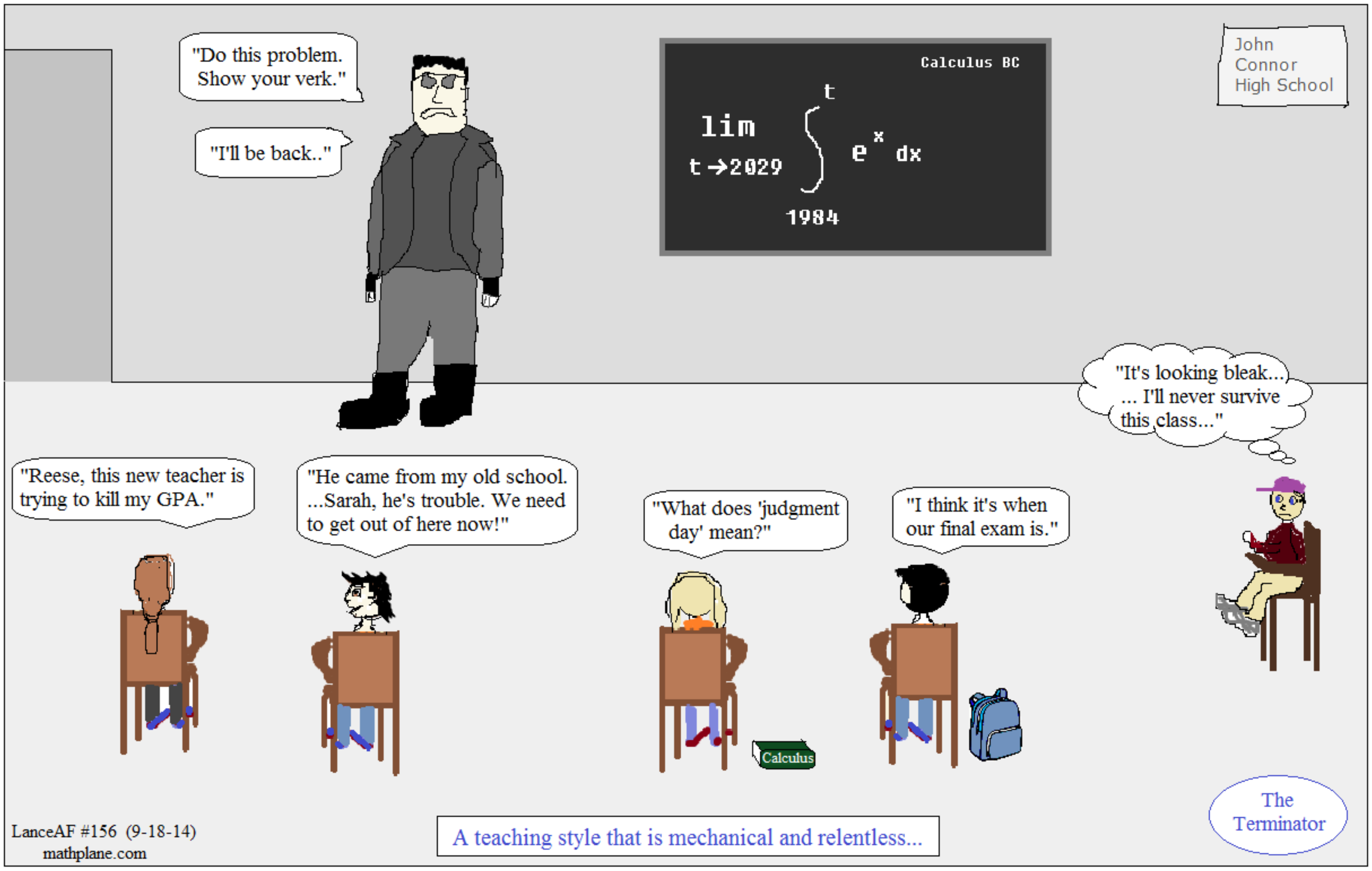
CALCULUS AB:

Multiple Choice Questions 2

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Topics include limits, continuity, differentiation, second derivatives, mean value theorem, implicit differentiation, related rates, and more…


"Do this problem. Show your verk."
"I'll be back.."
Calculus BC
lim
t→2029
t
1984
e^x dx
John Connor High School
"It's looking bleak... ... I'll never survive this class..."
"Reese, this new teacher is trying to kill my GPA."
"He came from my old school. ...Sarah, he's trouble. We need to get out of here now!"
"What does 'judgment day' mean?"
"I think it's when our final exam is."
Calculus
LanceAF #156 (9-18-14)
mathplane.com
A teaching style that is mechanical and relentless...
The Terminator

