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Business Mathematics – Vol 2

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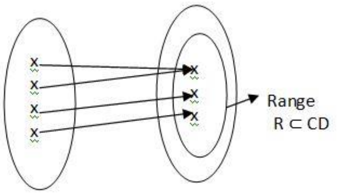
Content

1. Functions

Functions

The Function, $f: A \rightarrow B$, such that f(x) = y

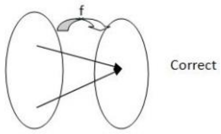
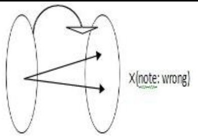
A, Domain – Set of 'x' values / pre-image of y B, Co – Domain – Set of 'y' values	Range – Set of f(x) values (image of x values) [Range is subset of Co - Domain] f(x) is called as the functional value

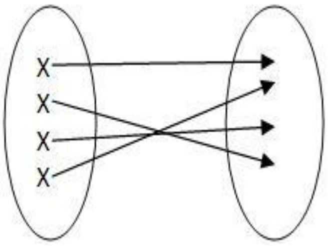


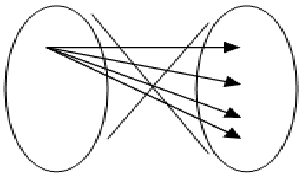
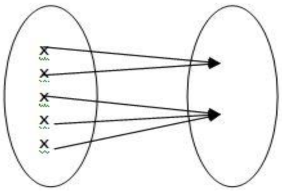
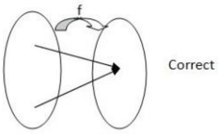
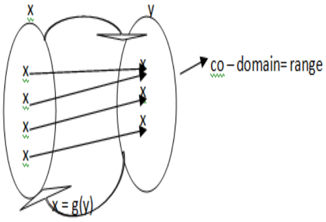
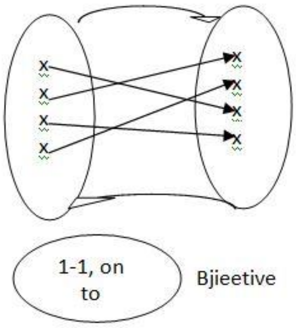
Points to Ponder:

1. All the values in the Domain should be mapped

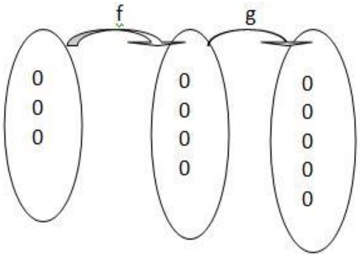
2. One value in the Domain should not have same image.

Function	Not a Function
	

Sl. No	Name of the Function	Mappings $f: A \rightarrow B$	Points to Ponder	Examples
1	Identity function (I) **I: A ⟶ *A*,** I(x) = x		1.Domain = Range 2. 1-1function 3. If, Range = Co-Domain, then Domain = Range = Co-Domain and 1-1 and onto function	**I: *N* ⟶ *W*,** I(x) = x Here, f(1) = 1 & f(2) = 2

2	One to Many		It is not a function, as one 'x' value cannot have more than one f(x) value, which means the function is not properly defined	$f: A \to E$ f(x) = x^2
3	Many to one		A properly defined Function	
4	All to one		Constant Function f(x) = k, k – constant	f: A⟶B f(x) = x^2, $1 \epsilon B$
5	Onto, function		Range = Co – Domain	**f: $W \longrightarrow N$,** f(x) = x + 1
6	Inverse Function		Only a function which is 1-1 and onto shall have an Inverse function	

Specific Functions

No	Function	Mapping	Examples	Remarks
1	Single-valued function	$y = f(x), for\ all\ x$	$f(x) = x^2, x \in N$	Only one functional value for all x
2	Multi-valued function	$f(x) = \begin{cases} f_1(x), for\ some\ x \\ f_2(x), for\ someother\ x \end{cases}$	$f(x) = \begin{cases} x^2, for\ x < 0 \\ x^3, for\ x \geq 0 \end{cases}$	functional values depend on the values of 'x' falling in different range
3	Absolute / Modulus Function	$f(x) = \|x - a\|$ $f(x) = \begin{cases} x - a, x > a \\ a - x, x < a \\ 0, x = a \end{cases}$	$f(x) = \|x\| + \|x - 2\|$ $f(x) = \begin{cases} 2 - 2x, \quad x < 2 \\ 2, 0 \leq x \leq 2 \\ 2x - 2, x > 0 \end{cases}$	
4	Equal function	Two functions with same functional values	Let $f: N \rightarrow N, f(x) = x^2$ and $g: N \rightarrow W, g(x) = x^2$	f(x) = g(x) but co domain is different
5	Composite Function	fog= $f(g(x))$ 	1. f is onto 2. In general, fog ≠ gof 3. If g = f^{-1}, then fog = I	(g.f) (x) = g (f (x)), if g(x) = f inverse
6	Even	$f(-x) = f(x)$	$f(x) = x^2$	
7	Odd	$f(-x) = -f(x)$	$f(x) = x^3$	
8	Explicit	Variables are seperable	x + y = 2	y = 2 - x
9	Implicit	Variables are not seperable	$x^2y = 0$	y = 0/x^2 , indeterminate form, is not possible

Examples

Sl. No	Function	Type
	$\boldsymbol{f: A \to B, f(x) = x^2}$	
1	A = W & B = W	1 -1 & Onto
2	A = IN & B = W	1 -1
3	A = E & B = E	Not a function
4	A = _ & B= W	many -1 on to
5	A = _ & B = IN	Not a function
6	$f: N \to N, f(x) = x$	Identity function / 1 – 1 / Onto
7	$f: N \to W, f(x) = x$	Identity function / 1 – 1 , but not onto
8	$f: W \to N, f(x) = x$	Not a function

2. Relations

Relations

Let S = $\{a, b, c\}$ and Relation R – Subset of the set $S \times S$

Sl.No.	Relation	Explanation	Examples		
			"Is equal to"	"Is parallel to"	"Is perpendicular to"
1	Reflexive 'a is relate ϕ to it self'	(a,a) ϵ R	a = a	a ∥ a	a⊥a is not true → Not Reflexive
2	Symmetric if (a,b) ϵ R then	(b,a) ϵ R	a =b ⇒ b=a	a ∥ b ⇒ b ∥ a	a⊥b b⊥a → Bymetric
3	Transitive	if (a,b) ϵR & (b,c) ϵ R ⇒ (a,c) ϵR	a=b, b=c ⇒ a=c	a ∥ b and b ∥ c ⇒ a ∥ c	a⊥b, b⊥ c a⊥c → not transitive
4	Equivalence	Reflexive + symmetry + Transitive	Equivalence	Equivalence	Not Equivalence

Cartesian Product (ordered pairs)

Let A = {1,2,3} and B = {2,4,6}

Then A × B = {(1,2), (1,4), (1,6), (2,2), (2,4), (2,6), (3,2), (3,4), (3,6)}

Points to Ponder:

By definition, Every subset of A × B is a relation from A to B.

