + x \cos x}{x^3}$

What about $\frac{d}{dx}\left(\frac{\sin x}{x^2+1}\right)$?

Quotients

$$\frac{d}{dx}\left(\frac{f}{g}\right) \stackrel{?}{=} \frac{f'}{g'}$$

$$\frac{d}{dx}\,\frac{x^3}{x^2} \stackrel{?}{=} \frac{3x^2}{2x} = \frac{3}{2}x$$

But $\frac{d}{dx}(x) = 1$ Not the same

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Check

$$\frac{d}{dx}\left(\frac{x^3}{x^2}\right) \qquad f = x^3 \quad g = x^2$$

$$f' = 3x^2 \quad g' = 2x$$

$$\frac{f'g - fg'}{g^2} = \frac{3x^2(x^2) - x^3(2x)}{x^4} = \frac{3x^4 - 2x^4}{x^2} = \frac{x^4}{x^4} = 1 \checkmark$$

ex $\frac{d}{dx}\left(\frac{e^x}{x^2+1}\right)$ $\qquad f = e^x \quad g = x^2+1$

$\qquad\qquad\qquad\qquad f' = e^x \quad g' = 2x$

$$= \frac{f'g - fg'}{g^2} = \frac{e^x(x^2+1) - e^x \cdot 2x}{(x^2+1)^2}$$

$$= \frac{e^x(x^2-2x+1)}{(x^2+1)^2}$$

ex Find the eq^n of the tangent to

$$y = \frac{x^2-1}{x^2+1} \quad \text{at } x = 1$$

$$y' = \frac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2}$$

$$= \frac{4x^3+2x-4x^3+2x}{(x^2+1)^2}$$

$$= \frac{4x}{(x^2+1)^2}$$

Now $y = 0$ when $x = 1$

Need y' here

$$y'\Big|_{x=1} = \frac{2(2)-0}{2^2} = 1$$

here

tangent $y - 0 = 1(x-1)$

Remaining Trig functions

$\tan x = \frac{\sin x}{\cos x}$ $f = \sin x$ $g = \cos x$

$f' = \cos x$ $g' = -\sin x$

$$\frac{d}{dx}\frac{\sin x}{\cos x} = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

so $\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x}$ $f = 1$ $g = \cos x$

$f' = 0$ $g' = -\sin x$

$$= \frac{0 \cos x - 1(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$\frac{d}{dx}\sec x = \sec x \cdot \tan x$

$\frac{d}{dx}\csc x = -\csc x \cdot \cot x$