QUESTIONS-→

1) $\lim_{x \to \infty} \dfrac{x^2 - 1}{3 + 4x - x^2}$

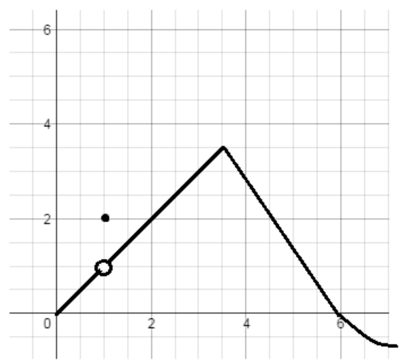
a) -1
b) -1/3
c) -1/4
d) 0
e) ∞

2) For the function $f(x)$ in the graph,

I. $\lim_{x \to c}$ exists for all c in the interval [2, 6]

II. the function is continuous on the interval [2, 6]

III. the function is differentiable on the interval [2, 6]



a) I only
b) II only
c) I and II
d) I and III
e) I, II, and III

3) What value of c makes this function continuous?

$$\begin{cases} \dfrac{2x^2 + 7x + 3}{x + 3} & \text{if } x \neq -3 \\ C & \text{if } x = -3 \end{cases}$$

a) -3
b) -5
c) 2
d) -1/2
e) 0

4) For the following function $s(t) = 2t^3 - t^2 + 8t - 4$, where t = seconds, what is the displacement over the first 4 seconds?

a) 35
b) 36
c) 88
d) 140
e) 144

Calculus Multiple Choice Questions

5) A rational function of the form $y = \dfrac{ax}{x+b}$ has a vertical asymptote at $x = 5$ and a horizonal asymptote at $y = -3$

Which is a possible function?

a) $\dfrac{5x}{x-3}$ b) $\dfrac{3x}{x+5}$ c) $\dfrac{-3x}{x-5}$ d) $\dfrac{-5x}{x+3}$ e) $\dfrac{-3x}{x+5}$

6) Let p(x) be a cubic polynomial function, where $p(3) < 0$, $p(7) > 0$, and $p(9) < 0$,
Which statements are true?

statement I: there are 3 zeros
statement II: a zero exists at $x < 3$ OR $x > 9$
statement III: for $p(x) = 0$, there are 2 solutions between 3 and 9

a) I
b) I and II
c) I and III
d) II
e) I, II, and III

7) $\lim\limits_{x \to 3} 9 =$

a) 3
b) 9
c) Does not exist
d) 0
e) 27

8) Find the value of k so g(x) is continuous:

$$g(x) = \begin{cases} k + x & x < 10 \\ xk & x \geq 10 \end{cases}$$

a) 10
b) 0
c) 10/9
d) 1
e) no solution

9) What are the minimums of $6x^4 - 48x^2$?

a) 0

b) -2, 2

c) -2, 0, 2

d) -4,

e) -4, 4

10) What is the derivative of $x^2 \sin(5x)$?

a) $2x\cos(5x)$

b) $10x\cos(5x)$

c) $2x + 5\cos(5x)$

d) $2x\sin(5x) + x^2\cos(5x)$

e) $2x\sin(5x) + 5x^2\cos(5x)$

11) Find the slope of the line tangent to the curve $y = x^3 - 3x^2$ *at the point of inflection.*

a) -3

b) -1

c) 0

d) 1

e) 3

12) As x increases to infinity, the function $f(x) = 2e^{-x}$ gets closer to

a) 0

b) 1/2

c) 2

d) e

e) infinity

13) If $y = \sin(x)\cos(y)$, then @ $(\pi, 0)$ $\frac{dy}{dx} =$

a) -1

b) 0

c) 1

d) π

e) 2π

14) If $x^2 + 2y^2 = 22$, what is the behavior of the graph at $(-2, 3)$

a) increasing, concave up

b) increasing, concave down

c) decreasing, concave up

d) decreasing, concave down

e) increasing, point of inflection

15) Find the equation of the line tangent to $x^3 + y^3 = 3xy + 4x - 5y$ @ $(2, 1)$

a) $y = 1$

b) $5x + 2y = 12$

c) $2y - 5x = -8$

d) $5x - y = 9$

e) $x = 2$

16) $f(x) = x^2 + 1$ on the interval $[0, 2]$ $\boxed{\text{Integral Mean Value Theorem}}$ $\boxed{\text{Calculus Multiple Choice Questions}}$

I. Find the average value of the function (on the given interval)

a) 2

b) 5/2

c) 7/3

d) 14/3

e) 5

II. Determine the value "c" guaranteed by the 'Mean Value Theorem'

a) -1.15

b) -.57

c) .57

d) 1.15

e) 2.3

17) $h(x) = x^3 - 2$ on the interval $[-1, 3]$ $\boxed{\text{Derivative Mean Value Theorem}}$

I. Find the Average Rate of Change (AROC) on the interval

a) 2

b) 4

c) 13/2

d) 7

e) 11

II. Find the value "c" to satisfy the 'Mean Value Theorem"

a) -2.33

b) -1.32

c) 1

d) 1.53

e) 2.11

18) Let x and y be functions of time t related by the equation $y^2 = xy + 8$

at $t = 1$, $y = 3$ and $\frac{dy}{dt} = 2$ Find $\frac{dx}{dt}$

a) 3

b) $\frac{34}{9}$

c) -5

d) 0

e) $\frac{1}{3}$

19) What is the y-intercept of the line that is tangent to $2\sqrt{x} + 4\sqrt{y} = x + y + 3$ at $(4, 9)$?