Consider the product set X × Y = {(1,3), (2,3), (4,3), (2,2), (3,2), (4,2), (1,1), (2,1), (3,1), (4,1)}

Subset	**Relation**
{(1,1), (2,2), (3,3)}	'Is equal to'
{(1,3), (2,3), (1,2)}	'Is less than'
$\{(\mathbf{4},\mathbf{3}),(\mathbf{3},\mathbf{2}),(\mathbf{4},\mathbf{2}),(\mathbf{2},\mathbf{1}),(\mathbf{3},\mathbf{1}),(\mathbf{4},\mathbf{1})\}$	'Is greater than'
$\{(\mathbf{4},\mathbf{3}),(\mathbf{3},\mathbf{2}),(\mathbf{4},\mathbf{2}),(\mathbf{2},\mathbf{1})(\mathbf{3},\mathbf{1}),(\mathbf{4},\mathbf{1}),(\mathbf{1},\mathbf{1}),(\mathbf{2},\mathbf{2}),(\mathbf{3},\mathbf{3}))\}$	'Is greater than or equal'

Identity and Inverse Relation

1. Identity Relation: The relation I = {(a, a): a∈ A} is called the identity relation on A.

Illustration: Let A = {1, 2, 3} then I = {(1,1),(2,2),(3,1)}

2. Inverse Relation: Let R be a relation on A, then the relation R^{-1} on A, defined by

R^{-1} = {(b,a): (a,b) ∈R} is called an iverse relation on A.

Here, Dom (R^{-1}) = Range (R) & Range(R^{-1}) = Dom (R)

Then R being a subset of A× A, it is a relation on A. Dom (R) = {1, 2, 3} and Range (R) = {2,1}

Now, (R^{-1}) = {(2,1),(2,2),(1,3),(2,3)} Here, Dom (R^{-1}) = {2,1} = Range (R) and

Range (R^{-1}) = {1, 2, 3} = Dom (R).

Illustration: Let A = {1, 2, 3}, then

R1 = {(1,1),(2,2),(3,3),(1,2)}	Reflexive and transitive but not symmetric, since (1,2) ∈ R_1 but (2,1) does not belongs to R_1.
R2 = {(1,1), (2,2),(1,2), (2,1)	Symmetric and transitive but not reflexive, since (3,3) does not belong to R_2.
R3 = {(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)}	Reflexive and symmetric but not transitive, since (1,2)∈ R3&(2,3) ∈ R3 but (1,3) does not belong to R3.

3. Equations

Equation: An algebraic expression equated to a constant / zero

Example: $x^2 = 4$ or $x^2 - 4 = 0$

Equation – A mathematical statement of equality

Conditional Equation	**Identity**
The equality is true for certain values of the variable involved Example: **$\frac{x+2}{3} + \frac{x+3}{2} = 3$ is true for $x = 1$ only**	The equality is true for all the values of the variable Example: $(a+b)^2 = a^2 + b^2 + 2ab$ $\frac{x+2}{3} + \frac{x+3}{2} = \frac{5x+13}{6}$ are true all values of x
Solution / Root – The determined value that satisfies the equation. Example: $x^2 - 4 = 0$ has roots $x = 2$ & $x = -2$	

Types of Equation

Sl.No	Equation	Highest power of the variable	Example
I	Linear / Simple	1	$8x+17(x-3) = 44(4x-9)+12$
II	Simultaneous Linear Equations	a. Two variables	$x+2y = 1$ & $2x+3y = 2$
		b. Three variables	$2x - y + z = 3$ $x + 3y - 2z = 11$ $3x - 2y + 4z = 1$
III	Quadratic	2	$3x^2 + 5x + 6 = 0$
IV	Cubic	3	$4x^3 + 3x^2 + x - 7 = 0$

I Simple Equation

1. General Form $ax + b = 0$. Here a($\neq 0$) and b – Constants

2. If a=0, then b=0 is not possible in all case

Example: if a=0, b=1, then

$ax + b = 0\,(x) + 1 = 1 \neq 0$

3. It has only one root

Simple Problems – Set 1

Sl. No	**Question**	**Answer**
1	Find x for $\frac{4x}{3} - 1 = \frac{14}{15}x + \frac{19}{5}$	By transposing the variables in one side and the constants in other side we have $\frac{4x}{3} - \frac{14x}{15} = \frac{19}{5} + 1 \rightarrow \frac{(20-14)x}{15} = \frac{19+5}{5}$ $\frac{6x}{15} = \frac{24}{5} \rightarrow x = \frac{24\times15}{5\times6} = 12$
2	The denominator of a fraction exceeds the numerator by 5 and if 3 be added to both the fraction becomes $\frac{3}{4}$. Find the fraction	Let x be the numerator and the fraction is $\frac{x}{x+5}$. By the question $\frac{x+3}{x+5+3} = \frac{3}{4}$ $4x + 12 = 3x + 24$ or $x = 12$ The required fraction is $\frac{12}{17}$
3	If thrice of A's age 6 years ago be subtracted from twice his present age, the result would be equal to his present age. Find A's present age.	Let x years be A's present age. By the question, $2x - 3x + 18 = x$ $2x = 18 \rightarrow x = 9$ years, A' s present age
4	A number consists of two digits, the digit in the ten's place is twice the digit in the units place. If 18 be subtracted from the number the digits are reversed. Find the number.	Let x be the digit in the unit's place. So the digit the ten's place is $2x$. Thus the number becomes$10 \times (2x) + x$. By the question $20x + x - 8 = 10x + 2x \rightarrow x = 2$ The number is $10 \times (2 \times 2) + 2 = 42$
5	For a certain commodity the demand equation giving demand 'd' in kg, for a price 'p' in rupees per kg is $d = 100(10 - p)$. The supply equation giving the supply s in kg for a price p in rupees per kg. is $s = 75(p - 3)$. The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold.	Given $d = 100(10 - p)$ and $s = 75(p - 3)$ Since the market price is such that demand (d) = supply (s) we have 100 $(10 - p) = 75(p = 3)$ 1000 – 100p = 75p – 225 → -175 p = -1225 $\therefore p = \frac{-1225}{-175} = 7$, the market price of the commodity $\therefore$ Thus the required quantity, bought = 100(10 − 7) = 300kg and sold = 75 (7 − 3) = 300kg

II a. Simultaneous Linear Equations in two unknowns

General form of Linear Equation	**Simultaneous Equation in x & y**
$ax + by + c = 0$ Here, a($\neq 0$) & $b(\neq 0)$ are co efficients of x and y respectively c is a Constant	$a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ Here, $a_1(\neq 0)$ & $a_2(\neq 0)$ - co-efficients of x $b_1(\neq 0)$ & $b_2(\neq 0)$ - co-efficients of y c - Constant

Method of solutions of Equations $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$