a) 3

b) 6

c) 9

d) 12

e) 15

20) If $x^2 - y^2 = 16$ find $\frac{d^2y}{dx^2}$

a) $\frac{x^2 - y^2}{y^2}$

b) $\frac{y^2 - x^2}{y^3}$

c) $\frac{1}{y^2}$

d) $\frac{16x}{y^2}$

e) $\frac{x^2}{y^2}$

21) If $f(x) = x^3 + x^2 + x + 3$ and $g(x) = f^{-1}(x)$

what is the value of $g'(6)$?

a) -1/6
b) 1/6
c) -6
d) 6
e) 121

22) g is differentiable and $g(x) = f^{-1}(x)$ for all x

$f(-4) = 12$ $f(9) = -4$ $f'(4) = -6$ $f'(9) = 3$

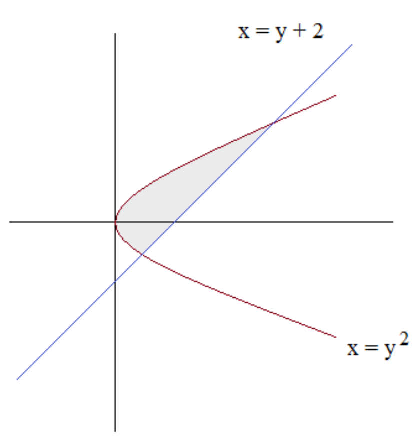
what is $g'(-4)$?

a) 1/3
b) -1/4
c) 1/9
d) -1/6
e) need more information

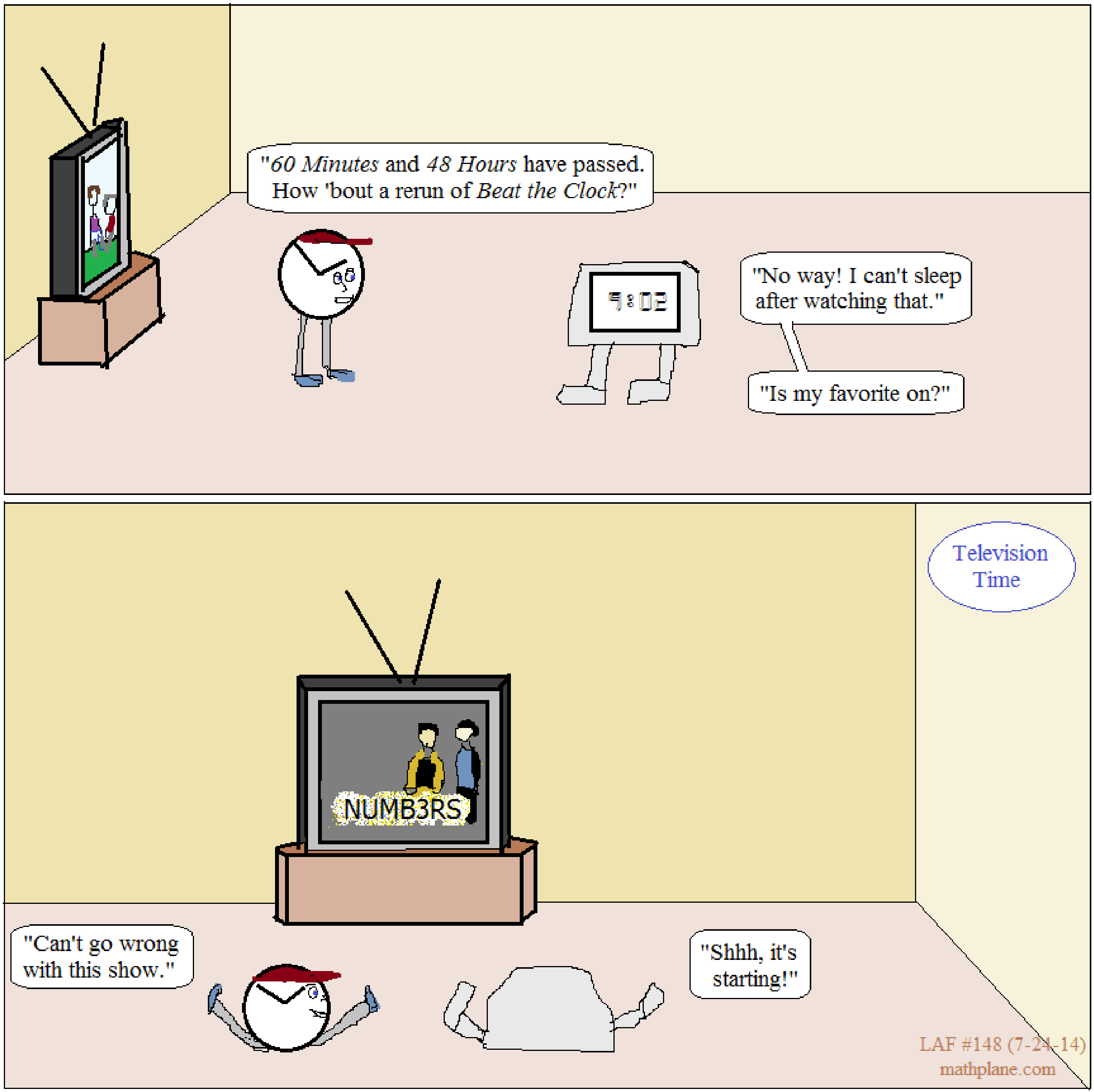
23) Find the area of the region bounded by