	Method	**Explanation**	**Example: Solve** $3x + 2y + 17=0,\ 5x - 6y - 9 = 0$
1	Elimination	Two equations reduced to one linear equation with one unknown	Multiply $3x + 2y + 17 = 0$ by 3 $9x + 6y + 51 = 0$ Add with $5x - 6y - 9 = 0$ we get, $14x + 42 = 0$ → -x = $-\frac{42}{14}$ = -3 Putting x = -3 in first equation, we get $3(-3) + 2y + 17 = 0$ → y= -4 Hence, x = -3 and y = -4
2	Cross Multiplication	b_1 ⤨ c_1 ⤨ a_1 ⤨ b_1 b_2 c_2 a_2 b_2 $\frac{x}{b_1c_2-b_2c_1} = \frac{y}{c_1a_2-c_2a_1} = \frac{1}{a_1b_2-a_2b_1}$ $x = \frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}$ and $y = \frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$	2 ⤨ 1 ⤨ 3 ⤨ 2 b_1 - 7 5 - b_2 6 -9 6 $\frac{x}{2(-9)-17(-6)} = \frac{y}{17\times5-3(-9)} = \frac{1}{3(-6)-5\times2}$ $\frac{x}{84} = \frac{y}{112} = \frac{1}{-28}$ $\frac{x}{3} = \frac{y}{4} = \frac{1}{-1}$ → $x = -3$ and $y = -4$

II b. Method of solving simultaneous Linear Equation with 3 variables

Simple Problems – Set 2

1	$2x-y+z=3$, $x+3y-2z=11$, $3x-2y+4z=1$	
	Method of elimination (Any two of 3 equations can be chosen for elimination of one for the variables)	**Method of cross Multiplication**
	$2x-y+z=3$ …….. (i) $x+3y-2z=11$ …….. (ii) $3x-2y+4z=1$ …….. (iii) By (i) ×2 we get $4x-2y+2z=6$ ……..(iv) By (ii) +(iv), $5x+y=17$ ………(v) (the variable z is thus eliminated) By (ii) × 2, $2x+6y-4z=22$ ……..(vi) By (iii) + (vi), $5x+4y=23$ …….. (vii) By (v) – (vii) , $-3y=-6$ or $y=2$ Put $y=2$ in (v), $5x+2=17$ → $5x=15$ → $x=3$ Put $x=3$ and $y=2$ in (i) 2×3 – 2 + z =3 → 6 – 2 + z = 3 → z = -1 So $x=3$, $y=2$, $z=-1$ is the required solution	We write the equations as follows; $2x-y+(z-3)=0$ $x+3y+(-2z-11)=0$ By cross multiplication $\frac{x}{-1(-2z-11)-3(-z-3)}=\frac{x}{(z-3)-2(-2z-11)}=\frac{1}{2\times3-1(-1)}$ $\frac{x}{20-z}=\frac{y}{5z+19}=\frac{1}{7}$ → $x=\frac{20-z}{7}$, $y=\frac{5y+19}{7}$ Substituting above values for x and y in equation (iii) i.e $3x-2y+yz=1$, we have $3\left(\frac{20-z}{7}\right)-2\left(\frac{5z+19}{7}\right)+4z=1$ ➔ 60 -3z -10z – 38 +28z = 7 → 15z = 7-22 ➔ 15z = -15 → z = -1 Now $x=\frac{20-(-1)}{7}=\frac{21}{7}=3$, $y=\frac{5(-1)+19}{7}=\frac{14}{7}=2$ Thus $x=3$, $y=2$, $z=-1$
2	$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=5$, $\frac{2}{x}-\frac{3}{y}-\frac{4}{z}=-11$, $\frac{3}{x}+\frac{2}{y}-\frac{1}{z}=-6$ Take $u=\frac{1}{x}$; $v=\frac{1}{y}$ $w=\frac{1}{z}$ and get u + v + w = 5 ………(i) 2u – 3v – 4w = -11 ……..(ii) 3u + 2v – w = -6 ……. (iii) By (i) + (iii) 4u +3v = -1 …….(iv) By (iii) ×4 12u + 8v -4w ……(v) By (ii) – (v) -10u -11v = 13 Or, 10u +11v = -13 ……(vi)	By (iv) ×11 $44x+33v=-11$ ……(vii) By (vi) ×3 30u + 33v = -39 …….(viii) By (vii) – (viii) 14u = 28 or u =2 Putting u = 2 in (iv) 4×2 +3v = -1 Or 8 + 3v = -1 Or 3v = -9 or v = -3 Putting u =2, v=-3 in (i) or 2-3 +w =5 Or -1 + w = 5 or w = 5+1 or w =6 Thus $x=\frac{1}{u}=\frac{1}{2}$, $y=-\frac{1}{v}=\frac{1}{-3}$, $z=\frac{1}{w}=\frac{1}{6}$ is the solution.

III Quadratic Equation

General form: $ax^2 + bx + c = 0$ Here x – variable a($\neq$ 0), b, c – constants Degree – 2 →The equation will have two roots	**Pure quadratic equation,** b = 0 Example: $x^2 - 4 = 0$ **Affected Quadratic Equation,** b≠0 Example $x^2 - 5x + 6 = 0$

Methods of solving: Solve $x^2 - 5x + 6 = 0$

Factorisation Method $\boldsymbol{(x - a)(x - b) = 0}$	**Discrimination Method (Formula Method)** $\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$
$x^2 - 5x + 6 = 0$ → $x^2 - 2x - 3x + 6 = 0$ → $x(x - 2) - 3(x - 2) = 0$ → $(x - 2)(x - 3) = 0$ → $x = 2$ or 3	Here, a=1, b=-5, c=6 (Comparing the equation with $ax^2 + bx + c = 0$) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-5) \pm \sqrt{25-24}}{2} = \frac{6}{2}$ and $\frac{4}{2}$ $\therefore x = 3\ and\ 2$

Remarks

1. Let the roots be $\propto and\ \beta$. Then

Sum of the roots = $\propto + \beta = \frac{-b}{a} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ $\propto + \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-2b}{2a} = \frac{-b}{a}$	Product of the roots =$\propto \beta = \frac{c}{a} = \frac{constant\ term}{coefficient\ of x^2}$ $\propto \beta = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) = \frac{c}{a}$

Simple Problems – Set 3

1. If α and β be the roots of $x^2 + 7x + 12 = 0$ find the equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$

Solution:

Sum of the roots of the required equation $=(\alpha + \beta)^2 + (\alpha - \beta)^2$ $= (-7)^2 + (\alpha + \beta)^2 - 4\alpha\beta$ $= 49 + (-7)^2 - 4 \times 12 = 50$	Product of the roots of the required equation = $(\alpha + \beta)^2(\alpha - \beta)^2$ = 49 (49-48) = 49
The required equation is $x^2 - (sum\ of\ the\ roots)x + product\ of\ the\ roots = 0$ $x^2 - 50x + 49 = 0$	

2. If α, β be the roots of $2x^2 - 4x - 1 = 0$ find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Solution: $\alpha + \beta = \frac{-(-4)}{2} = 2$, $\alpha\beta = \frac{-1}{2}$

$\therefore \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta} = \frac{2^3 - 3\left(-\frac{1}{2}\right).2}{\left(-\frac{1}{2}\right)} = -22$

3. If α and β are the two roots of the equation $x^2 - px + q = 0$ form the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

Solution: As α, β are the roots of the equation $x^2 - px + q = 0$

$\alpha + \beta = -(-P) = p$ and $\alpha\beta = q$.