$x = y^2$

$x = y + 2$



a) 7/2
b) 4
c) 9/2
d) 8
e) 9


"*60 Minutes* and *48 Hours* have passed. How 'bout a rerun of *Beat the Clock*?"
"No way! I can't sleep after watching that."
"Is my favorite on?"
Television Time
NUMB3RS
"Can't go wrong with this show."
"Shhh, it's starting!"
LAF #148 (7-24-14)
mathplane.com

SOLUTIONS-→

Calculus Multiple Choice Questions

SOLUTIONS

1) $\lim\limits_{x \to \infty} \dfrac{x^2 - 1}{3 + 4x - x^2}$

$\boxed{\text{a) } -1}$

b) -1/3

c) -1/4

d) 0

e) ∞

rewrite: $\lim\limits_{x \to \infty} \dfrac{x^2 - 1}{-x^2 + 4x + 3}$

since degree of numerator (2) and degree of numerator (2) are the same, look at the lead coefficients...

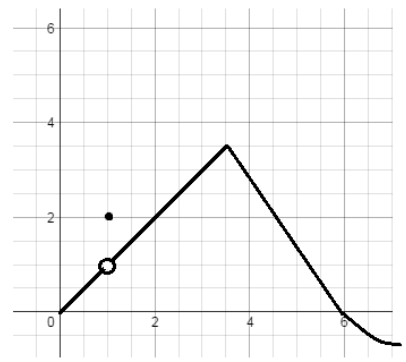
$\dfrac{1}{-1} = -1$

2) For the function $f(x)$ in the graph,

I. $\lim\limits_{x \to c}$ exists for all c in the interval [2, 6]

II. the function is continuous on the interval [2, 6]

III. the function is differentiable on the interval [2, 6]



a) I only

b) II only

$\boxed{\text{c) I and II}}$

d) I and III

e) I, II, and III

I. limit does exist (in fact, it exists between [0, 6]

II. function is continuous on [2, 6] (It is not continuous at x = 1)

III. It is not differentiable at x = 3 3/4

3) What value of c makes this function continuous?

$$\begin{cases} \dfrac{2x^2 + 7x + 3}{x + 3} & \text{if } x \neq -3 \\ C & \text{if } x = -3 \end{cases}$$

a) -3

$\boxed{\text{b) } -5}$

c) 2

d) -1/2

e) 0

the rational expression is a line with a 'hole'

To fill that hole, we find the limit as x approaches -3

$\lim\limits_{x \to -3} \dfrac{(2x + 1)(x + 3)}{(x + 3)} \quad \lim\limits_{x \to -3} (2x + 1) = -5$

4) For the following function $s(t) = 2t^3 - t^2 + 8t - 4$, where t = seconds, what is the displacement over the first 4 seconds?

a) 35

b) 36

c) 88

d) 140

$\boxed{\text{e) } 144}$

The displacement is the "net change"...

@ $t = 0$, $s(0) = -4$

@ $t = 4$, $s(4) = 140$

The displacement/net change is 144 units...

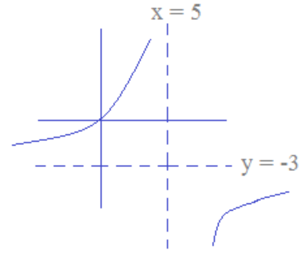
Calculus Multiple Choice Questions

SOLUTIONS

5) A rational function of the form $y = \dfrac{ax}{x+b}$ has a vertical asymptote at x = 5 and a horizonal asymptote at y = −3

Which is a possible function?