Now $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}. = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{p^2-2q}{q}$; and $\frac{\alpha}{\beta}.\frac{\beta}{\alpha} = 1$

$\therefore$ Required equation is $x^2 - \left(\frac{p^2-2q}{q}\right)x + 1 = 0$ → $q\,x^2 - (p^2 - 2q)x + q = 0$

2. Construction of a Quadratic Equation

$x^2 - (sum\ of\ the\ roots)x + product\ of\ the\ roots\ = 0$

Explanation:

$ax^2 + bx + c = 0$

$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$(dividing the full equation by a)

$\Rightarrow x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$

$\Rightarrow x^2 - \ (sum\ of\ the\ roots)x + product\ of\ the\ roots = 0)$

3. Nature of the roots: Discriminant , $b^2 - 4ac$ discriminates between the roots

Sl.no	$b^2 - 4ac$	Roots	Example
1	$= 0$	Real & equal	$x^2 - 8x + 16 = 0$ [a =1, b= -8, c= 16] $b^2 - 4ac = (-8)^2 - 4 \times 1 \times 16 = 64 - 64 = 0$ The roots are real and equal.
2	< 0	Imaginary	$5x^2 - 4x + 2 = 0$ [a = 5, b = -4, c = 2] $b^2 4ac = (-4)^2 - 4 \times 5 \times 2 = 16 - 40 = -24 < 0$ The roots are imaginary and unequal
3	>0 & Perfect square	Real, Rational & Unequal (distinct)	$3x^2 - 8x + 4 = 0$ [a = 3, b = -8, c = 4]

			$b^2 - 4ac$ = $(-8)^2 - 4 \times 3 \times 4 = 64 -$ 48 = 16> 0 and a perfect square The roots are real, rational and unequal
4	>0 & not a perfect square	Real, Irrational & unequal	$2x^2 - 6x - 3$ = 0 $b^2 - 4ac$ = $-(6)^2 - 4 \times 2 \times (-3)$ = 60 >0 The roots are real and unequal

Points to Ponder

1. Irrational roots occurs in conjugate pairs : $(m + \sqrt{n})$&$(m - \sqrt{n})$ are the roots.

Example: If one root of the equation is 2 - $\sqrt{3}$ from the equation given that the roots are irrational

Solution: other roots is 2 + $\sqrt{3}$ ∴ sum of two roots =2 – $\sqrt{3} + 2 + \sqrt{3}$ =4

Product of roots = $\left(2 - \sqrt{3}\right)\left(2 + \sqrt{3}\right)$ = 4 - 3 = 1

∴ Required equation is: $x^2 - (sum\ of\ roots)x + (product\ of\ roots)$ = 0 → $x^2 - 4x$+1 = 0.

2. Reciprocal roots - $\propto \beta = 1 \implies \frac{c}{a} = 1$ c=a

3. Roots are equal but opposite in sign, ∝ = - β

Then $\propto + \beta = 0$, $\frac{-b}{a} = 0 \implies$ b = 0.

Conceptual Problems

1	**Solve x: $4^x - 3.2^{x+2} + 2^5 = 0$**	
	$4^x - 3.2^{x+2} + 2^5 = 0$ $(2^x)^2 - 3.2^x.2^2 + 32 = 0$ $(2^x)^2 - 12.2^x + 32 = 0$ $y^2 - 12y + 32 = 0$ (taking $y = 2^x$) $y^2 - 8y - 4y + 32 = 0$	$y(y-8) - 4(y-8) = 0$ $\therefore (y-8)(y-4) = 0$ Either $y - 8 = 0$ or $y - 4 = 0$ $\therefore y = 8$ or $y = 4$ $\implies 2^x = 8 = 2^3$ or $2^x = 4 = 2^2$ Therefore $x = 3$ or $x = 2$.
2	Solve: $\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right)$ = $7\frac{1}{4}$	
	$\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) = \frac{29}{4}$ $\left(x + \frac{1}{x}\right)^2 - 4 + 2\left(x + \frac{1}{x}\right)$ = $\frac{29}{4}$ (as $(a-b)^2$ = $(a+b)^2 - 4ab$) $p^2 + 2p - \frac{45}{4} = 0$ Taking p = $x + \frac{1}{x}$ 4 $p^2 + 8p - 45$ = 0	$4p^2 + 18p - 10p - 45 = 0$ $2p(2p+9) - 5(2p+9) = 0$ $(2p-9)(2p+9)$ =0 ∴ Either 2p + 9 =0 or 2p – 5 = 0 ⇒ p = -$\frac{9}{2}$ or p = -$\frac{5}{2}$

3	If the roots of the equation $p(q-r)x^2+q(r-p)x+r(p-q)=0$ are equal show that $\frac{2}{q}=\frac{1}{p}+\frac{1}{r}=0$	
	Since the roots of the given equation are equal the discriminant must be zero $q^2(r-p)^2-4.p(q-r)r(p-q)=0$ $q^2\,r^2+q^2p^2-2q^2rp-4pr(pq-pr-q^2+qr)=0$ $p^2q^2+q^2r^2+4p^2r^2+2q^2pr-4p^2qr-4pqr^2=0$	$(pq+qr-2rp)^2=$ $pq+qr=2pr$ $\frac{pq+qr}{2pr}=1$ $\frac{q}{2}.\frac{(p+r)}{pr}=1$ $\frac{1}{r}+\frac{1}{p}=\frac{2}{q}$
4	Solve for x,y and z: $\frac{xy}{x+y}=70$ $\frac{yz}{y+z}=140$	
	we can write as $\frac{x+y}{xy}=\frac{1}{70}$ or $\frac{1}{x}+\frac{1}{y}=\frac{1}{70}$ ………(i) $\frac{x+z}{xz}=\frac{1}{84}$ or $\frac{1}{z}+\frac{1}{x}=\frac{1}{84}$ …….(ii) $\frac{y+z}{yz}=\frac{1}{140}$ or $\frac{1}{y}+\frac{1}{z}=\frac{1}{140}$ ……(iii) By (i) + (ii) + (iii), we get $2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\frac{1}{70}+\frac{1}{84}+\frac{1}{140}=\frac{14}{420}$ $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{7}{420}=\frac{1}{60}$ ……. (iv)	, By (iv) – (iii), $\frac{1}{x}=\frac{1}{60}-\frac{1}{140}=\frac{4}{420}\rightarrow x=105$ By (iv) –(ii), $\frac{1}{y}=\frac{1}{60}-\frac{1}{84}=\frac{2}{420}$ → y = 210 By (iv) – (i) $\frac{1}{z}=\frac{1}{60}-\frac{1}{70}$ → z = 420 Thus $x=105, y=210\; z=420$
Statement Problems		
4	Difference between a number and its positive square root is 12. Find the numbers.	
	Let the number be x. Then $x-\sqrt{x}=12$………(i) $x-\sqrt{x}-12=0$ Taking $y=\sqrt{x}$, $y^2-y-12=0$ $(y-4)(y+3)=0$	∴ Either $y=4$ or $y=-3$ Either $\sqrt{x}=4$ or $\sqrt{x}=-3$ If $\sqrt{x}=-3\rightarrow x=9$ It does not satisfy equation (i) so $\sqrt{x}=4$ or $x=16$
5	A piece of iron rod costs ₹60. If the rod was 2 metre shorter and each metre costs ₹1.00 more, the cost would remain unchanged. What is the length of the rod?	
	Let the length of the rod be x metres. The rate per meter is ₹ $\frac{60}{x}$ As given $\frac{60}{x-2}=\frac{60}{x}+1\rightarrow\frac{60}{x-2}-\frac{60}{x}=1$ $\rightarrow\frac{120}{x(x-2)}=1\rightarrow x^2-2x=200$	$x^2-2x-120=0$ $(x-12)(x+10)=0$ Either $x=12$ or $x=-10$ (not possible) ∴ Hence the required length = 12m
6	Divide 25 into two parts so that sum of their reciprocals is $\frac{1}{6}$	
	Let the parts be x and $25-x$ By the question $\frac{1}{x}+\frac{1}{25-x}=\frac{1}{6}$	$x^2-15x-10x+150=0$ $x(x-15)-10(x-15)=0$ $(x-15)(x-10)=0$

	$\frac{25-x+x}{x(25-x)} = \frac{1}{6}$ $150 = 25x - x^2 \rightarrow x^2 - 25x + 150 = 0$	$x = 10$ and $x = 15$ So the parts of 25 are 10 and 15

IV Cubic Equation – An equation with degree 3

Simple Problems – Set 4

1	Solve $x^3 - 7x + 6 = 0$	
	Putting $x = 1$ L.H.S is Zero. So $(x-1)$ is a factor of $x^3 - 7x + 6$ We write $x^3 - 7x + 6 = 0$ in such a way that $(x-1)$ becomes its factor. This can be achieved by writing the equation in the following form. $x^3 - x^2 - x - 6x + 6 = 0$	$x^2(x-1) + x(x-1) - 6(x-1) = 0$ $(x-1)(x^2 + x - 6) = 0$ $(x-1)(x^2 + 3x - 2x - 6) = 0$ $(x-1)\{x(x+3) - 2(x+3)\} = 0$ $(x-1)(x-2)(x+3) = 0$ $\therefore\ x = 1, 2, -3$
2	Solve for real x: $x^3 + x + 2 = 0$	
	By trial we find that $x = -1$ makes the LHS zero. So $(x+1)$ is a factor of $x^3 + x + 2$ We write $x^3 + x + 2 = 0$ as $x^3 + x^2 - x^2 - x + 2x + 2 = 0$ $x^2(x+1) - x(x+1) + 2(x+1) = 0$ $(x+1)(x^2 - x + 2) = 0$	Either x + 1 =0; x = -1 $x^2 - x + 2 = 0$ i.e. $x = \frac{1\pm\sqrt{1-8}}{2} = \frac{1\pm\sqrt{-7}}{2}$ As $x = \frac{1\pm\sqrt{-7}}{2}$ is not real, $x = -1$ is the required solution.

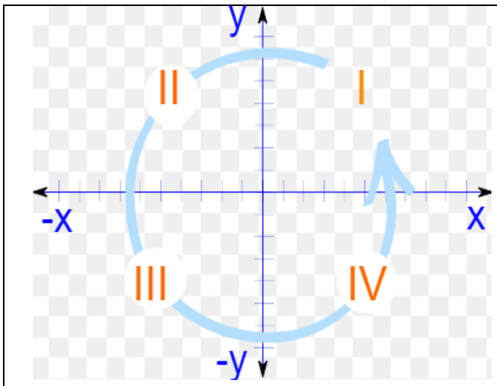
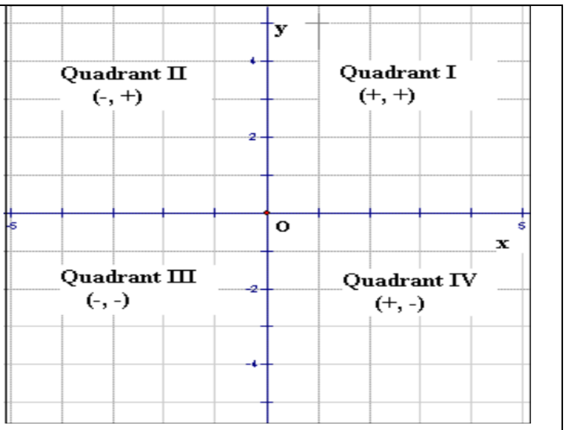
4. Inequalities

Inequality – A relationship between two unequal quantities

Example:

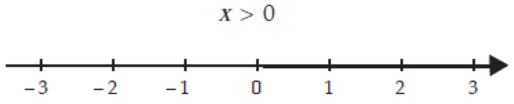
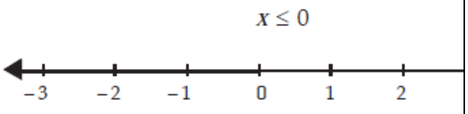
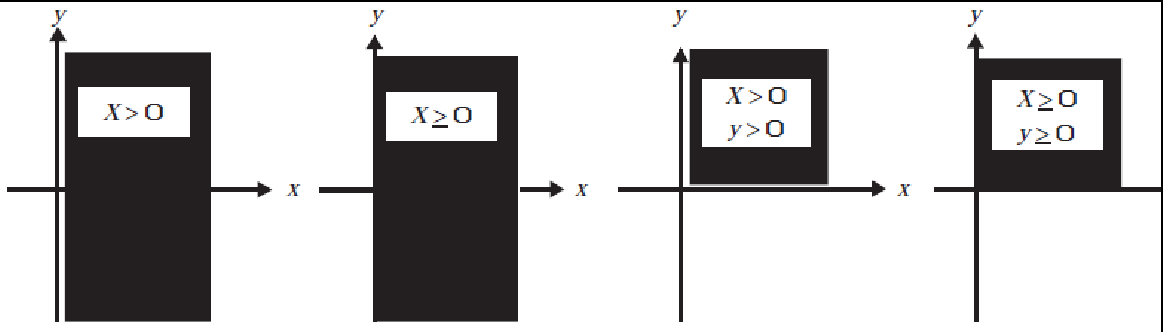
Real Life Situation	Inequality
A lift shall escalate the floors with a maximum capacity of 1000 pounds	$x \leq 1000$ $(x - Total\ weight\ in\ pounds)$
Doctor advices to take atleast an apple a day	$x \geq 1,$ $(x - Number\ of\ apples)$