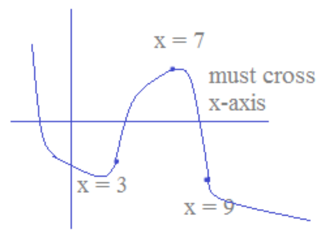
a) $\dfrac{5x}{x-3}$ b) $\dfrac{3x}{x+5}$ c) $\dfrac{-3x}{x-5}$ d) $\dfrac{-5x}{x+3}$ e) $\dfrac{-3x}{x+5}$



6) Let p(x) be a cubic polynomial function, where p(3) < 0, p(7) > 0, and p(9) < 0 ,
Which statements are true?

statement I: there are 3 zeros
statement II: a zero exists at x < 3 OR x > 9
statement III: for p(x) = 0, there are 2 solutions between 3 and 9

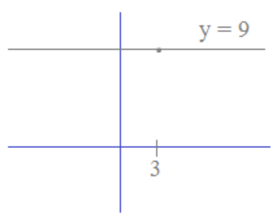
(possible sketch)



polynomial function is continuous...

a) I
b) I and II
c) I and III
d) II
e) I, II, and III

7) $\lim_{x \to 3} 9 =$ 9



a) 3
b) 9
c) Does not exist
d) 0
e) 27

8) Find the value of k so g(x) is continuous:

$$g(x) = \begin{cases} k + x & x < 10 \\ xk & x \geq 10 \end{cases}$$

to be continuous, each part of the piecewise function must meet:

k + x = xk

at x = 10:

10 + k = 10k
10 = 9k
k = 10/9

a) 10
b) 0
c) 10/9
d) 1
e) no solution

9) What are the minimums of $6x^4 - 48x^2$?

a) 0

$\boxed{\text{b) -2, 2}}$

c) -2, 0, 2

d) -4,

e) -4, 4

SOLUTIONS

Find derivative: $24x^3 - 96x$

Set equal to zero: $24x^3 - 96x = 0$

(critical values) $24x(x^2 - 4) = 0$

$x = -2, 0, 2$

Determine max/min:

$$\begin{array}{ccccccc} - & & + & & - & & + \\ \hline & -2 & & 0 & & 2 & \end{array}$$

decreasing increasing decreasing increasing

-2 and 2 are minimums
(0 is a relevant maximum)

second derivative: $72x^2 - 96$

@ x = -2, posittive (concave up)
minimum

@ x = 0, negative (concave down)
maximum

@ x = 2, positive (concave up)
minimum

10) What is the derivative of $x^2\sin(5x)$?

a) $2x\cos(5x)$

b) $10x\cos(5x)$

c) $2x + 5\cos(5x)$

d) $2x\sin(5x) + x^2\cos(5x)$

$\boxed{\text{e) } 2x\sin(5x) + 5x^2\cos(5x)}$

product rule $x^2 \quad \sin(5x)$

$f'(x)g(x) + g'(x)f(x)$ $\quad 2x \cdot \sin(5x) + \cos(5x) \cdot 5 \cdot x^2$

$2x\sin(5x) + 5x^2\cos(5x)$

11) Find the slope of the line tangent to the curve $y = x^3 - 3x^2$ *at the point of inflection.*

$\boxed{\text{a) -3}}$

b) -1

c) 0

d) 1

e) 3

First, where is the point of inflection? Where 2nd derivative equals zero.

$y' = 3x^2 - 6x$

$y'' = 6x - 6$ $\quad y'' = 0$ when $x = 1$

Therefore, point of inflection is (1, -2)

$-2 = (1)^3 - 3(1)^2$

Now, find the slope at $x = 1$

$y' = 3(1)^2 - 6(1) = -3$

12) As x increases to infinity, the function $f(x) = 2e^{-x}$ gets closer to

$\boxed{\text{a) 0}}$

b) 1/2

c) 2

d) e

e) infinity

rewrite function as $\frac{2}{e^x}$

as x gets infinitely larger, e^x goes to infinity...

therefore, $\frac{2}{e^x}$ gets smaller and smaller, approaching 0

Calculus Multiple Choice Questions

SOLUTIONS

13) If $y = \sin(x)\cos(y)$, then @ $(\pi, 0)$ $\frac{dy}{dx} =$

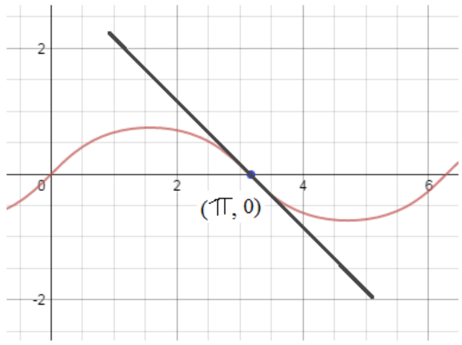
a) $\boxed{-1}$

b) 0

c) 1

d) π

e) 2π



Use implicit differentiation to find the derivative (instantaneous rate of change)

product rule

$$1 \cdot \frac{dy}{dx} = \cos(x)\cos(y) + \left(-\sin(y)\frac{dy}{dx}\right)\sin(x)$$

to find IROC at point, substitute $(\pi, 0)$

$$\frac{dy}{dx} = \cos(\pi)\cos(0) - \sin(0)\frac{dy}{dx} \cdot \sin(\pi)$$

$$= (-1)(1) + (0)\frac{dy}{dx}(0) = \boxed{-1}$$

14) If $x^2 + 2y^2 = 22$, what is the behavior of the graph at $(-2, 3)$

a) increasing, concave up

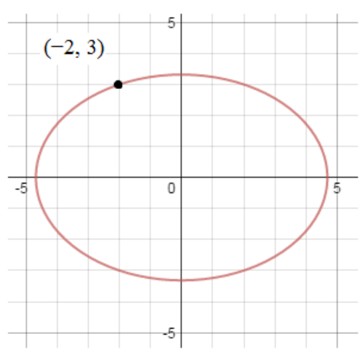
b) $\boxed{\text{increasing, concave down}}$

c) decreasing, concave up

d) decreasing, concave down

e) increasing, point of inflection

Notice, the graph is an ellipse!