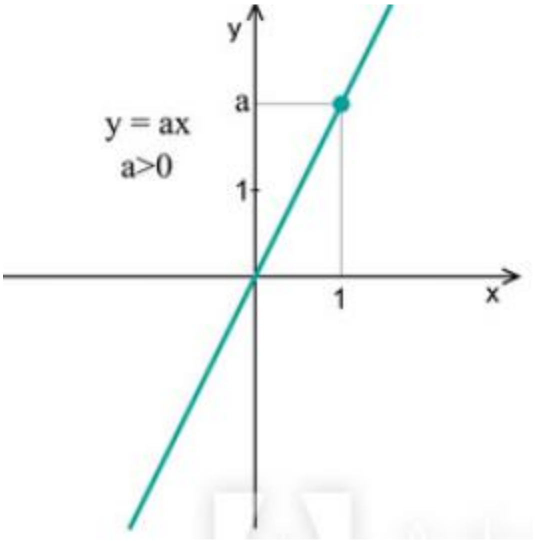
Note: Quadrants on a graph

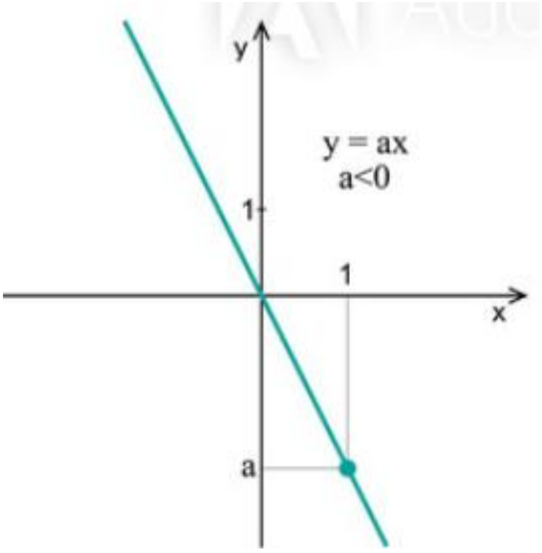
	

Types of Inequalities

I Linear Inequalities with one variable

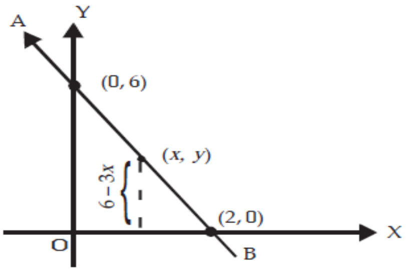
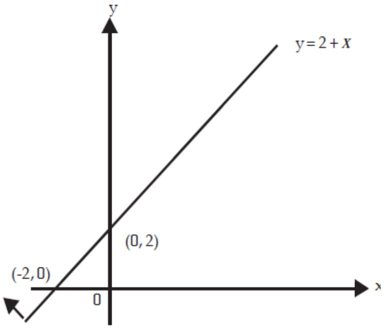
One – Dimensional (x – axis)	
	
Two – Dimensional (x and y – axis)	
	
$y = ax$, a > 0	$y = ax, a < 0$





Simple Problems

1. **Find the common region $3x + y < 6$ and $x - y \leq -2$**

Steps	Line 1	Line 2
	Consider the line, $3x + y = 6$	Consider the line,$x - y = -2$
	For $x = 0$, y= 6 : The line $3x - y = 6$ crosses the y-axis at (0, 6) For y= 0, x= 2 : The line $3x - y = 6$ crosses the x-axis at (2, 0)  	For $x = 0$, y= 2 : The line $3x - y = 6$ crosses the y-axis at (0, 2) For y= 0, x= -2 : The line $3x - y = 6$ crosses the x-axis at (-2, 0) 

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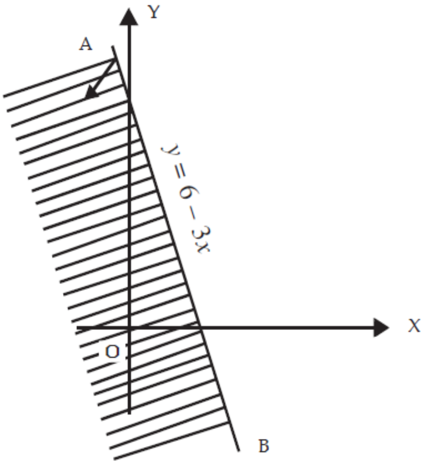
Graphing of Linear Inequality



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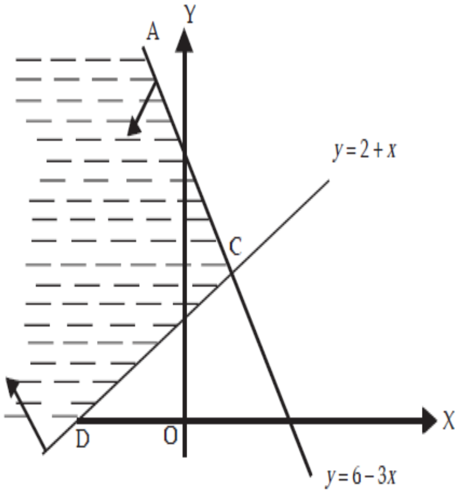

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2. **Descriptive Problem**: A manufacturer produces two products A and B, and has his machines in operations for 24 hours a day. Production of A requires 2 hours of processing in Machine M_1 and 6 hours in Machine M_2. Production of B requires 6 hours of processing in Machine M_1 and 2 hours in Machine M_2. The manufacturer earns a profit of ₹ 5 on each unit of A and ₹ 2 on each unit of B. How many units of each product should be produced in a day in order to achieve maximum profit?
Solution:

Steps	Procedure	Remarks
1	**Development of inequalities from the Descriptive Problem** Let x – Number of Units of A y – Number of units of B w.r.t profit, $5x + 2y$ w.r.t $M_1, 2x + 6y \leq 24$ w.r.t $M_2, 6x + 2y \leq 24$	Transformation of Descriptive to Mathematical Form
2	**Formulation of Linear Inequalities in one variable and solution space with various conditions** Maximise Z = $5x + 2y$ Subject to constraints, w.r.t $M_1, 2x + 6y \leq 24$ w.r.t $M_2, 6x + 2y \leq 24$ x, y ≥ **0**	Linear Programming Problem (LPP), a standarised form (Z – the objective function)
3	**Graphing of linear inequalities and Determination of common region**	Solving the two lines w.r.t. M_1