To determine increasing or decreasing, find first derivative...

$$2x + 4y\frac{dy}{dx} = 0$$

$$4y\frac{dy}{dx} = -2x \qquad \text{at } (-2, 3) \quad \frac{dy}{dx} = \frac{-(-2)}{2(3)} = \frac{1}{3} > 0$$

$$\frac{dy}{dx} = \frac{-x}{2y}$$

increasing...

To determine concavity, find second derivative...

$$\frac{dy}{dx} = \frac{-x}{2y} \qquad \text{Quotient Rule}$$

$$\frac{d}{dx} \cdot \frac{dy}{dx} \longrightarrow \frac{d^2y}{dx^2} = \frac{-1(2y) - 2\frac{dy}{dx}(-x)}{(2y)^2} = \frac{-2y + 2x\frac{dy}{dx}}{4y^2}$$

$$\frac{-y + x\frac{dy}{dx}}{2y^2} = \frac{-y + x\left(\frac{-x}{2y}\right)}{2y^2} \qquad \text{at } (-2, 3) \quad \frac{d^2y}{dx^2} = \frac{-(3) + (-2)\frac{1}{3}}{2(3)^2}$$

$$= \frac{-11/3}{18} < 0$$

concave down...

15) Find the equation of the line tangent to $x^3 + y^3 = 3xy + 4x - 5y$ @ $(2, 1)$

a) $y = 1$

b) $\boxed{5x + 2y = 12}$

c) $2y - 5x = -8$

d) $5x - y = 9$

e) $x = 2$

$$3x^2 + 3y^2\frac{dy}{dx} = 3\left((1)(y) + (x)(1)\frac{dy}{dx}\right) + 4 - 5\frac{dy}{dx}$$

substitute $(2, 1)$

$$12 + 3\frac{dy}{dx} = 3\left(1 + 2\frac{dy}{dx}\right) + 4 - 5\frac{dy}{dx}$$

$$2\frac{dy}{dx} = -5$$

slope of tangent line at $(2, 1) = \frac{-5}{2}$

$$y - 1 = \frac{-5}{2}(x - 2)$$

$$\boxed{\begin{array}{l} y = \frac{-5}{2}x + 6 \\ 5x + 2y = 12 \end{array}}$$

16) $f(x) = x^2 + 1$ on the interval $[0, 2]$ — Integral Mean Value Theorem

I. Find the average value of the function (on the given interval)

a) 2

b) 5/2

c) $\boxed{7/3}$

d) 14/3

e) 5

To find the average value...

$$\int_0^2 x^2 + 1 \, dx = \left. \frac{x^3}{3} + x \right|_0^2 = \frac{8}{3} + 2 - (0/3 + 0) = \frac{14}{3}$$

area under the curve
(i.e. total value on interval [0, 2])

average value: $\dfrac{\frac{14}{3}}{(2 - 0)} = \dfrac{7}{3}$ average value

II. Determine the value "c" guaranteed by the 'Mean Value Theorem'

a) -1.15

b) -.57

c) .57

d) $\boxed{1.15}$

e) 2.3

since the function is continuous and closed on the interval,
there must be a value "c" such that $f(c)$ = average value

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

so, where does the function equal $\frac{7}{3}$?

$$\frac{7}{3} = x^2 + 1$$

$$x = \frac{2\sqrt{3}}{3} \quad \text{approx. } 1.15$$

We don't include -1.15 (because it is not in the interval)

17) $h(x) = x^3 - 2$ on the interval $[-1, 3]$ — Derivative Mean Value Theorem

I. Find the Average Rate of Change (AROC) on the interval

a) 2

b) 4

c) 13/2

d) $\boxed{7}$

e) 11

Average
Rate
Of
Change (slope) $\dfrac{25 - (-3)}{3 - (-1)} = 7$

If function is continuous and differentiable over interval [a, b]
there exists at least one point c where

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

instantaneous
rate of change
at c

average rate
of change
between a and b

II. Find the value "c" to satisfy the 'Mean Value Theorem"

a) -2.33

b) -1.32

c) 1

d) $\boxed{1.53}$

e) 2.11

Instantaneous
Rate
Of
Change

$h'(x) = 3x^2 - 0$

at point "c" $h'(c) = 7$

$$3c^2 = 7$$

$$c = \cancel{-1.53} \text{ or } 1.53$$

SOLUTIONS

18) Let x and y be functions of time t related by the equation $y^2 = xy + 8$

at $t = 1$, $y = 3$ and $\frac{dy}{dt} = 2$ Find $\frac{dx}{dt}$

a) 3

b) $\boxed{\frac{34}{9}}$

c) -5

d) 0

e) $\frac{1}{3}$

Using implicit differentiation and related rates....
derivative *with respect to t*

$$2y\frac{dy}{dt} = (1)\frac{dx}{dt}(y) + (x)(1)\frac{dy}{dt} + 0$$

direct substitution...

$$2(3)(2) = (1)\frac{dx}{dt}(3) + (1/3)(1)(2)$$

$$12 = 3\frac{dx}{dt} + 2/3$$

$$\frac{34}{3} = 3\frac{dx}{dt}$$

$$\frac{34}{9} = \frac{dx}{dt}$$

Since $y = 3$,

$$(3)^2 = x(3) + 8$$

$$1 = 3x$$

$$x = 1/3$$

19) What is the y-intercept of the line that is tangent to $2\sqrt{x} + 4\sqrt{y} = x + y + 3$ at $(4, 9)$?

a) 3

b) 6

c) 9

d) 12

e) $\boxed{15}$

First, to find the tangent line,
we need the slope and a point...

The point is (4, 9) (the point of tangency)
the slope is the IROC at (4, 9)

$$2x^{\frac{1}{2}} + 4y^{\frac{1}{2}} = x + y + 3$$

(implicit differentiation to find dy/dx)

$$x^{\frac{-1}{2}} + 2y^{\frac{-1}{2}}\frac{dy}{dx} = 1 + 1\frac{dy}{dx} + 0$$

Find dy/dx at (4, 9)

$$(4)^{\frac{-1}{2}} + 2(9)^{\frac{-1}{2}}\frac{dy}{dx} = 1 + 1\frac{dy}{dx}$$

$$\frac{1}{2} + \frac{2}{3}\frac{dy}{dx} = 1 + 1\frac{dy}{dx}$$

$$-\frac{1}{2} = \frac{1}{3}\frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{3}{2}$$

slope is -3/2
point is (4, 9)
then, equation of the line is

$$y - 9 = \frac{-3}{2}(x - 4)$$

therefore, the y-intercept is

$$y - 9 = \frac{-3}{2}(0 - 4)$$

(0, 15)

20) If $x^2 - y^2 = 16$ find $\frac{d^2y}{dx^2}$

a) $\frac{x^2 - y^2}{y^2}$

b) $\boxed{\frac{y^2 - x^2}{y^3}}$

c) $\frac{1}{y^2}$

d) $\frac{16x}{y^2}$

e) $\frac{x^2}{y^2}$

first, find $\frac{dy}{dx}$

$$2x - 2y\frac{dy}{dx} = 0$$

$$-2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

then, find second derivative $\frac{d^2y}{dx^2}$ where $\frac{dy}{dx} = \frac{x}{y}$

(quotient rule)

$$\frac{(1)(y) - (1)\frac{dy}{dx}(x)}{y^2}$$

$$\frac{y - x\frac{dy}{dx}}{y^2} = \frac{y - \frac{x^2}{y}}{y^2}$$

$$= \frac{y^2 - x^2}{y^3}$$

Calculus Multiple Choice Questions

SOLUTIONS

21) If $f(x) = x^3 + x^2 + x + 3$ and $g(x) = f^{-1}(x)$

what is the value of $g'(6)$?

a) -1/6

b) $\boxed{1/6}$

c) -6

d) 6

e) 121

$g'(x) = \dfrac{1}{f'(y)}$

Slope at (6, 1) is the reciprocal of the slope at (1, 6)

For y = 6,

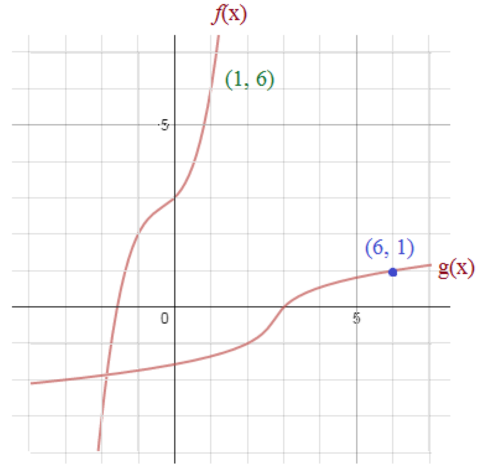
$6 = x^3 + x^2 + x + 3, \quad x = 1$

So, the slope at (1, 6) will be the reciprocal of the slope at (6, 1)!

$f'(x) = 3x^2 + 2x + 1 + 0$

then, $f'(1) = 3 + 2 + 1 = 6$

therefore, $g'(6) = \dfrac{1}{f'(1)} = \dfrac{1}{6}$



Inverses reflect over the line y = x, and the coordinates are reversed...