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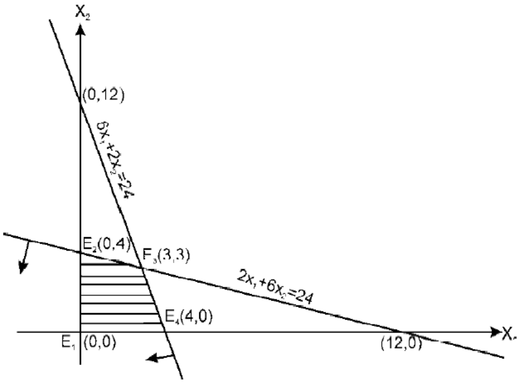
<td>
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<td>For $x = 0$, y= 4 : The line $2x + 6y = 24$ crosses the y-axis at (0, 4)
For y= 0, x= 12 : The line $2x + 6y = 24$ crosses the x-axis at (12, 0)
w.r.t. M_2
For $x = 0$, y= 12 : The line $6x + 2y = 24$ crosses the y-axis at (0, 12)
For y= 0, x= 4 : The line $6x + 2y = 24$ crosses the x-axis at (4, 0)</td>
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<td>4</td>
<td>Feasible Region and Feasible Points
Here
Feasible Region – Area $E_1E_2E_3E_4$
Feasible Points - E_1, E_2, E_3 andE_4</td>
<td>The common region</td>
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<td>5</td>
<td>Optimal Solution

$$E = \begin{array}{cc} \begin{matrix} X_1 & Y_2 \end{matrix} & \\ \begin{bmatrix}0&0\\0&4\\3&3\\4&0\end{bmatrix} & \begin{matrix}E_1\\E_2\\E_3\\E_4\end{matrix} \end{array} \quad C = \begin{bmatrix}5\\2\end{bmatrix}\begin{matrix}X_1\\X_2\end{matrix}$$

The values are

$$EC = \begin{bmatrix}0&0\\0&4\\3&3\\4&0\end{bmatrix}\begin{bmatrix}5\\2\end{bmatrix} = \begin{bmatrix}0\times5+0\times2\\0\times5+4\times2\\3\times5+3\times2\\4\times5+0\times2\end{bmatrix} = \begin{bmatrix}0\\8\\21\\20\end{bmatrix}\begin{matrix}E_1\\E_2\\E_3\\E_4\end{matrix}$$

Here, At E_3- EC is at the maximum.
Thus, $x = 3$ and $y = 3$</td>
<td>In Matrix form</td>
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<td colspan="3">Thus, the manufacturer has to produce 3 units of A and 3 units of B to get the maximum Profit</td>
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Exercises

1. A company produces two products A and B , each of which requires processing in two machines. The first machine can be used at most for 60 hours, the second machine can be used at most for 40 hours. The product A requires 2 hours on machine one and one hour on machine two. The product B require s one hour on machine one and two hours on machine two. Express above situation using linear inequalities.

Solution:

Let the company produce, x number of product A and y number of product B. As each of product A requires 2 hours in machine one and one hour in machine two, x number of product A requires 2x hours in machine one and x hours in machine two. Similarly, y number of product B requires y hours in machine one and 2y hours in machine two. But machine one can be used for 60 hours. Hence 2x+y cannot exceed 60 and x+2y cannot exceed 40. In other words, $2x + y \leq 60\ and\ x + 2y \leq 40$. Thus, the conditions can be expressed using linear inequalities.

2. A fertilizer company produces two types of fertilizer called grade I and grade II. Each of these types is processed through two critical chemical plant units. Plant A has maximum of 120 hours available in a week and plant B has maximum of 180 hours available in a week. Manufacturing one bag of grade I fertilizer requires 6 hours in plant A and 4 hours in plant B. Manufacturing one bag of grade II fertilizer requires 3 hours in plant A and 10 hours in plant B. Express this using linear inequalities.

Solution: Let us denote by x_1, the number of bags of fertilizer of grade I and by x_2, the number of bags of fertilizers of grade II produced in a week. We are given that grade I fertilizer requires 6 hours in plant A and grade II fertilizer requires 3 hours in plant A and plant A has maximum of 120 hours available in a week. Thus $6x_1 + 3x_2 \leq 120$.

Similarly grade I fertilizer requires 4 hours in plant B and grade II fertilizer requires 10 hour in Plant B and Plant B has maximum of 180 hours available in a week. Hence, we get the inequality

$$4x_1 + 10x_2 \leq 180$$

3. Graph the inequalities $5x_1 + 4x_2 \geq 9, x_1 + x_2 \geq 3$, $x_1 \geq 0$ and $x_2 \geq 0$ and mark the common region.

Solution:

We draw the straight lines $5x_1 + 4x_2 = 9\ and\ x_1 + x_2 = 3$

$\mathbf{5x_1 + 4x_2 = 9}$		
x_1	0	9/5
x_2	9/4	0

$\mathbf{x_1 + x_2 = 3}$		
x_1	0	3
x_2	3	0

Now, if we take the

point(4,4), we find

$5x_1 + 4x_2 \geq 9$ →i.e. 5.4 + 4.4 ≥ 9

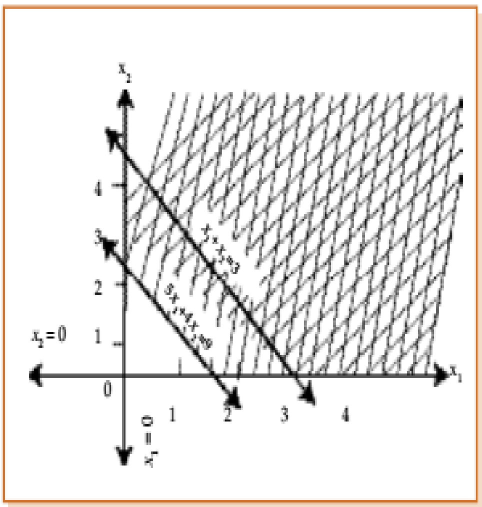
→ $36 \geq 9(True)$

Hence (4, 4) is in the region which satisfies the inequalities.

$x_1 + x_2 \geq 3 \rightarrow\ 4 + 4 \geq 3$

$8 \geq 3(True)$

We mark the region being satisfied by the inequalities and note that the cross-hatched region is satisfied by all the inequalities.



4. Draw the graph for the following $x + 2y = 4$ and $-y \leq 3$. Mark the common region.

Solution:

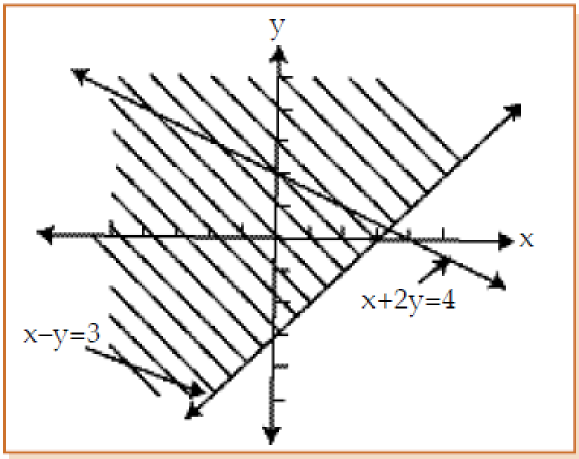
The solution set of system is that portion of the graph of $x + 2y = 4$ that lies within the half-plane representing the inequality $x - y \leq 3$

For $x + 2y = 4$

x	4	0
y	0	2

For $x - y = 3$

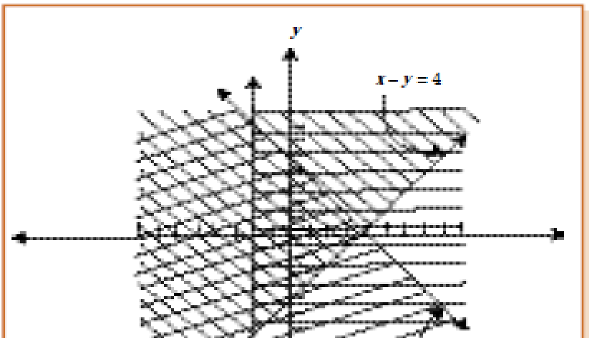
X	3	0
y	0	-3



5. Draw the graphs for the following $x + y \leq 4,\ \ x - y \leq 4$ and $x \geq -2$. Mark the common region.

For $x - y = 4$

X	4	0
Y	0	-4



For $x + y = 4$

x	0	4
y	4	0

The common region is the one represented by overlapping of the shadings.

6. Draw the graphs: $5x + 4y \leq 100,\ \ 5x + y \geq 40,\ 3x + 5y \leq 75, x \geq 0\ and\ y \geq 0$. Mark the common region.

Solution:

$5x + 4y = 100\ or, \frac{x}{20} + \frac{y}{25} = 1$

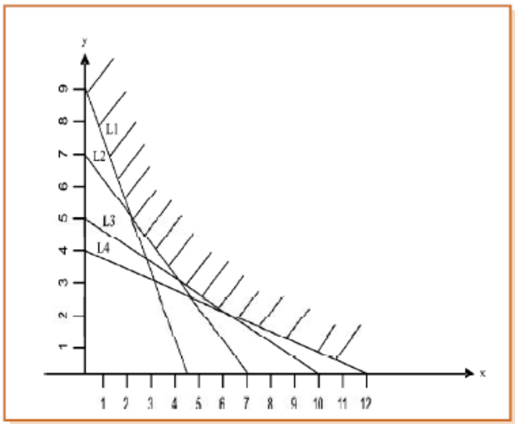
$3x + 5y = 75\ or, \frac{x}{25} + \frac{y}{15} = 1$

$5x + y = 40\ or, \frac{x}{8} + \frac{y}{40} = 1$

Plotting the straight lines on the graph paper

we have the above diagram;

The common region is the shaded portion ABCD.



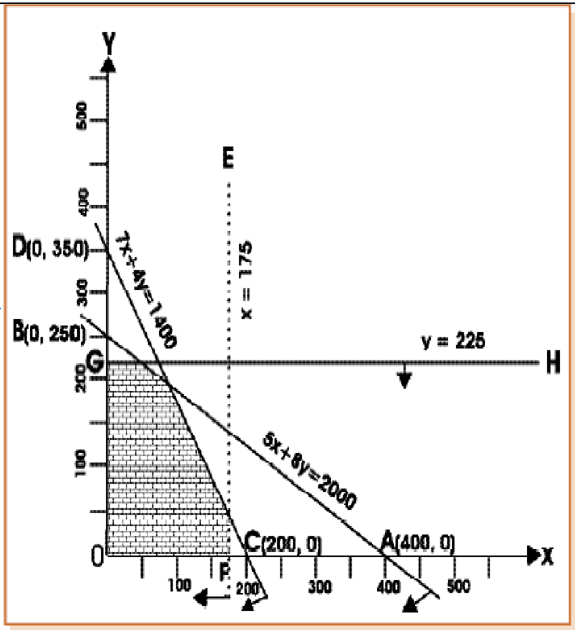
7. Draw the graphs: $5x + 8y \leq 2000, x \leq 175, x \geq 0,\ 7x + 4y \leq 1400, y \leq 225\ and\ y \geq 0$. Mark the common region;

Solution:

Let us plot the line $AB(5x + 8y = 2{,}000)$ by joining the points $A(400{,}0) and\ B(0{,}250)$.

X	400	0
Y	0	250

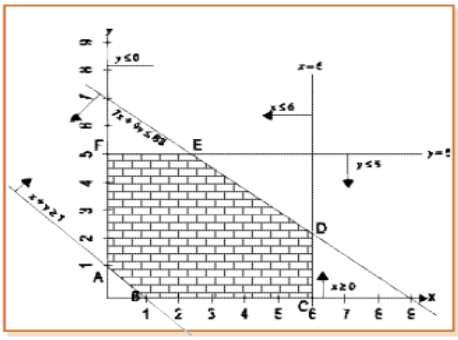
x	200	0
y	0	350



Similarly, we plot the line $CD(7x + 4y = 1400)$ by joining the points $C(200{,}0) and\ D(0{,}350)$ Also, we draw the lines $EF(x = 175)$ and $GH\ (y = 225)$	

8. Draw the graphs: $x + y \geq 1,\ 7x + 9y \leq 63, y \leq 5, x \leq 6, x \geq 0\ and\ y \geq 0$

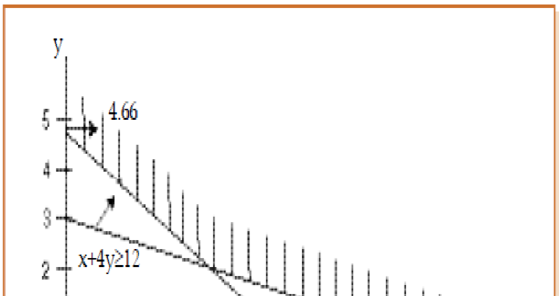
Solution:

$x + y = 1;\ \frac{x}{y}\vert\frac{1}{0}\vert\frac{0}{1};\ 7x + 9y = 63, \frac{x}{y}\vert\frac{9}{0}\vert\frac{0}{7}$ We plot the line $AB(x + y = 1)$, $CD(y = 5), EF\ (x = 6), DE(7x + 9y = 63)$ Given inequalities are shown by arrows. Common region is ABCDEF	

9. Two machines (I and II) produce two grades of plywood, grade A and grade B. in one hour of operation machine I produces two units of grade A and one unit of grade B, while machine II, in one hour of operation produces three units of grade A and four units of grade B. The machines are required to meet a production schedule of at least fourteen units of grade A and twelve units of grade B. Express this using linear inequalities and draw the graph.

Solution: Let the number of hours required on machine I be x and that on machine II be y. Since in one hour, machine I can produce 2 units of grade A and one unit of grade B, in x hours it will produce 2x and x units of grade A and B respectively. Similarly, machine II, in one hour, can produce 3 units of grade A and 4 units of grade B. Hence, in y hours, it will produce 3y and 4y units Grade A & B respectively.

The linear inequalities as follows: $2x + 3y \geq 14(Requirement\ of\ grade\ A)$ $x + 4y \geq 12(Requirement\ of\ grade\ B)$	The shaded portion is moving towards infinity on the positive side. Thus the result of these inequalities is unbounded.



Moreover x and y cannot be negative, thus $x \geq 0 \; and \; y \geq 0$ (Hence only the 1st Quadrant is considered)

For $2x + 3y = 14$

x	7	0
y	0	4.66

For $x + 4y = 12$

x	0	12
y	3	0