22) g is differentiable and $g(x) = f^{-1}(x)$ for all x

$f(-4) = 12 \qquad f(9) = -4 \qquad f'(4) = -6 \qquad f'(9) = 3$

what is $g'(-4)$?

a) $\boxed{1/3}$

b) -1/4

c) 1/9

d) -1/6

e) need more information

g(x) and f(x) are inverses...

So, if $f(9) = -4$, then $g(-4)$ must equal 9.

Then, $f'(9) = 3$... therefore, the slope of the inverse $g'(-4) = 1/3$

23) Find the area of the region bounded by

$x = y^2$

$x = y + 2$

a) 7/2

b) 4

c) $\boxed{9/2}$

d) 8

e) 9

$y + 2 = y^2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = -1$ and 2

(1, -1)

(4, 2)

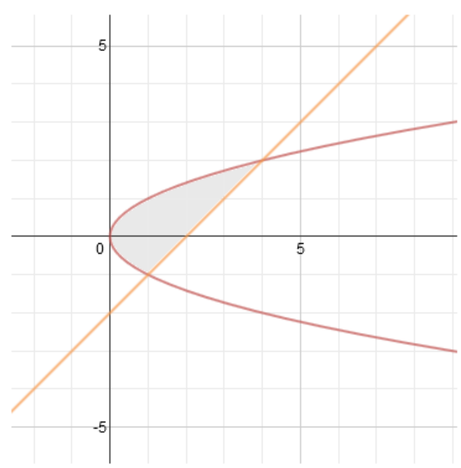
area between line and y-axis

area between curve and y-axis

$$\int_{-1}^{2} y + 2 - (y^2)\ dy$$

$$\frac{y^2}{2} + 2y - \frac{y^3}{3} \Bigg|_{-1}^{2}$$

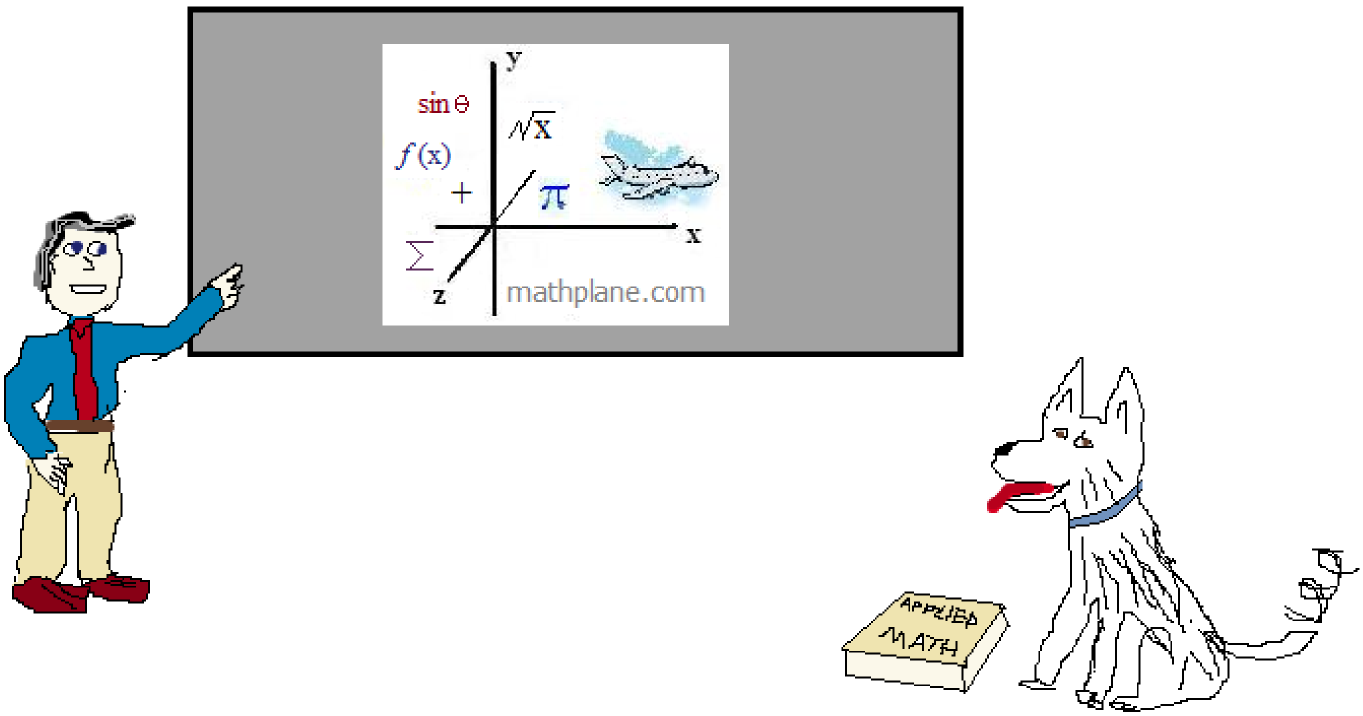
$$2 + 4 - \frac{8}{3} - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = \boxed{\frac{9}{2}}$$



Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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